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Aerodynamic control analysis in a canard configuration for a surface-to-surface missile

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The modern battlefield requires highly accurate missiles, which has led to the modification of unguided missiles into guided versions. This paper presents the process of adapting an unguided missile with a range of 40 km into a guided version using a canard control system. A key aspect of the upgrade was the development of a control system that allows the trajectory to be corrected after crossing the apex of the flight path, particularly during the descent phase. This paper discusses the design details and application of a two-channel control system (pitch and yaw) in which the control signals are synchronized with the speed of the projectile. Mathematical modelling and numerical simulations have shown that, with appropriate control parameters, a zero mean control force can be achieved, leading to trajectory stabilization and minimized aiming errors. The proposed solution provides a basis for further research and dynamic field tests and can contribute to the development of precision guidance technology for surface-to-surface missiles.

1. Introduction

On modern battlefields, the most commonly used type of rocket ammunition remains unguided artillery rockets due to their low production and operational costs, relative to the value of targeted objectives. They are particularly effective when fired in salvoes, as they provide a large coverage area with their destructive power. However, their drawback is a large dispersion of impact points, which can result in damage to unintended targets, including civilian objects. Consequently,

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modern militaries impose increasing demands on weapon technology, emphasizing not only versatility where possible but also the ability to perform complex tasks and improve hit effectiveness through guidance precision.

An example of this trend is the development plans for precision munitions described in document [1], section 4.3, which indicate that guided weapons are a rapidly advancing field in the armament of leading armies worldwide. This intensive development is confirmed by numerous studies [2–4] which present various approaches to control systems and explore new methods and issues related to missile guidance.

The Polish Army currently operates unguided ground-to-ground rocket missiles launched from various launchers. In line with global trends in guided rocket weapon development, it is necessary to introduce guided rocket systems that are sufficiently effective to ensure that, despite the inaccuracy of a single missile strike, the target lies within the warhead's lethal radius. Implementing an onboard control system will minimize unintended collateral damage.

As demonstrated, modern military operations require significantly increased missile effectiveness through the development of precision guidance technologies. A key achievement is converting unguided missiles into guided systems, enabling precise navigation to the target.

This article presents the adaptation process of an unguided missile with a range of approximately 40 km in a ground-to-ground configuration into a guided version through the introduction of an aerodynamic control system in a canard layout. The implementation of this solution involved designing a control system capable of trajectory correction after reaching the apogee, with particular emphasis on the terminal phase. Mathematical modeling and numerical simulations showed that properly selected control parameters allow trajectory stabilization and minimization of aiming errors by achieving zero average control force.

A significant design challenge was adapting the control system to a tube launcher, which imposed spatial constraints on installing control mechanisms. To accommodate the control system, the missile's aerodynamic front section was extended by approximately 200 mm, allowing the mounting of canard control surfaces fitting within the missile caliber. Due to dimensional limitations, the use of canard control surfaces was the only feasible aerodynamic control solution.

The main focus of the work is on utilizing missile understeer as a method to stabilize the trajectory after the apex of flight. Controlled understeer improves missile stability during the descent phase, allowing more precise guidance and increasing the missile's range by about 20% due to more controlled descent and reduced trajectory errors. The use of linear control characteristics, understood as the proportional relationship between control surface deflection and aerodynamic response (lift force and moment), ensures precise missile response to control surface deflections, which is critical for maintaining flight stability under varying aerodynamic conditions.

Similar solutions can be found in the Israeli AccuLAR missile, caliber 122 mm, produced by Israel Military Industries. It features an aerodynamic canard control system on the front section and rear rotating control surfaces for stabilization. The missile carries a warhead weighing 20–35 kg up to a distance of 40 km, achieving accuracy with a CEP error not exceeding 10 m using satellite navigation via GPS or GLONASS [5].

Other advanced guided missiles include:

The Italian-German Vulcano missile, caliber 127 mm, capable of striking targets up to 100 km away, utilizing aerodynamic control and GPS/INS navigation systems, suitable for firing from both artillery guns and rocket launchers [6];

The American 270 mm EX-171 Extended Range Guided Munition (ERGM), aerodynamically stabilized with GPS and an inertial navigation system (INS), capable of ranges exceeding 60 km with potential for further extension [7];

The Ukrainian Vilkha-M missile employs a hybrid gas-dynamic and aerodynamic control system, achieving precision strikes at distances of up to 130 km. It is equipped with GPS and INS navigation, enabling rapid trajectory corrections, especially useful under disturbances and changing atmospheric conditions [8].

All these examples confirm the dynamic development of guided missile technology, combining long range, high accuracy, and reduced collateral effects in combat operations.

In conclusion, the presented approach to modernizing unguided missiles addresses key design, aerodynamic, and control system aspects, enabling the creation of a more precise, effective, and long-range missile system. Such solutions meet contemporary battlefield requirements while minimizing the risk of collateral damage.

2. Canard system, operating principle

The generic missile Fig. 1 is currently an unguided missile throughout its entire flight path, moving in an open-loop control system. Its stabilization during flight occurs both aerodynamically and rotationally.

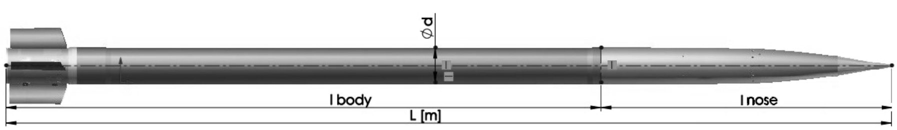


Fig. 1. The generic unguided missile adopted for analysis

The subject of the work is the analysis of the understeering system for a generic missile. This system aims to increase the missile's accuracy by introducing flight correction after the apex of the trajectory, i.e., during the descending phase. The implementation of this system changes the missile's character from unguided to guided, operating in a 'canard' configuration Fig. 2.

Fig. 2. The front part of the missile in the canard configuration

The new missile is a system composed of the following elements: a cruise engine, a warhead, stabilizers, and a control understeering system. The control understeering system includes components such as: a programmable flight system (computer), IMU (Inertial Measurement Unit), battery, steering machine, and control surfaces.

The entire system can be represented schematically as shown below in Fig. 3. Before launch, the computer has calculated and loaded a reference flight trajectory to the target. The flight trajectory is provided to the Control Signal Generation System [CSGS]. CSGS transmits control information to the Control Actuation System [CAS]. The CAS enforces the application of aerodynamic forces on the missile during flight to achieve the desired motion parameters. Simultaneously, based on the deflection of the control surfaces, feedback information is sent to the summation node, where, along with spatial position data of the missile obtained from the IMU, the control surface deflection error is corrected. This correction is necessary because the missile experiences disturbances during flight. In the closed-loop system, the missile's actual position and motion, described by kinematic relations obtained from IMU measurements, are continuously fed back to the computer. These measured values are compared with the reference values predicted by the mathematical model of the missile's dynamics. The difference between the measured (kinematic) state and the model-based prediction is treated as the control error, which is then minimized by the guidance algorithm (Fig. 3).

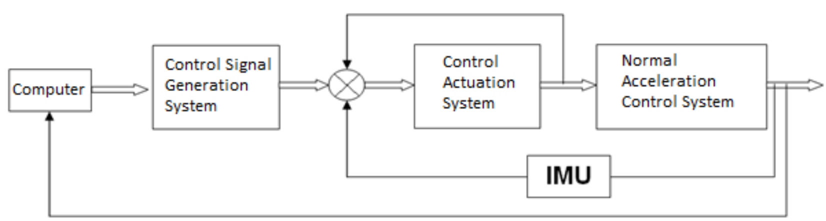


Fig. 3. Simplified block diagram of the programmable missile control system

Achieving the desired flight trajectory of a missile requires the use of a control system that will precisely determine the extent of the disturbances acting on the missile due to external perturbations. This system aims to generate appropriate control forces that will influence the missile, correcting its flight path. The mutual interaction between the missile and its control system involves changes in the forces and moments acting on the missile, thus modifying the normal acceleration interactions. This process is carried out in a closed-loop system, in which the missile is governed by control signals, and the missile's responses to external disturbances are minimized.

Using the aerodynamic canard configuration, the system requirements can be grouped into two categories:

Primary requirements:

1) ensure stabilization of the missile's motion, regardless of flight conditions and the missile's inherent stability;
2) continuously adjust flight parameters to follow the reference trajectory;
3) maintain the necessary dynamic accuracy of motion parameters despite continuous external disturbances.

Supporting functions enabling the above requirements:

1) measurement of selected flight parameters and comparison with required values;
2) conversion of measured deviations into proportional electrical signals and amplification of these signals according to the adopted guidance method;
3) transformation of control signals into the deflection of control surfaces.

In this regard, the missile guidance system consists of a set of elements, which can be represented using the following block diagram Fig. 4.

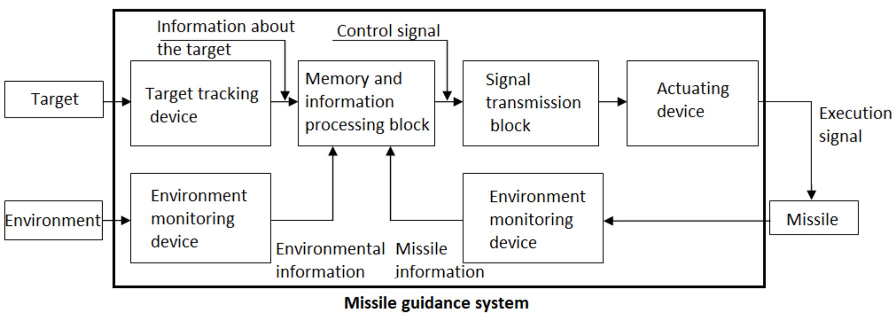


Fig. 4. General block diagram of the missile control system

To change the flight direction of the missile's center of mass, it is necessary to apply forces normal to the tangent of the flight path. The element generating these forces is the control surfaces. In the applied control system Fig. 5, the change in flight direction should occur in a rectangular coordinate system, as this is the most advantageous configuration from the perspective of design parameters and control quality, especially for an actuator system based on two-position control surfaces operating in a stepwise-variable mode, meaning that these surfaces can only switch between two basic positions (e.g., deflected left or right), with transitions occurring in discrete steps rather than continuously. In Fig. 5, P – denotes the aerodynamic force, Ps – the lift force component generated by the control surfaces, and δ – the deflection angle of the control surfaces. The time-history represents the variation of the lift component Ps – as a function of time during the missile's flight, from launch to the terminal phase, illustrating how the stepwise control inputs affect the lift dynamics. This design simplifies the mechanism but at the expense of smooth control.

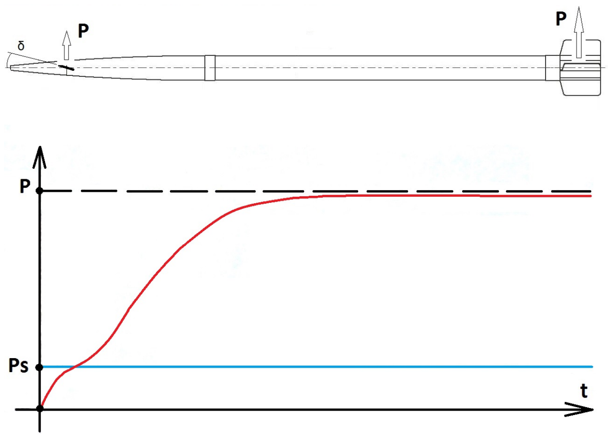


Fig. 5. Change of lift force (Ps) generated by the control surfaces as a function of time

This rectangular configuration is considered the most advantageous in this design context because it allows a clear separation of pitch and yaw control channels, simplifying both the actuator system and control law implementation. Moreover, it provides predictable aerodynamic response with stepwise-variable surfaces, which is essential for a spinning missile. Finally, given the spatial constraints of the canard placement and the limited number of control surfaces, this configuration minimizes interference effects while ensuring sufficient control authority.

3. Assumptions and modeling of aerodynamic forces

The basic data assumed for the analysis are presented in Table 1.

In the analysis of the forces acting on a projectile, their sum is considered, reduced to the center of mass, along with the stabilizing moments resulting from the interaction of forces in a coordinate system related to the center of pressure and the center of mass. The stabilizing moment is generated by forces acting on the projectile that are not directed along its symmetry axis, leading to rotation of the projectile. These moments can be controlled by appropriately positioning the fins, which affect flight stability. The action of these moments allows controlling the orientation of the projectile, preventing uncontrolled changes in its trajectory. To ensure stability, the control moments must satisfy a specific condition, balancing the forces that cause instability. Proper modeling of stabilizing moments is crucial for precise projectile flight and target impact.

Table 1. Basic data about the missile assumed for the analysis

Missile flight characteristics	Values	Physical units
Average thrust of the missile engine	20.3	kN
Missile engine operating time	3	s
Span of the stabilizers including the fuselage-mounted section	0.254	m
Angular velocity of the projectile	62.8	rad/s
Initial mass of the projectile	60.6	kg
Average flight speed	3.2	Ma
Geometric location of the center of gravity (measured from the missile's nose along the longitudinal axis)	1.621	m
Assumed maximum angle of attack of the projectile	5	deg
Area of one fin	0.0117	m^2
Area of the fin including the fuselage-mounted section	0.0186	m^2
Thickness of the fin	0.0015	m
Length of the fin	0.198	m
Body cross-sectional area	0.342	m^2

Since the analyzed projectile is a spinning projectile, the key moments for analysis are the pitching moment M_θ and the rolling moment M_φ. The yawing moment M_ψ describes the possibility of the projectile performing oscillations in the appropriate plane (vertical plane) around its axis. These oscillations have a limited range, resulting from the aerodynamic characteristics and design of the projectile. The analysis of these moments is essential for evaluating the stability of the projectile's flight. A precise understanding of these forces allows the optimization of the trajectory and improvement of accuracy.

In the case of this projectile, the mathematical determination of the forces acting on the projectile during flight, the moments of rotation, and the center of pressure based on the literature [8], where the aerodynamic model of the control surfaces is described by the lift force Y which is typically described in classical aerodynamics by the following equation:

$$Y = c_y q S, \tag{1}$$

where c_y is the lift force coefficient, q is the dynamic pressure, S is the lift surface area.

In practice, the lift force is treated as a single resultant value because the aerodynamic contributions of the wings, body, and stabilizer act together to produce a combined upward force. Unlike drag, which is primarily additive and can be calculated separately for each component, lift is strongly influenced by the mutual interaction of these elements and must therefore be considered as a total value. Consequently, the drag generated by each part of the missile is computed individually, while lift is modeled as the net effect of all contributing components. The calculations also include interference effects, since interactions between the elements can modify both lift forces and drag forces.

For small angles of attack, the lift coefficient c_y can be approximated using a series expansion, which takes into account the influence of the angle of attack and the deflection of the control surface: Eq. (2):

$$c_y = c_{y0} + c_y^{\alpha}\alpha + c_y^{\delta}\delta. \tag{2}$$

In this equation, c_{y0} represents the initial value of the lift coefficient at zero angle of attack, α is the angle of attack, and δ is the deflection of the control surface. The coefficients c_y^{α} and c_y^{δ} correspond to the influence of changes in the angle of attack and the deflection of the control surface on the value of the lift force. For small angles of attack, this relationship allows the precise determination of the lift force during various phases of flight, such as launch, mid-course, and terminal guidance.

Assumptions made in the project:

- The aerodynamic characteristics have been determined within the range of small angles of attack, where their linearity is assumed.
- Structural deformations have been neglected, assuming their minimal impact on the aerodynamic results. In situations where deformations are significant, they should be considered in the analyses.
- The results present an approximation and constitute the first iteration of the design process.
- The aerodynamic characteristics of the lifting surfaces were initially determined for isolated cases. The aerodynamic coefficients for the isolated wing were corrected by the appropriate interference coefficients and the ratio of the wing area to the reference area.

When designing the control surfaces, the following factors should be taken into account:

- Increasing the control surface area leads to an increase in the control force, which is associated with higher aerodynamic drag. Therefore, it is necessary to find an optimal solution in terms of both of these parameters.
- The optimal shape of the control surfaces minimizes aerodynamic drag, ensuring high sensitivity to changes in the angle of attack.
- The distance of the control surfaces from the missile's center of mass affects the efficiency of the rotational moment generated by the control surfaces.

Based on the assumptions made and the modeling of aerodynamic forces, a detailed geometric analysis of the missile's control surfaces was conducted. The basic data regarding the missile, such as the launch mass, stabilizer span, angle of attack, and average flight speed, are presented in Table 1. The overall geometric model of the missile's control surfaces is shown in Fig. 6, which illustrates the

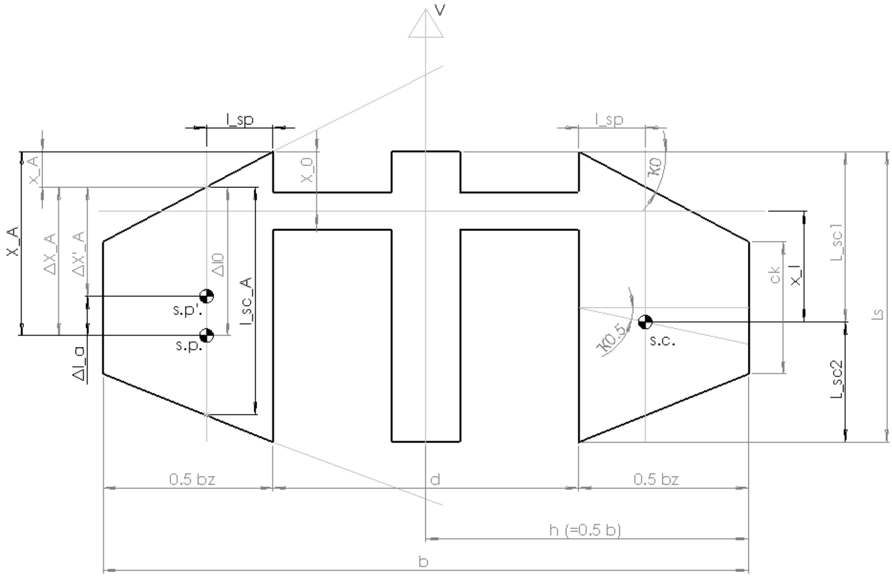


Fig. 6. The geometric model of the pair of control surfaces

positioning of the control surfaces relative to the fuselage and their symbolic dimensions.

The analysis took into account both the size of the control surfaces and their position relative to the missile's center of mass, which is crucial for the effectiveness of the generated rotational moments. The applied model allowed the determination of the optimal shape of the control surfaces, minimizing aerodynamic drag while ensuring adequate sensitivity to changes in the angle of attack. The final geometry of the control surfaces is presented in Fig. 7 in the context of the computational results, which consider the impact of various parameters on the stability and trajectory control of the missile.

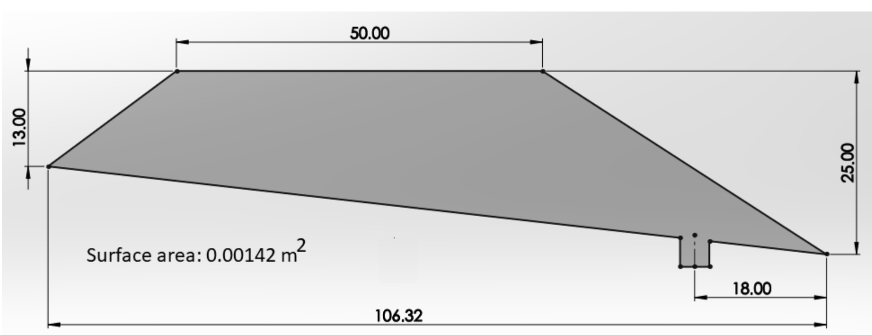


Fig. 7. The geometric model of the control surface obtained based on analytical analyses

The final control surface geometry presented in Fig. 7 was obtained through an iterative parametric optimization process. The primary objective was to achieve sufficient control authority while minimizing additional aerodynamic drag. The sensitivity study considered variations in the control surface area, span, chord length, and the distance from the missile's center of gravity. Among these, the surface area and longitudinal placement were found to have the strongest influence on the generated control moments. The final configuration represents a compromise between control effectiveness, structural feasibility, and aerodynamic efficiency, ensuring stable guidance while keeping the control surfaces compact.

In the analyzed guidance system of a missile projectile, both the design and geometry of the control surfaces play a key role in the effectiveness of guiding the missile to the target. The control of the missile is based on the proper selection of the geometry of the control surfaces and aerodynamic parameters, which influence the control moments generated during flight. In particular, the geometry of the control surface must be designed in such a way as to provide optimal conditions for generating the required control moment with minimal aerodynamic drag, which contributes to improving flight stability and reducing energy consumption.

In the context of the previously discussed issues, the next chapter will discuss the general characteristics of the guidance system for a surface-to-surface missile. The missile projectile, moving along its flight trajectory, responds to changing conditions that depend on the missile's intended purpose, technical parameters, flight dynamics, and guidance methods to the target. The main goal is to implement a guidance method based on a programmed trajectory, which allows full control over the flight, ensuring precise task execution. The guidance system of the missile projectile is responsible for generating the appropriate control signals that enable trajectory maintenance and minimize deviation.

A fundamental advantage of the adopted solution is the alignment of the control surface's lift force direction with the direction of the resultant lift force of the entire projectile. This concept allows the reduction of the size of the control surfaces and stabilizers, which in turn reduces aerodynamic drag and increases the effectiveness of the control system. As a result, the need for large control moments is minimized, leading to more precise guidance of the projectile to the target. An important aspect is also the appropriate geometry of the control surfaces, enabling effective guidance of the projectile in two planes (pitch and yaw). The design of the control surfaces should take into account changes in deflection angles depending on the projectile's rotational speed and flight trajectory requirements.

The conducted simulations confirmed that the geometry of the control surfaces has a decisive impact on the effectiveness of the guidance system. Proper selection of the shape and placement of the control surfaces enabled flight path stabilization and a significant reduction of aiming errors. These results demonstrate that taking into account both aerodynamic requirements and the projectile's rotational dynamics leads to the development of a highly effective guidance system that ensures high target accuracy.

4. General characteristics of the guidance system for a surface-to-surface missile

The missile projectile moves along a trajectory that depends on technical conditions and design requirements, such as the purpose of the missile, technical parameters, and guidance methods. In the analyzed case, a guidance method based on a programmed flight plan was applied, resulting from the missile's intended purpose and design. The flight program calculates the reference trajectory based on input data, and the deviations are calculated using the IMU, which are then used to generate control signals.

To achieve flight along a given trajectory, onboard control actuators must be used, which are a set of devices that execute a given control law. One of the components of the control actuator system is the previously proposed control surface. The goal of this system is to influence the projectile with control forces and moments to ensure that it follows the desired path. In general, the control system determines the degree to which external and internal disturbances violate the motion constraints of the object while simultaneously generating appropriate signals to ensure the missile reaches its target.

The problem formulation can be reduced to the following task: the missile projectile, with a variable mass, is treated as a system with a finite number of real variables, whose values at any given moment in time t define its state. This state can be described at any point x_n in an n-dimensional Euclidean space E^n called the state space. Therefore, the dynamic behavior of the projectile is governed by a control algorithm, referred to as a rule r, where θ^1 denotes the vector of state variables that describe the angular position of the projectile in space. This rule comes from a set of other rules whose behavior can be described by dynamic equations, which change over time, and their time-course is the control – the behavior of the projectile during flight. Therefore, given a rule, we describe the state of the projectile moving along a certain path p in the state space. In the subsequent design of the path, only those rules from the set that belong to the true set, satisfying the assumptions, will be considered. In particular, we are interested in the rules that guide the projectile to the target with minimal targeting error.

$$\left[x^0 \to x^1(t)\right] \in \theta^1. \tag{3}$$

The designed algorithm must feature a clear assignment of numbers for a given flight path, described by a specific law. This assignment is referred to as a functional, and the values themselves are the costs. When guiding the projectile along a given path, the cost of transition depends on the control algorithm and the trajectory the projectile is to follow. The functional can describe various parameters of the projectile, but the important aspect is the optimization of the control law, i.e., meeting the condition [10]:

$$V_{\text{opt}}\left(x^0, x^{1*}, r^*, p^*\right) \leqslant V\left(x^0, x^1, r, p\right). \tag{4}$$

It should be noted that there may be more than one optimal control law for the projectile in the set, and this law may either result in hitting the target or missing it. Therefore, particular attention must be given to the initial assumptions in order to determine the correct approach to the guidance law for the flying object toward the target.

To reduce the risk of undesirable guidance outcomes, including complete target miss, the cost functional was extended to a weighted form with additional barrier functions. The proposed functional accounts for both trajectory deviation and control effort, while also enforcing physical constraints:

$$
\begin{aligned}
J = {} & \left(r(T) - r_{\text{target}}\right)^T Q_T \left(r(T) - r_{\text{target}}\right) \\
& + \int\limits_{t_0}^{T} \left[(x - x_{\text{ref}})^T Q \,(x - x_{\text{ref}}) + u^T R u + \lambda \left\| \overline{P_S}(t) \right\|^2 + \Phi_{\text{bar}}(x, u) \right] \mathrm{d}t,
\end{aligned} \tag{5}
$$

where $\overline{P_S}(t)$ denotes the mean control force per revolution, and λ is its weighting coefficient. The barrier functions are introduced in logarithmic form to reflect actuator and aerodynamic constraints:

$$
\Phi_{\text{bar}}(x, u) = \mu_\delta \sum_k -\log\left(\delta_{\max} - |\delta_k|\right) + \mu_\alpha \left[-\log\left(\alpha_{\max} - |\alpha|\right)\right] \tag{6}
$$

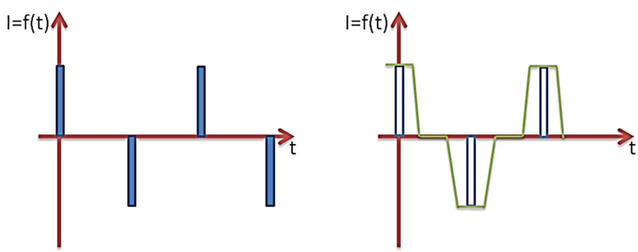
with δ_k denoting control deflections and α the angle of attack. This compact formulation balances guidance precision with feasibility and reduces the likelihood of mathematically “optimal” but physically unacceptable solutions.

The cost functional J is an extended version of the classical quadratic cost used in LQR. It combines a terminal cost measuring the distance to the target at the end of the prediction horizon, running costs penalizing state and control deviations, and an additional term $\lambda \left\| \overline{P_S}(t) \right\|^2$, which enforces the zero-mean control force condition characteristic of spinning projectiles. Furthermore, logarithmic terms $\Phi_{\text{bar}}(x, u)$ act as barrier functions that grow rapidly near physical limits (e.g., maximum actuator deflections or allowable angle of attack), effectively discouraging the optimizer from violating constraints. As a result, the functional provides a compromise between guidance accuracy and control feasibility, reducing the risk of mathematically “optimal” solutions that would otherwise result in target miss or infeasible trajectories [11].

From a design perspective, the interactions between the projectile and the UAV (Unmanned Aerial Vehicle) are determined through control forces and moments, which change the values of normal accelerations. To alter the direction of the projectile’s flight, or more simply, the trajectory of its center of mass, normal forces are applied using control surfaces tangential to the direction of the missile’s velocity vector. In the analyzed case, the guidance is dependent on the aerodynamic design, which is based on the canard system described earlier in the article. The advantage of this solution is the alignment of the lift force direction of the control surface with the direction of the resultant lift force of the entire projectile, which increases the effectiveness of the control surfaces. This allows the reduction of both the control surfaces and stabilizers, thus reducing the overall drag of the projectile.

Additionally, the deflection angle of the control surfaces is effective, as it increases the leverage of the control force relative to both the center of gravity and stabilizers, which reduces the forces and moments needed to guide the missile. However, it should also be noted that this system is not without its drawbacks, such as long transition processes and a lack of smooth control due to the creation of a positive control moment.

Furthermore, the airstream flowing from the control surfaces, when stabilizers are large, can affect them, reducing their effectiveness. These drawbacks result from the generation of a positive aerodynamic moment. Based on the adopted aerodynamic system, it follows that the missile control in this specific case must be carried out in a rectangular system, meaning that the control system should be divided into two independent channels: pitch and yaw. Analyzing the above assumptions from the perspective of both the design capabilities and the quality of control – understood here as the precision, stability, and responsiveness of the control system – a solution has been proposed for the control actuator system based on a pair of differential control surfaces, controlled in the first version of the missile in a stepwise manner Fig. 8.



(a) (b)

Fig. 8. Impulse description of the control signal: (a) shows an ideal symmetric time-history of the control signal, whereas (b) presents the actual signal including phase shifts that introduce asymmetry

The control scheme for the projectile in two channels(pitch and yaw) is presented in Fig. 9.

The two-channel control system consists of two control signals: Us_{poch} and Us_{odch}. These signals are fed into a summing junction where the control signals generated in the stabilization circuits are added to the signals produced in the missile guidance circuits. These signals are then split as current signals and distributed to the corresponding control actuators responsible for generating forces and moments in each channel. For clarity, the symbol i_{poch} denotes the derivative of the control

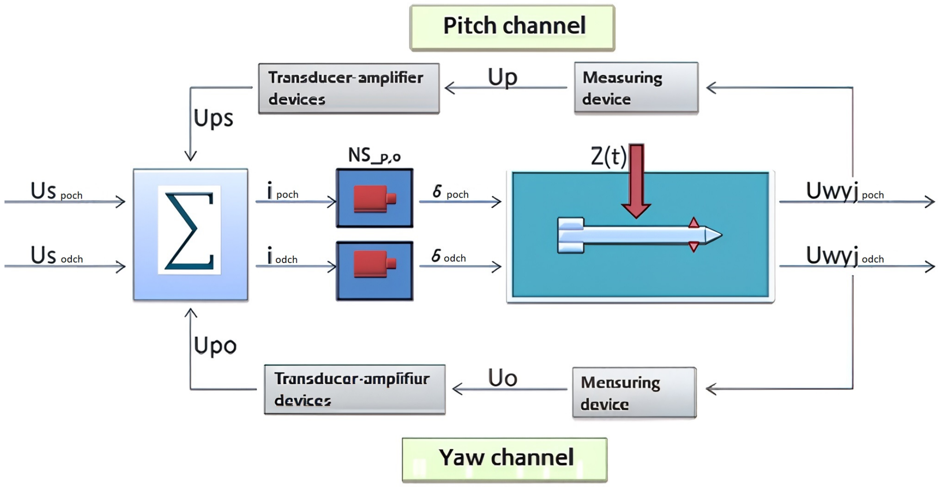


Fig. 9. Two-channel control scheme. Two-channel control scheme. Symbols: 'wyj' = output, 'poch' = derivative. All signals are shown in normalized form

signal, whereas I_{poc} appearing in equation (7) refers to the actuator input signal. The separated currents, according to the given control law, are sent to the respective control surface actuators *NS*, which deflect the control surfaces by an angle δ, directing the control force vector in the phase plane.

After passing through the entire system, the signals return via measurement devices, which measure the actual parameters of the signals along with their corresponding signs. The measured values, along with the sign of the deviation, are processed into proportional electrical voltage signals. The information then passes through a converting-amplifying device, which transforms and amplifies the control signals.

5. Analysis of periodically varying control of a rotating missile projectile

The behavior of the missile projectile during flight can be observed based on the performance characteristics of a pair of control surfaces operating according to the adopted control law, which generate the control force and control moment. Therefore, starting from the flight trajectory simulation, a control algorithm can be designed, which in the final phase of the project should be verified through dynamic tests using live firing [12].

In a rotating missile projectile, both the force and the moment lie in the same plane, while the control of the projectile involves changing the signs of the control signals generated by the control force and moment at the frequency of the projectile's rotation. According to the assumptions of the task, the control of the projectile begins when its rotational speed decreases from the initial speed of

14.5 rad/s to 1.047 rad/s, i.e., at the moment when the projectile passes the apex of its flight trajectory.

It is assumed that the average value of the control force during one rotation depends on the time spent at the control surface's position. In the case of a symmetric signal waveform, the average control force is zero, while for an asymmetric signal, it is not zero. Consequently, by appropriately shifting the signal in time, we achieve the tilting of the average control force P_{mean}, which allows control in two flight planes.

Given the deviation of the actual trajectory relative to the reference trajectory, specific signal values appear in both channels of the control system. These signals need to be reduced to zero according to the adopted control law so that the missile can guide itself toward the target, i.e., minimize the error. Therefore, if the missile is correctly guided, it means that in both channels: pitch (horizontal axis Y) and deviation (vertical axis Z), continuous zero control signals $S_{\text{Od}} = S_{\text{Po}} = 0$ appear. This implies that the components of the average control force S_{mean} are also zero.

The average values of the projections of the instantaneous control force are also zero during one rotation of the missile when the frequency of signal change I_{poc} is an even multiple of the frequency of the missile's rotational motion around the longitudinal axis x.

$$\omega = \omega_x 2n. \tag{7}$$

Thus, with zero control signals $S_{\text{Od}} = S_{\text{Po}} = 0$, the symmetric signal waveform should follow the function below:

$$U_{\text{sym}} = \text{sign}\{\sin[(2n)\omega_x t]\}. \tag{8}$$

In the equation, it can be seen that with a given angular velocity of the rotating projectile and time, we can calculate the rotation angle ($\omega = \psi/t$) in the following plane.The simulation of the projectile's rotation at 62.8 rad/s is shown in Fig. 10.

$$U_{\text{sym}} = \text{sign}\{\sin\,[(2n)\psi_x]\}. \tag{9}$$

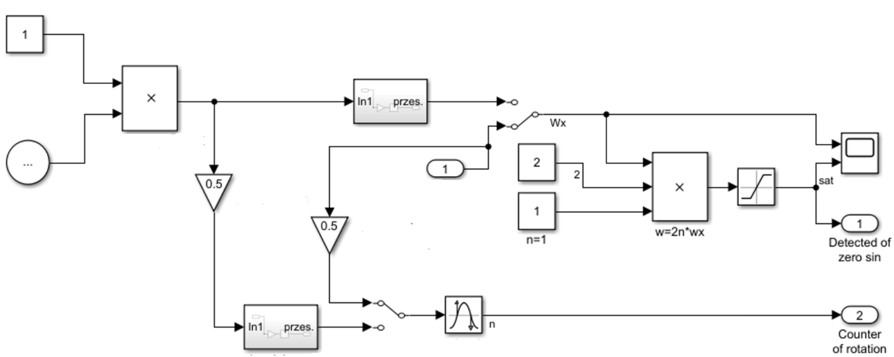


Fig. 10. Simulation of the missile's rotation at a speed of $\omega = 62.8$ rad/s revolutions per second during flight time $t = 1$ second

Additionally, the formula implements the sign function, which, by its definition in the set of real numbers, causes a change in the sign of the control signal. If we now assume that $n = 1$ the sign function, by definition, only takes values from the set $\{-1, 0, 1\}$, then the following function can be derived. This representation allows quantization in both the time and frequency domains.

$$U_{\mathrm{sym}} = \mathrm{sign}\{\sin[2\psi_x]\}, \tag{10}$$

where the sign function is transformed into U_{sign}, which represents the amplitude of the symmetric signal. This allows quantization both in the time and frequency domains in further steps of the algorithm.

Next, if either of the signals S_{Od} or S_{Po} has a value different from zero, the control force will be zero. This results from unequal filling times, meaning the asymmetry of the signal waveform. This asymmetry can be achieved by adding to the sinusoidal component I_{poc} a sinusoidal component of the resulting control signal with half the frequency of the signal I_{poc}. Here, I_{poc} denotes the actuator control command implemented as a switching (square-wave) signal synchronized with the missile's roll motion.

$$\sqrt{(S_{\mathrm{Od}})^2 + (S_{\mathrm{Po}})^2}\,\sin(\omega_x t). \tag{11}$$

In connection with the above, the missile control signal:

$$I_{\mathrm{poc}} = I_A \mathrm{sign}\left(U_s \sin\left(2\varphi - \varphi_o\right) + \sqrt{(S_{\mathrm{Od}})^2 + (S_{\mathrm{Po}})^2}\,\sin\left(\varphi - \varphi_o\right)\right), \tag{12}$$

where: $W_S = \sqrt{(S_{\mathrm{Od}})^2 + (S_{\mathrm{Po}})^2}$ – the resulting control signal, $\varphi = \arctan\left(S_{\mathrm{Po}}/S_{\mathrm{Od}}\right)$ – the phase shift I_{poc}.

It is worth noting, however, that neglecting the dynamics of the actuator system is a simplifying assumption, which may affect the accuracy of the system's behavior representation, especially in dynamic testing. Nevertheless, assuming an appropriate amplitude of the symmetric signal, the proportionality between the average control force P_{mean} and the resultant signal W_S can still be maintained.

However, to determine this value, it is necessary to establish the relationship $P_{\mathrm{mean}} = f(W_S)$ in both channels: pitch (S_{Po}) and deviation (S_{Od}).

The phase shift φ_o of the signal I_{poc} during the cyclic operation of the control surfaces (during one rotation) does not affect the average control force, so it can be assumed that the control occurs in the vertical plane $S_{\mathrm{Po}} = 0 \longrightarrow W_S = S_{\mathrm{Od}}$, $\varphi_o = 0$. Therefore, the sinusoidal function takes the form:

$$U_{s1} = W_S \sin(\varphi) + U_s \sin(2\varphi). \tag{13}$$

During one rotation of the missile, the vector of the instantaneous control force rotates with the missile at its angular velocity, following the direction of its rotation, describing the following work path relative to the vertical phase plane. As a result, the average value of the projection of the instantaneous control force onto the vertical phase axis, lying in the X, Z plane of the Earth's coordinate system for $\varphi_o = 0$ and within the range $\varphi \in \langle 0, 2\pi \rangle$, will not be equal to zero. The conclusion from this is that the vector of the average control force has a line of action along the vertical phase axis, meaning it acts in the vertical plane.

The results shown in Fig. 11 illustrate the movement of the control surfaces on the projectile [13]. The CoR (Center of Rotation) chart displays the rotational velocity of the projectile as a function of time. The horizontal axis represents time, with a value of 1 second simulated in Matlab.

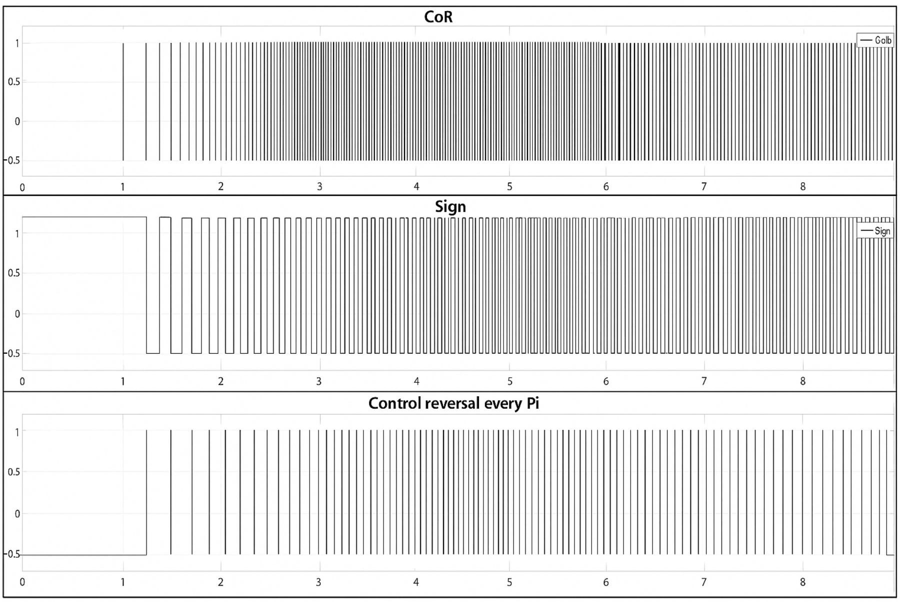


Fig. 11. Shift of control by π

Next, using the signum function, the rotation was converted from peaks to a square wave signal, and then the falling edge of the signum signal was detected. As a result, a control surface shift was obtained every half rotation of the missile.

6. Aerodynamic control system in a surface-to-surface missile with low-rate spin

6.1. Control algorithm description

The analyzed missile rotates during flight at a certain angular velocity around its longitudinal axis. Because of this rotational motion, the missile is equipped with a single pair of fins located at the front of the missile [14]. This configuration allows the generation of control torque and force in the plane of the fins. However, because the rocket rotates, it can be controlled in any spatial plane.

The control algorithm is based on the deflection of the fins as a function of the rocket's angle of rotation [15]. Depending on the position of the rocket's line of sight relative to the fin plane, the control surfaces are deflected in a positive or negative direction.

Due to the dynamics of the movement of the control surfaces and in order to improve control efficiency, fin deflection occurs over a wider range of rocket angles. This range is referred to as the control activation range. In other words, the fins are deflected to a certain position only when the angle between the line of sight to the target and normal to the fin plane is within certain limits (Fig. 12).

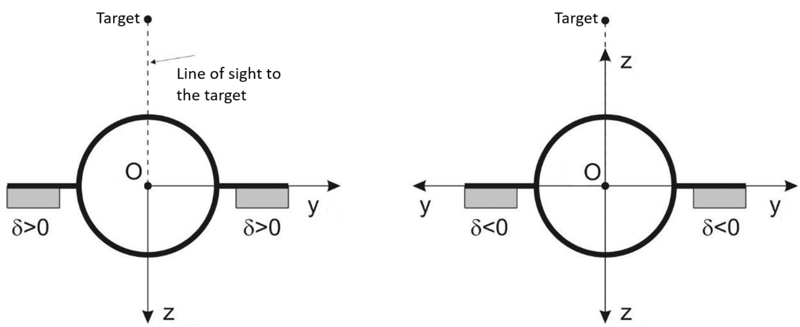


Fig. 12. Example of the deflection of the missile's control surfaces when the line of sight to the target is perpendicular to the control surface plane

As shown in Fig. 12, control activation occurs when the angle between the line of sight and the plane of the control surface falls within the specified activation range. Consequently, control is possible only within a limited spatial sector around the missile's roll axis, as illustrated in the figure. This mechanism clarifies the relationship between the line of sight and the activation range.

6.2. Control signal generation and servomechanism control

The algorithm determines the deflection angle of the control surfaces, which is then transmitted to the control system actuators – the servomechanisms. This value is calculated as the product of the desired deflection angle of the control surfaces and a variable that defines the direction of deflection. This variable is generated in the "Flip-Flop" block.

If the difference between the rocket's roll angle and the target bearing angle falls outside the allowable control activation range, the direction variable is set to zero. This means that, despite the non-zero desired deflection angle, the control surface is not actuated. Otherwise, the variable takes a positive or negative value, causing the control surface to deflect in the corresponding direction. A schematic representation of the algorithm's operation is shown in Fig. 13.

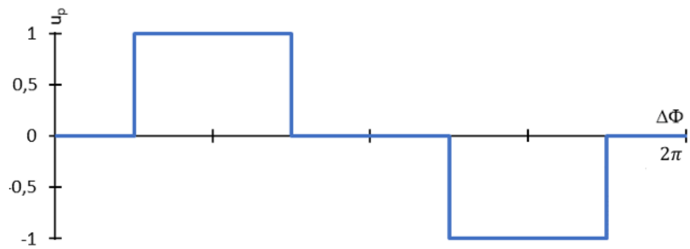


Fig. 13. Control signal trajectory. The "Flip-Flop" block is responsible for changing the sign of the signal depending on the missile's roll angle, implementing the phase-shift algorithm. This ensures synchronization of the canard deflections with the missile's angular position, guaranteeing that the control force is applied in the desired plane

7. Simulation results

The figure below shows the results of the rocket control simulation for a target line-of-sight angle of 40°, a control activation range of 80° (i.e., ±40°), and a commanded control surface deflection angle of 3°, Fig. 14.

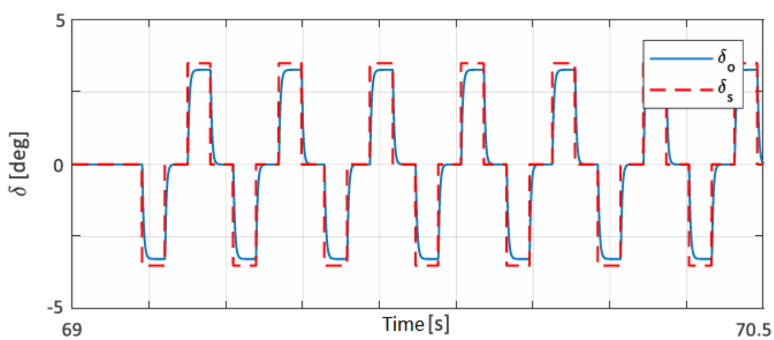


Fig. 14. Simulation results – control surface deflection angle over time

Fig. 14 shows the first few cycles of changes in the command signal sent to the servomechanisms and the corresponding response over time. Fig. 15 presents a single cycle of these signals as a function of the rocket's rotation angle. Additionally, the angle at which the line of sight to the target becomes perpendicular to the orientation of the control surfaces is indicated. As can be observed, the center of the positive and negative active control range aligns precisely with these angles, which in this case are A° and B°, respectively.

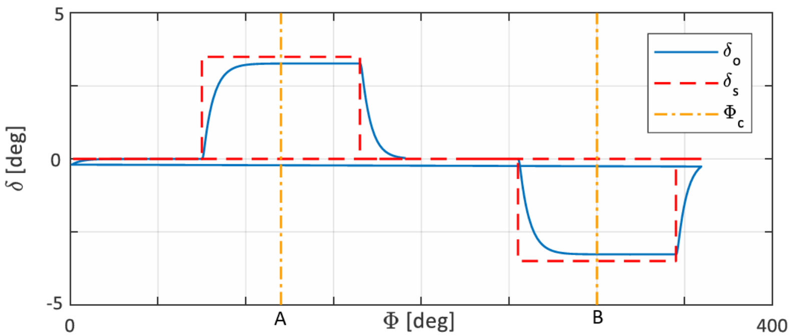


Fig. 15. Simulation output – control surface deflection depending on roll angle

7.1. Trajectory deviation and control performance

The developed control algorithm for the control surfaces was employed to evaluate the performance of aerodynamic surfaces in altering the rocket's flight trajectory [15]. The primary objective of the tests was to quantify the extent to which the rocket's impact point could be displaced relative to the ballistic trajectory, in four directions: left, right, forward, and backwards. The control system was activated after the rocket reached the apex of its trajectory and remained active throughout the descent phase. The control surfaces were deflected up to a maximum of $\pm 5^\circ$ in the specified direction.

Tests were conducted for three distinct launch angles $(55^\circ, 40^\circ, 20^\circ)$ and three different control activation ranges $(\pm 80^\circ, \pm 45^\circ, \pm 20^\circ)$. The following charts present the results of these tests, illustrating the variations in the rocket's trajectory, as well as the pitch and yaw angles. The black circles on the XY plane represent equidistant intervals from the impact point of the uncontrolled rocket, spaced at 500-meter increments, Fig. 16.

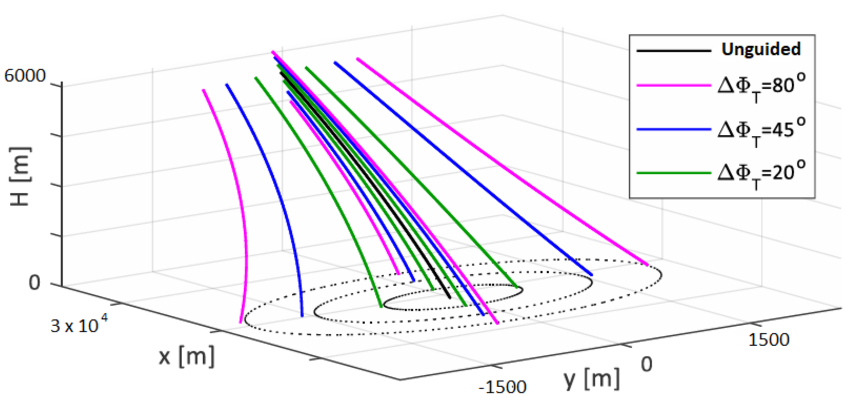


Fig. 16. Approaching the final phase of the trajectory

7.2. Analysis of results

Analysis of the results revealed that the trajectory deviation of the controlled rocket is primarily influenced by the launch angle, which determines both the range and the duration of the controlled flight phase, as well as the activation range of the control surfaces. For longer ranges and larger activation ranges, deviations of over 1000 meters in the trajectory were observed. The maximum pitch and yaw angle deviations were also reported [16].

This section describes an algorithm for controlling a projectile rotating about a longitudinal axis, equipped with a single pair of forward control surfaces. Due to the rotation of the projectile, control is possible in any spatial plane, despite the limited arrangement of the control fins. The control algorithm is based on deflecting the fins within a certain angular range (called the control activation range), which depends on the angle between the line of sight and the fin plane. Simulations confirmed the effectiveness of the proposed solution – the control system enabled a significant deviation of the projectile's point of impact from the ballistic trajectory, especially for higher launch angles and wider activation ranges. The results also show the effect of the control on the changes in the angles of pitch and yaw of the projectile during the descent phase.

The oscillations observed near the third tick on the x-axis in Figs. 17 and 18 are a direct consequence of the applied control algorithm. Since the missile rotates about its longitudinal axis, the canard deflections are synchronized with the roll frequency, which leads to periodic variations in pitch and yaw angles. These oscillations therefore do not indicate instability but rather reflect the cyclic nature of the control action. It should also be emphasized that the propulsion system is inactive during this phase of flight (after motor burnout), so the oscillations do not result in additional propellant consumption. Their impact on range is negligible, as the average control force over one rotation tends toward zero, minimizing extra aerodynamic drag. On the contrary, the algorithm improves overall trajectory accuracy and, under certain conditions, can even extend the effective range.

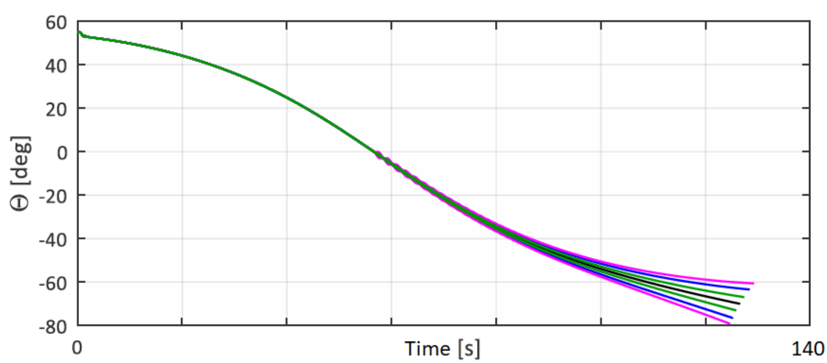


Fig. 17. Effect of control on the rocket's pitch angle

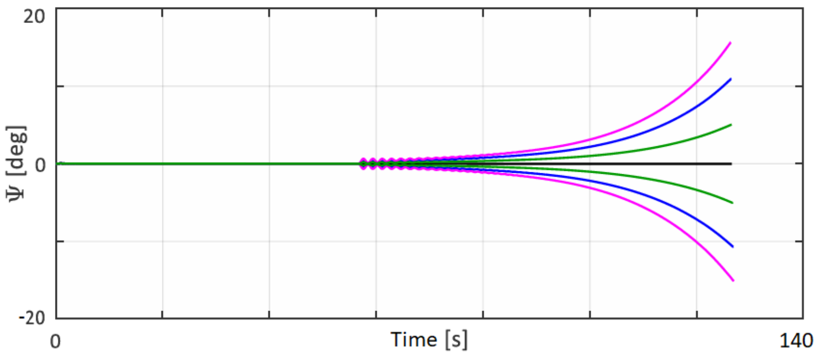


Fig. 18. Effect of control on the rocket's yaw angle

8. Conclusions

The analysis of the developed dual-channel control system for the spinning missile demonstrated that synchronizing the control signals with the missile's rotational frequency – ranging approximately from 1 to 14 rad/s – enables effective adjustment of control moments and forces. This synchronization allows reducing the guidance error by several tens of percent compared to the ballistic trajectory. Simulations confirmed that when the proportionality condition between the control signals and the average control force (which tends toward zero for symmetric signals) is met, the missile's trajectory remains stable, ensuring precise control. It is worth noting that the results regarding the impact error are similar to those obtained in simulations of bomb flight control in [17], which further confirms the validity of the adopted modeling approach.

Tests conducted for launch angles of 20°, 40°, and 55°, combined with control activation ranges between ±20° and ±80°, showed that maximum deviations of the impact point relative to the ballistic trajectory exceeded 1000 meters. The largest deviations were observed for higher launch angles and wider activation ranges, highlighting the significant influence of these parameters on control effectiveness. Furthermore, the maximum pitch and yaw angle deviations reached several degrees, confirming the control system's ability to correct the missile's trajectory in both flight planes.

The use of cyclic changes in control signals, synchronized with the missile's rotation, maintains a stable proportionality between control signals and the generated control force. This feature enables precise three-dimensional guidance of the missile despite the limited number of control surfaces (a single pair). This leads to effective error balancing in both control channels (pitch and yaw), as demonstrated in simulations lasting approximately one second of flight time.

Simulation results and trajectory analysis indicate that, with a properly selected frequency of control signal changes, the average control force approaches zero, and the control signals become symmetrical. This condition allows minimizing guidance error and enables the missile to self-correct toward the target, optimizing control in both flight planes. Practically, this results in impact point shifts of hundreds of meters relative to the ballistic trajectory.

In conclusion, the proposed control algorithm proves highly effective in precise guidance of a spinning missile, even with a limited actuator configuration. Dual-channel control combined with rotationally synchronized signal switching reduces trajectory errors and enhances guidance accuracy. However, final validation of the system's performance requires dynamic firing tests to verify the simulated benefits under real flight conditions and to allow further refinement of the control algorithm and parameters.

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