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Natalia FRANKOWSKA [1], Agnieszka OZGA [1]

Application range of a mathematical model computing distributions of random impulse excitations

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The paper describes the model of an oscillator with damping, whose vibrations are forced by a random series of impulses. The mathematical model of the inverse problem used to calculate the distributions can only be applied when the values of the random impulses are known. If impulse values cannot be estimated based on the vibration signal, machine learning algorithms and feature engineering should be used to determine their distribution. In the discussed paper, unsupervised machine learning (specifically, the agglomerative hierarchical clustering) is employed to evaluate the applicability of the algorithms to the problem of recognizing the magnitudes of random impulses and characterizing their distributions.

1. Introduction

In this paper, we advance the study of stochastic mechanics [1–4] by incorporating unsupervised learning methods [5, 6] in the analysis of stochastic dynamical systems. The clustering of distributions was carried out using hierarchical clustering. This approach makes it possible to detect hidden structures and similarities within the data. Stochastic mechanics provides theoretical and mathematical tools [7–9] that enable the analysis [10, 11] and prediction [12, 13] of the behavior of systems subject to random influences [14, 15]. In works published in the 21st century, the approaches introduced by Sobczyk [16] and Soong [17] established the theoretical framework for random differential equations and their applications in engineering. Contemporary research introduces advanced studies

✉ Natalia FRANKOWSKA, email: frankows@agh.edu.pl
[1] AGH University of Krakow, Krakow, Poland

of first-order differential equations with Dirac impulses, in which all parameters (initial condition, equation coefficients) are random variables with continuous probability densities [18]. These works can be applied to modeling phenomena with parametric uncertainty, where impulses represent external disturbances or interventions. Recent advances in uncertainty quantification for stochastic differential equations have been significantly enhanced by the work of Bevia et al., who developed forward uncertainty quantification methods for random differential equation systems with delta-impulsive terms [19]. This approach directly addresses the computational challenges in quantifying uncertainty for general classes of random differential equations with Dirac impulses at finite time instances. Studies devoted to tools for solving inverse problems have also been published. In [20], the authors presented a comprehensive treatment of the RVT (Random Variable Transformation) technique for determining the first probability density function of solutions to linear random initial value problems, which provides a useful tool in inverse problems, especially when the realizations of impulses are unknown.

The aim of the work presented in this article, as in other studies involving random systems [21–26], is to solve a technical problem. The discussed problem is connected with an attempt at designing and constructing a measuring device controlling the homogeneity of granularity of the medium in a dust pipe. To be more precise, the device had to signal the appearance of big or small particles in excessive quantity in the transported dust, at a given mean input value in a real technological system.

The degree of coal fragmentation and the type of fuel used have a significant impact on the efficiency of the combustion process and the level of pollutant emissions, including harmful compounds such as PAHs [27]. The literature discusses the influence of coal particle size, combustion conditions, and the composition of fuel mixtures on the quantity and composition of emitted pollutants [28, 29]. Moreover, studies have shown that processes such as granulation can effectively reduce the dusting of fine coal fractions [30].

To solve the problem, a mathematical model of responses of discrete systems and continuous ones to forcing with a random series of impulses was presented in accord with the system Definition – Theorem – Proof. Theoretical studies on systems forced by a random series of impulses were started in the second half of the 20th century [31–33]. Research was also carried out by [34]. Using a set of stochastic differential equations [35], the transformations taking place on an oscillator with damping and on a string in damping were described.

The problem of control of medium homogeneity may be solved by examining the motion of a vibrating system that is influenced by this medium. The motion of such a system X is a stochastic process that can be mathematically described in the following way:

$$X(t,x) = \sum_{0<t_i<t} G(x,\eta_i,\epsilon_i,t-t_i), \tag{1}$$

where G is a function dependent on the choice of the vibrating system, x are coordinates of the point of the vibrating system, η_i are the values of i-th impulse, and η_i is a sequence of independent identically random variables with finite expectation, where each η_i is a discrete random variable taking values $\eta_1,\eta_2,\ldots,\eta_n$ with probabilities $p_1,p_2,\ldots,p_n$, ϵ_i are the places where the i-th impulse acts, t_i are i-th moment of excitation of the movement.

$$\tau_i = (t_i - t_{i-1}). \tag{2}$$

τ_i is a sequence of independent identically distributed random variables with an exponential distribution:

$$t(\tau) = \begin{cases} \lambda e^{-\lambda\tau} & \text{for} \quad \tau \geqslant 0, \\ 0 & \text{for} \quad \tau < 0. \end{cases} \tag{3}$$

The constant λ is the intensity of impulse occurrence. The intervals between the impulses and values of the impulses are independent random variables.

Continuing the research, this work presents verification of the stochastic model. The idea of the described studies relates to non-deterministic mechanics – the state of the system observed at a chosen moment does not determine the state at any subsequent time, which follows from the stochastic character of the process. We were seeking research methods that would allow for analyzing an inverse problem [36] while taking into account the uncertainties included in the computed loads. Analysis techniques applied in stochastic mechanics are characterized by the fact that with the help of models it is possible to obtain the value of the random variable recorded in the form of a temporal series describing the position of the system. The presented approach includes analyses of thousands of samples for different parameters of random forcing.

2. Mathematical model of an oscillator forced by a random series of impulses

A mathematical model for the inverse identification problem that allows one to determine the distribution of the values of impulses forcing the vibrations of the system was developed [35] in several stages and was constructed on the basis of linear differential equations using ergodic theory together with the basics of the theory of dynamic systems, measure theory, group theory, probability calculus, and the theory of stochastic processes based on it. This universal mathematical model will enable us to perform a statistical interpretation of the measurement data. In the article, the mathematical model will be used in an analysis of vibrations of an oscillator. Removal of a random variable, that is, the location of the impulse hit, allows simplification of the model for an analysis of the impact of two random variables in the vibrations of the system. When the vibrations are forced in the form of a random series of impulses described with the equation:

$$\frac{\mathrm{d}^2X}{\mathrm{d}t^2} + 2b\frac{\mathrm{d}X}{\mathrm{d}t} + a^2X = \sum_{t_i<t} \eta_i \delta(t - t_i), \tag{4}$$

where η_i are random value of the i-th impulse, $\delta(t - t_i)$ is the Dirac distribution at the moment of excitation t_i and b and c are parameters of the vibrating system the damping coefficient b and the frequency $c = \sqrt{a^2 - b^2}$ [35].

The solution of this problem (4) using the superposition method at zero initial conditions $X(0) = 0$ and $X'(0) = 0$ for each stochastic impulse is the equation:

$$X(t) = \frac{1}{c} \sum_{0<t_i<t} \eta_i e^{-b(t-t_i)} \sin(c(t - t_i)). \tag{5}$$

Computation of the distribution of impulse values η_i, that is, the solution of an inverse problem, is possible through the application of the estimators of k-th raw moments calculated from $X(t)$.

$$\hat{m}_k = \frac{1}{[t/h]} \sum_{n<t/h} X^k(nh), \tag{6}$$

where h is the sampling period [35].

3. Numerical details

In this case, the mathematical model of an oscillator is understood in accord with the terminology used for defining statistical models as a formalized description of a certain theory or causal situation that is assumed to generate the observed data.

In this article, we introduce the limitations of the mathematical model used to solve the problem of recognizing the distribution of value of impulses generated for seven different distributions:

1. $\phi_1 : p(\eta_1 = 70) = 0.5, \ \ p(\eta_2 = 80) = 0.5$
2. $\phi_2 : p(\eta_1 = 140) = 0.5, p(\eta_2 = 10) = 0.5$
3. $\phi_3 : p(\eta_1 = 130) = 0.5, p(\eta_2 = 20) = 0.5$
4. $\phi_4 : p(\eta_1 = 85) = 0.5, \ \ p(\eta_2 = 65) = 0.5$
5. $\phi_5 : p(\eta_1 = 145) = 0.5, p(\eta_2 = 5) = 0.5$
6. $\phi_6 : p(\eta_1 = 120) = 0.5, p(\eta_2 = 30) = 0.5$
7. $\phi_7 : p(\eta_1 = 110) = 0.5, p(\eta_2 = 40) = 0.5$

The distributions were characterized by two events of different forces of impact. Distributions were selected so that the mean value was the same in all seven cases, stochastic raw moments of the second order (and subsequent orders) are different in most cases. The first and fourth distributions represent the situation which is the most desirable from the point of view of technological application – we have two impulses of similar values. The remaining distributions represent the situation which includes errors in the granulation process, since there occur large and small particles in the distribution.

Studies on the model should be appropriately designed so that step changes do not occur in the computed raw moments some time after the start. Earlier analyses have shown [35, 37–40] that oscillators with strong damping should be used and that impulses should occur frequently enough for values of estimators of raw moments calculated from the equation to change to the least extent. The parameters used in the simulations, which satisfy the above mentioned assumption, have been shown in Table 1.

Table 1. Parameters used in simulations

Parameter	Parameter's value
λ	10
c	20
b	10
period of time	0 to 3600 s
number of random samples generated for each distribution	1000

For such selected parameters of the vibrating system, the calculation of probabilities with an error of less than one percent is possible in the first few minutes [40] from the equations:

$$\sum_{i=1}^{q} \overline{p}_i \left[(m_n m_1 - m_{n+1})\eta_i + \sum_{j=1}^{n} \binom{n}{j} m_{n-j} m_1 \eta_i^{j+1} \frac{C(j+1)}{C(1)c^j} \right] = 0, \tag{7}$$

$$\sum_{i=1}^{q} \overline{p}_i = 1, \tag{8}$$

where $C(j)$ are constants to be determined: for even j from equation (9), and for odd $j > 0$ from equation (10):

$$C(j) = \frac{1}{\omega^{j+1}} \frac{j!}{\prod\limits_{r=1}^{j/2} ((jb/\omega)^2 + (2r)^2)} \frac{\omega}{jb}, \tag{9}$$

$$C(j) = \frac{1}{\omega^{j+1}} \frac{j!}{\prod\limits_{r=0}^{(j-1)/2} ((jb/\omega)^2 + (2r+1)^2)}, \tag{10}$$

which, for the distribution composed of two equations, takes the form (11) and (12):

$$\overline{p}_1(t) = \frac{-\left(4b\left(\hat{m}_1^2(t) - \hat{m}_2(t)\right)\eta_2 + \hat{m}_1(t)\eta_2^2\right)}{(\eta_1 - \eta_2)\left(4b\left(\hat{m}_1^2(t) - \hat{m}_2(t)\right) + \hat{m}_1(t)(\eta_1 + \eta_2)\right)}, \tag{11}$$

$$\overline{p}_2(t) = \frac{4b\left(\hat{m}_1^2(t) - \hat{m}_2(t)\right)\eta_1 + \hat{m}_1(t)\eta_1^2}{(\eta_1 - \eta_2)\left(4b\left(\hat{m}_1^2(t) - \hat{m}_2(t)\right) + \hat{m}_1(t)(\eta_1 + \eta_2)\right)}. \tag{12}$$

The values of the impulses η_1 and η_2 are of fundamental importance in these equations. With simulation parameters selected to resemble those present in the technical problem considered, no individual impulse is visible in the response signal (Fig. 1). In the conducted simulation, impulses most frequently occurred before the vibrations induced by previous impulses had subsided. As a result, estimating the values of the impulses η_1 and η_2 present in the distribution is unattainable, and the use of equations (11) and (12) is not possible.

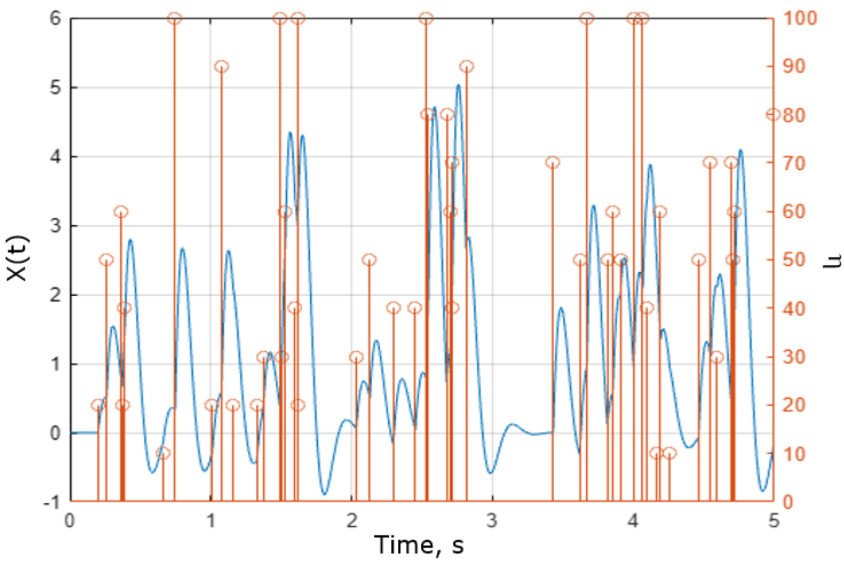

Fig. 1. The course of $X(t)$ for a selected sample

Therefore, only the values of the moments calculated from equation (6) are subject to analysis.

The aim of the research is to solve the inverse problem, that is, to determine the values of η present in the distribution under the assumptions given at the beginning of this section.

To achieve our goal, we pose the research question:

Which statistical parameters describing time series allow for distinguishing of distributions?

In the search for the answer to this research question, we are going to apply hierarchical analysis executed in Python environment. For seven distributions generated during simulation studies, three consecutive stochastic moments were calculated (5). The data were imported into the Python environment [41, 42], where features of the calculated moments [43] were extracted using the Time Series Feature Extraction Library (TSFEL) [44]. This library was developed for the analysis of time series and enables the determination of 65 different statistical, temporal, and spectral features. To reduce the number of data points, a significance test of the features' importance was conducted. Two supervised learning algorithms, decision trees and random forests, were used for permutation [45]. Based on the obtained results, four features were selected for analysis [46]. During the analysis of the first moment, one significant feature was identified – the sum of absolute differences. The remaining features were determined based on the analysis of the second and third stochastic moments.Due to the similar distribution of most generated features, resulting from their strong correlation, three representative features were selected for further analysis to reduce dimensionality: the minimum, the median, and the area under the curve. The selection of these three features was sufficient, and the results obtained were close to those achieved when analyzing all generated features.

4. Results

As it has already been stated, the study involves 1000 trials for every course. As we can see in Fig. 2 [47], using the first raw moment in the analysis will not allow for a division of the distributions. As regards the second moment, from the 2000th second, we can distinguish three groups of distributions characterized by similar values of η_1 and η_2 (ϕ_1, ϕ_4 and ϕ_2, ϕ_5 and ϕ_3, ϕ_6, ϕ_7) on the basis of the computed mean value for any arbitrarily long time interval. In the case of the third moment, three groups of distributions can be distinguished even before the 1000th second, and the distributions ϕ_1 and ϕ_4 are completely distinguishable.

For study the method of unsupervised learning [48, 49], connected with data clustering, was selected. Single observations are grouped into increasingly larger clusters, until a cluster consisting of the maximum number of elements is formed. Similarities between elements – expressed with the help of metrics like Manhattan, Euclidean, Chebyshev or Minkowski distance, etc., were adopted as the basis for clustering. Apart from the metric, the algorithm requires also selection of the method of clustering. In Python environment, in which the study was carried out, agglomerative analysis can use the following methods: average, centroid, complete, median, single, ward and weighted [50]. The previously prepared data were imported into the Python environment and then normalized. Dendrograms were generated for all possible combinations of distance metrics and clustering methods, based on which the number of separated clusters was determined. To evaluate the results, external validation was conducted. Answering the research question posed in the paper we obtain the following results. Using the hierarchical analysis to distinguish time series computed for the second raw moment between the 1500th second and 1800th one we can see two different classifica-

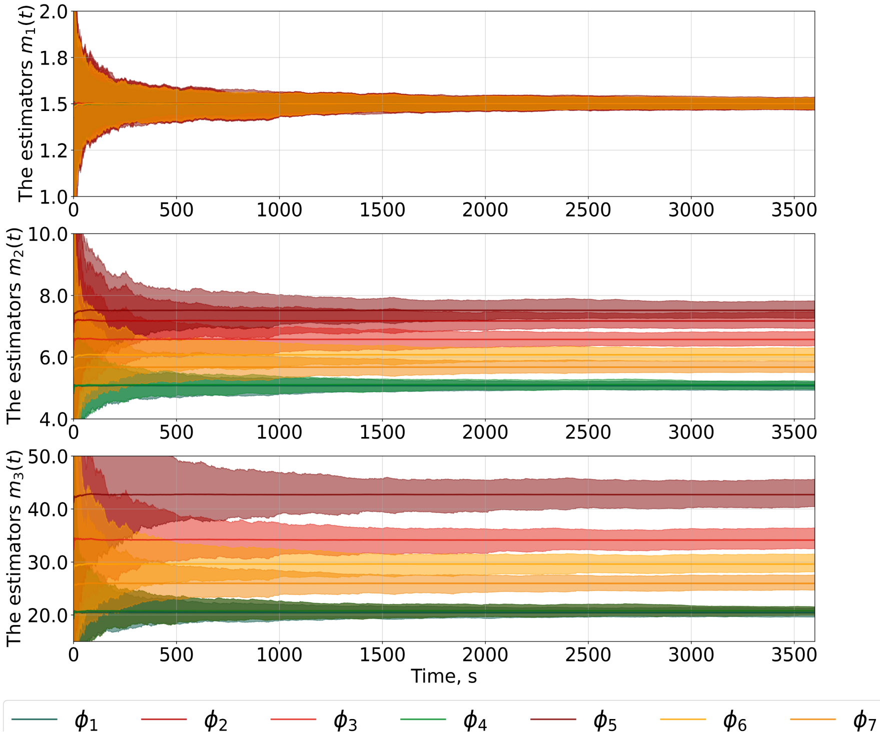

Fig. 2. The first, the second and the third stochastic raw moments $\hat{m}_k$ calculated from the location $X(t)$ for a thousand different courses

tions. On the one hand, the division into two classes seems natural – the first group, characterized by the correct technological process responsible for the granulation of dust, and the second group is connected with two faulty processes. On the other hand, however, it should be checked whether it is possible to distinguish all seven distributions of impulses. In the best case, based on the features of the second moment, it was possible to distinguish 4 groups of distributions. However, adding the features of the third moment to the dataset allowed for the differentiation of 5 groups (Fig. 3). The only indistinguishable distributions were the very similar ϕ_1 and ϕ_4, as well as ϕ_6 and ϕ_7. External validation, involving the comparison of predicted and actual clusters, was conducted using two metrics: adjusted Rand index (ARI) [51] and Folkes-Mallows index (FMI) [52] (Table 2). In both cases, the obtained results were compared with the actual labels. The Rand index (RI) [53] can be described by the following equation:

$$\mathrm{RI} = \frac{a+b}{\binom{n}{2}}, \tag{13}$$

where: a are correct similar pairs, b are correct dissimilar pairs, n is the total number of elements, $\binom{n}{2}$ is the total number of possible pairs of elements. Its adjusted version is given by the following equation:

$$\text{ARI} = \frac{\text{RI} - \text{RI}_{\text{expected}}}{\text{RI}_{\text{max}} - \text{RI}_{\text{expected}}}, \tag{14}$$

where: $\text{RI}_{\text{expected}}$ is the expected value of the Rand index for random clusterings, RI_{max} is the maximum possible value of the Rand index.

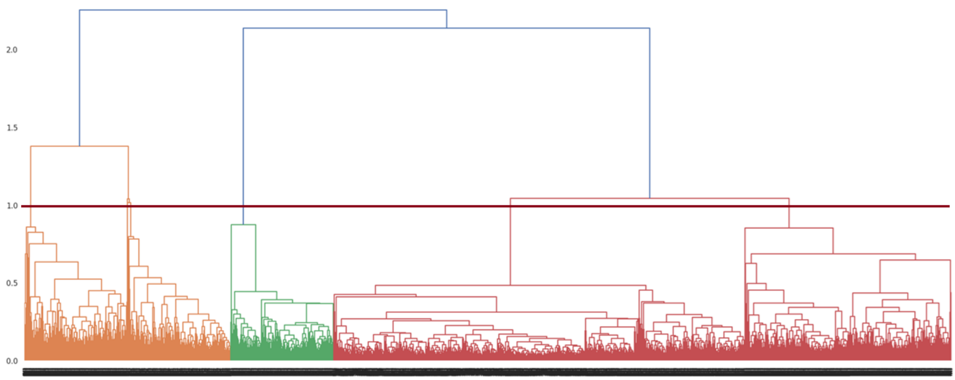

Fig. 3. Dendogram for the second and third stochastic moment

Table 2. Results of external validation

moment	second		second and third	
number of clusters	seven	four	seven	five
Fowlkes-Mallows index	0,84	0,99	0,84	0,99
Rand index	0,71	0,99	0.71	0,99

Fowlkes-Mallows index, which is the geometric mean between precision and recall is given by:

$$\text{FM} = \frac{\text{TP}}{\sqrt{(\text{TP} + \text{FP})(\text{TP} + \text{FN})}}, \tag{15}$$

where: TP – true positive, TN – true negative, FP – false positive, FN – false negative.

Thanks to the fact that in the time series we applied four statistics: minimum, median, area under the curve, and sum of absolute differences, for the second stochastic moment we obtain the division into four groups. Adding features of the third moment to the analysis allows for the classification of 5 groups. Distributions in which η_1 i η_2 have similar values are well distinguishable from distributions where the values of impulse are significantly different. Distributions ϕ_1 and ϕ_4, as well as ϕ_6 and ϕ_7, are indistinguishable at this stage. The data come from a simulation and therefore it is possible to verify the obtained results. By grouping distributions into groups with similar values of the specified impact force, we are able to distinguish them with almost 100% accuracy, as indicated by the metrics used for external validation.

5. Conclusions

The paper describes the model of an oscillator with damping, whose vibrations were forced by a random series of impulses. Analysis techniques applied in stochastic mechanics is characterized by the fact that with the help of models it is possible to obtain the random variable recorded in the form of a temporal series describing the position of the system. On the other hand, the mathematical model of the inverse problem used to calculate the distributions can only be applied when the values of the random impulses are known. If impulse values cannot be estimated based on the vibration signal, machine learning algorithms and feature engineering should be used to determine the distribution of impulses. In the paper discussed, one makes use of unsupervised machine learning to determine the potential of application of the algorithms for solution of the recognition of value of random impulses. Research has shown great potential in using artificial intelligence algorithms to solve the inverse problem.

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