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THE SECRET OF SÉGUIN'S RODS: RATIO AS A CAPSTONE IN MARIA MONTESSORI'S INTEGRATION OF ARITHMETIC AND GEOMETRY IN CHILDHOOD

Summary: Maria Montessori built an integrated approach to arithmetic and geometry in childhood, in the two stages of infant school (presented in *The method of scientific pedagogy applied to infant education* (1909)) and primary school (presented in the twin books *Psycho Geometry* and *Psycho Arithmetic* (1934)). The crucial role of the concept of ratio is discussed, as embedded in the educational materials designed by her, especially in the bicoloured rods inspired by her reading of Édouard Séguin. The rods were a materialization of the conceptual network connecting measure, rational numbers, and arithmetical/geometrical ratio and proportion.

Keywords: history of mathematics education, Montessori, ratio, Séguin's rods

Maria Montessori and mathematics: from *The method of scientific pedagogy applied to infant education* (1909) to the twin books *Psycho Geometry* and *Psycho Arithmetic* (1934)[1]

This article examines how Maria Montessori (1870–1952) integrated arithmetic and geometry, emphasizing the central role of ratio embodied in Édouard Séguin's (1812–1880) rods. An outstanding physician, educator, and creator

[1] In the first Spanish edition of the two books, the titles appear with separated words: *Psycho Arithmetic* and *Psycho Geometry*. In the 1934 English edition, however, the titles are written as single words. In our text, we have chosen to adopt the original Spanish spelling.

of a new pedagogical system, Montessori was born in a little town of Chiaravalle, near Ancona, soon after the proclamation of the United Kingdom of Italy. In 1875, she moved with her family to Rome, where in 1883–1886 she attended the Michelangelo Buonarroti vocational middle school, and in 1886–1890 the physico-mathematical branch of the Leonardo da Vinci vocational high school (an 'istituto tecnico' in Italian, that is, technical institute). Maria Montessori dealt with the child's approach to mathematics (in infant schools and primary schools), supported by a solid mathematical education[2]. The national movement that led to the unification of Italy in 1861 was marked by post-Enlightenment political-cultural ideals that saw science as a sure support for the construction of a strong and modern country from an institutional, social, and economic point of view. A national mathematical community was formed that was increasingly recognized abroad. Special attention was paid, in particular, to geometry. Italian mathematicians, led by Luigi Cremona (1830–1903) in Rome, had an important role in guiding the extension of a liberal bourgeois educational system where geometry had a central role in both general secondary schools and, even more so, in vocational schools. Montessori was trained in two vocational schools[3]. In these institutes, students got a solid knowledge of classical Euclidean geometry (closely following the *Elements*): without the use of coordinates to 'algebraize' geometric theorems, and without the use of measures (the study of segments without reference to length, of plane figures without reference to area, and so on). Later, modern synthetic approaches to descriptive and projective geometry were introduced[4]. After these schools, and before initiating her undergraduate studies at the Medicine and Surgery Faculty in 1892–1896[5], Montessori attended courses in the Mathematical,

[2] A. Boscolo, M. Crescenzi, B. Scoppola, *Sulla genesi e lo sviluppo del pensiero matematico di Maria Montessori*, "Rivista di Storia dell'Educazione" 2021, vol. 8, no. 2, p. 9–23. On her secondary and higher education, see A. Matellicani, *La "Sapienza" di Maria Montessori. Dagli studi universitari alla docenza 1890–1919*, Aracne, Roma 2007. On Maria Montessori's life and works and the sources and evolution of scholarship on this regard, see the entry in the national Italian biographical dictionary, F. De Giorgi, *Montessori Maria*, [in:] *Dizionario Biografico degli Italiani*, vol. 76, Istituto dell'Enciclopedia Italiana, Roma 2012, p. 166–172.

[3] See F. Furinghetti, *Mathematics education for young women and girls from the birth of the Kingdom of Italy to Fascism*, "The Journal of Mathematical Behavior" 2025, vol. 78, Art. 101210. Furinghetti points out that in Italy only in 1883 were girls officially allowed to enroll in the classical middle school (Gymnasium), yet the silence of the first state 1861 educational law regarding the admission of girls made it possible to enroll in high school (either general or vocational) on a case-by-case basis.

[4] See S. Di Sieno, *Luigi Cremona e la formazione tecnica pre-universitaria nella seconda metà dell'Ottocento*, [in:] *Da Casati a Gentile. Momenti di storia dell'insegnamento secondario della matematica in Italia*, ed. by L. Giacardi, Pubblicazioni del Centro Studi Enriques – Agorà Edizioni, La Spezia 2006, p. 99–124.

[5] She attended the Faculty of Philosophy and Letters in years 1902–1904. In December 1904, she became 'libero docente' in Anthropology at the Faculty of Sciences.

Physical and Natural Sciences Faculty at the Rome University La Sapienza in the years 1890–1892.

After graduating, her main interest was in the medical care of children with intellectual difficulties ('fanciulli deficienti'), a subject she further studied during a stay in Paris between 1897 and 1898. Montessori started her pedagogical journey in infant schools in 1907, and her overall vision of the pedagogy of mathematics – part of an overall pedagogical vision – was built starting from the pilot teaching in some school units for early childhood in Rome starting from 1907. This experience was discussed in her essay *The method of scientific pedagogy applied to infant education in the Children's Houses* (1909, with new editions in 1913 and 1926)[6]. The two books *Psycho Geometry* and *Psycho Arithmetic*, published in Spanish in Barcelona, focused on primary school and constitute a point of arrival, condensing 25 years of experimental practice in the development of her vision.

In Spain, the reform of the normal schools for the training of teachers in the early 1930s included changes in the methodology of mathematics[7], which prompted the publication of several books by Spanish authors focused on mathematics, including numbers and geometry – such as the *Metodología de la matemática elemental* (1933) by Julio Rey Pastor and Pedro Puig Adam and *Metodología de la aritmética y de la geometría* (1932) by Margarita Comas Camps. In the program established by the Department of Education, Montessori was mentioned among the scholars who had designed special methods for mathematics together with Friedrich Fröbel (1782–1852) and her contemporary Ovide Decroly (1871–1932). This created favorable conditions for the publication of her twin books in 1934. In thanking her editor, Ramón de San Nicolas Araluce (1865–1941)[8] at the end of her Foreword to *Psycho Arithmetic*, she recalled the cumbersome reference – for any book like hers – of the established praxis in primary school:

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[6] Below, quotations are from M. Montessori, *Il metodo della pedagogia scientifica applicato all'educazione infantile nelle case dei bambini. Edizione critica*, ed. by P.Trabalzini, Opera Nazionale Montessori, Roma 2000. After the war, the book was published with a different title, *La scoperta del bambino* [The discovery of the child].

[7] M. Sierra Vázquez, C. López Esteban, *Innovaciones en la formación en matemáticas y su didáctica de los maestros en el primer tercio del siglo XX: aportación del movimiento normalista español (1923–1936)* [Innovations in primary teacher training on mathematics and its didactics in the first third of the twentieth century: contributions of the Spanish «Normalista» movement (1923–1936)], "Historia de la educación" 2011, vol. 29, p. 179–193. The program of the course on Methodology of mathematics is included in the appendix to the paper.

[8] Born in the north of Spain, after an experience in Mexico, Araluce settled in Barcelona at the end of the 19th c., and published literature and essays, particularly for the younger readers, see J. Chumillas i Coromina, *Semblanza de Publicaciones de la Casa Editorial Araluce*, Biblioteca Virtual Miguel de Cervantes – Portal Editores y Editoriales Iberoamericanos (siglos XIX–XXI) – EDI-RED, Alicante 2016. Araluce had already published several translations from Italian of Montessori's books, including *The method of scientific pedagogy*: Montessori's pedagogy had been very well received in Catalonia.

> I would like to extend my heartfelt thanks to the publisher Araluce, which has published these works: the psycho-geometry and the psycho-arithmetic, the result of much work brought to completion slowly and with great care. It was not easy to find a publisher brave enough to launch books for elementary school that lie outside the traditional conventions of teaching and place the psychic development of the child above school disciplines. Moreover, these books go well beyond the usual limit in terms of the care dedicated to the printing of the text and the illustrations. Because of this, only a person who had conviction and was capable of great generosity could have published them. These conditions explain why the only books of psycho-geometry and psycho-arithmetic are being published for the first time in Spanish.[9]

The books stood out for their editorial care, including hundreds of colour illustrations. Her focus, according to what she stated in the first chapter of *Psycho Geometry*, was not to clarify the logical structure of elementary mathematical contents (the 'sequence of ideas'), but rather the psychological conditions of the student's encounter with mathematics:

> The words *difficulty – obstacle – pons asinorum – stumbling block* – are applied to a regrettable failure to teach elementary mathematics, which represents the very first steps of education. The collection of problems faced by educators cannot be resolved through the logical study of the sequence of problems. […] His [the pupil] *psychic activity* is the sine qua non for success. Everything that is boring, discouraging and interrupts becomes an obstacle that no logical teaching preparation can overcome. We therefore need to study the conditions necessary for the unfolding of spontaneous individual activities, and develop the art of allowing joy and enthusiasm for work to spread.[10]

The first chapter of *Psycho Geometry* presented general aspects and an introduction to the period of primary school. Additionally, *Psycho Arithmetic* starts with a summary of activities with words and symbols for numbers in the pre-elementary period.

The question arises as to how she envisaged the interconnections between arithmetic and geometry. The relevance of the overarching aspects of interaction between the two 'sister sciences' in Montessori's pedagogy of mathematics, as it evolved over 25 years of practice and research, has been put forward by Bene-

[9] M. Montessori, *Psico Aritmética. La aritmética desarrollada con arreglo a las directrices señaladas por la psicología infantil, durante veinticinco años de experiencia*, Araluce, Barcelona 1934. All the quotations in English translations are from M. Montessori, *Psycho arithmetic. Arithmetic developed under the guidelines outlined by child psychology*, ed. by B. Scoppola, K.M. Baker, Montessori-Pierson Publishing Company, Amsterdam 2016, p. xix.

[10] M. Montessori, *Psico Geometría. El estudio de la geometría basado en la psicología infantil*, Araluce, Barcelona 1934. The quotation is from the English translation from the original Italian manuscript and the original editions: M. Montessori, *Psychogeometry*, ed. by B. Scoppola, Montessori-Pierson Publishing Company, Amsterdam 2011.

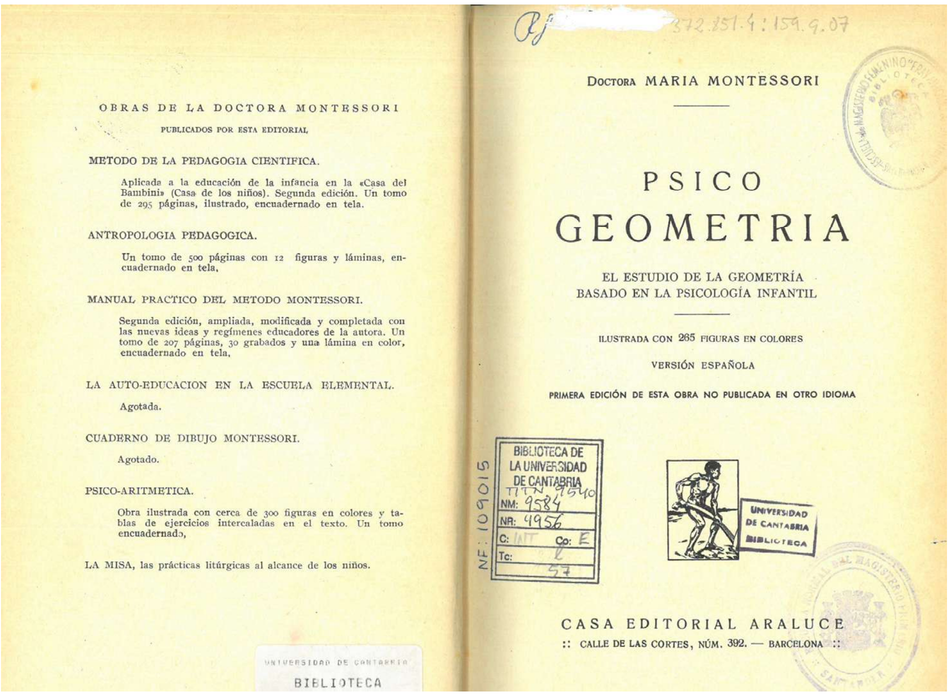

Fig. 1. *Psycho Geometry*: cover of the original 1934 edition in Spanish. From the university library of the Universidad de Cantabria, Santander (Spain).

detto Scoppola, who has traced her 'inspiring principles' in a testimony from a series of lectures addressed to teachers held in Rome in 1931[11]. The following reflection appears crucial:

> Up to a certain point, arithmetic and geometry progressed together, but then it became necessary to separate them. However, the simplest and clearest thing is the origin of things: as I always say, the child must be given the origin of things because the origin is clearer and more natural for their mind. We just need to find *a material that makes the origin accessible*.[12]

[11] B. Scoppola, *Lezioni di Maria Montessori*, "Annali di storia dell'educazione" 2011, vol. 18, p. 413–433. The paper has as an archival source (partially transcribed), the handwritten notes by Flaminia Guidi (1905–2006) of the XVI Corso internazionale di formazione [International training course], now at the Library of the Montessori Primary school in Via Lemonia, Rome (founded in 1957 by Guidi).

[12] Lesson 31, Rome, 5.05.1931, B. Scoppola, *Lezioni di Maria Montessori*, p. 414, our translation. This reference to the origin of things brings to mind Mary Everest Boole's (1832–1916) reference to the 'Recovered Past: the main reason why arithmetic seems dull is that teachers fail to put pupils in possession of that knowledge of subtle forces, 'which gives its possessor the key of the future' (M. Everest Boole, *Lectures on the logic of arithmetic*, Clarendon Press, Oxford 1903, p. 7).

This primordial connection between arithmetic and geometry is thus embedded in her educational aids or devices – the materials offering a 'materialized mathematics' (using her own words)[13]. Geometrical ideas such as addition, decomposition, additive comparison, and ratio underlie the conception and design of her teaching materials, some of them addressing sensorial knowledge (offering visual-tactile-motor perceptions) and focused on shape and size, and the arithmetic materials.

Our focus in this paper is on the concept of ratio (arithmetical ratio and geometrical ratio) as a pivotal conception in the linkage between geometry and arithmetic. Next, we consider the role of geometrical *ratio* and the geometrical notion of *part* in Montessori materials, and especially in the rods (since *The method of scientific pedagogy*) and in the decomposition material for the circle in congruent circular sections introduced in the chapter on the circle in *Psycho Geometry* (under the title 'Fractions'). Then, in the third section, we turn to the presentation of ratio and proportion involving integers in the final chapter of *Psycho Arithmetic*. We shall see that she dealt with numerical ratios and with the solving of proportions leaning on a geometrical vision of ratio that has been worked since early childhood by means of her sensorial/geometrical material.

Exploiting the geometrical meaning of Séguin rods

In Montessori's *The method of scientific pedagogy*, a section was devoted to arithmetic instruction, while there was no such section on geometry. However, geometry was embedded in the pages devoted to what Montessori described in 1909 as 'education of the visual sense', where she gave room to sensorial (visual, tactile, and motion) experiences with sets of wooden objects of regularly increasing size in a single-colour (activity focused on comparisons 'greater than', 'equal to', and 'less than'). The section, and, in fact, the whole structure of the essay, is deeply inspired by Jean Itard (1774–1838) and Édouard Séguin's (1812–1882) approach to education, with which she came into contact during his stay in Paris shortly after graduating, in 1897–1898 (possibly through the advice by Désiré-Magloire Bourneville (1840–1909))[14]. Séguin's educational ideas stemmed from his work with 'extraordinary children', as he called them (then referred to mainly with the word 'idiot'), but he himself proposed their more general validity for the education of children.

The idea of developing sets of wooden objects as an educational device had its roots in one set designed in the 1840s by Séguin, as Montessori herself acknowl-

[13] M. Montessori, *Psychoarithmetic*, p. xix.

[14] The titles of the paragraphs of the book followed Séguin terminology (Muscular education and sensorial education). Montessori herself has described her exploration of Séguin's contributions in Paris.

edged: 10 rods (parallelepipeds of equal square section and growing edge). He did not intend the rods at all as a teaching aid for geometry, but as starting point of his educational work, regarding the interaction between the 'me' and the 'not me'[15]. Yet Séguin explicitly mentioned the primordial nature of geometrical 'formulas', as the first postulate in Euclid's *Elements*, Book I, and, after experiments with geometrical, linear drawings, he turned to decomposition/composition and comparison ('greater than') to be carried out with wooden objects. He first designed a set of bricks (congruent solids) and then the rods (increasingly large solids, evoking increasingly greater segments because of the invariant section).

Montessori shared Séguin's view on the applicability of his 'physiological' methods to any child. The use of materials in early childhood education was entirely natural for her, given the widespread dissemination of Fröbel's gifts, also in Italy[16]. She wrote that 'the same material *makes it possible education* for the disabled [deficienti]; *causes self-education* for the normal'[17]; and contrasted her approach – inspired by Séguin – with the Fröbelian:

> Therefore it is not a question of teaching the child *knowledge* about dimensions, by means of objects, nor is the aim of inducing the child to know how to use the material presented to him *without error*, performing an exercise *well.*
>
> This would put our material on par with any other – for example that of Fröbel; and would require the *active* work of the teacher, who would act by providing *knowledge* and hurrying to correct every error […].[18]

Montessori's materials involving geometrical ratio

Montessori designed two sets of rods (both with edges from 1 decimeter to 1 meter), one of them single-coloured and one bicoloured, with the colour identifying alternating in segments of 1dm, that she often named Séguin rods. Furthermore, she also expanded the idea to other sets of wooden objects of the same shape and different size following precise ratios, both blocks (solids: all of them parallelepipeds) and cylinders, and cutouts (plane figures: rectangles, circles) to

[15] Séguin introduced this material in an essay published in 1843, after the set of bricks was designed in 1839. See E. Gil Clemente, A. Millán Gasca, *Geometry as "forceps of intelligence": lines, figures and the plane in Édouard Séguin's pedagogical thought*, "Bollettino di storia delle scienze matematiche" 2021, vol. 41, no. 2, p. 315–339.

[16] Ratio was deeply present in Fröbel's gifts; see M. Friedman, J. Muñoz Alvis, *Haüy, Weiß, Fröbel: the influence of nineteenth-century crystallography on the mathematics of Friedrich Fröbel's kindergarten. Part 1: the published materials*, "Paedagogica Historica" 2023, vol. 59, no. 2, p. 191–211.

[17] M. Montessori, *Il metodo della pedagogia scientifica applicato all'educazione infantile nelle case dei bambini. Edizione critica*, p. 335, our translation.

[18] Ibidem, p. 340–341, our translation.

be inserted on boards[19]. She did not discuss in detail the role of ratio in the first edition of *The method of scientific pedagogy*, perhaps because a typical question about those materials was, 'Which one is bigger?'[20]. However, there are explicit references to ratio both in the 1926 edition and in the above-mentioned 1931 series of lectures.

Thus, in the 1926 edition, the following sets of blocks were described: the single-coloured set of rods, a second set of prisms with increasing square bases but the same height, and a set of cubes of increasing edge[21]. Those devices had both a general educational meaning and a specifically mathematical meaning. Even if all of them were (solid) blocks, they were intended to represent segments (length), surfaces (area), and solids (volume). The mathematical meaning of the three sets of wooden objects is addressed explicitly in the 1926 edition of *The method of scientific pedagogy*:

> If the relative differences among the three series of blocks, a *mathematical proportionality is found in them*.
>
> In fact, the ten rods are in a ratio to one another as the series of numbers:
>
> $$1 : 2: 3 : 4 : 5 : 6 : 7 : 8 : 9 : 10.$$
>
> The ten prisms of the same length, but varying according to their square section, are in a ratio to one another as the squares of the numbers:
>
> $$1 : 2^2 : 3^2: 4^2 : 5^2 : 6^2 : 7^2 : 8^2 : 9^2 : 10^2$$
>
> Finally, the ten cubes, with all three dimensions varying, are in a ratio to one another as the cubes of the numbers:
>
> $$1 : 2^3 : 3^3 : 4^3 : 5^3 : 6^3 : 7^3 : 8^3 : 9^3 : 10^3.$$
>
> These proportions are accessible to the child only sensorially, it is true, but his mind is exercised on exact foundations – such to *prepare mathematical attitudes*.[22]

In a lecture of the 1931 training series, regarding the wooden cutouts in boards, she discussed the two boards presenting six rectangles sharing one side (10 cm)

[19] Boards or trays with knobbed insets were drawers in the 'geometry cabinet'. They were inspired by Itard and Séguin's boards to compare shape, as she recalled in *The method of scientific pedagogy*.

[20] M. Montessori, *Il metodo della pedagogia scientifica applicato all'educazione infantile nelle case dei bambini. Edizione critica*, p. 390. In Italian 'Quale è più grosso?', an informal word (a standard word would be 'grande') referring to the bulk of something.

[21] In the first edition of *Il metodo della pedagogia scientifica*, there was a single set of bicoloured rods that was used both in the education of the visual sense and in the introduction of number (see below); there were four different sets of blocks, including the rods, in the section 'Education of the visual sense'. In the 1926 edition, this section was named 'Visual distinctions', and there were three sets of blocks: the red rods, the pink cubes, and the brown prisms of square section (parallelepipeds). Those materials are still used nowadays.

[22] M. Montessori, *Il metodo della pedagogia scientifica applicato all'educazione infantile nelle case dei bambini. Edizione critica*, p. 393 (our translation, our emphasis).

and with decreasing second side (10, 9,…, 5 cm) and six circles of increasing diameter (5, 6,…, 10 cm)[23]. She observed that what made the comparison between those 'resembling' sets interesting was the 'reciprocal relationship of measure [*rapporti di misura*]' so that, for example, the last rectangle is half the square[24].

The transversal role of Séguin's rods in Montessori's psychomathematics*:* geometry, arithmetic, measure

The set of rods – including both the red rods and the bicoloured rods – was used by Montessori in multiple contexts regarding different stages and concepts:

1) (more/less) geometrical comparisons, included in 'sensorial education' (mono-colour rods) in infant school, where they were used together with the other two sets of blocks (see above);
2) the introduction of counting numbers from one to ten (the two-colour rods) in infant school;
3) geometrical decomposition and parts/fractions/ratio (the two-colour rods) in primary school;
4) algebraic relationships (the two-colour rods) in primary school;
5) the ratio of the meter to the decimeter, in the context of the presentation of the metrical decimal system in primary school.

Cases 1) and 2) are described in the first edition of *Il metodo della pedagogia scientifica*; while 3) is discussed in *Psycho Geometry* and 4) and 5) in *Psycho Arithmetic*.

This transversal use in the stages of infant and primary school gave a kind of continuity of the child's experience in mathematics, as the rods were presented to very young children who could turn to this familiar material again and again, always finding inspiration for action and understanding. In the 1926 edition of *The method of scientific pedagogy*, when discussing the mathematical proportionality to be found in the sets of blocks, she wrote: 'However, when in elementary school he becomes *interested in arithmetic and geometry*, he takes up the blocks of his early childhood and studies them again in *relative proportions*, applying the science of numbers'[25].

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[23] Those two boards are part of a family discussed in the section on 'Differential visual perception of shapes and visual-tactile-muscular perceptions' in the first edition of *The method of scientific pedagogy* (M. Montessori, *Il metodo della pedagogia scientifica applicato all'educazione infantile nelle case dei bambini. Edizione critica*, p. 393 ff). In *Science and hypothesis* (1902), Henri Poincaré discussed the three aspects visual, tactile and motor of the representative space as an image of geometrical space in Chapter 4, Space and geometry.

[24] 20th Lecture, Rome, 24.03.1931; B. Scoppola, *Lezioni di Maria Montessori*, p. 416 ff. Quoted expressions in p. 419.

[25] M. Montessori, *Il metodo della pedagogia scientifica applicato all'educazione infantile nelle case dei bambini. Edizione critica*, p. 393, our emphasis. This 'return' to memories of infant

The use of the rods as a device for the introduction of counting numbers from one to ten was described in the pages devoted to 'Teaching numeration and introduction to arithmetic' in *The method of scientific pedagogy* from its first edition onwards. Before introducing numerals, she suggested oral activities using collections of things (such as the spindles designed by her, or other available little objects such as tokens and Fröbelian little cubes)[26], but the main role was played by the rods. In 1926, she added an explanation regarding the choice of the rods (and, incidentally, she explicitly mentioned ratio as a property embodied in the rod design):

> The first material used for numeration is the series of ten rods for length, already used for sensory education: the rods *are in a ratio from one to ten*. In fact, the shortest rod is ten centimeters long, the second one is twenty, and so on, up to the tenth, which is one hundred centimeters, or one meter. However, when used for numeration, the rods are no longer all of one colour, as they were when they served as sensory material for appreciating graduated lengths visually. Here, instead, the various ten-centimeter segments are alternately coloured red and blue, making it possible to distinguish and count them on each rod. If the first rod represents the quantity 1, the others successively represent the quantities: 2, 3, 4, 5, 6, 7, 8, 9, 10.
>
> The advantage of this material is that it can present, united yet distinct and countable, the component units of each number it represents. For example, the rod for five is one single piece corresponding to the number 5, but the five units are distinguished by the colours. This method overcomes a significant difficulty: that which arises in numeration when adding units separately, one after the other.[27]

In one of the 1931 lectures, she explicitly discussed side by side the mono-colour and the bicolour rods, and included a hint of a possible use for them in introducing the concept of ratio that is not to be found explicitly in the twin books:

school" is mentioned explicitly by her when going ahead with advanced topics such as algebra or the measurement of lengths: M. Montessori, *Psychoarithmetic*, p. 221, 356.

[26] Ibidem, p. 623.

[27] Ibidem, p. 618, note 4. Thus, a geometrical object helps to understand the paradox of the multiple 'ones': 'If we use to count any small objects, such as, say, small identical cubes, why is it said "one" placing the first [cube] down, while placing another [cube] down it is said "two" and so on? The young child tends to say "one" in relation to each new object that is added, that is, to say: "One, one, one, one, one", rather than, "One, two, three, four, five". The fact that by adding a new unit, a group is enlarged; and that this growing set must be considered, constitutes precisely the obstacle that opposes numbering, when dealing with children of three and a half or four years of age. The grouping of units that are actually separated from each other into a set, is a mental task initially inaccessible to the child. In fact, many little children count by reciting the natural series of numbers by memory, but remain confused in front of the quantities corresponding to them. Counting fingers, hands and feet is already something more concrete for the child, because he can always find the same objects invariably gathered in that specific quantity. He will know that he has two hands and two feet'.

> As for the fact of penetrating the mathematical concept of ratio, this concept is further facilitated by the alternating colours, which *precisely demonstrate the ratio* in which the quantities are. To a child who already has a confident well-defined sensory preparation, we can teach this concept with great clarity and ease.[28]

The 'demonstration of the ratio in which the quantities are' was reached, therefore, through the exhibition – by means of the alternated colours – of the parts forming any rod, each part being congruent to the smallest (red) block; that is, it was offered to the child the decomposition of any rod as geometrical iterated addition of the red rod. Notice that the rods could be used to show ratios between solid bodies, segments and surfaces.

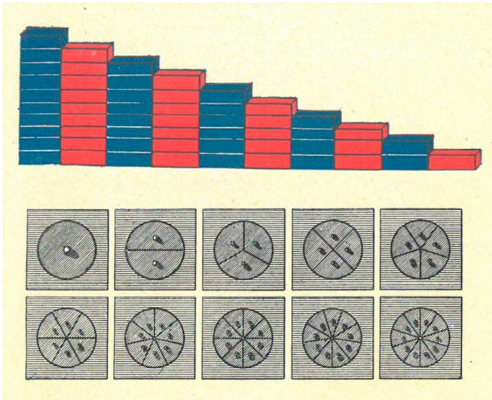


Fig. 2. Parts of the circle and parts of the rods in *Psycho Geometry*.

Source: M. Montessori, *Psicogeometria*, Araluce, Barcelona 1934, p. 191.

A mention of the bicoloured rods as an aid for introducing geometrical ratio appeared in *Psycho Geometry*, somewhat hidden in one paragraph of the chapter on the circle entitled 'Fractions' (see Fig. 2). There the set of rods was coupled with a wooden cutout board consisting of ten circles, nine of them divided into equal pieces (2, 3,…,10 circular sectors). In fact, the section was a hybrid where the word 'fraction' was used both to indicate an arithmetical object made up of two natural numbers (numerator and denominator) and a geometrical *part* or 'fragment' (of a surface, of an angle, of a segment). Thus, in some sense, the chapter addressed the ratios 1 : 2, 1 : 3,…, 1 : 10 together with the rational numbers $\frac{1}{n}$, $n = 1,\ldots, 10$.[29]

This same device was used in *Psycho Arithmetic* to support the understanding of the ratio of the meter to the decimeter (see Fig. 3), a capstone in the chapter on the Decimal Metric System. The concept of ratio in measure was not directly addressed in that chapter. A cross-reference was included instead to the chapter on the equivalence of plane shapes in *Psycho Geometry*: there, several examples of splitting a square into equal pieces were discussed, using the words 'half',

[28] 25th lecture, Rome, 20.04.1931, see B. Scoppola, *Lezioni di Maria Montessori*, p. 425 (our translation).

[29] 'Such fractions, that are quantities – and in our case, also figures, can be added', M. Montessori, *Psicogeometría*, Araluce, Barcelona 1934, p. 191: the paragraph goes ahead showing some examples regarding additions and fractions of fractions, closing with some examples of decimal positional notation of such numbers.

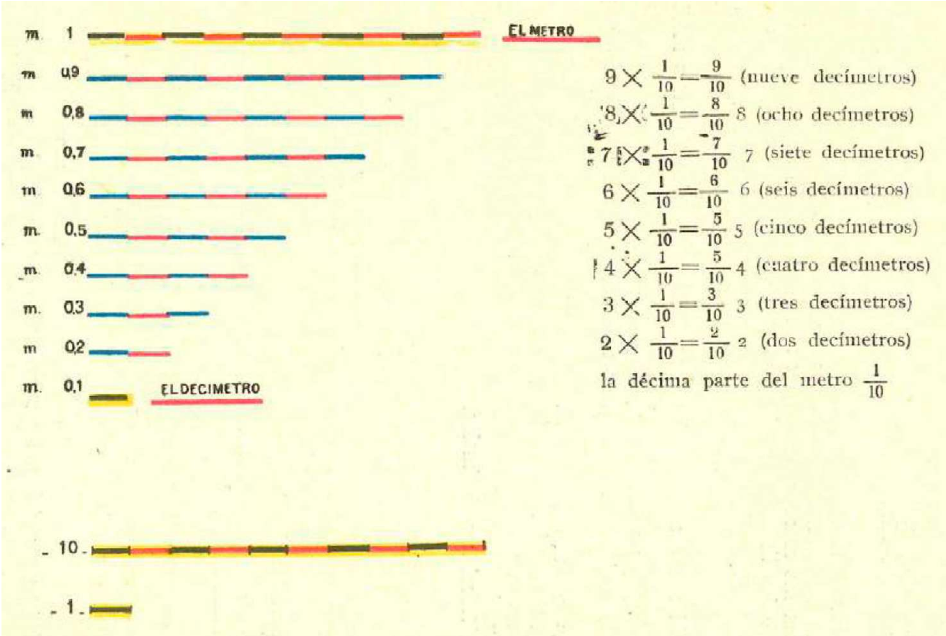


Fig. 3. The red small rod and the largest rod become the decimeter and the meter in the chapter on the Decimal Metrical System in *Psycho Arithmetic*. Notice that the rational numbers are used in the diagram, but the notation was not explained in the preceding chapters of the book.

Source: M. Montessori, *Psico Aritmética,* Araluce, Barcelona 1934, p. 345.

'quarter'; and the calculation of the area of a surface was explained, without any mention of the word 'ratio'[30].

Notice that she was far from considering ratios only as quotients of numbers. Montessori did not give up using the geometric concept of ratio: on the contrary she introduced it from early childhood, embedded in materials of which Séguin's rods were central – this constitutes the 'secret' of Séguin's rods, to which the title of this paper refers, because of course Séguin did not consider ratio when working with disabled boys, but only comparisons ('greater than'). Book V of Euclid's *Elements* loosely defines ratio (Def. III) as 'a sort of relation in respect of size between two magnitudes of the same kind', thus to convey what sort of relations it is she designed materials embodying the idea of part/multiple of a segment, surface, angle etc., as described by Def. I ('A magnitude is a part of a magnitude, the less of the greater, when it measures the greater'); and II ('The greater is a multi-

[30] The number of unit squares in a rectangle and the difficulty of putting many square meters on a plot of land were vaguely mentioned in *Psycho Geometry*.

ple of the less when it is measured by the less')[31]. She was thus directly facing the crucial educational issue of the connection between arithmetic and geometry, an issue deeply involved in the role of intuition in the learning of mathematics.

Thus, rather than addressing the issue of geometrical ratio with definitions and in connection with the measurement of length, surface, and volume, Montessori built a 'sensory preparation' – geometrical visual, manual, and motor sensations based on a variety of materials – that would have made the teaching of the concept of ratio possible 'with great clarity and ease'.

Let us turn finally to the chapter on ratio and proportion in *Psycho Arithmetic* (the last chapter), where she tried to do so, thus entering the territory of secondary education.

'In search of a fourth unknown companion': ratio and proportion in *Psycho Arithmetic* (1934)

The production of texts on teaching arithmetic in the various national languages continued steadily between the late 19th and early 20th c. 'Arithmetic' as a literary genre has a long editorial history in Europe, dating back to the beginning of printing, and the market continued to demand them[32]. The power of tradition was strong regarding its contents, that is, the traditional corpus of practical reckoning now taught in the primary schools. Typical contents included: writing numbers, both integers and broken numbers [fractions], using the decimal positional system; measurement and money (the so called 'denominated' quantities); written calculations; and the solving of practical problems for commerce, accounting, or crafts, mainly in situations involving proportionality and thus applying multiplication/division coupled in the so called 'Rule of Three'. Measurement was presented as an area of numerical procedures, concealing the geometrical ideas of *part* and of *ratio*.

In the same period, 'methodological' essays on the specific task of starting arithmetic with children, such as Montessori's, were published, discussing the actual goal of it in current times and the suitable methods for such goals to be fulfilled (suggestions of activities, educational aids and toys, learning paths). The development of institutions for the training of future primary school teachers

[31] In fact, modern mathematics has arithmetized the concept of ratio to avoid Euclid's definitions, and thus its introduction in school as a division between measures. But there is a circularity in doing so, as for obtaining measures it is necessary to have a definition of ratio. See A. Malet, *Measuring goods, or the social origins of the early modern "arithmetization" of ratio and proportionality*, [in:] *Report No. 58/2017, Mathematical instruments between material artifacts and ideal machines: Their scientific and social role before 1950*, ed. by M. Bullynck et al., Oberwolfach 2017, p. 17–23.

[32] See K. Bjarnadóttir, *History of teaching arithmetic*, [in:] *Handbook of the history of mathematics education*, ed. by A. Karp, G. Schubring, Springer, Cham 2014, p. 431–457.

and the sustained trend of proposals for innovation in practice may explain the presence of such essays from a market perspective as well as from a cultural one. There was a growing awareness of the need to address the specificity of the effective practice of teaching children arithmetic, and this prompted the exploration of insights coming from different areas, such as child psychology, experimental pedagogy, as well as historical or social frameworks[33].

Such a reflection was framed by some scholars in a larger picture: the 'introduction of children to the mathematical world, far beyond the tradition of pure numerical literacy'. In Europe, the book by the French mathematician and MP Charles-Ange Laisant (1841–1920), *Initiation mathématique* [Mathematical initiation] (1906), adopted this approach. With 65 short chapters, it started with the young child counting little sticks (drawn strokes or physical matches), discovering figures, and then placing sticks end to end to build the straight line, thus entering the kingdom of geometry[34]. He emphasized the concept of ratio as a cornerstone of children's mathematical initiation (integrating number and geometry), which could quickly become perfectly familiar to the pupil through examples. In paragraph 15 of *Initiation mathématique*, he forcefully criticized the conventional place of ratio at the end of arithmetic:

> *[Ratio] is the root of all calculation and all measurement, but, by some strange hallucination, in academical teaching it is put at the end of arithmetic*[35]. It is not possible to count two beans without having this idea of the ratio of two to one; nor to measure a length of three yards without comparing the length with that of one yard (ratio of three to one), and so on.

[33] Women appear as authors of such kind of educational essays in mathematics: see A. Millán Gasca, *Educational writing by women in the early 20th century: mathematics for children in Mary Everest Boole, Maria Montessori, Grace Chisholm Young and Margalida Comas Camps*, [in:] *Women on women. De-gendering perspectives*, ed. by R. Leproni, Franco Angeli, Milano 2021, p. 141–153. There were contrasting approaches, for example, between those from members of the mathematical environment and those from pedagogues, but also some shared ideas; see A. Millán Gasca, *A hidden thread. Ideas and proposals on children's mathematics education in the Late Modern Age*, [in:] *History and epistemology in mathematics education. Trends, practices, future developments*, ed. by E. Barbin, M. Fried, M. Menghini, F. S. Tortoriello, Springer, Cham 2025, forthcoming, and the references herein.

[34] Among essays on mathematical education, Benchara Branford's *A study of mathematical education* (1908, 2nd ed. 1921, regarding all the grades of education) advocates the integration of arithmetic and geometry, while Alexander McLellan and John Dewey's *The psychology of number and its applications to methods of teaching arithmetic* (1895), obviously focusing on number, starts from the consideration of segments and lengths. Strikingly, the British mathematicians Grace E. Chisholm Young and William H. Young published a book entirely devoted to geometry, *The first book of geometry* (1905), proposing as in the case of Laisant a proper, accurate educational itinerary to be started in early childhood.

[35] Our emphasis.

> At this stage it will be desirable to show the pupil, without any theoretical explanation, without any definition, without any appeal to his memory, the commonest measures, weights and coins which we find ready to our hand, yards, quarts, ounces, cents, etc.
>
> We will give him exercises in making use of them, accustoming himself to them to measure and to count, and the idea of ratio will insensibly grow in his mind, will associate itself indissolubly with that of number, with which is essential for the day in the future when we will pass from play to work. And this work can become not only interesting but amusing, instead of being a wearisome task, if not a torture.[36]

The presence of a final chapter in *Psycho Arithmetic* (1934) devoted to 'Ratio and proportion' could be seen – following Laisant's views – as a rather conventional choice, with Montessori addressing the crowning issue of practical, 'commercial' arithmetic. It is a very concise chapter (7 pages out of 386 in the original Spanish edition), involving no materials and thus no illustrations. No geometrical representations based on the concept of ratio referred to segments, angles, plane figures, or solids are given there[37]. But the concept of ratio was dealt with by leaning on previous geometrical experience developed around the Séguin rods, and the rest of the materials considered above.

The chapter is divided into two parts:

- the first part (the initial pages of the chapter) focuses on: first, introducing in a few lines the concept of ratio for pairs of counting numbers; and secondly, discussing and giving a definition of the concept of proportion, and linking it to arithmetic fractions;
- the second part addresses solving proportions (two methods are briefly described) and includes, for the first time in the book, two practical problems (on the motion of a train and the fabric used to make curtains) of the usual kind in traditional arithmetic textbooks.

Let's quote the first lines of the chapter:

> If I compare the metre to the decametre, I can infer that a metre is one tenth of a decametre. Indeed, its very name suggests this. I shall therefore state the obvious fact that the *ratio* between a decametre and a metre is 10. If I then compare the decimetre to the metre, we encounter the same concept, although the terms of the comparison have changed: the *ratio* between a metre and a decimetre is also 10. I thus reach the conclusion that the same relationship exists between a metre and decametre as between a decimetre and metre, and also, that these measurements share the *same ratio*.

[36] C.A. Laisant, *Mathematics*, Doubleday, Pace & Co, Garden City, New York 1914, p. 32 (our emphasis; the French word rapport has been translated by ratio (instead of proportion in the original American English translation).

[37] A preliminary study of this chapter was presented in L. Parenti, *La proporzionalità come introduzione al pensiero scientifico in età infantile* [Proportionality as introduction to scientific thought in childhood], unpublished Master thesis, Roma Tre University, Rome 2016.

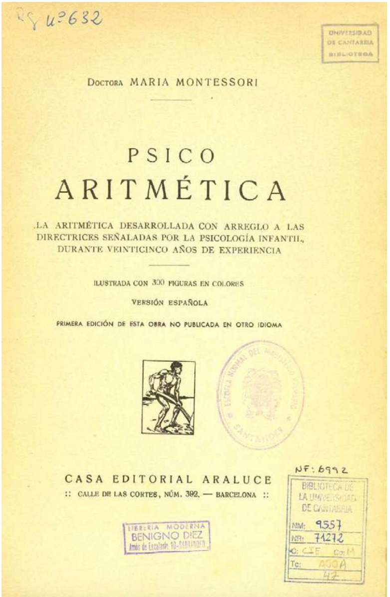


Fig. 4a. *Psycho Arithmetic*: cover of the original 1934 edition in Spanish. From the university library of the Universidad de Cantabria, Santander (Spain).

If I were to say that the metre is to the decametre as the decimetre is to the metre, I would be saying the same thing, but in such a way that the two ratios are now being compared.[38]

So the presentation does not start from arithmetic and does not start with a definition of ratio as division, but it starts from an example of the ratio of one segment to another, the meter and the decimetre, using the verbal expression 'one tenth' (as in 'double', 'twice', 'half', 'triple', or 'one third'). The 'sort of relation in respect of size' between the meter and the decimetre had been considered in detail in the preceding chapter, 'Decimal metric system', leaning heavily on the rods (see above Fig. 3), and building on the preceding work on the blocks and boards (see Fig. 4a and Fig. 4b).

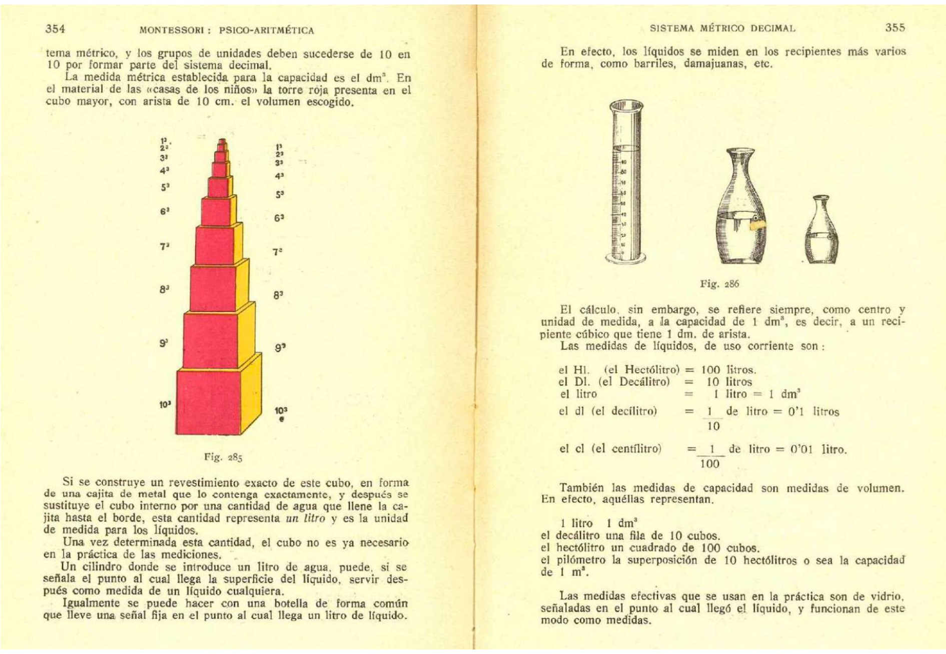
The leap from segments to numbers was somewhat natural, by thinking about segments as measures, and then from denominated numbers to abstract numbers. Furthermore, the ratio could be linked to the 'broken' numbers introduced in *Psycho Geometry* (see above and Fig. 2):

> We could make similar comparisons using abstract numbers. If, for example, we compare 2 to 6, we could say that the ratio between them is $\frac{1}{3}$, because 2 is one third of 6.
>
> This 'ratio between the first number and the second' is the same as the ratio of 5 to 15, because 5 is one third of 15.
>
> We can therefore repeat that which we have already said: 2 is to 6 as 5 is to 15 […] 'as' in the sense of 'in the same way that'.[39]

The proportion was defined as the equality of two ratios, introducing letters to symbolize its four terms, and attention was drawn to the infinite pairs of counting numbers with equal ratios. Thus, the ratio was not fully merged with a division

[38] M. Montessori, *Psychoarithmetic. Arithmetic developed under the guidelines outlined by child psychology*, p. 389 (some minor corrections in the English translation).

[39] Ibidem, p. 389. Denominated numbers (that is, numbers referred to money or lengths and so on) as opposed to abstract or pure numbers, were an important reason for the complexities in teaching arithmetic.



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tema métrico, y los grupos de unidades deben sucederse de 10 en 10 por formar parte del sistema decimal.

La medida métrica establecida para la capacidad es el dm³. En el material de las «casas de los niños» la torre roja presenta en el cubo mayor, con arista de 10 cm. el volumen escogido.

Fig. 285

Si se construye un revestimiento exacto de este cubo, en forma de una cajita de metal que lo contenga exactamente, y después se sustituye el cubo interno por una cantidad de agua que llene la cajita hasta el borde, esta cantidad representa *un litro* y es la unidad de medida para los líquidos.

Una vez determinada esta cantidad, el cubo no es ya necesario en la práctica de las mediciones.

Un cilindro donde se introduce un litro de agua, puede, si se señala el punto al cual llega la superficie del líquido, servir después como medida de un líquido cualquiera.

Igualmente se puede hacer con una botella de forma común que lleve una señal fija en el punto al cual llega un litro de líquido.

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En efecto, los líquidos se miden en los recipientes más varios de forma, como barriles, damajuanas, etc.

Fig. 286

El cálculo, sin embargo, se refiere siempre, como centro y unidad de medida, a la capacidad de 1 dm³, es decir, a un recipiente cúbico que tiene 1 dm. de arista.

Las medidas de líquidos, de uso corriente son:

el Hl. (el Hectólitro) = 100 litros.
el Dl. (el Decálitro) = 10 litros
el litro = 1 litro = 1 dm³
el dl (el decílitro) = $\frac{1}{10}$ de litro = 0'1 litros
el cl (el centílitro) = $\frac{1}{100}$ de litro = 0'01 litro.

También las medidas de capacidad son medidas de volumen. En efecto, aquéllas representan.

1 litro 1 dm³
el decálitro una fila de 10 cubos.
el hectólitro un cuadrado de 100 cubos.
el pilómetro la superposición de 10 hectólitros o sea la capacidad de 1 m³.

Las medidas efectivas que se usan en la práctica son de vidrio, señaladas en el punto al cual llegó el líquido, y funcionan de este modo como medidas.

Fig. 4b. The cubic blocks in the study of volume in *Psycho Arithmetic* (currently known as 'Pink tower').

(or to a fraction) between numbers in the sense that it regarded it in the same manner as infinite pairs of numbers (where this 'same manner' had been explored perceptually, geometrically, with rods, blocks, and boards). Some manipulation of symbols leads to the property of ratios $a : b = c : d$ means that $a \times d = b \times c$; the geometrical counterpart of continuous proportion between segments regarding equivalence of surfaces was mentioned ('we have geometrically the equality of a square and a rectangle') but not exhibited with a drawing.

For solving proportions ('If 2 is one third of 6, 5 will be one third of what number?')[40], she introduced the little script of three people who know each other and go out in search of a fourth unknown companion, thus arising mimesis in pupils. She encouraged thus the idea that calculating such a number is simple, 'because the fourth is in the same relation as shared by two known terms'[41], yet showed the complexities of this search with two examples regarding distance and time, and length of fabric and number of windows. She introduced two methods and showed in one example that both lead to the solution, involving either a mechanical procedure (with multiplication and division) or some reasoning on finding the ratio to 1 (e.g., 1 hour, 1 window) in the case of the reduction to the unit.

[40] Ibidem, p. 393.

[41] Ibidem, p. 393.

In those final pages, a clear, but somewhat unresolved, effort to avoid the reduction of ratio to a purely arithmetical notion (the quotient in a division, or plainly a rational number) can be discerned: Montessori appears as struggling to reach a point of arrival and showing the challenges of mathematics for teenagers in secondary school. The path was traced, and it asked to explore the connection between number – the capstone of modern mathematics – and 'old' geometrical concepts deeply rooted in the human experience, from the basic concepts of equality and decomposition to the startling, demanding, yet so natural, early childhood concept of ratio.

In conclusion, it may be noted once again that the concept of geometrical ratio was embedded – hidden, as it were, like a secret – in Séguin's wooden rods. Through a partly implicit process, the adoption of these rods made ratio a crucial concept in Montessori's pedagogy of mathematics, also because geometrical ratio was central to her own Italian mathematical background, in which geometry was highly valued. In fact, the entire set of her educational materials carried, as a hidden core, the idea of ratio and geometrical comparison; thus, geometry proved to be fundamental for the initiation into mathematics for millions of students.

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