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THE INTRODUCTION OF NUMBER SETS IN MODERN MATHEMATICS IN BELGIUM

Summary: During the 1960s and 1970s, a major curriculum change took place in mathematics education. In this so-called Modern Mathematics era, emphasis was placed on mathematical structures, and set theory was seen as the best basis for mathematics education. These beliefs also influenced the way in which number sets were introduced. Some extreme proposals were made, such as defining cardinals in primary education and introducing the Peano axioms in secondary education. In this paper, we examine the extent to which extreme ways of introducing number sets were actually implemented in the classroom. We analyse four textbook series for secondary education in the Flemish part of Belgium. We conclude that the Peano axioms were not introduced, but a formal definition of cardinals was. The four textbook series show several differences in the methods used to introduce the set of rational numbers and the set of real numbers, and in the order in which they are introduced. Limiting the current research to Flemish textbook series allows for more detailed work, but also shows the need to continue this research for the French-speaking part of Belgium. The same applies to extending the current research from the secondary level to the primary education level.

Keywords: history of mathematics education, Modern Mathematics, New Math, number sets, cardinal, Peano axioms

Introduction

In mathematics education, the sense of renewal and change that swept through society after World War II culminated in the Modern Mathematics or New Math movement¹. A collective of French mathematicians, the Bourbaki group, started in the mid-1930s by formalising all mathematical knowledge using set-theory and so-called ‘mother structures’. The Swiss psychologist Jean Piaget recognized in these mother structures the mental structures that play a role when learning mathematics. As a result, ‘A model for the science of mathematics was promoted as a model for mathematics education’². In the 1960s and 1970s, mathematics curricula around the world were changed to introduce set theory, mathematical structures such as groups, and new areas such as logic, analysis, and statistics. But as early as the 1950s, there had been some experimentation with a modernized version of school mathematics. In the USA, for example, the University of Illinois Committee on School Mathematics (UICSM) set up an experimental mathematics curriculum for the secondary school that was affiliated with the University of Illinois³. In the UICSM material, a one-to-one correspondence of sets is defined, which leads to the definition of natural numbers as classes of sets that are in one-to-one correspondence: ‘The number two is the class of all sets that match $(\Delta \Delta)$ ’⁴. Integers are then defined as equivalence classes of ordered pairs of natural numbers ‘The positive integers were represented by pairs (a, b) with $a > b$, while the negative integers were represented by pairs where $a < b$. The class of all pairs (a, a) was defined as zero’⁵.

An early experiment in teaching Modern Mathematics can also be found in Belgium: Frédérique Lenger (later Papy-Lenger) and Willy Servais developed an experimental programme for teaching Modern Mathematics to future kindergarten teachers⁶. Lenger and Madeleine Leprope taught the programme in two schools during the school year 1958–1959. It started with set theory, followed by ‘Operations on integers and relation with the operations on sets’⁷. Servais briefly refers to these classroom experiments as ‘some isolated tests’⁸ at the Royaumont seminar of 1959.

¹ See, e.g., D. De Bock, *Modern mathematics. An international movement?*, Springer, Cham 2023, p. 1–9.

² Ibidem, p. 46.

³ See D.L. Roberts, *The rise of the American New Math movement: How national security anxiety and mathematical modernism disrupted the school curriculum*, [in:] *Modern mathematics. An international movement?*, ed. by D. De Bock, Springer, Cham 2023, p. 13.

⁴ Ibidem, p. 24. Underlining in the original.

⁵ Ibidem.

⁶ D. De Bock, G. Vanpaemel, *Rods, sets and arrows. The rise and fall of modern mathematics in Belgium*, Springer, Cham 2019, p. 96–98.

⁷ F. Lenger, M. Leprope, *Le programme B des écoles normales gardiennes*, “*Mathematica & Paedagogia*” 1958–1959, vol. 16, p. 71.

⁸ D. De Bock, G. Vanpaemel, *Rods, sets and arrows*, p. 98.

The Royaumont seminar brought together European and American mathematicians, educators, and mathematics teachers under the theme ‘New thinking in school mathematics’⁹. One of the participants was the Bourbakist Jean Dieudonné. His outcry, ‘Euclid must go!’¹⁰, during his proposal for the reform of school mathematics, caused a lot of controversy and was taken home by the other participants. But Dieudonné also shared his vision of what he thought a Modern Mathematics curriculum should be. He mentioned, for pupils up to the age of 14:

If it is thought advisable, psychologically speaking, to start some axiomatics at that level, then, according to our general principle, we should look to the part of mathematics with which the children have had the most protracted contact on the “experimental” side, namely elementary arithmetic. [...] Indeed, it is one of the simplest and most beautiful exercises in logic to develop the usual laws of arithmetic starting from the “Peano axioms”, and I see no reason why this should not be attempted at the earliest possible moment.¹¹

About the age of 14, Dieudonné said: ‘the time is ripe for an axiomatic description of real numbers. [...] by saying that the real numbers form an Archimedean ordered field, in which the principle of nested intervals holds’¹².

The French mathematician Gustave Choquet shared his proposal for a Modern Mathematics reform of the curriculum at the Royaumont seminar. It included a clear statement on how number sets should be introduced. According to Choquet, the notion of cardinals and operations with them could be introduced at the primary education level:

Exercises with finite collections clarify the concept of one-to-one correspondence which, in turn, leads to the concept of cardinal. [...] Addition and multiplication are introduced by the union of finite disjoint sets and the product of finite sets, respectively. [...] The introduction of positive and negative integers at the age of nine raises no difficulty.¹³

At the level of secondary education, Choquet proposed in his outline of a syllabus in arithmetic:

The set of integers developed by means of the Peano axioms (or an equivalent); definition of operations, either by recurrence or by union and product of two sets; the ring of integers Z ; field of rationals, Q , as a sub-field of the set of real numbers, R .¹⁴

⁹ OEEC, *New thinking in school mathematics*, OEEC, Paris 1961.

¹⁰ Ibidem, p. 35.

¹¹ Ibidem, p. 41. The Peano axioms are a formal way to build the set of natural numbers starting from the number 0 using a successor function S . That is, $1 = S(0)$, $2 = S(S(0))$ and so on. The equality relation and the successor function satisfy certain axioms.

¹² Ibidem, p. 42.

¹³ Ibidem, p. 65. A cardinal is the number of elements in all finite sets that are in one-to-one correspondence.

¹⁴ Ibidem, p. 67.

Moreover, Choquet proposed to have children do arithmetic in other number systems than the decimal number system:

It will be far more profitable and pleasant for the pupil to do a few calculations in the binary, octad, or duodecimal systems, than to practice endlessly in the system of decimal numeration.¹⁵

Finally, Servais stressed the importance of a thorough knowledge of complex numbers for students who want to study physics or technology¹⁶. He suggested introducing complex numbers by extending calculations with real numbers to expressions of the type $a + bi$, where $i^2 = -1$.

Several of these proposals can be seen as extreme alternative ways of introducing number sets. One might therefore wonder which of them made it into the classroom in the era of Modern Mathematics. Therefore, in this paper, we aim to answer the research question: ‘To what extent were extreme proposals for the introduction of number sets actually been implemented in secondary education during Modern Mathematics?’.

This paper is structured as follows: first, we give an overview of the existing literature on the implementation of alternative methods for introducing number sets during the period of Modern Mathematics. Then we present the theoretical framework used in this paper, followed by a presentation of the results and a conclusion.

Overview of some international situations

We present the known international research on the different methods used to introduce the number sets in Modern Mathematics in the 1960s and 1970s. A search was performed on Google Scholar using the keywords ‘modern mathematics’ or ‘new math’ and ‘number sets’, ‘cardinal’, ‘Peano axioms’, and translations (in French, German, Portuguese, and Spanish) and combinations thereof.

Because it was considered important to stay up to date with what was happening internationally, the Dutch journal for mathematics teachers “Euclides” published the West German curriculum for Modern Mathematics, which was introduced in 1968. Apart from the directive to introduce natural numbers as cardinals in primary education, no other extreme proposal for the introduction of number sets is taken up¹⁷. The positional number system must be explained, and as an example of a number system without position, the Roman number system can be used. Examples of arithmetic in the binary number system must be given. It is no-

¹⁵ Ibidem.

¹⁶ Ibidem, p. 72.

¹⁷ *Richtlinien und Rahmenpläne für den Mathematikunterricht*, “Euclides” 1969, vol. 44, no. 6, p. 177.

table that non-negative rational numbers are introduced before negative numbers appear. Further, it is prescribed to embed the set of natural numbers in the set of positive rational numbers and to introduce the real numbers using nested intervals of rational numbers¹⁸. Complex numbers are not mentioned.

Evidence of a (partial) implementation of Choquet's proposal made at the Royaumont seminar comes from the Nordic countries. In Icelandic textbooks and in translations of Danish textbooks, used in primary education, one finds illustrations of the implementation of Choquet's proposals. For example, 7-year-olds were taught the concept of numbers by establishing a visual one-to-one correspondence between a set of points and the number line. At the age of 8, addition and subtraction of natural numbers on the one hand and multiplication on the other hand were visually represented by drawings of the union of disjoint finite sets and of the product set of finite sets¹⁹. Similar illustrations were found in Norwegian textbooks where 7-year-olds were asked to represent one-to-one correspondences on visualisations of finite sets and to add numbers by considering the visualisation of the union of two disjoint finite sets²⁰. However, Choquet's proposal to introduce the Peano axioms in secondary education was not followed in Iceland²¹. Arithmetic in non-decimal number systems was implemented in textbooks of the Modern Mathematics period in both Iceland and Norway, and this was for both primary and secondary education, although sometimes only as optional material.

In Spain, set theory was introduced in primary education as part of a reform in 1965. This included the concept of a one-to-one correspondence between two sets and the abstract concept 'natural number is what all the equivalent sets share in common'²². Throughout the first three years of primary school, these concepts were repeated in an ever-increasing formal way, culminating in the formal definition of a cardinal in the third year. Also, from an analysis of textbooks used in primary education in Buenos Aires during the period 1967–1987, it is clear that

¹⁸ Ibidem, p. 178–180.

¹⁹ K. Bjarnadóttir, *Recommendations of the Royaumont Seminar on primary school arithmetic. Influences in the Nordic countries*, [in:] "Dig where you stand" 4. *Proceedings of the fourth International Conference on the History of Mathematics Education*, ed. by K. Bjarnadóttir, F. Furinghetti, M. Menghini, J. Prytz, G. Schubring, Edizioni Nuova Cultura, Rome 2017, p. 52–53.

²⁰ H. Opsal, B. Smestad, *Concepts introduced in Norwegian textbooks before, during and after the New Math period*, [in:] *Proceedings of the Twelfth Congress of the European Society for Research in Mathematics Education*, ed. by J. Hodgen, E. Geraniou, G. Bolondi, F. Ferretti, Free University of Bozen–Bolzano, Italy and ERME, Bozen–Bolzano 2022, p. 2089–2096.

²¹ K. Bjarnadóttir, *Influences from the 1959 Royaumont seminar. Proposals on arithmetic and algebra teaching at lower-secondary level in Iceland*, [in:] *Researching the history of mathematics education*, ed. by F. Furinghetti, A. Karp, Springer, Cham 2018, p. 18.

²² A.M. Oller-Marcén, *The transition to modern mathematics in Spanish primary education: The 1965 syllabus*, [in:] *Advances in the history of mathematics education*, ed. by A. Karp, Springer, Cham 2022, p. 188–189.

the natural numbers were introduced as cardinals of finite sets and calculations with natural numbers were derived from operations on finite sets²³. The same is found in an Argentine textbook series, intended for use in the first two years of secondary education²⁴. After presenting the integers on the number line, the rational numbers are defined as equivalence classes of ordered pairs of integers, and their properties are studied on the number line. Finally, the set of real numbers is introduced using a bijective map onto the points of the number line.

In Colombia, the textbooks that were used at the start of the implementation of Modern Mathematics were translations from, for example, material of the School Mathematics Study Group in the USA²⁵. During the second half of the 1970s, the textbook series *Matemática Moderna Estructurada* [Modern Structured Mathematics] appeared. In the first volume, natural numbers are defined using cardinals of equivalence classes of sets that are in one-to-one correspondence with each other. The equivalence class of the empty set is not considered, as a consequence, the set of natural numbers does not contain 0²⁶. The second volume of the textbook series contains the introduction of negative numbers and of rational numbers. The set of natural numbers is first extended to contain 0. Then, on the cartesian product of the natural numbers with itself, $N \times N$, an equivalence relation is defined using the pairs (x, y) with $(x, y) = x - y$ if $x \geq y$ and $(x, y) = -(y - x)$ if $x < y$. The first part in the definition of (x, y) leads to the equivalence classes representing natural numbers, and the second part to the equivalence classes representing negative integers. Then, first the positive rational numbers are defined using equivalence classes of the set $F \times F$ with $F = \{a/b \mid a, b \in N, b \neq 0\}$. After that, negative rational numbers and their properties are introduced. Real numbers and complex numbers appear in volumes three and four, respectively. The set of real numbers is formed by uniting the set of rational numbers and the set of irrational numbers, and complex numbers are introduced to be able to solve all quadratic equations²⁷.

One of the main actors in the era of Modern Mathematics in Belgium was Georges Papy²⁸. Especially during the 1960s, Papy was extremely active in the dissemination of Modern Mathematics in Belgium and abroad. Often in collaboration with others, including his wife Papy-Lenger, Papy carried out classroom

²³ A. Deriard, *Manuales en Buenos Aires (1967–1987) en la búsqueda de una “vulgata escolar”*: *Racconto de un proceso de iniciación a la investigación*, “Revista História da Educação” (Online) 2020, vol. 24, no. e99373, p. 17.

²⁴ See P. Herbst, *The Number-Line Metaphor in the Discourse of a Textbook Series*, “For the Learning of Mathematics” 1997, vol. 17, no. 3, p. 36.

²⁵ M.A.S. Gómez, *Implementación de la matemática moderna en Colombia: serie matemática moderna estructurada*, “Palobra” 2021, vol. 21, no. 1, p. 30.

²⁶ *Ibidem*, p. 33.

²⁷ *Ibidem*, p. 34–36.

²⁸ D. De Bock, G. Vanpaemel, *Rods, sets and arrows*, p. 98.

experiments, organised teacher (re)training courses, participated in (international) meetings and organisations, and wrote a textbook series, *Mathématique Moderne* (MM)²⁹. The first volume, MM1, was published in 1963. After the treatment of set theory, natural numbers are defined as cardinals of finite sets, and operations on natural numbers result from operations with finite sets. Papy writes in the introduction to MM1:

Experience has shown that children enjoy studying addition and multiplication in connection with the theory of sets and relations and love to establish the links between their earlier knowledge of elementary arithmetic and their more recently acquired knowledge of sets.³⁰

Papy introduces the binary representation of natural numbers in MM1 using dots on an abacus. Integers are then obtained by representing two natural numbers on this abacus by red and blue dots, which cancel each other out if they are in the same place on the abacus³¹.

In MM2, published in 1965, the set of real numbers is first introduced using a binary graduation of a straight line. The order and addition of vectors on the straight line are transferred to the set of real numbers. The multiplication of real numbers is defined by the composition of two homotheties³². The rational numbers are then introduced as an ordered subfield of the real numbers. At the end of MM6, published in 1967, complex numbers are introduced using so-called direct similarities, that is, specific transformations of the Euclidean vector plane. The properties of these direct similarities and isomorphisms allow Papy to show that the complex numbers form a field which extends the field of the real numbers³³.

Apart from some experimental classes, Papy's MM series was never really used in classrooms because it was not aligned with the Modern Mathematics curriculum that was implemented from 1968 onwards. However, Papy's MM series was translated into several languages and thus influenced Modern Mathematics reforms in many countries³⁴.

²⁹ D. De Bock, M. Roelens, G. Vanpaemel, *Mathématique moderne: A pioneering Belgian textbook series shaping the New Math reform of the 1960s*, [in:] "Dig where you stand" 5: *Proceedings of the 5th International Conference on the History of Mathematics Education*, ed. by K. Bjarnadóttir, F. Furinghetti, J. Krüger, J. Prytz, G. Schubring, H. J. Smid, Freudenthal Institute, Utrecht 2019, p. 129–131.

³⁰ G. Papy, *Mathématique moderne I*, Didier, Brussels–Paris 1963, p. vii.

³¹ D. De Bock, M. Roelens, G. Vanpaemel, *Mathématique moderne: A pioneering Belgian textbook series*, p. 134.

³² A homothety is a transformation that rescales the space with a fixed factor.

³³ D. De Bock, M. Roelens, G. Vanpaemel, *Mathématique moderne: A pioneering Belgian textbook series*, p. 134–135.

³⁴ *Ibidem*, p. 138.

Theoretical framework

The research question addressed in this paper relates to classroom practices; the aim is to examine how number sets have actually been implemented. This study is limited to the Modern Mathematics period, which was a curriculum reform. Therefore, we first present a theoretical framework for studying such processes and then we complement this with an account of how the classroom practice on a specific topic can be studied.

A curriculum reform takes place over a period of time and concerns a number of stakeholders, which Tanja Hamann has termed the ‘multi-dimensionality of reform’³⁵. To structure the stakeholders, Hamann adapted a model by Helmut Fend to obtain the following layers: the theoretical scientific discipline, the curriculum, the classroom concept, and the practical schooling. On each of these layers, institutions and protagonists, as well as their products, are identified. The top layer, the theoretical scientific discipline, consists of mathematicians and educationalists who produce scientific theories. At the level of the curriculum, we find governments and administration who ordain curricula and syllabuses. The layer of the classroom concept is populated by textbook authors and didacticians who produce textbooks, manuals, and teaching methods. The final layer of the practical schooling is populated by schools, teachers, students, and parents who teach or go through learning processes. The different layers of the educational reform model can influence each other downwards or upwards, and one person can act on different layers, of course adapting their functioning accordingly. Hamann applies the model to the case of Modern Mathematics in primary schools in West Germany³⁶.

Historical research often lacks empirical data on the layer of practical schooling. While there are reports by inspectors or about experiments, for example, these are often broadly formulated or focus on one specific topic. A proxy for what actually happened in the classroom is the study of textbooks. According to John A. Nietz: ‘an analysis of old textbooks throws much light on what was taught in the past’³⁷. From the introduction or preface of a textbook, one can learn about the objectives of teaching certain topics. The presentation and organisation of the content can also give an insight into the teaching methods used. In addition, ‘a text unfolds a distinct regime of mathematical discourse’³⁸.

³⁵ T. Hamann, *New Math at primary schools in West Germany – a theoretical framework for the description of educational reforms*, [in:] *Proceedings of the Eleventh Congress of the European Society for Research in Mathematics Education*, ed. by U. T. Jankvist, M. van den Heuvel-Panhuizen, M. Veldhuis, Utrecht University and ERME, Utrecht 2019, p. 2118.

³⁶ *Ibidem*, p. 2117–2124. See also A.M. Oller-Marcén, *The transition to modern mathematics in Spanish primary education*, p. 174–175.

³⁷ J.A. Nietz, *The contribution of an analysis of old school textbooks to the history of education in United States*, “The Journal of Educational Research” 1941, vol. 35, no. 3, p. 207.

³⁸ P. Herbst, *The number-line metaphor in the discourse of a textbook series*, p. 36.

In what follows, we first apply Hamann's theoretical four-layered framework and then analyse a selection of textbook series to answer our research question about how number sets were introduced during the era of Modern Mathematics. We restrict our analysis to four textbook series³⁹ published in Flanders, the Dutch-speaking community of Belgium. Limiting the analysis to one community allows for a more detailed investigation of the selected textbook series and also includes a broader range of textbook series, which increases the representativeness of teaching practices in this community.

Results

We have already discussed the first layer of Hamann's theoretical framework, the layer of the theoretical scientific discipline, in the introduction. The main protagonists were Dieudonné and Choquet, both research mathematicians at the university, but not directly involved at primary or secondary education levels. They proposed to define the natural numbers and computations with them using cardinals of finite sets and operations on finite sets in primary education and to introduce the Peano axioms in secondary education. Since it is difficult to retrospectively measure the activities and learning processes of teachers and students on the topic of the introduction of number sets during the Modern Mathematics era, the fourth layer of the framework is not considered. Therefore, in this section we first focus on presenting the relevant curriculum, i.e., the second layer of the theoretical framework. We then examine the classroom concept, the third layer, in detail through an analysis of four textbook series. As Modern Mathematics was new to most secondary school teachers, it is reasonable to assume that they relied heavily on the newly developed textbooks. This makes the textbooks a valid proxy for actual classroom activities.

The 1968 mathematics curriculum

We do not repeat the genesis of the first Modern Mathematics curriculum in Belgium, which was introduced for 12-year-olds in 1968⁴⁰. Only the curriculum for the general division of secondary education is considered here, not that for the technical schools, where Modern Mathematics was introduced only in 1969.

³⁹ The analysed textbook series are *Opbouw* [Construction], *Wis en Kundig* [Wise and knowledgeable], *Wiskunde De rij* [Mathematics: the sequence] and *Wiskunde '68* [Mathematics '68]. See the list of analysed textbooks at the end of the paper for details.

⁴⁰ For that we refer to D. De Bock, G. Vanpaemel, *Rods, sets and arrows*, p. 137–140. The curricula (four slightly different versions for the two dominant linguistic communities, the Flemish and French, and for two networks, state secondary schools and Catholic secondary schools) are published in full in "Mathematica & Paedagogia" 1968, vol. 34, p. 7–34.

The curriculum for the first year of secondary education (12-year-olds) consisted of the topics: set theory, relations, natural numbers, integers, and geometry. We read that children are already used to doing arithmetic with natural numbers from primary school, so it is enough to point out the equivalence of the statements ‘the number of elements of a set A is four’ and ‘there exists a bijection of the set A to $\{1, 2, 3, 4\}$ ’⁴¹. This awareness will lead children to associate the operations on natural numbers with the operations on finite sets. The curriculum explicitly mentions that the use of the term cardinal is not required. The need to make students aware that the set of natural numbers is a subset of the set of integers is emphasised. Solving first-degree equations with one unknown and word problems are also considered very important. It is suggested to temporarily replace $-z$ by \bar{z} , a notation that was introduced by Papy in MM1⁴². It is not necessary to practice arithmetic extensively, but the positional system must be explained, and to do so, one may consider non-decimal number systems, in particular, the binary number system.

Textbook analysis

For the *Opbouw* textbook series, already in 1964, the first book of the series was published. This first edition of *Opbouw 1*, with the subtitle *Nieuwe wiskunde* [New mathematics], thus appeared before the implementation of the Modern Mathematics curricula of 1968. However, it clearly formed a basis for the second edition of *Opbouw 1* published in 1969, and because of certain similarities with other textbook series, we include it in the analysis. For each of the textbook series under study, we first discuss the relevant information that is found in the introduction or footnotes in the text, which gives a general flavour of how number sets are introduced in that textbook series. Then we examine in detail the introduction of number sets in the textbook series.

The first edition of *Opbouw 1* does not contain an introduction, so we cannot give a direct insight into the ideas of the authors. However, further on, it will become clear that the authors strived for completeness, for example, by discussing a geometrical representation of operations with natural numbers. In the second edition of *Opbouw 1*, there is an introduction in which the authors address the students and urge them to put a lot of faith in their teachers. But they also write that ‘mathematics does not have a good name. Some of your predecessors certainly

⁴¹ *Leerplan wiskunde voor het eerste leerjaar van het rijksmiddelbaar onderwijs van het lagere graad Programme de mathématique pour la première année d’études de l’enseignement moyen du degré inférieur*, “Mathematica & Paedagogia” 1968, vol. 34, p. 11 and p. 18, and *Leerplan wiskunde voor de zesde van de humaniora Programme de mathématique pour les classes de 6° des humanités*, “Mathematica & Paedagogia” 1968, vol. 34, p. 25.

⁴² G. Papy, *Mathématique moderne 1*, p. 319.

have told you how daunting difficult this subject is'⁴³. The authors refer to their experiments with the new curriculum, 'originating from the Belgian professor G. Papy, who is since many years a worldwide authority in the area of school mathematics'⁴⁴. In this context, they also warn that the mathematics in this textbook is totally different from the mathematics that the students' parents learned. In addition, it is mentioned that this edition of *Opbouw I* is adapted to the curriculum of 1968. About natural numbers, the authors write: 'You are obviously already accustomed with these numbers from primary school: there you learned how to do calculations with these, here you find the explanation why it must be done that way and not differently'⁴⁵.

The authors of the textbook series *Wis en Kundig* refer to their experiments with Modern Mathematics over the past years and mention that their books can be used by both state schools and Catholic schools. Moreover, they were clearly inspired by Papy and his MM textbook series. On the one hand, they write, 'It [this textbook] would like to pay a modest tribute to them who through their groundbreaking work with an unwavering faith sent to us the new spirit: Professor PAPPY and Madame PAPPY'⁴⁶. On the other hand, they construct the ordered field of the real numbers before the ordered field of the rational numbers, and this using a graduation of a line as Papy did in his MM series⁴⁷. Similarly, the introduction of the complex numbers is presented following 'the beautiful example of MM6 of PAPPY'⁴⁸, at least for students in the options with the highest number of weekly hours of mathematics. For all other students the field of the complex numbers is introduced by defining a multiplication on $R \times R$ together with a bijection from $R \times R$ to a new set which then results in the set of the complex numbers⁴⁹.

The new curriculum is also followed in *Wiskunde De rij*, while it is stressed that freedom is given to teachers, who can choose the order of the different subjects and the time devoted to them, and which teaching methods to use. According to its author, the textbook series *Wiskunde De rij* potentially distinguishes itself from other textbook series, since it adopts a moderate position, as opposed to the extreme position some other authors choose. This should make it easier

⁴³ R. Bens, E. Bouqué, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw I*, Wesmael–Charlier 1969, p. xiv.

⁴⁴ Ibidem.

⁴⁵ Ibidem, p. xv.

⁴⁶ R. Verhulst, M. Verwulgen, F. Van Roey, *Wis en Kundig 1A*, Standaard Uitgeverij 1968, p. 5. Captions in the original.

⁴⁷ R. Verhulst, M. Verwulgen, F. Van Roey, *Wis en Kundig 2*, Standaard Uitgeverij 1969, p. 156, and R. Verhulst, M. Verwulgen, F. Van Roey, *Wis en Kundig 3*, Standaard Uitgeverij 1970.

⁴⁸ R. Verhulst, F. Van Roey, K. De Munter, P. Bockstaele, *Wis en Kundig 4B*, Standaard Uitgeverij 1971, p. 6. Captions in the original.

⁴⁹ R. Verhulst, F. Van Roey, K. De Munter, P. Bockstaele, *Wis en Kundig 4A*, Standaard Uitgeverij 1971, p. 247.

for inexperienced, mistrustful, or discouraged teachers who have to start with the new curriculum⁵⁰. Notably, the geometrical topics prescribed by the curricula are treated throughout, together with the topics on sets. Already on page 17, straight lines in the plane are defined and studied. In this textbook series, no direct reference to other mathematicians or textbook authors is made.

As its title suggests, the textbook series *Wiskunde '68* is also based on the curriculum of 1968. It is written in the introduction that the series is suited for both the state schools and the Catholic schools and that the structure of the books follows the curriculum and the pedagogical suggestions therein. The only reference to other mathematicians or textbook authors is one to colleagues of a working group of 'experimenting teachers'⁵¹.

As the main focus of this paper is to investigate which extreme proposals for teaching natural numbers were actually implemented during the Modern Mathematics period, Table 1 presents the topics that are related to this issue. In Table 1, set theory contains: sets, subsets, intersection, union, difference, Cartesian product of sets, partition, relation, equivalence relation, order, function, injection, surjection, and bijection (one-to-one correspondence). We indicate not only which topics are treated, but also the order in which they are presented, by denoting the respective page numbers. From Table 1 it is clear that the *Opbouw* and *Wis en kundig* textbook series formally define cardinals, although the 1968 curriculum explicitly mentions that it is not required to use that term. In *Wiskunde De rij* and in *Wiskunde '68* the term 'cardinal' is not used, it is only defined what it means for two sets to be equivalent. In *Wiskunde De rij 1*, the number of elements of a set is already given very early in the book by counting the number of elements of the set using the non-zero natural numbers. Later in the book, a second definition for the number of elements of a set using a one-to-one map between sets is given, and this definition is also valid for infinite sets. The set of natural numbers is just given by enumerating them in the set notation, and only then it is said that a natural number n is the number of elements of a set A if and only if there exists a one-to-one map from A to $\{1, 2, 3, \dots, n\}$ ⁵².

In the *Wiskunde De rij* and *Opbouw* series, the properties of the operations on natural numbers are not formally proven, 'You find these properties in the following comparative table, which does not need much more explanation'⁵³. About the multiplication of natural numbers, it is written 'The way in which we present the multiplication here, does not resemble that of primary school at first sight. Still, it will soon become clear that there is no essential difference between the two repre-

⁵⁰ R. Broeckx, *Wiskunde De rij 1*, De Nederlandsche Boekhandel 1968, p. 4.

⁵¹ E. Jennekens, *Wiskunde '68 Deel 1*, De Sikkel 1969, p. 3–4.

⁵² R. Broeckx, *Wiskunde De rij 1*, p. 16, 66, 112–115.

⁵³ R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw Nieuwe wiskunde 1*, Wesmael–Charlier 1964, p. 64.

sentations'⁵⁴. In *Wis en Kundig 1B*⁵⁵ and in *Wiskunde '68*, however, the properties of the operations on the natural numbers are formally proven using the cardinals of finite sets. In *Wiskunde '68* these proofs are marked as optional material.

Table 1. Relevant subjects in the first books of the analysed textbook series.

Textbook Topic	<i>Opbouw 1</i> (edition 1964)	<i>Opbouw 1</i> (edition 1969)	<i>Wis en Kundig 1</i>	<i>Wiskunde De rij 1</i>	<i>Wiskunde '68</i> (Part 1)
Set theory	p. 9–53	p. 1–64	p. 7–160	p. 7–98 Including part of geometry	p. 7–88
Equivalent sets Being equivalent is equivalence relation	p. 57–58	p. 70–150, p. 155–156	p. 161–164	p. 17, p. 66 –	p. 91 –
Cardinal of a set	p. 59	p. 156	p. 164–165	– p. 16, number # of elements set	– p. 92, number # of elements set
Natural number as cardinal	p. 60	p. 157–158	p. 167	– p. 115 natural number as #	– p. 93 natural number as #
Computations with natural numbers using operations on sets	p. 62–84, p. 93–97	p. 168	p. 182–217	p. 117–119, p. 122–126	p. 96–128
Number systems including binary number system	p. 88–91, p. 283–287	p. 201	p. 175–181, p. 188–189, p. 196–197, p. 208	p. 115–116, p. 120, p. 129, p. 132–134	p. 129–135
Integers	p. 109–117	p. 230–240	p. 224–247	p. 152–171	p. 139–185
Rational numbers	p. 120–143	–	–	–	–

It is of particular interest to consider the geometrical representation of the operations on natural numbers in the 1964 edition of *Opbouw 1*, which is not present anymore in the 1969 edition, see Fig. 1, 2 and 3. The sum of two natural numbers a and b is obtained geometrically from the point (a, b) by rotating it clockwise around the point $(a, 0)$ over 90° or by projecting the point (a, b) on the horizontal axis in the direction $(1, -1)$, see Fig. 1 left. That is, the addition of two natural numbers is a map from $N \times N$ to N . Subtracting two natural numbers is the inverse operation of adding two natural numbers, hence it is represented using

⁵⁴ Ibidem, p. 87.

⁵⁵ R. Verhulst, M. Verwulgen, F. Van Roey, *Wis en Kundig 1B*, Standaard Uitgeverij 1968.

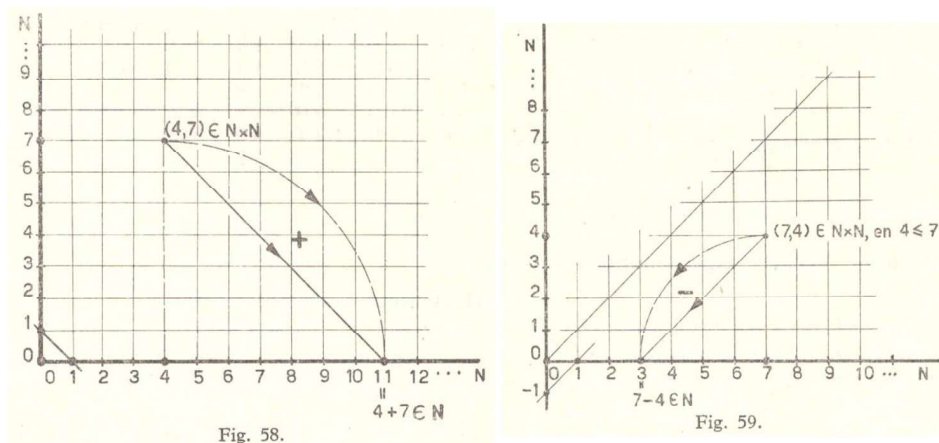


Fig. 1. R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw Nieuwe wiskunde 1*, Wesmael-Charlier 1964, p. 66 (left) and p. 71 (right).

a counterclockwise rotation over 90° or as a projection on the horizontal axis in the direction $(1, 1)$, see Fig. 1 right. By extending the number line, the integers are introduced in the 1964 edition of *Opbouw 1*. To define the addition of integers, the authors extend the addition of natural numbers, see Fig. 2 left. From this, they deduce the rules for adding two integers. They note the same process could be followed for subtracting two integers, but that is not needed since $a - b = a + (-b)$ for $a, b \in \mathbb{Z}$, as they make clear in Fig. 2 right.

Similar drawings as in Fig. 1 are given in *Wiskunde '68* for the addition and subtraction of natural numbers and of integers, however, nothing is said about rotating or projecting a point, nor is the addition or the subtraction of two integers formalised as a map from $\mathbb{Z} \times \mathbb{Z}$ to \mathbb{Z} ⁵⁶. In a later chapter in *Wiskunde '68* the integers and operations on them are introduced using translations on the number line⁵⁷. Also, in *Wiskunde De rij*, similar drawings as in Fig. 1 are given, again without referring to rotations or projections, but the direction of the arrows is stressed and called the 'addition direction' or the 'subtraction direction', and the formal notation using a map from $\mathbb{Z} \times \mathbb{Z}$ to \mathbb{Z} is used⁵⁸.

Similarly, the geometrical representation of the quotient of two natural numbers, see Fig. 3 left, allows the authors in the 1964 edition of *Opbouw 1* to introduce the positive rational numbers, see Fig. 3 right. These positive rational numbers are then defined as equivalence classes of equal fractions, the latter be-

⁵⁶ E. Jennekens, *Wiskunde '68 Deel 1*, p. 148–150.

⁵⁷ Ibidem, p. 224–229.

⁵⁸ R. Broeckx, *Wiskunde De rij 1*, p. 118, 122, 156, 160.

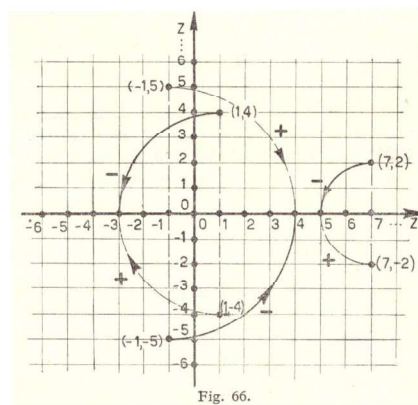
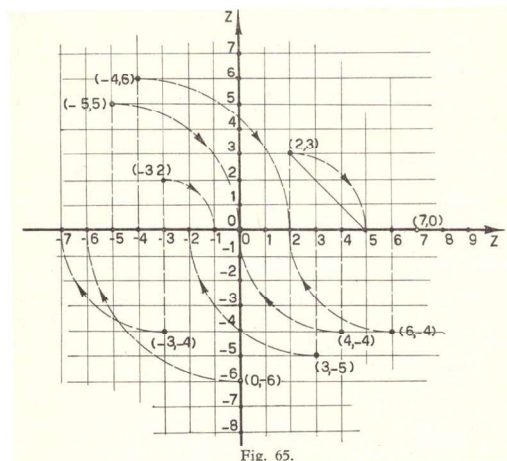


Fig. 2. R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw Nieuwe wiskunde 1*, Wesmael–Charlier 1964, p. 111 (left) and p. 112 (right).

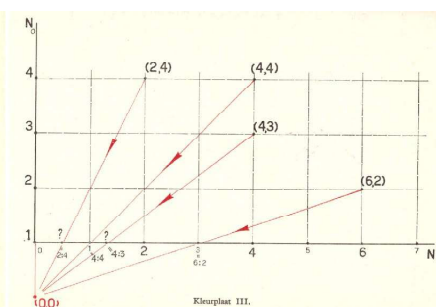
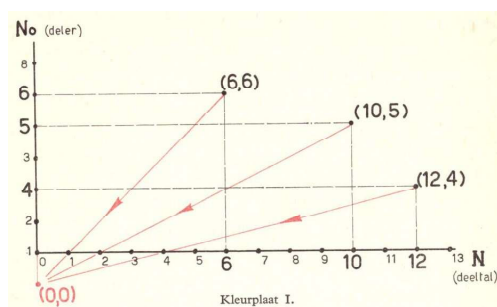


Fig. 3. R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw Nieuwe wiskunde 1*, Wesmael–Charlier 1964, p. 95 (left) and p. 121 (right). Note the typing mistake in the drawing on the left, on the vertical axis 8 should be 7 and 3 and 4 are switched, resulting also in a mistake in the couple (12, 4) which should be (12, 3).

ing defined by ordered pairs of natural numbers, that is, $a/b = (a, b)$ with $a \in N$ and $b \in N_0^{59}$.

While the 1964 edition of *Opbouw 1* defines the integers and positive rational numbers, as well as their properties, the 1969 edition only provides an introduction to the integers and a recapitulation of operations involving fractions, in

⁵⁹ R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw Nieuwe wiskunde 1*, p. 123–125.

the context of solving equations and word problems. In *Opbouw 2*⁶⁰ the rational numbers are constructed using a graduation of a straight line, after which the set of rational numbers is presented as an ordered field. Then the real numbers are defined as either rational numbers or as non-repeating decimal numbers, after which the ordered field of the real numbers is studied further in *Opbouw 3*⁶¹ and *Opbouw 4a*⁶².

Out of all the analysed textbook series, *Wiskunde De rij* and *Wiskunde '68* contain the least extreme methods for introducing number sets. As already mentioned, the term 'cardinal' is not used in these textbook series, and the integers are defined using only everyday examples, like a thermometer, or using translations. In *Wiskunde De rij*, rational numbers are defined as equivalence classes of equal fractions. Here, the equality of fractions is based on the representation of fractions on the number line using vectors. Two fractions are equal if they have the same image on the number line under this representation⁶³. Also for the field of real numbers and its properties the representation by vectors is used. Moreover, it is explicitly mentioned that the use of homotheties, as Papy did, is avoided⁶⁴.

In *Wiskunde '68*, the set of the rational numbers is introduced by looking for a set for which division of two elements is an internal operation. Only at a second stage the rational numbers are represented on a number line. After representing the rational numbers as finite decimal numbers or as infinite but repeating decimal numbers, the set of real numbers is formed by adding to the set of the rational numbers the non-repeating infinite decimal numbers, that is, the irrational numbers⁶⁵. The properties of the set of real numbers are then proven using the properties of vectors in the vector plane⁶⁶. Similarly, the set of complex numbers is obtained by looking for a set of numbers in which every number has a square root. Complex numbers are considered as pairs of real numbers, operations on these are defined, and their properties are proven. Only then is the notation using the complex unit given⁶⁷.

⁶⁰ R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw 2*, Wesmael-Charlier 1977.

⁶¹ R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw 3*, Wesmael-Charlier 1972.

⁶² R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, G. Verschuere, *Opbouw 4a*, Wesmael-Charlier 1971.

⁶³ R. Broeckx, *Wiskunde De rij 2*, De Nederlandsche Boekhandel 1969, p. 124–126.

⁶⁴ R. Broeckx, E. Van Eyck, *Wiskunde De rij 3*, De Nederlandsche Boekhandel 1970, p. 5, 30–66.

⁶⁵ E. Jennekens, *Wiskunde '68 Deel 2 Deel A relaties groepen getallen*, De Sikkel 1969, p. 109, 114–115, 213.

⁶⁶ E. Jennekens, *Wiskunde '68 Deel 3 Deel A algebra*, De Sikkel 1970, p. 47.

⁶⁷ E. Jennekens, G. Deen, *Wiskunde '68 Deel 5 Deel A matrices – complexe getallen*, De Sikkel 1972, p. 301–310.

In the 1964 edition of *Opbouw 1* and in *Wiskunde '68*, Roman numbers are used as an example of a non-positional number system, as suggested in the West German curriculum⁶⁸. In the 1969 edition of *Opbouw 1*, the example of the Roman number system is still given, but there it is not explicitly written anymore that it is a non-positional number system⁶⁹. Only in *Wiskunde De rij* is the suggestion to temporarily replace $-z$ by \bar{z} , made by the curricula of 1968, followed⁷⁰.

All textbook series under examination present the decimal and the binary number system. In *Wiskunde De rij*, the senary (base 6) number system is also presented, and converting a number from the decimal number system to the binary or senary number system is explained⁷¹. In the 1964 edition of *Opbouw 1*, some 'games' with binary numbers are given in an appendix⁷², this is not present anymore in the 1969 edition. A short history of the concept of numbers is present in both editions⁷³.

Conclusion

From our overview of the international approaches to the introduction of number sets, we found no evidence that Dieudonné's and Choquet's extreme proposal to introduce the Peano axioms in secondary education has been adopted anywhere. However, their proposal to work with cardinals has been partially implemented in both Europe and South America. As for Belgium, the 1968 curriculum for the first year of secondary education prescribes introducing the set of natural numbers using cardinals without explicitly using the term. Our study of four Flemish textbook series shows how the introduction of natural numbers as cardinals was implemented. In *Wiskunde De rij* and in *Wiskunde '68*, the term 'cardinal' is not mentioned; in *Opbouw* and *Wis en Kundig*, it is. One could possibly see Papy's influence here: the authors of the latter two textbook series explicitly refer to him.

We limited our analyses to textbook series published in Flanders, leaving the need for further research on the French-speaking part of Belgium. As the 1968 curricula in the two linguistic communities were broadly similar, and Papy also had an extensive influence in Flanders, it is not expected that the results will dif-

⁶⁸ R. Bens, Bosteels, E. Bouqué, De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw Nieuwe wiskunde 1*, p. 91 and E. Jennekens, *Wiskunde '68 Deel 1*, p. 131.

⁶⁹ R. Bens, E. Bouqué, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw 1*, p.206.

⁷⁰ R. Broeckx, *Wiskunde De rij 1*, p. 152–161.

⁷¹ R. Broeckx, *Wiskunde De rij 1*, p. 115–116, 132–133.

⁷² R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw Nieuwe wiskunde 1*, p. 283–287.

⁷³ R. Bens, G. Bosteels, E. Bouqué, W. De Roover, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw Nieuwe wiskunde 1*, p. 275–283 and R. Bens, E. Bouqué, W. Dewilde, F. Smislaert, A. Snauwaert, *Opbouw 1*, p. 216–219.

fer greatly. However, it would be interesting to detect more detailed differences, such as the use of \bar{z} , instead of $-z$ for negative numbers. Moreover, we focused on secondary education, so one might still wonder what happened in primary education. At the end of the 1960s, some experiments with Modern Mathematics in primary education started, followed by the introduction of formal curricula in the 1970s⁷⁴. Finally, we have only considered the Modern Mathematics period in this paper, but a comparison with how number sets were introduced before and after this period might also be of interest for further research.

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⁷⁴ D. De Bock, G. Vanpaemel, *Rods, sets and arrows*, p. 153–158.

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