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GEOMETRIC CONSTRUCTIONS IN SECONDARY SCHOOLS IN PRUSSIAN POLAND

Summary: In this article, the status, pedagogical functions, and evolution of compass-and-straightedge constructions in the secondary schools of Prussian Poland are examined. Focusing on the provinces of West Prussia and the Province of Posen, it traces the early and sustained integration of synthetic tasks into final examinations – from Elbing's 1799 locus-asymptote problem to 20th c. exercises, such as Bromberg's 1904 task of constructing a parallelogram given the sum of the squares of two adjacent sides, the included angle and the length of the opposite diagonal. In West Prussia, teacher-authors F. Buchner, F. Strehlke, W.A. Förstemann, O. Reichel, K. Güßlaff, O. Herweg, and F. Kronke shaped classroom practice and examinations through graded problem collections and the fusion of planimetry with descriptive geometry. In the Province of Posen, synthetic tasks appeared with some regularity in final examinations from the late 1860s onward, supported, among other factors, by J. Schacht's *Raumlehre* programme and methodological contributions by W. Jaehnike, R. Heffter, and H. Kiehl. Compliance with ministerial decrees and those of the *Provinzialschulkollegium* ensured that *Abitur* papers faithfully mirrored classroom instruction. By 1914, candidates in both provinces were expected to execute and justify constructions of triangles, quadrilaterals, selected loci and conics, illustrating how grassroots innovation and examination-driven feedback harmonised curricular standards across regions.

Keywords: Prussian Poland, geometric constructions, secondary education, mathematics curriculum, West Prussia, Province of Posen

Introduction

The development of mathematics since the 17th c. was marked by a rise in new areas of research, based on analytic methods. Most notable was the elaboration of infinitesimal calculus, due in particular to the work of Gottfried W. Leibniz, Isaac Newton, Leonhard Euler and Joseph-Louis Lagrange. These algebraising approaches were characterised in particular by their use of symbolic notation. Research on geometry became affected by these analytic methods, too: a new branch of geometry began to develop, thanks to René Descartes' research: analytic geometry.

Classical synthetic geometry was not abandoned and continued to be studied, constructing the figures with a straightedge and compass, so that one can observe not only parallel developments but also a certain competitiveness between synthetic and analytic geometry. Solving construction tasks in synthetic geometry was conceived of as applying a pair of methods: 'geometrical analysis' and 'algebraic analysis': combining the solution by means of geometric constructions with their treatment by means of algebra. This coupling within traditional Euclidean geometry is expressed in titles of various geometry textbooks, notably in Simon Antoine Jean L'Huilier's 1809 textbook *Élémens d'analyse géométrique et d'analyse algébrique*[1].

What was the impact of this changed 'landscape' of mathematical research branches upon school mathematics? One remarks telling differences between various European countries. England is well-known as the country with the strictest adherence to Euclidean synthetic construction procedures in schoolbooks. Advocates such as Thomas Simpson (1710–1761) and Robert Simson (1687–1768) repeatedly extolled the formative value of rigorous synthetic construction[2]. The title of John Leslie's 1809 textbook signalled the classical approach: *Elements of Geometry, Geometrical Analysis, and Plane Trigonometry*[3]. On the 18th c. Continent, the stance was different. Mathematicians in France, Germany, and many other European countries did not abandon classical Euclidean, construction-based geometry, but they layered onto it algebraic and analytic formulations for prob-

[1] S.A.J. L'Huilier, *Éléments d'analyse géométrique et d'analyse algébrique*, Paschoud, Paris–Geneve 1809.

[2] According to Florjan Cajori, in the first half of the 19th c. the leading books in England were those written by Thomas Simpson (e.g. T. Simpson, *Elements of Geometry, etc. with the Construction of a Great Variety of Geometrical Problems*, 4th ed., J. Nourse, London 1780) and John Bonnycastle (J. Bonnycastle, *Elements of Geometry containing the principal propositions in the first six, and the eleventh and twelfth books of Euclid. With notes critical and explanatory*, 4th ed., Printed for J. Johnson, London 1808) (F. Cajori, *A History of Mathematics*, 2nd ed., The Macmillan Company, New York 1910, p. 193).

[3] J. Leslie, *Elements of Geometry, Geometrical Analysis, and Plane Trigonometry*, James Ballantyne and Co., Edinburgh, 1809.

lems involving curves, loci, and motion, up to including analytic geometry into their textbooks. Adrien-Marie Legendre's 1794 textbook *Éléments de Géométrie, avec des Notes*[4] became the internationally leading modern geometry textbook, not hesitating to use algebraic methods for solving geometrical tasks.

Prussia in the 19th c. marks a turning point. The first period of the neo-humanistic education reform of 1810 showed a dominance of the analytic approach in the mathematics curriculum[5]. However, from the late 1820s, the Ministry of Education and the regional authorities (*Provinzialschulkollegien*) increasingly reduced the teaching of analytic geometry and stressed the Euclidean synthetic methods in teaching geometry[6]. Mathematics teachers at the reformed secondary schools (*Gymnasien*) backed this tendency, praising geometry as 'one of the most admirable products of the human mind […] the foundation of all mathematical education and an indispensable instrument of intellectual cultivation, which must be faithfully and conscientiously taught and practiced in all higher and middle schools'[7]. In this context, 'higher' schools were secondary institutions preparing pupils for final examinations, granting them access to university studies. Friedrich Buchner, an exponent of this tendency, argued that those seeking immediate results might rely on newer algebraic and analytic methods, whereas those aiming to educate thoughtful young mathematicians must not neglect the old synthetic ones[8].

Teachers adhering to this view of mathematics teaching conceived the tasks for the final examinations of their Gymnasium students according to a four-stage scheme – analysis, synthesis (construction), proof, and determination of the number of solutions – which mirrored the classical analysis-synthesis couple. This scheme is spelled out by Friedrich Buchner and reiterated by teachers such as Karl Güßlaff and Otto Herweg. In the analysis phase, four techniques were admitted: loci, auxiliary constructions, similarity-based arguments, and – where useful – elementary algebraic reasoning; the core of instruction and examination, however, remained synthetic ('Greek').

This article examines the status, pedagogical functions, and evolution of geometric construction problems in secondary schools of Prussian Poland, preparing

[4] A.M. Legendre, *Éléments de Géométrie, avec des Notes*, Chez l'Imprimeur de l'Institut, Paris 1794.

[5] See Peter Ullrich's contribution in this issue of "Analecta", *About the Time When Calculus Was Banned in Prussian Gymnasia*, for a discussion of the role of calculus teaching across the various periods of the 19th c.

[6] G. Schubring, *Mathematics Education in Germany (Modern Times)*, [in:] *Handbook on the History of Mathematics Education*, ed. by A. Karp, G. Schubring, Springer, New York, NY 2014, p. 243.

[7] F. Buchner, *Beitrag zur Methode des Unterrichts in der Geometrischen Analysis*, Gedruckt bei August Albrecht, Elbing, 1829, p. 3. All translations in this article are by the author.

[8] Ibidem, p. 5.

pupils for the final examinations. The term 'Prussian Poland' is employed here in its narrow sense, restricted to the provinces of West Prussia and the Province of Posen – which formed the core of the Prussian partition after 1815 – while smaller, detached territories annexed by Prussia fall outside the study's scope.

The basic sources for this investigation are annual school reports, the so-called *Schulprogramme* (also called *Jahresberichte*), a publication form established by the Prussian government in 1824. Published yearly by each Gymnasium and destined primarily for the local public, they used to invite the public to the school year's final examination; the reports consisted of two parts:

- notices of school life during the last year, in particular about the taught curricula, the teachers, the students and examinations;
- to encourage Gymnasium teachers to continue studying and remain in contact with scientific life, the second part comprised a text by one of the teachers, chosen by the staff[9].

Final examinations regulations and the secondary-school network

In 1788, Prussia issued a decree introducing the world's first final secondary-school examination intended as a gateway to university study[10], designed both to standardise entrants' knowledge and to harmonise secondary-school curricula. The first examinations were held in 1789 in 35 Prussian schools, including two in Prussian Poland[11]: the *Stadtschule* in Marienburg (Malbork) and the Gymnasium in Elbing (Elbląg). By 1805, the system had been extended to Jenkau (Jankowo Gdańskie), Culm (Chełmno), Marienwerder (Kwidzyn), and Thorn (Toruń). Although mathematics was not formally compulsory under the 1788 regulations, it often appeared in practice. An analysis of the sets of final-examination problems shows that some tasks involved geometric constructions, often related to conic sections. For example, a problem from Elbing in 1799 asked students: 'Construct and analyze the position of the asymptotes of a hyperbola – do they touch the hyperbola, and if so, where?'[12].

[9] See G. Schubring, *Analysen der Profile der Mathematik-Schulprogramme in den verschiedenen deutschen Staaten des 19. Jahrhunderts*, [in:] *Schulprogramme Höherer Lehranstalten. Interdisziplinäre Perspektiven auf eine wiederentdeckte bildungs- und kulturwissenschaftliche Quellengattung*, ed. by N. Ächtler, Wehrhahn, Hannover 2021, p. 305–327.

[10] P. Schwartz, *Die Gelehrtenschulen Preußens unter dem Oberschulkollegium (1787–1806) und das Abiturientenexamen*, vol. 1, Weidmannsche Buchhandlung, Berlin 1910, p. 71–72.

[11] The details of the first final examinations in Prussian Poland have been compiled on the basis of materials contained in: P. Schwartz, *Die Gelehrtenschulen Preußens unter dem Oberschulkollegium (1787–1806) und das Abiturientenexamen*.

[12] K. Karpińska, S. Domoradzki, *O egzaminie maturalnym z matematyki na obszarze zaboru pruskiego od XVIII do początku XX wieku*, "Antiquitates Mathematicae" 2017, vol. 11, no. 1, p. 169.

A decisive step came with the regulation of 25 June 1812, which made mathematics obligatory in the gymnasial final examination and introduced the term *Abitur* as its official designation[13]. In geometry, candidates were expected to demonstrate knowledge of plane and solid geometry (based on Books I–VI and XI–XII of Euclid's *Elements*). The 1834 guidelines specified that the written mathematics examination should contain four problems – two arithmetic and two geometric.

In the mid-19th c., the examination landscape broadened with the rise of the *Realschulen*, general-education secondary schools with a stronger emphasis on mathematics and the natural sciences. Thereafter, final examinations were administered chiefly in gymnasia (*Abitur*) and selected types of *Realschulen*. Under the 1859 regulations[14], final examinations were introduced in *Realschulen erster Ordnung*, where the written mathematics paper comprised four tasks: one on quadratic equations, one on planimetry or analytic geometry, one on plane trigonometry, and one on stereometry or conic sections.

In the 1860s, there were 18 gymnasia and 9 real schools in Prussian Poland authorized to conduct final examinations: 1. Protestant gymnasia located in Danzig (Gdańsk), Neustadt in Westpreußen (Wejherowo; founded in 1861), Elbing, Marienburg, Marienwerder, Thorn, Posen (Poznań; *Friedrich-Wilhelm-Gymnasium*), Lissa (Leszno), Krotoschin (Krotoszyn), and Bromberg (Bydgoszcz); 2. Catholic gymnasia operated in Culm, Konitz (Chojnice), Deutsch Krone (Wałcz), Braunsberg (Braniewo), Posen (*Marien-Gymnasium*), Ostrowo, and Tremessen (Trzemeszno; closed in 1863); 3. Non-denominational gymnasium (*Simultan-Gymnasium*) in Inowrazlaw (Hohensalza; Inowrocław; founded in 1863); 4. *Realschulen* in Elbing, Thorn, Posen, Meseritz (Międzyrzecz), Fraustadt (Wschowa), Rawitsch (Rawicz), Bromberg, the *Johannisschule* and the *Petrischule* in Danzig.

Although regulations and school types continued to evolve, the mathematics problems for the final examinations were always prepared autonomously by each school. The exam questions were written by the school's teachers but required approval from the Royal Commissioner, who also served as a member of the examination committee. Consequently, the content of the exams reflected the specific instructional practices of each institution. The character of mathematical tasks set in final examinations was largely shaped by the implemented curricula and the content of the textbooks in use.

[13] J.C. Kröger, *Denkschrift über den Gymnasial-Unterricht im Königreich Preussen von V. Cousin*, Johann Friedrich Hammerich, Altona 1837, p. 87.

[14] *Unterrichts- und Prüfungsordnung der Realschulen und der höheren Bürgerschulen*, Wiegand und Gieben, Berlin 1859.

West Prussia: a construction-centred culture

In gymnasia in Prussian Poland, the most widely used textbook was *Die Elementar Mathematik* by Ludwig Kambly[15], and in *Realschulen*, *Anfangsgründe der reinen Mathematik für den Schul- und Selbst-Unterricht* by Karl Koppe[16]. Other widely respected texts included *Hauptsätze der Elementar-Mathematik zum Gebrauche an Gymnasien und Realschulen* by Ferdinand Gustav Mehler[17] and *Methodisch geordnete Aufgabensammlung* by Ernst Bardey[18]. These textbooks were commonly used, e.g., in schools in Thorn, Culm, and Inowrazlaw, and were among the most popular across Prussia, as reported in the "Zeitschrift für das Realschulwesen" in 1880[19]. All of these textbooks included construction problems. Kambly and Koppe, in particular, introduced the concept of the geometric locus of points, solved several example problems, and included sets of similar problems for students to solve independently; however, these tasks were mainly related to the construction of triangles with the required properties.

In 1910, the historian of mathematics Florian Cajori evaluated these textbooks. He described Kambly's as 'clever but unscientific' and considered Koppe's to be even weaker.

> Schlegel[20] says that 'the quality of the most widely used books allows us to draw conclusions about the scientific level of teaching generally achieved in this subject'. […] Could it be that the masses of teachers have no feel or insight into textbooks that contain questions of intuition or other psychological issues overlooked by the authors of more scientific books? Simon[21] points out that until recently, the German teacher, unlike the French, enjoyed complete freedom in teaching, and that small texts like Kambly's allow for a broader individual input from teachers.[22]

[15] L. Kambly, *Die Elementar Mathematik*, vols. 1 (21 ed.) – 4 (10th ed.), F. Hirt, Breslau 1876–1878.

[16] K. Koppe, *Anfangsgründe der reinen Mathematik für den Schul- und Selbst-Unterricht*, vols. 1 (4th ed.) – 4 (5th ed.), G.D. Bädeker, Essen 1852–1871.

[17] F.G. Mehler, *Hauptsätze der Elementar-Mathematik zum Gebrauche an Gymnasien und Realschulen*, 4th ed., G. Reimer, Berlin 1869.

[18] E. Bardey, *Methodisch geordnete Aufgabensammlung*, B.G. Teubner, Leipzig 1888.

[19] *Statistik der Lehrbücher für die höheren Lehranstalten Preußens*, "Zeitschrift für das Realschulwesen" 1880, vol. 5, p. 729.

[20] H. Schotten, *Inhalt und Methode des Planimetrischen Unterrichts*, B.G. Teubner, Leipzig 1890, p. 21.

[21] M. Simon, *Ueber die Entwicklung der Elementar-Geometrie im XIX Jahrhundert*, B.G. Teubner, Leipzig 1906, p. 25.

[22] F. Cajori, *A History of Mathematics*, p. 190.

Teacher-authored loci problems: Otto Reichel's response to curricular shortfalls

It turns out that the material in these textbooks was insufficient – especially in the area of geometric constructions – to properly prepare students for university studies. This was confirmed by former students themselves[23]. In response to these shortcomings, many publications by secondary school teachers emerged to fill this educational gap. One such contribution was made by Otto Reichel, a teacher at the Thorn Gymnasium from 1864 to 1869 – where Kambly's manual was in use. He developed additional problems focusing on geometric loci, which were not included in any existing textbooks. Reichel incorporated these problems into his teaching and published them in 1866 in his article *Beiträge für den Unterricht in der Geometrie*. His initiative is an example of a teacher going beyond the official curriculum to better prepare students for their final examinations, university studies, and future professions.

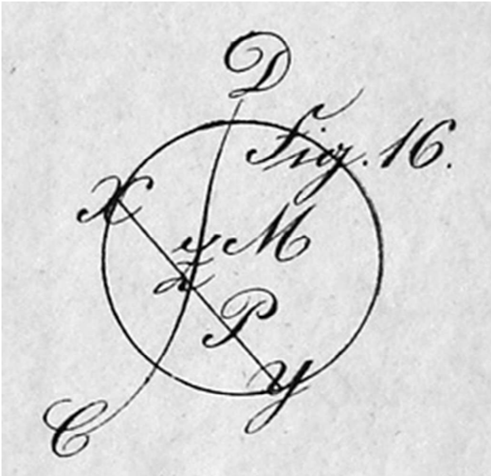

Fig. 1. Illustration of task no. 26 from O. Reichel's article.

Source: O. Reichel, *Beiträge für den Unterricht in der Geometrie*, [in:] *Königliches evangelisches Gymnasium und Realschule erster Ordnung zu Thorn*, Gedruckt in der Rathsbuchdruckerei, Thorn 1866, p. 17].

Reichel included a list of 28 tasks and exercises in the article, without providing solutions. The tasks were arranged so that their level of difficulty progressively increased. Three of them were theorems that formed the basis for solving subsequent tasks. The solutions often required performing complex, multi-step geometric constructions. Their level of difficulty is well illustrated by task no. 26 from the list: 'Given a circle *M*, a point *P* located inside this circle, and a curve *CD* intersecting the circle. Construct a chord *XY* within circle *M* such that the distance from point *P* to point *Y* (located on the circle) is equal to the distance from point *X* (also lying on the circle) to point *Z*, which is the intersection of the chord with the curve *CD*'[24] (see Fig. 1).

[23] O. Reichel, *Beiträge für den Unterricht in der Geometrie*, [in:] *Königliches evangelisches Gymnasium und Realschule erster Ordnung zu Thorn*, Gedruckt in der Rathsbuchdruckerei, Thorn 1866, p. 1.

[24] Ibidem, p. 16.

Otto Reichel's article introduced problems that could not be found in contemporary school textbooks. In this respect, his work constituted a unique and noteworthy contribution. At the time, the vast majority of independent publications by teachers in Prussian Poland focused primarily on compiling and organizing problems already discussed in existing literature, occasionally supplementing them with a few original tasks. These publications typically took the form of textbooks, which the authors used as teaching aids in their own classrooms. None of them came into widespread use, but some were discussed in the specialist literature, and some were officially approved for school use by ministerial decrees.

Systematizing synthetic tasks: Buchner, Strehlke and Förstemann's contributions

One of the most significant works of this kind was *Beitrag zur Methode des Unterrichts in der Geometrischen Analysis* by Franz Buchner[25], a teacher at the Elbing Gymnasium. Buchner systematized triangle-construction teaching by listing 34 essential theorems and compiling 245 problems which, in his view, ought to form part of the school curriculum – although most did not. The work also included exercises developed by the author together with colleagues and students during regular classes[26].

He explicitly presented his compilation as a comprehensive synthesis of earlier problems and theorems. On this basis, Buchner drew his problem set from material taken from 18 works, among them:

1. Thomas Simpson, *A Treatise of Algebra: Wherein the Principles Are Demonstrated, and Applied*, 3rd ed., Printed for J. Nourse, London 1767.
2. Meier Hirsch, *Sammlung Geometrischer Aufgaben*, vol. 1, Heinrich Frölich, Berlin 1805.
3. Simon Antoine Jean L'Huilier, *Éléments d'analyse géométrique et d'analyse algébrique: à l'usage des élèves de l'École militaire de Thoune*, Paschoud, Paris 1809.
4. Adrien-Marie Legendre, *Éléments de Géométrie*, 10th ed., Courcier, Paris 1813.
5. D.C.L. Lehmus, *Lehrbuch der Geometrie*, vol. 1, G. Reimer, Berlin 1816.
6. Ernst Gottfried Fischer, *Lehrbuch der ebenen Geometrie für Schulen*, Duncker und Humblot, Berlin 1820.
7. Wilhelm Adolph Diesterweg, *Geometrische Aufgaben nach der Methode der Griechen bearbeitet*, vol. 1, Reimer, Berlin 1825; vol. 2, Büschler, Elberfeld 1828.
8. Friedrich Strehlke, *Aufgaben über das geradlinigte Dreieck geometrisch und analytisch gelöset*, Gebrüder Bornträger, Königsberg 1826.

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[25] F. Buchner, *Beitrag zur Methode des Unterrichts in der Geometrischen Analysis*, p. 8.

[26] Ibidem, p. 10.

9. Wilhelm August Förstemann, *Lehrbuch der Geometrie*, vol. 1, Kafemann, Danzig 1827.
10. Karl Friedrich Pfleiderer, *Scholien zu Euclids Elementen*, ed. by J.M. Hauber, Metzler, Stuttgart 1827.

Many of the most advanced problems in Buchner's collection derive from W.A. Diesterweg's *Geometrische Aufgaben nach der Methode der Griechen* (vol. 1). For example: 'Construct a triangle given the sum of the base and the altitude to that base, the angle opposite the base, and the sum of the square of the base and the square of the sum of the other two sides'[27].

Among the problems solved by Buchner was also the following: 'Assume the notations from the diagram below [see Fig. 2 – K.K.]. Construct a triangle given $a + c$, h, $A - C$'[28].

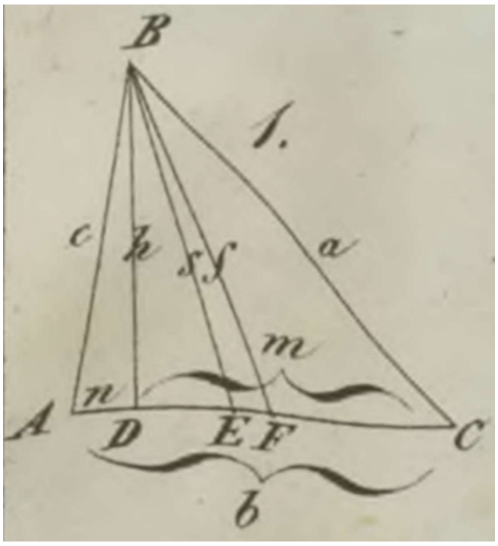

Fig. 2. Illustration of a task from F. Buchner's collection of problems.

Source: F. Buchner, *Beitrag zur Methode des Unterrichts in der Geometrischen Analysis*, Gedruckt bei August Albrecht, Elbing 1829, p. 22].

Solutions to this problem can be found in the aforementioned algebra textbook by T. Simpson (1), in the treatise by S.A.J. L'Huilier (3), and in the work of F. Strehlke (8). Of particular importance for the present article are the publications by Strehlke and W.A. Förstemann (9), as both authors were teachers in Danzig.

F. Strehlke's *Aufgaben über das geradlinigte Dreieck geometrisch und analytisch gelöset* presents 49 triangle-construction problems solved by the method of geometrical analysis. In an appended 'Supplement', he adds algebraic–analytic discussions, including trigonometric solutions to selected earlier problems, and treats several theorems on the circle and on conic sections, offering proofs that he regarded as simpler than those found in other textbooks[29]. In preparing the work, he drew on problems by T. Simpson, M. Hirsch, S.A.J. L'Huilier, D.C.L. Lehmus, E.G. Fischer, W.A. Diesterweg, and K.F. Pfleiderer.

[27] W.A. Diesterweg, *Geometrische Aufgaben nach der Methode der Griechen bearbeitet*, vol. 1, Reimer, Berlin 1825, p. 65–70, F. Buchner, *Beitrag zur Methode des Unterrichts in der Geometrischen Analysis*, p. 17.

[28] Ibidem, p. 22.

[29] F. Strehlke, *Aufgaben über das geradlinigte Dreieck geometrisch und analytisch gelöset*, Gebrüder Bornträger, Königsberg 1826, p. 5.

W.A. Förstemann's *Lehrbuch der Geometrie* consisted of two volumes. Vol. 1 treated elementary plane geometry and solved construction problems by the method of geometrical analysis – for example: dividing a segment in prescribed ratios; constructing triangles and polygons; transforming polygons into area-equivalent figures; and constructions involving circles. Vol. 2 focused on applying algebra to the solution of geometric problems[30]. Förstemann's textbook also had a compilatory character. As an example, one may cite a problem included in his textbook, which had earlier been solved by M. Hirsch and subsequently adopted by Förstemann, but not addressed by Strehlke: 'Construct a triangle given its base, the difference between the base angles, and the length of the median to the base'[31].

M. Hirsch's textbook was used, for example, at the Gymnasium in Culm[32]. Hirsch solved construction problems not only involving the construction of triangles with given properties (approximately ten such problems), but also tasks such as dividing a line into segments with specified proportions. Most of them were later reproduced and solved in the works of Förstemann, Strehlke, and Buchner. Hirsch's collection of problems was one of the most widely used at the time and was later reprinted many times[33]. On the other hand, the historian of mathematics Moritz Cantor wrote that the '*Sammlung geometrischer Aufgaben*, published in two volumes in 1805–7, was probably not widely distributed because it was often too difficult for beginners'[34]. Nevertheless, M. Cantor's assessment may be questioned, as the construction tasks in Hirsch's textbook were not particularly advanced; rather, they reflected the typical level of difficulty expected at the time.

Although Buchner, Strehlke, and Förstemann prepared works related to geometric construction problems, it remains unclear to what extent the material they published was actually implemented in classroom instruction. What is certain is that construction problems were indeed taught, as they regularly appeared in the curricula outlined in school reports. For instance, the 1830 report of the Danzig Gymnasium states that Förstemann taught methods for solving construction problems using algebra during his lessons, and the school library held the second volume of his textbook *Lehrbuch der Geometrie* (1829), in which this subject was thoroughly discussed[35]. Förstemann's textbook was highly praised: 'This is his most magnificent work, the idea of which was clearly conceived as early as in

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30 W.A. Förstemann, *Lehrbuch der Geometrie*, vol. 2, Anbuth, Danzig 1829.

31 M. Hirsch, *Sammlung Geometrischer Aufgaben*, vol. 1, Heinrich Frölich, Berlin 1805, p. 247; W.A. Förstemann, *Lehrbuch der Geometrie*, vol. 1, Kafemann, Danzig 1827, p. 200.

32 *Nachrichten über das Königliche katholische Gymnaisum zu Culm*, Culm 1843, p. 45.

33 *Statistik der Lehrbücher für die höheren Lehranstalten Preußens*, p. 729.

34 M. Cantor, *Hirsch, Meyer*, [in:] *Allgemeine Deutsche Biographie* (ADB), vol. 12, Duncker & Humblot, Leipzig 1880, p. 467–468.

35 *Nachricht von dem Zustande des städtischen Gymnasiums zu Danzig*, Danzig 1830, p. 7, 12.

1818, and is intended to be a source of information for both teachers and students, especially the former. […] in writing this book, he subjected to rigorous examination everything that was in earlier textbooks, and especially the newer research of French mathematicians'[36].

The situation with Strehlke was similar. Between 1840 and 1842 Friedrich Strehlke – first a teacher at the Gymnasium in Danzig and later head of the *Petrischule* (a higher-grade *Realschule*) – added a section titled "Pädagogische Mitteilungen" to the school's annual reports[37]. Unusual for the time, it circulated problems actually tried in class and found intellectually fruitful. Most were geometric construction tasks centered on conic sections, complemented by exercises from physics and chemistry. An example printed with a diagram in the 1842 report read: 'In a plane triangle, the following are given: the radius r of the incircle, the radius R of the circumcircle, and the height h; construct the triangle'[38].

Specialist journals welcomed the initiative. The "Archiv der Mathematik und Physik" praised the series and urged other teachers to publish similar collections[39]. Locally, however, the reception was lukewarm. In a memorial address (1886), *Victoriaschule* director Stephan Neumann suggested that this may have been 'perhaps because he also included many things that went beyond the immediate needs of the school, at least of the *Bürgerschulen*'[40].

Although short-lived, Strehlke's enterprise remains an early example of systematically sharing classroom-tested problems. It highlights both the pedagogical promise of such transparency and the tensions it could provoke within the administratively regulated Prussian school system of the mid-19th c.

Strehlke's brief yet firm experiment with "Pädagogische Mitteilungen" formed part of a West-Prussian debate on how mathematics should be taught in secondary schools and which problems best prepared students for the matriculation examination.

Ministerial endorsements and textbook changes

On 17 September 1845, the *Provinzialschulkollegium* of the Province of West Prussia issued an order recommending that private study for students in the two highest grades be based on Franz Luke's (a teacher at the Gymnasium in Culm) textbook: *(Sammlung von 100) geometrische Aufgaben, nach der Methode der*

[36] E.W. Förstemann, *Das Leben von Wilhelm August Förstemann*, Druck von C. Heinrich, Dresden 1891.

[37] *Programm, wodurch zu der öffentlichen Prüfung der Schüler der Petrischule*, Danzig 1840–1842.

[38] Ibidem, 1842, p. 18.

[39] *Literarischer Bericht*, Beilage zu: "Archiv der Mathematik und Physik" 1841, vol. 1, p. 66–67.

[40] S. Neumann, *Ein Lebensbild Friedrich Strehlke's. Gedächtniss-Rede gehalten in der Sitzung der Naturforschenden Gesellschaft zu Danzig am 3. Novbr. 1886*, [in:] *Schriften der Naturforschenden Gesellschaft in Danzig*, Wilhelm Engelmann, Danzig 1887, p. 173–191.

Alten für Schulen bearbeitet[41]. The phrase 'nach der Methode der Alten' (according to the method of the ancients) indicates a strictly Euclidean, synthetic approach, echoing Diesterweg's well-known preference for classical school geometry. The textbook was recommended specifically for the 'correctness of the solutions included therein'[42]. In Culm, teachers went a step further: in the school year 1850/1851, Luke used his textbook during lessons taught in the Secunda grade, noting that 'a great many of the tasks in this book are solved both in writing and orally'[43].

The tasks in Luke's textbook generally remain at a lower level of complexity than those found in Buchner's work (1829). Unlike Buchner, who focused primarily on triangle construction, Luke offered a more diverse selection of geometric constructions. His textbook contains 100 problems, including the construction of points that satisfy specified conditions; the construction of triangles under various constraints; the bisection and trisection of angles; the construction of circles with prescribed properties; and the construction of polygons (e.g., squares and rhombi).

The recommendation of this textbook for use in secondary schools preparing students for the final examinations was noted in the "Archiv der Mathematik und Physik". It was acknowledged that the book did not feature a large number of original exercises – a result of the fact that the topic of solving construction problems using classical ('Greek') methods had already been explored extensively in earlier literature. Nonetheless, it was stated: 'We are […] convinced that this collection of exercises will also be a valuable aid to many mathematics teachers during lessons'[44].

Luke's textbook was used as a textbook at the Gymnasium in Culm only in the 1850/1851 school year. Tracing subsequent textbook changes, one can observe that, among all the textbooks used there up to 1889, this one was the most advanced with regard to construction problems. In 1889, *Sammlung geometrischer Aufgaben* by M. Hirsch was added to the list of approved textbooks and was used there until the early 20th c.

Structured planimetric instruction by Güßlaff and Herweg

In 1852, the topic was taken up by Karl Güßlaff, a mathematics teacher at the Gymnasium in Marienwerder. In the school report, he published *Über das Auflösen planimetrischer Aufgaben*[45], a systematically structured article explicitly

[41] F. Luke, *Geometrische Aufgaben, nach der Methode der Alten für Schulen bearbeitet*, bei Lambeck, Thorn 1845.

[42] *Jahresbericht über das Königliche katholische Gymnasium zu Culm*, 1846, p. 25.

[43] *Jahresbericht über das Königliche katholische Gymnasium zu Culm*, 1851, p. 26.

[44] *Literarischer Bericht*, Beilage zu: "Archiv der Mathematik und Physik" 1845, vol. 6, p. 361.

[45] K. Güßlaff, *Ueber das Auflösen planometrischer Aufgaben*, [in:] *Jahresbericht über das königliche Gymnasium zu Marienwerder*, Gedruckt bei Friedr. Aug. Harich, Marienwerder 1852, p. 1–20.

aligned with ministerial guidelines. Güßlaff sharply criticised ‘university-style’ lecturing and rote learning, arguing instead for ‘conscious thinking’ and sustained pupil engagement; he also pointed to structural obstacles, notably the limited time allocation (three hours per week were ‘far from enough’) and the vulnerability of a curriculum dependent on a single specialist, since illness or travel could bring instruction to a halt.

As an alternative, he proposed a methodical programme for solving planimetric problems, illustrated by thirteen worked examples – some rarely discussed elsewhere – ranging from classical constructions of triangles and polygons to more advanced tasks, such as: ‘Given intersecting lines *MN* and *PQ* whose intersection is not constructible from the given data, draw through a point *G* between them a line that, when extended, passes through their (unknown) intersection’[46]. The article thus offered classroom-ready material calibrated to *Abitur* expectations. Whereas Strehlke approached such problems in an almost university manner, Güßlaff articulated a comprehensive secondary-school strategy: more instructional time, active student participation, and the consistent framing of every construction within the regulating cycle of analysis–synthesis–proof–determinacy. The practical impact of this approach is reflected in Marienwerder’s 1874 final examination problems, including: ‘Transform a given irregular pentagon into a regular pentagon of equal area’ and ‘Construct a triangle given its perimeter, one altitude, and the radius of its incircle’[47].

Otto Herweg, a mathematics teacher at the Gymnasium in Neustadt in Westpreußen, offered an even more didactic perspective on geometric constructions in his two-part essay *Kleinigkeiten aus dem mathematischen Unterricht. Teil II. Konstruieren*[48], published in the school reports of 1890 and 1891.

Comparative studies of Apollonius in gymnasium reports

In the upper-grade curricula of gymnasia and *Realschulen* in Prussian Poland, the classical problems of Apollonius appeared frequently. This topic was the focus of a booklet – issued as an appendix to the annual report of the *Friedrich-Wilhelms-Gymnasium* in Königsberg in der Neumark (Chojna) – by the mathematics teacher Friedrich von Lühmann, entitled *Die sectio rationis, sectio spatii und sectio determinata des Apollonius nebst einigen verwandten geometrischen*

[46] Ibidem, p. 12–13.

[47] *Königliches Gymnasium zu Marienwerder*, Marienwerder 1874, p. 22.

[48] O. Herweg, *Kleinigkeiten aus dem Mathematischen Unterricht. Theil II. Konstruieren (Erste Hälfte)*, [in:] *Beilage zum Programm des Königlichen Gymnasiums zu Neustadt in Westpreussen*, Druck von E.H. Brandenburg & Co., Neustadt WPr. 1890; O. Herweg, *Kleinigkeiten aus dem Mathematischen Unterricht. Theil II. Konstruieren (Zweite Hälfte)*, [in:] *Beilage zum Programm des Königlichen Gymnasiums zu Neustadt in Westpreussen*, Druck von E.H. Brandenburg & Co., Neustadt WPr. 1891.

Aufgaben (1882)[49]. In it, the author assembled what he called the 'most distinguished' solutions to three Apollonian problems (*sectio rationis*, *sectio spatii*, *sectio determinata*), provided each with detailed historical-critical commentary, and then presented his own concise constructions; he concluded with several related problems.

The booklet functions as a compact comparative compendium – from antiquity to 19th c. projective geometry – highlighting the strengths and limitations of each method and showing how the choice of analogous figures and a minimal set of auxiliary constructions can improve clarity. Although it was not explicitly designed for pupils or tied to a specific syllabus, its appearance in a school report meant that it could serve as advanced self-study material for final-year students.

Felix Kronke's 'Das Linearzeichnen': from planimetry to technical drafting

The next key source – no longer anchored in gymnasium mathematics lessons but in a technical drawing course at a *Realschule* – is Felix Kronke's school-report supplement *Das Linearzeichnen in der Realschule* (1901)[50]. Kronke outlines the content he actually taught at the *Oberrealschule* in Graudenz. He opens with a blunt diagnosis: the traditional drawing course 'no longer meets modern requirements'. His remedy is threefold: to expand the subject to four lessons per week, to place linear drawing under the responsibility of the mathematics teacher (since it rests on geometric theorems), and to define explicitly technical aims – training a precise hand, a sense of scale, and the spatial imagination expected of a modern educated person. This agenda underpins his strict insistence on ink work, a ban on templates, systematic practice with wooden models, and the relocation of formal proofs to the mathematics classroom.

In total, the supplement offers nearly 190 exercises, amounting to a comprehensive linear-drawing curriculum that fuses classical planimetry with elements of descriptive geometry – for example, orthogonal projections of prisms, pyramids, cylinders, cones, and spheres, as well as surface developments (nets). In doing so, it deliberately steers pupils toward technical practice and goes well beyond the methodical yet purely planimetric programmes proposed by Güßlaff and Herweg.

[49] F. von Lühmann, *Die sectio rationis, sectio spatii und sectio determinata des Apollonius nebst einigen verwandten geometrischen Aufgaben*, [in:] *Programm des Friedrich-Wilhelms-Gymnasiums zu Königsberg in der Neumark*, Königsberg in der Neumark 1882.

[50] F. Kronke, *Das Linearzeichnen in der Realschule. Ein Beitrag zur Frage über die Gestaltung des geometrischen Zeichenunterrichts an den höheren Lehranstalten*, [in:] *Städtische Oberrealschule zu Graudenz. Beilage zum Programm Ostern 1901*, Druck von Gustav Röthe's Buchdruckerei, Graudenz 1901.

In the secondary schools that prepared students for the final examinations in the Province of West Prussia, compass-and-straightedge constructions had long been regarded as one of the cornerstones of mathematical education. In the Province of Posen, despite a similar examination system, construction problems appeared less frequently in annual school reports, took up less room in the curricula, and were set on the final examinations far more rarely. At the same time, sources show that within the Province of Posen itself, there were large differences between individual institutions – from schools with a strong, synthetic tradition of constructive geometry, through centers that almost entirely ignored geometric drawings.

Province of Posen: the delayed shift toward geometric constructions

Polish schools in Posen, Tremessen and Ostrowo

Three institutions in the Province of Posen were officially classified as Polish schools: the St Mary Magdalene Gymnasium in Posen, the Gymnasium in Tremessen, and the Gymnasium in Ostrowo[51]. They tended to use the same textbooks and follow comparable curricula, and teachers frequently rotated among them. A review of their annual reports shows that, until roughly the 1870s, geometry was still framed under headings such as 'application of arithmetic to geometry'[52] or 'application of algebra to geometry'[53]. Although the geometry sections of the textbooks cited in these reports did include construction problems, these were few in number and strictly elementary.

While teaching the upper grades at the St Mary Magdalene Gymnasium in Posen, Philipp Spiller was already using a large problem collection of his own design. The first explicit reference to it appears in the school's teaching programme for 1842[54] and reappears in the 1845/1846 report[55]. Spiller had published the collection in 1839 under the title *3200 arithmetischen und geometrischen Rechnungsaufgaben aus dem durch das preußische Abiturienten-Reglement vorgezeichneten Gebiete der Elementar-Mathematik für Gymnasien, höhere Bürger-, Gewerbe- und Militair-Schulen*[56]. A year later, he issued a companion

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[51] K. Libelt, *Wykład matematyki dla szkół gimnazyalnych*, Nakładem i drukiem N. Kamieńskiego i Spółki, Poznań 1844, p. VII.

[52] *Programm des Königlichen Marien-Gymnasiums in Posen*, Posen 1845–1847.

[53] *Jahres-Bericht über das Schuljahr von Michaelis 1840. bis dahin 1841*, Posen 1841.

[54] *Zur öffentlichen Prüfung der Schüler des Königl. Marien-Gymnasiums zu Posen…*, Poznań 1842, p. 31.

[55] *Zur öffentlichen Prüfung der Schüler des Königl. Marien-Gymnasiums zu Posen…*, Poznań 1846, p. 6.

[56] P. Spiller, *3200 arithemtischen und geometrischen Rechnungsaufgaben aus dem durch das preußische Abiturientenreglement vorgezeichneten Gebiete der Elementarmathematik für Gymnasien, höhere Bürger-, Gewerbe- und Militair-Schulen*, E.S. Mittler, Posen und Bromberg 1839.

volume – *Resultate nebst Winken zu deren Auffindung…*[57] – containing complete solutions. Of the 3200 exercises, 605 concern plane geometry and trigonometry, yet none are construction problems; a substantial part of the geometry section is devoted to solving triangles by means of trigonometric computation. A representative example is: Given one of the equal sides of an isosceles triangle and one of the base angles, find the base[58].

On 10 April 1844, the *Provinzialschulkollegium* issued an order[59] recommending that every Catholic gymnasium in the Province of Posen adopt the Polish-language textbook *Wykład matematyki dla szkół gimnazjalnych*, prepared by Karol Libelt[60], then a teacher at the *Friedrich-Wilhelms-Gymnasium* in Posen. The textbook was introduced at the St Mary Magdalene Gymnasium in 1844/1845[61]), at the Tremessen Gymnasium in 1845/1846[62], and at the Ostrowo Gymnasium in 1845/1846 (and partly also in 1846/1847[63]). It treated only elementary geometric constructions.

During the 1846/1847 school year, the authorities ordered a switch to German-language instruction, which necessitated the adoption of German textbooks. All three Polish schools then introduced[64] Hans Brettner's (then director of the St Mary Magdalene Gymnasium in Posen) *Lehrbuch der Geometrie für Gymnasien, Realschulen und höhere Bürgerschulen*[65], which remained strongly calculation-oriented, with constructions serving mainly illustrative purposes. This approach continued the logarithmic–arithmetical tradition initiated by Spiller and consolidated by Strehlke.

At the St Mary Magdalene Gymnasium, Brettner's textbook was replaced in the 1876/1877 school year by Gustav Mehler's *Grundsätze der Elementar-Mathematik*. In 1895 the staff added Ernst Bardey's *Methodisch geordnete Aufgabensammlung, mehr als 8000 Aufgaben enthaltend über alle Teile der Elementar-Mathematik*. Neither work contained geometric-construction material,

[57] P. Spiller, *Resultate nebst Winken zu deren Auffindung zu den 3200 arithemtischen und geometrischen Rechnungsaufgaben aus dem durch das preußische Abiturientenreglement vorgezeichneten Gebiete der Elementarmathematik für Gymnasien, höhere Bürger-, Gewerbe- und Militair-Schulen*, E.S. Mittler, Berlin, Posen und Bromberg 1840.

[58] P. Spiller, *3200 arithemtischen und geometrischen Rechnungsaufgaben*, p. 202.

[59] *Jahres-Bericht des Königlichen Kathol. Gymnasiums zu Trzemeszno*, Trzemeszno 1844, p. 33.

[60] K. Libelt, *Wykład matematyki dla szkół gimnazyalnych*, p. VIII.

[61] *Zur öffentlichen Prüfung der Schüler des Königl. Marien-Gymnasiums zu Posen…*, Poznań 1845, p. 21.

[62] *Jahresbericht des Königlichen Gymnasiums zu Trzemeszno*, Trzemeszno 1846, p. 23. Reports are available for the years 1840 and 1842–1862.

[63] *Jahresbericht des Königlichen Gymnasiums zu Ostrowo*, Ostrowo 1846, p. 10; *Jahresbericht des Königlichen Gymnasiums zu Ostrowo*, Ostrowo 1847, p. 21.

[64] Ibidem, p. 21, 26.

[65] H. Brettner, *Lehrbuch der Geometrie für Gymnasien, Realschulen und höhere Bürgerschulen*, 4th ed., im Verlage bei Josef Max und Komp, Breslau 1847.

yet both remained standard well into the first decade of the 20th c. Although the textbooks offered no construction problems, such tasks began to appear on the final examinations. The first examination to include geometric constructions was held in 1888[66]; further instances are listed in the school reports for 1896, 1898, 1900, and every year from 1903 to 1913. Construction problems also found their way into the teaching programmes implemented at the time. On the *Abitur*, they typically involved constructing triangles from various given data or constructing quadrilaterals. One example reads: Construct a triangle with a given perimeter whose three medians are in the given ratio[67].

An important factor behind the appearance of construction problems in the curriculum and on the final examinations at this Gymnasium was, on the one hand, the 1901 ministerial decrees, which explicitly stressed such tasks, and on the other, a few committed teachers – foremost among them Julius Schacht.

In 1903[68], 1906[69], and 1908[70], Schacht published three articles in the school reports advocating a new approach to teaching geometry in secondary schools. He criticised the ministerial syllabi introduced in 1891 and 1901: the former did not include construction problems at all, while the latter did allow for them, yet – Schacht argued – 'The lack of any cultivation of spatial imagination in middle-form geometry lessons must be considered a serious deficiency in the harmonious education of students'[71].

In his articles, Schacht proposed a new solution – a new way of teaching geometry. He advocated a comprehensive *Raumlehre* rather than the conventional sequence 'first plane geometry, stereometry only in the senior grades', thus anticipating (in part) the later Meran Programme. His idea was to merge planimetry, stereometry and elementary trigonometry into a single, multi-year *Raumlehre* course (where trigonometry he treated as *rechnende Geometrie*) and he outlined in detail how this should be carried out from Quarta upward. A central feature of the proposal was the systematic, year-by-year solution of construction

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[66] *Jahresbericht des Königlichen Marien-Gymnasiums zu Posen*, Posen 1889, p. 13.

[67] *Jahresbericht des Königlichen Marien-Gymnasiums zu Posen*, Posen 1906, p. 13.

[68] J. Schacht, *Die Ausbildung des räumlichen Anschauungsvermögens im mathematischen Unterricht des Gymnasiums*, [in:] *Jahresbericht des Königlichen Marien-Gymnasiums zu Posen*, Marzbach'sche Buchdruckerei, Posen 1903, p. 1–12.

[69] J. Schacht, *Ein neuer Lehrgang für den Unterricht in der Raumlehre der höhren Lehranstalten. I. Teil. Die geradlinigen Figuren und die von Ebenen begrenzten Körper*, [in:] *Jahresbericht des Königlichen Marien-Gymnasiums zu Posen*, Marzbach'sche Buchdruckerei, Posen 1906, p. 1–21.

[70] J. Schacht, *Die methodische Verbindung der Planimetrie, Stereometrie und Trigonometrie zu einer einheitlichen Raumlehre. Ein skizzierter Leitfaden für den Unterricht in den mittleren Klassen der höheren Lehranstalten*, [in:] *Jahresbericht des Königlichen Marien-Gymnasiums zu Posen*, Marzbach'sche Buchdruckerei, Posen 1908, p. 1–16.

[71] J. Schacht, *Ein neuer Lehrgang für den Unterricht in der Raumlehre der höhren Lehranstalten*.

problems[72]: beginning with triangles and quadrilaterals, progressing to loci and constructions that exploit similar figures, and finally tackling more demanding tasks – for example, constructing triangles when the sums and differences of their angles or sides are given, or problems involving transversals. These constructions were to form a deliberately sequenced, methodical training tool: starting with simple similarity- and locus-based questions and leading to broad applications in the study of the circle and solids of revolution, with an ever-stronger emphasis on proof (logical reasoning) in the upper grades.

His 1903 article offered a broad outline; in 1906 and 1908 he expanded the idea, presenting a detailed syllabus, didactic methods, and specific problems to be solved in class. The final paper provided a complete methodological guide to implementing the *Raumlehre* programme, including a precise timetable of topics, the place and function of geometric constructions at every stage, a logical escalation of proof requirements, and the consistent use of models. Consequently, readers (also teachers) received a ready-made plan for a long-term, integrated *Raumlehre* course in which construction problems serve as a constant, carefully paced instrument for developing both students' spatial imagination and their logical thinking.

In 1875, by order of the *Provinzialschulkollegium* dated 15 July, geometry teaching in the top three grades at Ostrowo began to rely on Kambly's textbook[73]. A construction problem appeared on the final examination in Ostrowo earlier than in Poznań – already in 1882 – and was repeated many times thereafter, for instance in 1883, 1884, and so on. In 1884, Kambly's manuals for geometry and trigonometry were replaced by textbooks by Heinrich Wilhelm Lieber and Friedrich von Lühmann – most likely *Geometrische Constructionsaufgaben*[74] and *Leitfaden der Elementar-Mathematik*[75]. These books introduced a new, more advanced approach to geometric constructions.

In the introduction to *Geometrische Constructionsaufgaben*, Lieber and von Lühmann stressed the special importance of construction problems: the more practice pupils gain in solving them on their own, the more easily they will later tackle the harder trigonometric and stereometric tasks – assuming, of course, that they have acquired the necessary facility with algebraic operations. The collection contains roughly 3000 problems aligned with the secondary-school syllabus and stands out for its systematically graded, method-based organisation, which

[72] J. Schacht, *Die Ausbildung des räumlichen Anschauungsvermögens im mathematischen Unterricht des Gymnasiums*, p. 12.

[73] *Programm des Königlichen Gymnasiums zu Ostrowo*, Ostrowo 1875, p. 45.

[74] H. Lieber, F. von Lühmann, *Geometrische Constructionsaufgaben*, Verlag von Leonhard Simion, Berlin 1889.

[75] H. Lieber, F. von Lühmann, *Leitfaden der Elementar-Mathematik*, vol. 1: *Planimetrie*, Verlag von Leonhard Simion, Berlin 1894.

offers concise guidance and treats later topics more thoroughly than comparable manuals by Kambly and Koppe.

The collection devotes considerable space to the construction of quadrilaterals under specified conditions. It then turns to a broad set of circle-theory problems, ranging from basic chords and tangents and the construction of circles with a given radius to the classical tangency problems of Apollonius and Malfatti. The authors also analyse equivalent transformations of figures and their division into equal parts, and finally address constructions of algebraic expressions – covering geometric solutions of quadratic, biquadratic, and trigonometric equations (e.g., 'Construct an angle φ such that $a \cos \varphi + b \sin \varphi = c$'[76]). This extensive material forms the backbone of the textbook. It is rooted in earlier works – above all those of Karl Koppe, Johann August Grunert, Wilhelm Erler, J.O. Gandtner and K.F. Junghans, Wilhelm Brennecke, Christian Heinrich von Nagel, August Wiegand, B. Féaux, and F.H.T. Müller – yet it introduces its own thorough ordering and lucid structure.

Leitfaden der Elementar-Mathematik contains both theoretical discussions and model construction problems worked out with meticulous detail, already presented in the four classic stages – analysis, synthesis, proof, and determination. The authors construct triangles and polygons with specified properties, including constructions involving circles and algebraic expressions, and discuss loci of points. The problems themselves are elementary, while the textbook's chief content consists of theorems that provide the foundation for tackling the more complex tasks collected in the accompanying problem book.

In the 1884 final examination at Ostrowo, the following problem was set: 'Find a point on a given straight line from which the tangent to a given circle has a prescribed length'[77]. In the problem collection's appendix on loci, there is a theorem that yields an immediate solution to this task[78]. In a sense, this may be seen as a response to the *Provinzialschulkollegium*'s appeal of 1 December 1857, which reiterated the ministerial directive that *Abitur* questions should be drawn from material familiar to pupils from their lessons: topics chosen for written examinations are not to be remote and unfamiliar to the pupils' memory, but rather such as can certainly be assumed to be well known from classroom instruction, so as to remove the incentive for the frauds that still occur in the *Abitur*[79]. Thus, by this point, the curriculum at Ostrowo with respect to construction problems had been brought into line with the programmes used in the schools of the Province of West Prussia.

[76] H. Lieber, F. von Lühmann, *Geometrische Constructionsaufgaben*, p. 177.

[77] *Programm des Königlichen Gymnasiums zu Ostrowo*, Ostrowo 1884, p. 24.

[78] H. Lieber, F. von Lühmann, *Geometrische Constructionaufgaben*, p. 199.

[79] *Jahres-Bericht über das Königliche Katholische Gymnasium zu Trzemeszno*, Trzemeszno 1858, p. 23.

Bromberg's embrace of constructions: textbooks, teachers and final exams

A very similar strengthening of the role of construction problems can be traced at the Gymnasium in Bromberg[80]. In 1845, Brettner's textbooks were introduced there; ten years later, teaching was based on Koppe's manuals and Lorenz Wöckel's *Geometrische Aufgaben*[81]. In 1882, the school was using the books by Kambly, Bardey, and Wöckel, and in 1898, Wöckel's problem collection was replaced by that of Lieber and von Lühmann. Construction exercises sometimes appeared in the syllabi printed in the annual reports – e.g., in 1845, 1850, 1856–1863, 1885, and 1887. Nevertheless, the adoption of Wöckel's and later Lieber and von Lühmann's collections shows that from about 1855 the Bromberg Gymnasium attached considerable importance to construction problems. From 1879 onward, the school reports included the full sets of *Abitur* mathematics questions, and almost every year thereafter, these sets contained construction tasks (e.g., 1899 is an exception). Most concerned the construction of triangles; constructions of quadrilaterals also appeared fairly often – for example, the 1904 *Abitur* asks: 'Construct a parallelogram given the sum of the squares of two adjacent sides, the angle between them, and the diagonal opposite that angle'[82]. A new type of problem, seldom seen elsewhere but set several times in Bromberg, dealt with circle geometry – for instance: in 1887, 'In a circle of given radius, inscribe a quadrilateral whose two diagonals and the angle between them are given'[83]. Such problems can be found in Wöckel's problem collection.

The high regard for geometric constructions at the Bromberg Gymnasium is also borne out by teachers' papers published in the annual reports. One methodologically intriguing contribution is August Krüger's article *Über die Lehre von den Parallelen, namentlich in Bezug auf neuere Lehrbücher*[84]. It offers a concise review of contemporary secondary-school geometry textbooks, structured in two parts: a critique of the existing treatments of the parallel-line theory and a proposal for sequencing lessons that rests on precise definitions and a graduated scale of construction problems.

Two other treatises are also especially relevant to construction problems. The first is Willibald Jaehnike's paper *Die anbeschriebenen Kreise des bicentrischen*

[80] School reports are available for the years 1827–1830, 1832–1835, 1837–1875, 1878–1879, 1881–1891, 1893–1899, and 1904–1907.

[81] L. Wöckel, *Geometrie der Alten in einer Sammlung von 856 Aufgaben*, Verlag der Friedr. Korn'schen Buchhandlung 1880.

[82] *Königliches Gymnasium zu Bromberg*, Bromberg 1904, p. 7.

[83] *Königliches Gymnasium in Bromberg*, Bromberg 1888, p. 4.

[84] A. Krüger, *Über die Lehre von den Parallelen, namentlich in Bezug auf neuere Lehrbücher*, [in:] *Programm des Königlichen Gymnasiums zu Bromberg*, Buchdruckerei von F. Fischer, Bromberg 1852, p. 1–16.

oder Sehnen-Tangentenvierecks und die bicentrische Vierecksschar[85], published in the 1904 school report. Jaehnike wrote it to fill a gap in contemporary textbooks and problem collections: except for special cases such as regular quadrilaterals, bicentric quadrilaterals (that is, figures that can be both circumscribed about and inscribed in a circle) were virtually absent from teaching materials. Drawing on earlier studies by Oscar Schlömilch in the *Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht* and Josef Junker's *Geometrische Untersuchungen über bicentrische Vierecke* (*Realschule* in Crefeld report for 1891/1892[86]), Jaehnike proposed a unified, construction-oriented approach that would allow the topic to be introduced systematically into classroom practice. He analysed the properties of bicentric quadrilaterals, discussing polar and harmonic notions, devoted considerable attention to the excircles of such quadrilaterals and their properties, and linked these questions to the power of a point and to the similarity points of the excircles, offering recommendations for classroom use. The second part of the paper broadened the view to the entire family of quadrilaterals that are both inscribed and circumscribed, and tackled optimisation problems: within this family, the trapezoid has the smallest perimeter and area, while the kite has the largest. Taken together, the material forms a coherent set of tasks – from constructing a bicentric quadrilateral, through drawing its excircles and studying their properties, to exploring the whole Poncelet family of figures[87].

Another teacher, Robert Heffter, wrote an article entitled *Die Centralprojectionen des Kreises*[88], which was likely a direct report on his teaching practice at the Bromberg Gymnasium: the school reports for 1856–1863 repeatedly mention a 'new geometry of the circle' – precisely the material Heffter explains in detail in his paper. In the article, he shows how a single operation – the central projection of a circle onto a plane – can generate the constructions and properties of the circle, ellipse, parabola, and hyperbola, and how to turn these into classroom problems. He proposes a sequence of exercises that runs from chords and diameters, through harmonic division and the power of a point, to Pascal's hexagram. This sequence has a double didactic advantage: each problem flows naturally from the previous one, eliminating the need to treat each conic separately, and the end of each stage involves a simple graphical check (verifying equal segments, preserved ratios, etc.), allowing pupils to confirm a construction immediately without compromising the rigour of the proof. Thus, Heffter leads the reader effectively from intuitive circle geometry to the abstract properties of the conic

[85] W. Jaehnike, *Die anbeschriebenen Kreise des bicentrischen oder Sehnen-Tangentenvierecks und die bicentrische Vierecksschar*, Buchdruckerei von A. Dittmann, Bromberg 1904, p. 1–22.

[86] Ibidem, p. 3.

[87] Ibidem, p. 16.

[88] R. Heffter, *Die Centralprojectionen des Kreises*, [in:] *Programm des Königlichen Gymnasiums zu Bromberg*, Buchdruckerei von F. Fischer, Bromberg 1856, p. 1–22.

sections, harmoniously combining formal proof with direct visual verification at the blackboard.

This line of enquiry was taken up by Heinrich Kiehl, a teacher at the *Realgymnasium* in Bromberg, in his article *Die durch drei ähnliche Punktreihen erzeugten Dreiecke und Kegelschnitte*[89]. In this treatise in synthetic geometry, Kiehl worked exclusively with geometric methods. In the part on constructions, he showed, among other things, that the lines joining corresponding points of two similar rows on different perpendicular bisectors form parabolas (one for each side – three in all), thus translating the 'similarity of rows' directly into conic constructions. He then described their common tangents, points of tangency, directrices, and the associated loci[90]. By extending the discussion to carriers set at arbitrary angles, Kiehl generated entire families of parabolas with specified foci, tangents, and directrices, while consistently adhering to the principle of 'pure construction, no calculations'[91]. The paper contains no explicit teaching guidelines for use in lessons at the Bromberg *Realgymnasium*, yet the final examinations there almost always included construction problems involving synthetic work with conic sections[92]. An example from the 1906 examination reads: 'Given the two foci of an ellipse and a tangent, construct the point of tangency and the endpoints of the major and minor axes'[93].

Adoption of geometric construction in the gymnasia of the Province of Posen

In the annual reports of most other secondary schools in the Province of Posen, virtually no essays on geometric constructions appeared, and the weight given to such problems in their final examinations varied, even though they all used textbooks much like those already discussed.

At the *Friedrich-Wilhelm-Gymnasium* in Posen in 1872/1873 and between 1892 and 1901, construction problems figured explicitly in the syllabus and in almost every final examination, usually as tasks on constructing triangles or quadrilaterals. From 1892 onward, the school adopted the textbooks of Lieber and von Lühmann, and Bardey[94]. For the Gymnasium in Meseritz, extant reports cover 21 separate years between 1871 and 1915. In almost every one of those

[89] H. Kiehl, *Die durch drei ähnliche Punktreihen erzeugten Dreiecke und Kegelschnitte*, [in:] *Jahresbericht des städtischen Realgymnasium zu Bromberg*, Buchdruckerei von A. Dittmann, Bromberg 1888, p. 3–14.

[90] Ibidem, p. 7–8.

[91] Ibidem, p. 9–11.

[92] School reports are available for the years 1883–1884, 1886, 1888-1889, 1891, 1893–1899, 1903–1907.

[93] *Jahresbericht des städtischen Realgymnasium zu Bromberg*, Bromberg 1906, p. 7.

[94] *Programm des Königlichen Friedrich-Wilhelms-Gymnasiums zu Posen*, Posen 1892, p. 7.

years, the *Abitur* sets contained construction problems – ranging from the standard construction of triangles and quadrilaterals to tasks involving conic sections. A similar pattern was observed at the Gymnasium in Inowrazlaw: construction problems first appeared on the 1868 *Abitur* and were included almost continuously until 1905[95].

Construction problems appeared on most *Abitur* papers at the Gymnasium in Schrimm (Śrem)[96] as well, chiefly concerning triangle constructions and circle geometry – for example (1886): 'Construct a circle that is tangent to two given circles and whose centre lies on a given straight line'[97]. A strikingly similar theme turned up in the Gymnasium in Rogasen (Rogoźno)[98]. Construction problems were least frequent – only a handful of times – at the Gymnasium in Krotoschin[99], and they were of the traditional type. One of the more interesting examples, from the 1884 report, asked: 'Construct a triangle given its circum-radius r, the length of the bisector of angle α, and the altitude dropped to side a'[100].

Conclusions

The article demonstrates that geometric construction problems became not only a key pedagogical instrument but also a practical yardstick in final secondary-school examinations (*Abitur* and *Realschule* finals) in Prussian Poland from the late 18th to the early 20th c., with a focus on the provinces of West Prussia and Posen. In West Prussia, synthetic construction tasks entered examination practice remarkably early and, by the end of the 19th c., appeared almost routinely in written examinations, often in the mature four-part form of analysis, synthesis, proof, and determinacy. This development was driven less by ministerial syllabi than by the sustained activity of teacher-authors, who built graded canons of problems, broadened the repertoire of loci, systematised classroom methods, and linked plane constructions to descriptive geometry and the requirements of industrial-technical education, thereby compensating for the limits of widely used manuals that could not on their own meet university-entrance demands.

The Province of Posen charted a distinct trajectory. Until the mid-19th c., final-examination scripts from Posen (St Mary Magdalene Gymnasium), Ostrowo and Bromberg reflected the algebraic emphasis of Philipp Spiller's and Hans Brettner's textbooks, with constructions appearing only sporadically. Yet *Abitur* records indicate that synthetic construction tasks were valued well before

95 *Fünfter Jahresbericht des städtischen Gymnasiums zu Inowraclaw*, Inowraclaw 1868, p. 8.
96 School reports are available for the years: 1869, 1884–1886, 1903, 1914.
97 *Koenigliches Gymnasium in Schrimm*, Schrimm 1886, p. 7.
98 School reports are available for the years 1902–1908.
99 School reports are available for the years 1856–1858, 1863–1871, 1873–1875, 1882, 1884.
100 *Programm des Königl. Wilhelms-Gymnasiums zu Krotoschin*, Krotoschin und Ostrowo 1884, p. 16.

the 1901 ministerial syllabus, appearing across the province by the early 1870s (e.g., Meseritz) and in some schools even earlier (e.g., Inowrazlaw). Again, the decisive stimulus came from teachers: Julius Schacht's *Raumlehre* programme, published in the report of the St. Mary Magdalene Gymnasium in Posen, proposed integrating constructions into every grade from Quarta onward; Heinrich W. Lieber and Friedrich von Lühmann's 3000-problem *Geometrische Constructionsaufgaben*, first adopted in Ostrowo later in Bromberg and Posen, furnished staff with a comprehensive *Abitur* resource; and papers published in the Bromberg school reports by Willibald Jaehnike, Robert Heffter and Heinrich Kiehl illustrated extensions of synthetic techniques to bicentric quadrilaterals, circle projections and families of conics.

Across both provinces, final examinations largely mirrored what had actually been taught: the same teachers who delivered instruction set the papers, and regulations prohibited the introduction of material beyond the classroom programme. This makes clear the extent to which local examination committees and individual educators shaped the de facto curriculum. By the eve of the First World War, final examinations in both provinces routinely required students to complete and substantiate key construction types – from triangle and quadrilateral problems to loci and elementary conics – showing how teacher-led initiatives and the logic of high-stakes assessment gradually aligned curricular expectations across the region.

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