

*Frédéric Métin*

ORCID 0000-0002-8679-0490

University of Burgundy-Europe in Dijon, France

## CHARLES MÉRAY'S *GEOMETRY* TESTED IN THE CLASSROOMS

**Summary:** When he published his *Nouveaux Éléments de Géométrie* in 1874, Charles Méray had already written the book that made him the first mathematician to propose a coherent theory of irrational numbers. Had he continued on this path, he would be more famous than Cantor today.

A professor of advanced mathematics at the University of Dijon, he turned to elementary mathematics and focused on teaching geometry to beginners. According to him, plane geometry 'does not belong to the reality of things, as nature only offers space figures'. His view on old-fashioned geometry was cruel: '[geometry]... Finding it silent on three-dimensional space, the geometry believer willingly believes that it does not exist'.

So, the *Nouveaux Éléments* show another way of introducing and thinking about geometry, which can be linked to the fusion movement in Europe, including the use of algebra, especially for locus problems and conic sections. In 1874, classroom experiments had been rejected by academic authorities, but new attempts at the beginning of the 20th c. would give satisfaction to many teachers who were keen to test the new methods: starting with three-dimensional geometry dealing with everyday objects, use of visual intuition instead of abstract axioms, use of movement and geometric transformations, etc. Enthusiastic reports with many mentions of success were published in 1901, first in Burgundy, then in national academic newspapers.

**Keywords:** history of mathematics, fusion of space and plane geometry, mathematicians, Paris Congress in 1900, geometry education, French educational reform in 1902

Initially interested in the 1902 reform of the mathematics curriculum in France and the attempts to renew pedagogical methods in geometry teaching, we were pleased to discover unpublished letters and documents by Charles Méray, giving further evidence of the important role of his *Nouveaux Éléments de Géométrie* in these attempts. Further research led us to five main sources:

- The aforementioned private collection belonging to the heirs of Charles Méray and including many letters he received, most of them between 1906 and 1909. This collection may be inaccessible for the moment (perhaps definitively), but we had the opportunity to take pictures of the entire collection during a conference organized in Dijon in 2004; so, it now exists as a photographic collection (priv. coll. 1).
- A second private collection, comprising twelve letters from Charles Méray to Charles-Ange Laisant, dated 1900 (priv. coll. 2).
- Three letters from Charles Méray to Charles-Ange Laisant, dated 13 April 1900, 24 December 1900, and 10 January 1901 respectively, belonging to the Esperanto Museum of the Austrian National Library (ÖNB), and available on the website of this library.
- The correspondence between Charles-Ange Laisant and Henri Fehr, held by the Geneva Library (BGE).
- And finally, an important set of letters received by Charles-Ange Laisant, belonging to the French National Library (BnF) in Paris; many letters relate to Laisant's famous work *Initiation mathématique*, published in 1906.

Our aim here is to share our discoveries and to show how experiments with Méray's method in the classroom, both in secondary and primary education, contributed to legitimizing a new way of teaching geometry. After recalling the historical context in which the movement for the reform of geometry teaching was born in France, we will go through the years by reading the correspondences, in order to retrace the history of this movement up to its influence in the reform of the 1902 curricula in France, as well as its place in the debates that crossed the teachers' milieu at that time.

### **Historical background**

French Emperor Napoleon III lost the war against Prussia at the Battle of Sedan on 1 September 1870. In Paris, Léon Gambetta then founded the government of national defence of the Third Republic. Among the main ministers, Jules Ferry was of considerable importance for education, since from 1882 onwards he promulgated laws making education compulsory and making school public, free, compulsory, and secular. In addition, the Paul Bert law of 1879 decreed the creation of teacher training colleges in all French departments, not to mention teacher training colleges for women.

At that time, French public opinion was revanchist and anti-German. According to an idea put forward in 2004 by Michel Pauty, a Dijon historian, Charles Méray would have ended his international career after the 1870 war, to cut off all relations with the Germanic world and never speak German again. However, we found nothing in the correspondence we studied that confirms this point of view. Above all, Charles Méray was a Burgundian, and his attachment to his regional roots probably explains his isolation more than any Germanophobia.

### Biographical information

Méray was born in Chalon-sur-Saône on 12 November 1835 into a well-to-do family, his father being a notary in that city. From his correspondence, it seems that his studies at the college of Chalon-sur-Saône were a little difficult, as he himself confessed to Laisant his bad memories of ‘those private lessons in geometry that old Diard gave [him] in 1849 during each of which [he] did not stop crying because Diard knocked [him] out with imprecations: you Donkey, etc.’<sup>1</sup>

Méray entered the second year of the Lycée Carnot in Dijon in 1850, where he obtained his baccalaureate in literature at the end of the rhetoric class (now called *Première*), then moved to Paris for a year of elementary mathematics (now called *Terminale*) at the end of which he obtained his baccalaureate in science. After two years in the special mathematics section at the Lycée Saint-Louis in Paris, he was admitted in 1854 to both the École Polytechnique and the École Normale Supérieure<sup>2</sup> (ENS, rue d’Ulm). He chose to enter the École Normale Supérieure, probably because he was already more interested in mathematical research than in any military career, to which Polytechnique led directly.

From 1854 to 1857 Méray was constantly at the top of his class in mathematics, but his teachers complained: ‘A quick and penetrating mind, endowed with a great aptitude for the mathematical sciences, but capricious, disliking effort. Although he retains the 1st rank of his class, he has not done, by far, what he could have done’<sup>3</sup>; ‘Although recognized as the strongest by the lecturer in mathematics, he owes his second ranking in his specialty to the little care he takes in writing and developing his written assignments’<sup>4</sup>. This famous lecturer in mathematics was none else than Jules Vieille, whose footsteps would once again cross

<sup>1</sup> Austrian National Library [ÖNB], V40/B.Mer.3, letter from C. Méray to C.-A. Laisant, 28.12.1900. Diard was Méray’s mathematics teacher at the college, and, as it was usual at the time, he also gave private lessons to his pupil. All translations from French are by the author.

<sup>2</sup> For the unity of our text, we chose to write the names of these institutions in the modern way, that is, using the accented capital ‘É’ and capitals for adjectives (‘Normale’) where it wasn’t the usage in Méray’s time.

<sup>3</sup> French National Archives [AN], file 21 308A, Assessment of the Director of the School, 18.09.1855.

<sup>4</sup> *Ibidem*, Assessment of the Director of the School, 12.05.1856.

those of Méray, as we will see below. ‘This young man is destined for a bright future if his somewhat delicate health stands the fatigues of teaching’<sup>5</sup>. So he was a brilliant student, but terribly annoying for his teachers, since according to his own memories, he would have ‘made the study of the violin [his] main preoccupation’<sup>6</sup>.

After his success at the ENS, Méray was appointed mathematics teacher at the Lycée of Saint-Quentin (in northern France), where he taught for two years while preparing for the doctorate in mathematical sciences, which he obtained in 1858, as well as the *Agrégation* in mathematics in 1859. Exhausted and homesick, he was given a year’s leave to get married. He was then appointed to the Lycée of Nantes, where he stayed only a few days before resigning again and returning to Burgundy.

In his administrative files, Méray indicates that his resignation was due to health and family reasons, but the account of the Principal of Nantes Lycée refers to his difficult character and his bad manners: ‘I believe that Mr. Méray has something deranged in his brain. This is the only way to explain the strangeness of his conduct’<sup>7</sup>. The young teacher’s intransigent and procedural personality would often be criticized in the rest of his life and career.

Méray settled in the small village of Bourgneuf Val d’Or (today a hamlet of the wine-producing village of Mercurey, near Chalon-sur-Saône), where he remained from October 1860 to March 1866 outside academic activity. In October 1867, after having been a lecturer at the University of Besançon for a few months, he was appointed professor of calculus at the Faculty of Sciences of the University of Dijon, a position he held until his retirement in 1905.

From this date on, Méray began to publish papers and works on mathematics again, including the *Leçons nouvelles sur l’Analyse infinitésimal* (1892), which made him famous and caused him to be esteemed by Gino Loria as ‘the French Weierstrass’<sup>8</sup>. Interested in pedagogy, Méray had published the *Nouveaux Éléments de Géométrie* in 1874, but it had not met with the same success in the scientific world. It is tempting to see in the use of this term ‘Nouveaux’, a parallel with the same use by Antoine Arnauld for his own book as a reaction to Euclid’s ‘Old’ Elements. Méray’s *Éléments* may have been, in the same way, a reaction to the then widely used Legendre’s *Éléments*, but nothing in his own words legitimates this opposition, as Méray himself considers that nothing has changed since

<sup>5</sup> Ibidem, final year, 1857.

<sup>6</sup> J. Tannery, *L’enseignement des mathématiques à l’école*, [in:] *Le Centenaire de l’École Normale (1795–1895)*, ed. by R. Rémond et al., vol. 1, Éditions Rue d’Ulm, Paris 1994, p. 393.

<sup>7</sup> AN, file 21 308A, Letter from the chief education officer of Nantes Academy to the Minister of Public Instruction, 22.10.1860.

<sup>8</sup> G. Loria, *La fusione della planimetria con la stereometria. Una pagina di storia contemporanea*, “Periodico di Matematica” 1899, Serie 2, vol. 2, p. 4.

the times of Euclid<sup>9</sup>. The real opposition is found in Méray's proper words<sup>10</sup>: the book he has in mind is rather the one he calls 'the monster' or 'the Gargantua of geometry', that is, the 1242-page extensive *Géométrie* by Rouché and Comberousse, initially published in 1866<sup>11</sup>.

Published for the first time by Savy in Paris, then in Dijon by Jobard in 1903 and 1906, the *Nouveaux Éléments* were to have great importance in the pedagogical reform movement that would lead to the French reform of secondary education in 1902. However, the role of this work in the pedagogical thinking of its time can only be truly understood by reading the correspondence that Méray maintained with major players in the renovation movement, first and foremost Charles-Ange Laisant, who played a fundamental role in the rediscovery of the *Nouveaux Éléments*. For the most part, what we know of the correspondence between Méray and Laisant is dated 1900 and originates in three major European cities.

### The origin of the 1900 correspondence: three starting points

**Warsaw:** In this city, Ludwik Zamenhof invented Esperanto in 1887, at a time when, throughout Europe, people were trying to define an auxiliary language that could be used by (European) people. It was not conceived to replace all other languages, but as an international language that would allow communication, and therefore friendship between peoples, and which would facilitate commercial and scientific relations, etc. In 1887, under the pseudonym 'Dr Esperanto', Zamenhof published *Unua Libro*, the foundational textbook of the language, first in Russian, then in French, Polish, and finally in German. The first English translation, which later became the standard reference, was published in Warsaw in 1889.

The new language spread slowly in France, especially in wealthy circles, among academics and intellectuals. On the occasion of a lecture by his colleague Charles Lambert at the University of Dijon in January 1900, Charles Méray discovered Esperanto, for which he was immediately enthusiastic, to the point of spreading the good word among his colleagues and fellow mathematicians. He became one of the most active propagandists in France, and it was precisely on this occasion that he met Laisant for the first time<sup>12</sup>. In fact, Laisant served as an intermediary for the publication of an article by Méray on Esperanto in a scientific journal, and this is how their 1900 correspondence begins.

<sup>9</sup> C. Méray, *Nouveaux Éléments de Géométrie*, Gauthier-Villars, F. Savy éditeur, Paris, 1874, preface, p. XV.

<sup>10</sup> Méray archives, private collection no. 2 [priv. coll. 2], letters from Charles Méray to Charles-Ange Laisant, 06.12.1900; 10.01.1901, anonymous owner, Dijon (France).

<sup>11</sup> E. Rouché, C. Comberousse, *Traité de géométrie élémentaire*, Gauthier-Villars, Paris, 1866. At least seven editions before 1900, and a few more till the 1950s.

<sup>12</sup> C.-A. Laisant, *L'Espéranto et l'avenir du monde: suivi d'un exposé des éléments de la langue*, Internacia Asocio *Paco-Libereco*, Paris, 1908, p. 2.

**Paris/Geneva:** Laisant and Henri Fehr lived in Paris and Geneva, respectively, when they founded “L’Enseignement mathématique” at Fehr’s initiative. Laisant was 60 years old, Fehr was 30, and they had already crossed paths in Paris in 1894, when Fehr asked for Laisant’s help to find a job (math lessons, translations...) <sup>13</sup>. In 1898, Fehr proposed to Laisant to create an international journal of mathematical education <sup>14</sup>. Laisant hesitated because of his age and his unstable social position, then embarked on the adventure, because it would be an opportunity to publish articles exposing his conception of teaching. It was a period of intellectual effervescence that led to the famous Congresses organized on the occasion of the Universal Exhibition in Paris in 1900. Everything seemed possible, and a great desire for novelty appeared in France, which was entangled in the Dreyfus Affair. Progressives sought to innovate, fighting against a fossilised vision of education, at a very moment when education was becoming compulsory.

**Italy:** in fact, the real innovation came from Italy; it was the *fusion of plane and solid geometry*. The very first issue of “L’Enseignement mathématique” has a bibliographical section containing a presentation <sup>15</sup> by Léon Ripert of the second edition of the *Elementi di Geometria* by Lazzeri and Bassani, exposing the principles on which the didactic approach of these two authors is based: autonomy of geometry in relation to algebra, fusion of plane and solid geometry. Ripert points out that this new approach is widely shared in Italy, thanks to the work of Mathesis, the Italian association of mathematics teachers, unlike in France, where ‘we are less advanced, and the question does not seem to have penetrated’ <sup>16</sup>. He concludes: ‘The principle of the fusion of the two geometries, which was considered yesterday as a utopia, has become today an idea whose study is necessary; perhaps it is destined to be transformed, in the near future, into a classical method for teaching elementary geometry, pending its adoption in all branches of geometry’ <sup>17</sup>. We must remind readers that Lazzeri and Bassani’s book was not the first Italian one to deal with *fusion*, but as our purpose is limited to the French movement linked to Méray’s *Nouveaux Éléments*, we will refer readers to classical scholarly studies for the history of *fusion* in Italy <sup>18</sup>.

<sup>13</sup> See French National Library [BnF], NAF 28 336 (3), letter from H. Fehr to C.-A. Laisant, 08.01.1894.

<sup>14</sup> Library of Geneva [BGE], Ms fr 8178/13, letter from C.-A. Laisant to H. Fehr, 20.01.1898.

<sup>15</sup> L. Ripert, *Review of Lazzeri and Bassani’s Elementi di Geometria*, “L’Enseignement mathématique” 1899, vol. 1, no. 1, p. 62–66.

<sup>16</sup> *Ibidem*, p. 62.

<sup>17</sup> *Ibidem*, p. 65.

<sup>18</sup> See for example M.T. Borgato, *Il fusionismo e I fondamenti della geometria*, [in:] *Da Casati a Gentile. Momenti di storia dell’insegnamento secondario della matematica in Italia*, ed. by L. Giacardi, Lumière Internationales, Lugano 2006, p. 125–157.

Following this article, Laisant received several letters<sup>19</sup> expressing surprise that Ripert did not mention Méray's *Nouveaux Éléments*. He turned to Ripert, who justified himself as follows: 'I did not know Méray's *Nouveaux Éléments de Géométrie* (Savy, 1874); but even if I had known them, it is likely that I would not have quoted them. Indeed, I was reporting Lazzeri and Bassani, and not a paper on the fusion of the two geometries'<sup>20</sup>. He nevertheless concluded his letter with this statement: 'the main question (fusion of the plane and space) seems to me to deserve to be dealt with, if not immediately, at least in a few months [...] The bibliography of fusion seems to me to be at least as interesting to compile as that of the epicycloids'<sup>21</sup>.

Thus, at the beginning of 1899, Laisant's attention was drawn to Méray's *Nouveaux Éléments* and more generally to the teaching of geometry. This was the subject of a first article on fusion in issue 3 of "L'Enseignement mathématique" (May 1899) by Giacomo Candido<sup>22</sup>, who describes the situation in Italy after the creation of the Mathesis association, and the debates on the fusional approach. Then, in issue 5 of September 1899, Laisant shared ideas on the teaching of elementary geometry that he discussed with Amédée Mannheim, his colleague at the École Polytechnique, transcribing the conceptions that Mannheim had explained to him in May<sup>23</sup>: observation of the language difficulties of beginners, turn to drawing, construction of figures on a given scale, equality and similarity of figures, etc. As one can imagine, the observed pedagogical practices must have been very far from students experimenting and acting: on the contrary, they must have been rather dogmatic and theoretical, without appealing to the children's experience or their creative abilities.

Even before Laisant's realization, and perhaps partly at the origin of it, we must point out the role played by Gino Loria in the rediscovery of Méray's *Nouveaux Éléments*. As Italy was far ahead of the rest of Europe, Loria could already take a historical look at the recent development of fusion thinking. The dynamic Mathesis association organized conferences and published reports in its bulletin, the "Periodico di Matematica". The lecture that Gino Loria gave in Turin at the Mathesis Congress in September 1898 was published in booklet 1 of July–August 1899. Tracing the recent history of the fusion, Loria highlighted Méray's *Nouveaux Éléments de Géométrie*, published even before the first Italian treatise

<sup>19</sup> BGE, Ms fr 8178/23/no. 14, letter from E. Chailan (Paris) to C.-A. Laisant, 06.02.1899; BGE, Ms fr 8178/23/no. 21, letter from W.W. Beman (Ann Arbor, MI) to C.-A. Laisant, 20.02.1899.

<sup>20</sup> BGE, Ms fr 8178/23/no. 26, letter from L. Ripert to C.-A. Laisant, 04.03.1899.

<sup>21</sup> Ibidem.

<sup>22</sup> G. Candido, *Sur la fusion de la planimétrie et de la stéréométrie dans l'enseignement de la géométrie élémentaire en Italie*, "L'Enseignement mathématique" 1899, vol. 1, no. 3, p. 205–215.

<sup>23</sup> BGE, Ms fr 8178/23/no. 33–34, letter from A. Mannheim (Paris) to C.-A. Laisant, 04.03.1899.

promoting the fusionist approach. He gave a large excerpt from the preface to highlight Méray's innovative thought<sup>24</sup>:

I have abandoned the customary distinction between plane and space geometry. Apart from the fact that it is not in the reality of things, since nature offers us only spatial figures, it places a long interval between the theory of the straight line and that of the plane, each of which, however, is necessary for the perfect understanding of the other.<sup>25</sup>

How did Loria know about this book, which was distributed confidentially? We're given a clue by a small portrait of Méray belonging to Loria's photographic collection: the dedication bears the date of 28 February 1899, and Méray thanks Loria for his 'extreme benevolence'. They were therefore in contact before the publication of 1899, and it is conceivable that Loria informed Méray of his intention to quote him or that Méray had paid homage to his Italian colleague with his work, as he was accustomed to do, as we will discover from his archives.

### The Méray–Laisant correspondence

No letters from Laisant to Méray written in 1900 are known to us, but we are fortunate to have a good part of those from Méray to Laisant. In a paper in "L'Enseignement mathématique", Laisant mentions his meeting with Méray about Esperanto, and then 'in the last months of the year 1900, a fairly sustained correspondence between [them] two'<sup>26</sup>.

Indeed, after an exchange of eight letters on the subject of Esperanto and the holding of the Congress of Mathematicians, Méray returns to his *Nouveaux Éléments*<sup>27</sup>, after having enumerated all the tasks that exhaust him: 'not to mention my *Nouveaux Éléments de Géométrie*, which unexpectedly recover favour, etc., etc.' He specifies in a postscript:

My Geometry, so long disdained, although tried timidly and secretly with the greatest success in 1875-76-77, an attempt suspended by the brutal mania of the rector, the awful Vieille, whom you can hardly fail to know despite his eminent old fool nature, suddenly takes on a beginning of favour. A teacher at the École Normale in Auxerre, in agreement with its director and the present chief education officer, who is less foolishly stupid and malicious than Vieille, writes to me this morning that it is "definitively established" in this school, in which all the new pupils are successively equipped with it and satisfied to the last point. If you are interested, I will be happy to talk to you about it later<sup>28</sup>.

<sup>24</sup> G. Loria, *La fusione della planimetria con la stereometria. Una pagina di storia contemporanea*, p. 1–6.

<sup>25</sup> *Ibidem*, p. 3.

<sup>26</sup> C.-A. Laisant, *Une exhumation mathématique*, "L'Enseignement mathématique" 1901, vol. 3, no. 2, p. 98.

<sup>27</sup> Méray archives, priv. coll. 2, letter from C. Méray to C.-A. Laisant, 25.11.1900.

<sup>28</sup> *Ibidem*.

Jules Vieille, chief education officer of the Dijon Academy in the 1870s, had been Méray's lecturer at the *École Normale Supérieure*, and it seems likely that since that time, the two men had hated each other.

According to various letters and to an article that appeared in 1907 in the "Revue scientifique"<sup>29</sup>, we discover the course of the misadventures of Méray's method: the release of the *Nouveaux Éléments* was little noticed in Paris, but Augustin Chancenotte, a teacher at the *École Normale* and at the *École primaire supérieure* in Dijon, found the work very well suited to the needs of his students who were candidates for the *École des Arts et Métiers*, because of the significant time that could be saved by the method in studying descriptive geometry. The experiments had been undertaken from 1875 until Vieille forbade their continuation in 1879, as we have read earlier.

This abuse of power by the academic administration brought Méray and Laisant even closer together, when they gradually discovered that they had many points in common in their ways of thinking about teaching and that they both faced the immobility of the administration. Moreover, it seems that Méray's resentment was shared by Laisant, since the former wrote to the latter: 'How I like to hear you say that *education is given over to the beasts!*'<sup>30</sup> These 'beasts' are in fact the national and local education officials, ministers, academic officers, inspectors, etc. The two mathematicians would later even refer to the central administration as 'The Beast'. But conservatism, known as Euclidean, does not only concern civil servants, as Méray confirms:

If you really "delight" in reading my geometry, nothing can be more pleasant to me, for, already in agreement with me on the plan of construction, it would be, on your part, to tell me that its execution was not a failure. I have put all my care into it by employing 1½ to 2 years of my youth to do what others botch in 2 or 3 months. I was treated as nuts by those who at that time set the tone in Mathematics (notably by Briot, who declared that he did not admit motion in geometry, probably forgetting that the notion of the possible superposition of 2 figures implies a certain movement capable of giving rise to it, forgetting again that this famous rotation of a half-plane around the hinge is not precisely rest, forgetting the practical geometry of the surfaces of revolution, forgetting... everything).<sup>31</sup>

From December 1900 onwards, the correspondence only concerned experiments with Méray's method, because the new chief education officer in Dijon was Charles Adam (better known today for having been the editor of Descartes' works with Jules Tannery). Adam gave permission to use the *Nouveaux Éléments* in the normal schools. Méray then wrote:

<sup>29</sup> C. Méray, *Mes «Nouveaux Éléments de géométrie»*, "La Revue scientifique" 24 August 1907, 5th series, vol. 8, no. 8, p. 231–237.

<sup>30</sup> Méray archives, priv. coll. 2, letter from C. Méray to C.-A. Laisant, 06.12.1900.

<sup>31</sup> Méray archives, priv. coll. 2, letter from C. Méray to C.-A. Laisant, 08.12.1900.

At this very moment the reports on the old and new essays of my *Geometry* are being autographed, and their authors, Messrs. Chancenotte and Billiet, have authorized me to do what I wish; but you will have to wait for them for a few more days, as well as the one that M. Mironneau, Director of the École Normale in Auxerre, where M. Billiet teaches and for which I have good hope of obtaining the same permission, wanted to make.<sup>32</sup>

On several occasions, Méray pointed out that he made copies of these reports available to his correspondents, two hundred copies of which he had himself reproduced for distribution purposes. We have not yet found a single copy of these duplicates, but fortunately, the original reports were printed by the University of Dijon and are available in this version<sup>33</sup>.

### The 1900 and 1901 experimental reports

As Charles Adam had encouraged the experimental introduction of geometry lessons according to Méray's method, it was Pierre Billiet, an ambitious young teacher with expertise in mathematics, who committed his students to this path at the École Normale in Auxerre (Yonne department, Dijon Academy) from September 1900.

The previous year in Auxerre, Billiet and his colleagues had already initiated a pedagogical reflection on the method to be used with students in various subjects, including mathematics. The minutes of the session of Friday, 2 June 1899 state as follows:

The Director [...] invites the assembly of teachers to deliberate on the following question: Which method is best suited to the Écoles Normales? Lessons presented orally by the teacher and noted by the pupils – or books carefully chosen and placed in the hands of the pupils, with explanations and additions, readings and questions by the teacher – or mixed method?<sup>34</sup>

In response to this question, the teachers' council gave the following opinion, which is, in fact, Billiet's opinion, as he was the only mathematics teacher in the school:

The lessons very rarely take the form of uninterrupted exposition; most of the demonstrations are interspersed with questions; the teacher goes further only if he feels followed; once the demonstration is finished, it is briefly summarized, then repeated immediately by a student, who can ask for help from his classmates if necessary.

<sup>32</sup> Méray archives, priv. coll. 2, letter from C. Méray to C.-A. Laisant, 13.12.1900.

<sup>33</sup> P. Billiet, A. Chancenotte, A. Mironneau, *Rapports sur les résultats donnés dans l'enseignement primaire supérieur et dans celui des Écoles Normales d'Instituteurs par l'introduction des méthodes exposées dans les Nouveaux Éléments de Géométrie de M. Ch. Méray*, "Revue bourguignonne de l'Enseignement supérieur" 1901, vol. 11, p. 187–215.

<sup>34</sup> Yonne Departmental Archives, T supplement 7935 – 30, Register of the Council of Teachers of the École Normale in Auxerre 1881–1900, p. 1.

Under these conditions, it is difficult to take notes; moreover, the notebook is useless since it is most often only a repetition of the book, but less correct and less precise. It has the further disadvantage that the student is preoccupied with this written work and does not apply all his attention to explanations and demonstrations, and that he does not always follow the outline of the figures, or does not grasp the gesture which sometimes brings light into the mind.<sup>35</sup>

The other teachers follow similar methods, rejecting the old practice of dictation in lessons. And the teachers' council concludes: 'The method that all teachers propose to follow is the active method, the one that consists in resorting as often as possible to questions, in order to force the student to think with the teacher first and then to lead him to think without him'<sup>36</sup>.

As we read this pedagogical project, it is clear that the teachers in Auxerre wanted to break with the dogmatism they denounced and which seemed to be quite common in France at that time. Billiet was one of the leaders of this movement, but it must be said that he himself had followed Chancenotte's courses in Dijon and that he had been able to experience the benefits of Méray's method as a student twenty years earlier.

We will not mention Chancenotte's report, except to note his stress on the time saved by the use of the method in the study of descriptive geometry. This matter frightened many students who prepared for the entrance exams to the *École des Arts et Métiers*. The most interesting ideas came from Billiet's pen:

The study of geometry in the *Écoles Normales* must have a double aim: to cultivate the minds of the students and to put them, as early as possible, without loss of time, in possession of the necessary elements enabling them to pass from theory to practice. [...] It is therefore of the utmost importance to deprive the study of geometry of its scholastic character and to make this study true intellectual gymnastics.<sup>37</sup>

After describing some practical aspects of the implementation of the new method, he concludes this first report as follows:

To sum up, the new method presented by M. Méray saves a very significant amount of time on the total duration of geometry teaching; Now, time and effort at the age when one is studying, are so precious that no possible saving in this respect is negligible. It allows subjects to be studied in descending order of their relative usefulness, which is impossible with the traditional order. In many points, it re-establishes the concordance between various subjects of the mathematics curriculum and those of the courses related to them; it appeals to intelligence rather than to memory; finally, it accustoms the pupils to think for themselves and no longer only for their teacher or for a book.<sup>38</sup>

<sup>35</sup> Ibidem, p. 2.

<sup>36</sup> Ibidem, p. 9.

<sup>37</sup> P. Billiet, A. Chancenotte, A. Mironneau, *Rapports sur les résultats*, p. 193.

<sup>38</sup> Ibidem, p. 196.

To complete this report sent by Billiet to the chief education officer in December 1900, Mironneau collected the enthusiastic testimonies of some student teachers:

What strikes me in this geometry is the clarity of the proofs, their brevity, and at the same time, the links between theorems (Richardot).

The movements of rotation and translation, recently studied, bring to light in a single lesson, clearly and simply, all the theorems of space geometry related to the plane and the straight line (Flament).

Geometry in space, which had seemed so difficult to me, is very naturally based on plane geometry. From the beginning, this study became familiar to us (Richardot).

It is original and proceeds more logically than the other. Thanks to the union of plane geometry and geometry in space, applications can be reached more quickly; demonstrations are few and far between, and can be carried out materially with objects or instruments (Robin).<sup>39</sup>

On reading the reports and testimonies, Méray triumphed, as he immediately wrote to Laisant:

You must share my joy, because these pieces contain, to the ultimate point, the experimental justification of the correctness of our ideas, which we are probably the only ones to have put forward. The complete success of this attempt is now infallible, for the only point that was a little doubtful was the possibility of making children swallow the enormous sum of abstract and new things which are contained in our first chapters, after which the rest is nothing.<sup>40</sup>

By dint of perseverance and at the cost of intense propaganda, Méray and Laisant contributed to advancing reflection on education, against 'this odious administration of Public Instruction, a refuge for all pedants and all prigs, who only know how to strangle us with antediluvian programs, to demolish everything in order to rebuild and then demolish the whole thing again at the cost of hundreds of millions'<sup>41</sup>. French society was indeed permeated by new currents of thought on the subject of secondary education, and the work of a parliamentary commission led to the reform of secondary education in 1902, which, in a sense, enshrined Méray's ideas.

### From 1902 to 1905

In France, the 1902 reform showed a desire to develop secondary education for a wider public by giving greater importance to the sciences, to the detriment

<sup>39</sup> Ibidem, p. 197-198.

<sup>40</sup> ŌnB, Esperanto Museum, V40/B.Mer.2, letter from C. Méray to C.-A. Laisant, 24.12.1900. Emphasis added by Méray.

<sup>41</sup> Ibidem, V40/B.Mer.3, letter from C. Méray to C.-A. Laisant, 28.12.1900.

of Greek and Latin. The Curriculum Review Commission was chaired by Gaston Darboux, who participated in the work of the Mathematics Sub-Commission alongside esteemed professors such as Paul Appell and Jules Tannery.

Charles Méray did not remain inactive. In December 1900, he revealed to Laisant that he was in talks with publisher Gauthier-Villars for a second edition of the *Nouveaux Éléments* ‘which has become necessary since there are only about 80 copies left’<sup>42</sup>. It seems that these talks were not successful, since the second edition of the *Éléments* was published by Jobard in Dijon in 1903. Maybe the conditions offered by Gauthier-Villars were not enough for the author, as his correspondence shows his constant concern about the number of reprints (or even copyrights) he would be rewarded with.

But experiments continued, both in Auxerre and in the upper primary classes of Dijon, where François Monnot adopted Méray’s method from the beginning of the school year in October 1902 by using the pages of the forthcoming work, as they were printed. This may have been one of the reasons for Méray’s choice of a Dijon printer: the proximity of the printing press allowed the chapters to be gradually but immediately made available to the schools in Dijon. Méray kept in his papers a specimen of page 5 of his elements bearing the mention: ‘delivered by M. Jobard on 9 August 1902, as an appendix to the note containing the estimate of the second edition’<sup>43</sup>. As the estimate was drawn up in August and the first sheets of the book were used in October, we can better understand the benefit of proximity!

In agreement with Bailly, his school director, who was also in charge of mathematics, Monnot gradually advanced in the simultaneous study of the mixed geometries, while Bailly kept the Euclidean pedagogical tradition, in order to be able to compare the progress of their students, as they reported in August 1903:

Paying very little attention to “Fundamental Abstractions”, M. Monnot quickly addressed the first intuitions of the “First Properties of the Line and Plane” section. [...] M. Monnot had only a little patience to expend to succeed in getting them through this entrance door without stumbling; he succeeded in doing so by exhibiting to them with tact and skill all material objects that could provide them with the vision of these first intuitions, rarely figures on the blackboard, but rulers, slates, squares, the walls and floors of the room with their intersections, a moving drawer, a door turning on its hinges, etc. [...] When a pupil faces the blackboard, or rather his classmates, handling a ruler in front of them with one hand (the right one), a slate with the other one (the plane), it is pleasant to observe their attitude, the play of their faces; far from trying to escape from the classroom in thought, to abandon themselves to enthusiastic flights towards freedom and the games of the courtyard or the streets, we see them, down to

<sup>42</sup> Ibidem, V40/B.Mer.2, letter from C. Méray to C.-A. Laisant, 24.12.1900.

<sup>43</sup> Méray archives, private collection no. 1 [priv. coll. 1], correspondence concerning C. Méray’s *Nouveaux Éléments*, 1906-1909, in the possession of Charles Méray’s heirs, Mercurey (France).

the last pupil in the group, following with their eyes the drawing of the lines of space, the movement of objects moved in translation or rotation before them.<sup>44</sup>

This report appeared at the right time, specifically when the second edition of the *Nouveaux Éléments* was published (1903). Méray deployed an intense propaganda activity for his book, as is shown by the registers he kept of his mailings of the various reports (1901, 1902, 1903) and of more than 200 copies that he offered to his disciples or to personalities in the world of education<sup>45</sup>.

The reform of secondary education took effect at the beginning of the 1902 school year, and the new curricula were put in place, not without difficulty in the rather conservative world of education. At the instigation of Louis Liard, Deputy officer of the Academy of Paris and a supporter of Méray, the Paris Musée Pédagogique was entrusted with the organization of conferences on the teaching of science. The speakers were well-known scientists (Poincaré, Langevin, Borel, etc.) who presented their points of view on pedagogical methods and the epistemology of scientific disciplines<sup>46</sup>. But the sometimes-heated debates showed an opposition between the scholars and the classroom teachers, who complained of administrative constraints preventing them from setting up teaching that was truly based on experimentation in the real world. A few years later, in a letter to Laisant, Charles Bioche, a teacher at the Lycée Louis-le-Grand in Paris, looked back on his role in these debates and on the role of professional organizations in the revision of the reform in 1905:

If the reform (sorry, the retouching) of the curriculum was carried out in July 1905, it is because secondary school teachers led a very lively campaign and they succeeded in making themselves heard by the Administration; and the said teachers did not forget to mention M. Méray [...]. Obviously, we high school teachers did not make much noise; but we have sometimes done useful work; I had the opportunity to remind Borel of this in the discussion that followed his lecture.<sup>47</sup>

At the same time as the transcripts of the lectures and debates of the Musée Pédagogique appeared, Méray published in “L’Enseignement mathématique” a long article entitled *Justification des procédés et de l’ordonnance des ‘Nouveaux Éléments de Géométrie’*. The first half of it is a ferocious attack on the Euclidean pedagogical tradition and the demotivation it produces in students, especially because it presupposes ‘the existence of facts privileged enough to pos-

<sup>44</sup> J. Martin, *Rapport sur l’enseignement de la Géométrie par la Méthode de M. Méray*, “Revue bourguignonne de l’Enseignement supérieur” 1904, vol. XIV, fasc. 1, p. 191–201.

<sup>45</sup> Méray archives, priv. coll. 1.

<sup>46</sup> Musée pédagogique, *L’enseignement des sciences mathématiques et des sciences physiques. Conférences du Musée pédagogique*, Imprimerie nationale, Paris, 1904.

<sup>47</sup> BnF, NAF 28 336/93, letter from C. Bioche to C.-A. Laisant, 04.03.1906.

ness *immediate* obviousness that would be innate in any individual'<sup>48</sup>. However, as he stated a little later:

For those who have done a little Geometry, it is obvious, for example, that a mobile plane can be moved indefinitely while remaining applied to a fixed plane; but first, the notion of motion, that of geometrical coincidence, and finally that of the unlimited plane, must have been acquired, that is to say, that abstraction must have functioned, and this requires spending some effort and time.<sup>49</sup>

In an article in the "Revue bourguignonne de l'Enseignement supérieur", published almost at the same time as the two above, Henri Duport, a colleague of Méray's at the University of Dijon, praised the growing number of followers of the reform, who in turn experimented with Méray's method in their classes. Citing the recent work of Lobatchewski, Riemann, and Helmholtz, he stated: 'There is no longer any room for doubt: *The axioms of geometry are physical truths*, and it is useless to try to build the entire edifice on reasoning only'<sup>50</sup>. He concluded that Méray's approach was the best one, since 'first of all, the notion of motion is at the very basis of geometry; moreover, in order to arrive at real geometry, something must be added to the common basis of all geometries. This something can be taken with a certain arbitrariness, but the most natural choice is certainly the possibility for a body to move in such a way that the distance traveled by each point of the body is the same; we know that this property characterizes translational motions'<sup>51</sup>. Here is thus justified in a few words the most controversial point of Méray's approach, the choice of the movement of translation as the foundation and starting point of a geometry course focused on the practical sense and imagination of the students.

Duport then recounted his visit to Monnot's class and his surprise when he saw 12-year-old children dealing with questions of geometry in space with ease, or when he realized that instead of establishing the Pythagorean theorem, they were proving the law of cosines. The article ends with numerous testimonies collected from the experimenters of the method. It goes without saying that, as all these testimonies are more or less excerpts from letters addressed to him, Méray inspired a good part of the article. In 1906, the craze for the new method was revived by two publications of important works.

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<sup>48</sup> C. Méray, *Justification des procédés et de l'ordonnance des 'Nouveaux Éléments de Géométrie'*, "L'Enseignement mathématique" 1904, vol. 6, no. 5, p. 101.

<sup>49</sup> Ibidem, p. 102.

<sup>50</sup> H. Duport, *Les Nouveaux Éléments de Géométrie de M. Méray. Leur pénétration dans l'enseignement*, "Revue bourguignonne de l'Enseignement supérieur" 1904, vol. 14, fasc. 2, p. I.

<sup>51</sup> Ibidem, p. IV.

### Debates on teaching geometry from 1906 onwards

In 1906, there appeared both the third edition of Méray's *Nouveaux Éléments de géométrie* and *Initiation mathématique* by Laisant. Those two works would earn their authors many letters from supporters of a pedagogical renewal. Laisant's *Initiation* was clearly intended for beginners, thus rather for primary school teachers. In the same way, despite Méray's *Éléments* being planned for the Lycée, his most important correspondents were primary and middle school teachers. Moreover, Laisant admitted: 'The good elements of primary education are incomparably superior to those provided by the normal school of the same name'<sup>52</sup>. We find the same observation under Méray's pen in 1908, when he sent two copies of the *Nouveaux Éléments* to the library of the École Normale Supérieure de l'Enseignement primaire in Saint-Cloud; he kept a copy of the dedications: 'Offered to the École Normale Supérieure de Saint-Cloud by the author, deeply grateful to primary education for the warm welcome with which it alone has, so far, honored his theories, charmed by its firm hope now of triumphing someday over the whole line by and with it'<sup>53</sup>. On the copy of the 1874 edition, he wrote: 'an almost fossil copy of the first model of the weapon with which French primary education had the audacity to attack the geometric routine twenty-three centuries old and the skill to ensure its future victory now'<sup>54</sup>. He received the thanks of the director of the School, Albert Pierre, who stated in his letter: 'Your followers are increasingly numerous, and your theories are gradually triumphing over routine and inertia. I see it, like you do, almost everywhere'<sup>55</sup>.

The last public evidence of the implementation of Méray's method in geometry teaching appeared on the occasion of a survey by the "Revue de l'enseignement des sciences", founded by Francisque Marotte in 1907: *Enquête sur l'enseignement de la géométrie* [Survey on teaching geometry], that was even the first article in the first issue<sup>56</sup>. Marotte considered that the new curriculum of 1905 had generated a vast experiment, in which all teachers were invited to participate, and which should lead to a new method of geometry teaching. After exposing the current state of this teaching and describing the tensions between supporters of the Euclidean tradition and reformers, he calls on all mathematics teachers to answer several questions about the reform, its conditions of application in the field, and the use of motion in geometry.

<sup>52</sup> Méray archives, priv. coll. 1, letter from C.-A. Laisant to C. Méray, 22.04.1907. We must understand the word *superior* as a reference to the *École Normale supérieure*, and it is indeed the ENS located on Ulm Street in Paris, the most prestigious one.

<sup>53</sup> Méray archives, priv. coll. 1. Single sheet (back of an envelope dated 1908).

<sup>54</sup> Ibidem. Single sheet with black border (part of a death announcement).

<sup>55</sup> Ibidem, letter from A. Pierre to C. Méray, 26.01.1908.

<sup>56</sup> F. Marotte, *Enquête sur l'enseignement de la géométrie*, "Revue de l'enseignement des sciences" January 1907, vol. 1, no. 1, p. 4–10.

Méray probably encouraged his disciples to write to the “Revue de l’enseignement des sciences” on this subject, since Marotte postponed the examination of the replies to issue 9 of November, and he admitted that he had decided to concentrate his attention on only one of the questions: Indeed, from the first paragraph, taken from a letter from Vareil, a teacher at the École Normale in Melun (in the immediate surroundings of Paris), the tone is set. Vareil, who had adopted the method four years earlier, explained its main advantages:

1. The capital innovation of M. Méray consists of a new choice of the first intuitions;
2. M. Méray achieves the fusion of plane geometry and the geometry of space;
3. M. Méray makes frequent use of arithmetic.<sup>57</sup>

According to Vareil, the results obtained were much better than those obtained by his students with the classical method, because of the support of theorems compatible with common sense and the intuition of the students. We lack space to cite all the testimonies given in response to the inquiry, but it is remarkable that the names of almost all their authors are found in Méray’s correspondence...

## Conclusion

What can we learn from this story? A first impression is that the situation has not really changed in France, where the Central Administration of the Ministry of Education still decides on the content to be taught, and prohibits teachers from any deviation from what is stated. The weight of scientists and Inspectors is still as strong as ever, but scientists are no longer ‘pure’ scientists: the French Ministry of Education now draws much of its inspiration from neuroscience and its choices are guided more by the opinions of the best researchers in this field (in order to promote student success) than by the need to make tools available to as many people as possible, conditioning a certain vision of the world.

It must be said that our world has changed over the past hundred years, and that modern technologies will soon be able to replace human intelligence in many fields, perhaps including the teaching of mathematics. Educators at the beginning of the 20th c. believed in a science shared by everyone and remained optimistic about the future. The First World War made them disillusioned, and future curriculum reforms were even more marked by the need to surpass their German neighbours. Nowadays, the introduction of a very light form of algorithmic study at an early age is probably aimed at catching up with Silicon Valley.

The opposition between geometry in the manner of Euclid and an approach more faithful to intuition and based on everyday life remains relevant for analyzing the tensions that still run through the educational world today. What mathematics for the 21st c. human being? What knowledge in geometry? After the

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<sup>57</sup> Ibidem, November 1907, vol. 1, no. 9, p. 307–312.

reform of modern mathematics in the 1970s, geometry was the preferred field for learning mathematical demonstration. Things have changed a lot and the place of geometry in French school curricula has constantly decreased, often in favour of content related to computer science and modern technological innovations (algorithms, robotics, etc.).

To conclude, it should be remembered that the traditional (typically French?) hierarchy between primary, secondary, and higher education levels has remained very present and is still manifested in teaching competitions. But remembering Laisant's ideas about primary school teachers, we can ask: what could secondary school teachers learn from them?

Finally, it seems relevant to keep in mind the question that Laisant attributed to Charles Méray:

Should the purpose of mathematics education be to strengthen intelligence and prepare for applications? Or, on the contrary, is it a means of selection for examinations and competitions, an excellent means because such a study bristles with difficulties and requires prodigious efforts of memory?<sup>58</sup>

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**Frédéric Métin** is a teacher trainer at the University of Burgundy-Europe in Dijon, where he teaches mathematics and the history of mathematics to future primary and secondary school teachers. He earned his PhD in 2016, the subject of which was military architecture at the turn of the 17th c. His research includes experiments of classroom activities using sources from various times and contexts, from Antiquity to the 20th c.

email: frederic.metin01@u-bourgogne.fr