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GENESIS OF APPLIED MATHEMATICS: A SPECIFIC CASE STUDY BY REFLECTING EARLY MODERN SCHOLAR TREATISES

Summary: The emergence of applied mathematics as a distinct field is closely linked to the mathematisation of practical disciplines. This article investigates the development of scholarly textbooks on mine surveying as an early case of applied mathematics by analysing early modern scholarly treatises. It traces how mathematical methods were gradually incorporated into the literature on surveying, illustrating the evolving relationship between theoretical knowledge and practical applications. The study also considers the broader historical and institutional context, including shifts in academic classification and the role of philosophy in structuring scientific disciplines. By examining these aspects, the paper sheds light on the epistemological foundations of applied mathematics in the early modern period.

Keywords: applied mathematics, mining surveying, early modern sciences, mathematisation, scientific treatises, knowledge classification

Introduction

The question 'What is mathematics?'[1] is not just of lexical nature. Various attempts have been made to answer it at different times, and it is closely linked

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[1] '[I]t is first necessary to ask what is meant by mathematics in general. Illustrious scholars have debated this matter until they were blue in the face, and yet no consensus has been reached about whether mathematics is a natural science, a branch of the humanities, or an art form', see R. Tobies, *Iris Runge. A Life at the Crossroads of Mathematics, Science, and Industry*, Birkhäuser (Springer), Basel 2012, p. 9.

to the morphological question of associated fields of knowledge and is central to the (historical) investigation of the emergence of the modern system of scientific disciplines. According to the sociologist of science Rudolf Stichweh, a need for classification arises naturally – as in all fields of knowledge – from philosophy and teaching and their institutionalization[2]. A not insignificant but often trivialized factor here is the division of labor[3], which, however, also has its limits[4]. The medieval hierarchy of faculties – arts, medicine, law, and theology – in which the former had a more auxiliary function – in the form of a *studium generale* – was increasingly discredited theoretically, but was institutionally solid[5]. The 18th c. is eminent in this respect: it heralds the emergence of a concept of science that believes it can find the institutional embodiment of scientificity in the philosophical faculty – as the former faculties of arts were increasingly called philosophical faculties[6].

The term applied mathematics, as *mathesis applicata*, already appears in *Mathesis Juvenilis* (1699) by Johann Christoph Sturm (1635–1703) and was disseminated through *Institutiones Matheseos* (1718) by Johann Friedrich Weidler (1691–1755)[7]. In the beginning of the 18th c., *Markscheidekunst*, i.e., mine surveying, was just one topic among others in mathematics books, and at the end of the century, this subject was established as an academic engineering discipline.

The aim of this article is to study the scholarly literature, especially of two Wittenbergians, in the specific field of mine surveying as part of the diversifying field of applied mathematics in the early modern period.

Mathematics and mine surveying

In the past, *Markscheidekunst* or *geometria subterranea* was essentially understood as what is now known as mine surveying – i.e. underground surveying. The word nucleus *-kunst*, i.e. art, in *Markscheidekunst* ostensifies the understanding of knowledge that goes back to antiquity, which distinguished between the

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[2] R. Stichweh, *Zur Entstehung des modernen Systems wissenschaftlicher Disziplinen: Physik in Deutschland 1740–1890*, Suhrkamp Verlag, Frankfurt am Main 1984, p. 7 sqq.

[3] K. Marx, *Das Kapital. Kritik der politischen Ökonomie*, vol. 1: *Der Produktionsprozess des Kapitals*, Dietz Verlag, Berlin 1962 (Marx-Engels-Werke (MEW), vol. 23), p. 52 sqq.

[4] S.G. Becker, K.M. Murphy, *Social Economics. Market Behavior in a Social Environment*, The Belknap Press of Harvard University Press, Cambridge (MA), London 2000, p. 47 sqq.

[5] R. Stichweh, *Zur Entstehung des modernen Systems wissenschaftlicher Disziplinen: Physik in Deutschland 1740–1890*, p. 31 sqq.

[6] Ibidem.

[7] T. Reimers, *Der Beitrag des Wittenberger Mathematikers Johann Friedrich Weidler zur Begriffsgenese der Angewandten Mathematik*, [in:] *SieB. Siegener Beiträge zur Geschichte und Philosophie der Mathematik*, vol. 13, ed. by R. Krömer, G. Nickel, universi – Universitätsverlag Siegen, Siegen 2020, p. 33–55.

Seven Libearl and the Seven Practical Arts[8]. Like the Aristotelian categories, they were long regarded as a classical epistemological system of order. Judging by Lucius Annaeus Seneca's LXXXVIII letter to Lucilius Iunior, according to which '[...] are called liberal studies: because they are worthy of a free man'[9], the practical character of this discipline certainly predominates in the art of mine surveying to this day.

Mine surveying deals with measurements, calculations, and illustrations for mining purposes above and below ground. Mine-surveying *measurements* are recordings of existing or planned facilities. The results of the recording are primarily useful for illustration, whereby drafts drawn on maps or plans are transferred to the locality. On the one hand, *calculations* are used to transform measurement results for the production of sketches in accordance with the purpose. On the other hand, areas and masses are determined according to their content. *Illustrations* visualize the surface of the mine, mine workings and storage relations as well as the legal, contractual, and safety limits of the mine fields[10].

The three mathematical activities – measuring, calculating, and illustrating – are the main tasks of mine surveying. The Saxonian mining historian Hans Baumgärtel (b. 1927) correctly stated in his dissertation (1964) on the origin of the mining sciences that although mine surveying, even though it entirely belongs to the field of mining sciences – in contrast to geology and mineralogy, for example – it occupies a special position, in that it does not differ from land surveying in principle, but only partially through special working methods and instruments. Although mine surveyors were not only pure surveyors, they had to have mathematical skills in addition to mining skills[11].

Scholars and practitioners

Historians who dealt with the *mathematisation* of nature and technology have repeatedly sought to clarify the complex relationship between practitioners and scholars, sometimes also by questioning such labeling and its dichotomy[12]: The Austrian mathematician, philosopher, and victim of German fascism Edgar Rudolf Zilsel (1891–1944) argued that *superior artisans* – craftsmen who were not

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[8] The latter included different disciplines at different times.

[9] '[L]iberalia studia dicta sint vides: quia homine libero digna sunt', A.L. Seneca, *Epistulae morales ad Lucilium*, vol. 2, ed. by R. Nickel, De Gruyter, Berlin 201, p. 162.

[10] G. Schulte, W. Löhr, *Markscheidekunde für Bergschulen und für den praktischen Gebrauch*, 2nd ed., Springer-Verlag, Berlin 1941, p. 1.

[11] H. Baumgärtel, *Vom Bergbüchlein zur Bergakademie. Zur Entstehung der Bergbauwissenschaften zwischen 1500 und 1765/1770*, Dt. Verl. für Grundstoffindustrie, Leipzig 1965 (Freiberger Forschungshefte, Series D, no. 50), p. 93 sqq.

[12] See H.F. Cohen, *The Scientific Revolution: A Historiographical Inquiry*, University of Chicago Press, Chicago 1994.

afraid to take up a (new) idea in the early capitalist urban environment by scholastics and humanists and to try out or experiment – played a key role as technologists during the genesis of new sciences[13]. Zilsel's groundbreaking theory explains the socio-economic driving forces behind the developments of the early modern (natural) sciences, but the historian Hendrik Floris Cohen (b. 1946) rightfully raises the question:

> How is it that between 1550 and 1600 the social barrier between skilled artisans and the Latin-writing intellectuals suddenly began to break down?[14]

The historian Pamela Olivia Long (b. 1980) takes up the model of contact zones[15] in the context of authorship around the mining and metallurgical industry of the 16th c. and thus relativises the view of a hermetic guild and guild system in the urban milieu: She characterises the mine environment as so-called *traiding zones*, in which human and knowledge capital circulated and accommodated strongly, in which scholars also came together and occasionally collaborated with practitioners[16], thus showing that the social barrier was not impermeable to knowledge. She also advocates paying greater attention to practitioners in the social history of science[17]. In response to this demand, the historian of mathematics Thomas Timothée Morel (b. 1986) devotes himself to practitioners, especially mine surveyors, and refers to an opulent collection of early modern sources – from teaching letters and manuscripts to city chronicles and sermons – in order to study the socio-historical reality – practices, environment, values – and the circulation of knowledge among *practitioners*[18]. With regard to the geometry of mine surveying in the early modern period, he comes to the conclusion that

> it appears almost as if scholars and craftsmen were dealing with two different disciplines. [...] At the time, a growing number of areas of civil life came to use geometry, following an apparently large pattern. Virtually all of these uses relied on construc-

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[13] E.R. Zilsel. *The Social Origins of Modern Science*, ed. by D. Raven, W. Krohn, R.S. Cohen, Kluwer Academic Publishers, Dordrecht–Boston–London 2000 (Boston Studies in the Philosophy and History of Science, vol. 200), p. 7–21.

[14] H.F. Cohen, *The Scientific Revolution: A Historiographical Inquiry*, p. 340.

[15] P.L. Galison, *Abbild und Logik. Zur apparativen Kultur der Teilchenphysik*, [in:] *Analog/ Digital – Opposition oder Kontinuum? Zur Theorie und Geschichte einer Unterscheidung*, ed. by J. Schröter, A. Böhnke, transcript Verlag, Bielefeld 2004, p. 355–372; M.L. Pratt, *Imperial Eyes: Travel Writing and Transculturation*, 2nd ed., Routledge, London, New York 2008.

[16] P.O. Long, *Artisan/Practitioners and the Rise of the New Sciences, 1400–1600*, Oregon State University Press, Corvallis 2011.

[17] Eadem, *The Openness of Knowledge: An Ideal and Its Context in 16th-Century Writings on Mining and Metallurgy*, "Technology and Culture" 1991, vol. 32, no. 2 (Part 1), p. 318–355.

[18] T.T. Morel, *Underground Mathematics. Craft Culture and Knowledge Production in Early Modern Europe*, Cambridge University Press, Cambridge 2023.

> tive, elementary, and useful methods which [are] largely unrelated to the academic knowledge epitomized by Euclid's *Elements* [...].[19]

For the practitioner, the focus was on completing his tasks – in accordance with the respective mining regulations. Unsurprisingly, procedures were not tested on the basis of an abstract standard of validity in the sense of a mathematical proof, as it could be found at universities – if at all – but on the basis of common sense and – ultimately – on the basis of the specific outcome or result of the mining work[20].

A synoptical overview may be ventured: With a wealth of sources, Morel's studies underline the importance of superior *practitioners* and *artisans*[21], whose closer investigation Long demanded[22]. If one adapts Zilsel's model[23] for the development of science and technology in combination with the concept[24] of *Denkkollektive*, by the Polish microbiologist and epidemiologist Ludwik Fleck (1896–1961), to the mining sciences of the early modern period, then – together, as in Fig. 1 – a synoptic scheme emerges regarding the influence of the scholars on the mathematics of mine surveying[25].

The literary provenance of the art of mine surveying can be found in the representation of the art of field measurement[26]. This is depicted in the comprehensive writings of the Roman *argrimensores* and *gromatici*[27]. Heron of Alexandria (c. 10 – c. 70)[28] explicitly refers to the gromatic writings and takes up, among other things, their improved measurement and calculation methods compared to Egyp-

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[19] Ibidem, p. 82.

[20] Ibidem, p. 56–66.

[21] Ibidem.

[22] P.O. Long, *Artisan/Practitioners and the Rise of the New Sciences, 1400–1600.*

[23] E.R. Zilsel, *The Social Origins of Modern Science.*

[24] L. Fleck, *Entstehung und Entwicklung einer wissenschaftlichen Tatsache. Einführung in die Lehre vom Denkstil und Denkkollektiv*, ed. by L. Schäfer, T. Schnelle, Suhrkamp Verlag, Frankfurt am Main 2012.

[25] For the larger perspective on the mathematisation, see T. Reimers.*Geometria subterranea im Lehrbuchkanon der Universität Wittenberg im 18. Jahrhundert: Zur Herausbildung der Wissenschaft des Markscheidens*, Verlag Dr. Kovač, Hamburg 2025 (Mathematische Forschung und Lehre an der Universität Wittenberg, vol. 5).

[26] T. Reimers, *Wurzeln des Markscheidewesens im Spiegel gelehrter Schriften: Eine mathematikhistorisch-bibliographische Analyse*, [in:] *SieB. Siegener Beiträge zur Geschichte und Philosophie der Mathematik*, vol. 14, ed. by R. Krömer, G. Nickel, universi – Universitätsverlag Siegen, Siegen 2021, p. 93–127.

[27] See, e.g., M. Folkerts, *Die Mathematik der Agrimensoren – Quellen und Nachwirkung*, [in:] *In den Gefilden der römischen Feldmesser. Juristische, wissenschaftsgeschichtliche, historische und sprachliche Aspekte*, ed. by E. Knobloch, C. Möller, De Gruyter, Berlin, Boston 2013, p. 131–148.

[28] A.G. Drachmann. *Hero of Alexandria*, [in:] *Dictionary of Scientific Biography*, ed. by C.C. Gillispie, vol. 6, Charles Scribner's Sons, New York 1972, p. 310–315.

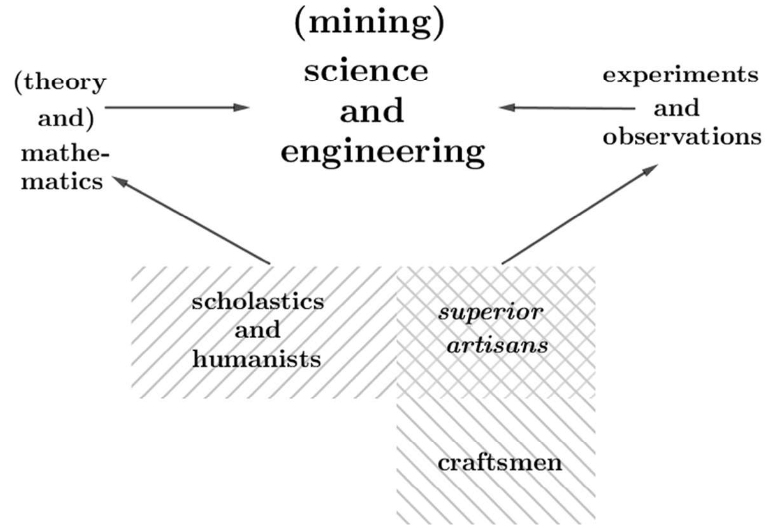

Fig. 1. Social origins of early modern mining science.
Note: Scheme of the author.

tian traditions[29]. The *Commentatio Dioptrica* of Heron contains several geodetic surveying problems that are genuinely dedicated to mine surveying[30]. He is the first to render outstanding services to a mathematical treatment of mine surveying problems by presenting a mathematical approach to mine surveying tasks from the point of view of a *mechanicus*, and he describes the counter-location tunnelling; this description is to be seen as a scholarly reconstruction of a practical procedure.

Erasmus Reinholdus and the humanists

According to the two West and East German historians of mathematics Christoph Joachim Scriba (1929–2013) and Peter Schreiber (b. 1938), the Renaissance is described as an epoch in the history of geometry by the combination of two characteristics[31]:

1. Practical orientation and identification of new fields of application,
2. Initialisation of progress often through amateurism and practice.

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[29] See, e.g., the district accounts in Alexandrinus Heron, *Rationes dimetiendi et commentatio dioptrica*, [in:] *Heronis Alexandrini opera quae supersunt omnia*, vol. 3, ed. by H. Schöne, in aedibus B.G. Teubneri, Lipsiae (Leipzig) 1903, p. 74.

[30] T. Reimers, *Wurzeln des Markscheidewesens im Spiegel gelehrter Schriften: Eine mathematikhistorisch-bibliographische Analyse*, p. 96–100.

[31] C.J. Scriba, P. Schreiber, *5000 Jahre Geometrie: Geschichte – Kulturen – Menschen*, 3nd ed., Springer-Verlag, Berlin, Heidelberg 2010, p. 245.

The latter once again takes up Zilsel's concept of *superior artisans* in the context of *mathematisation*. Ulrich Rülein von Calw (1465–1523)[32] and his *Ein Bergbüchlin*[33], Georg Agricola (1494–1555)[34] and his *De Re Metallica Libri Duodecim* (1556)[35] – here in particular his *Liber Quintus* – as well as Erasmus Reinholdus (1538–1592)[36] and his *Gründlicher vnd Wahrer Bericht. Vom Feldmessen* (1574)[37] with the appendix *Vom Marckscheiden, kurtzer vnd gründlicher vnterricht* should be mentioned[38]. Agricola structures his *De Re Metallica* based on the *Rei Rusticae Libri Duodecim* of the agrimensor Lucius Junius Moderatus Columellas (d. c. 70 CE)[39] and thus corresponds entirely to the humanistic ideal of the Renaissance, which, according to the contemporary understanding, was believed to be oriented towards classical models.

The latter is particularly eminent for the mathematics of mine surveying due to its authorship: after all, the younger Reinholdus was the son of the Wittenbergian mathematics professor of the same name, Erasmus Reinholdus (1511–1553)[40], who worked with Georg Joachim Rheticus (1514–1574)[41] – a German *patriarch* of trigonometry – at the University of Wittenberg – i.e. Leucorea.

While Baumgärtel devotes less detail to Reinholdus' work in his dissertation, the curator of the Mathematical and Physical Salon in Dresden, Gotthelf Ernst Herbert Wunderlich (b. 1905), praised it in particular as a textbook of the art of

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[32] F. Jentsch, *Rülein von Calw, Ulrich*, [in:] *Neue Deutsche Biographie (NDB)*, vol. 22, ed. by H.G. Hockerts, Duncker & Humblot, Berlin 2005, p. 222.

[33] W.L. Pieper, *Ulrich Rülein von Calw und sein Bergbüchlein*, Akademie-Verlag, Berlin 1955 (Freiberger Forschungshefte, Serie D, no. 7).

[34] H. Hartmann, *Georg Agricola (1494–1555) – Begründer dreier Wissenschaften: Mineralogie–Geologie–Bergbaukunde*, Wissenschaftliche Verlagsgesellschaft, Stuttgart 1953 (Große Naturforscher, vol. 13).

[35] G. Agricola, *De re metallica libri XII*, Hieronymus Froben, Nicolaus Episcopius, Basileae (Basel) 1556.

[36] A. Kühne, *Reinhold, Erasmus*, [in:] *Neue Deutsche Biographie (NDB)*, vol. 21, ed. by H.G. Hockerts, Duncker & Humblot, Berlin 2003, p. 367–368.

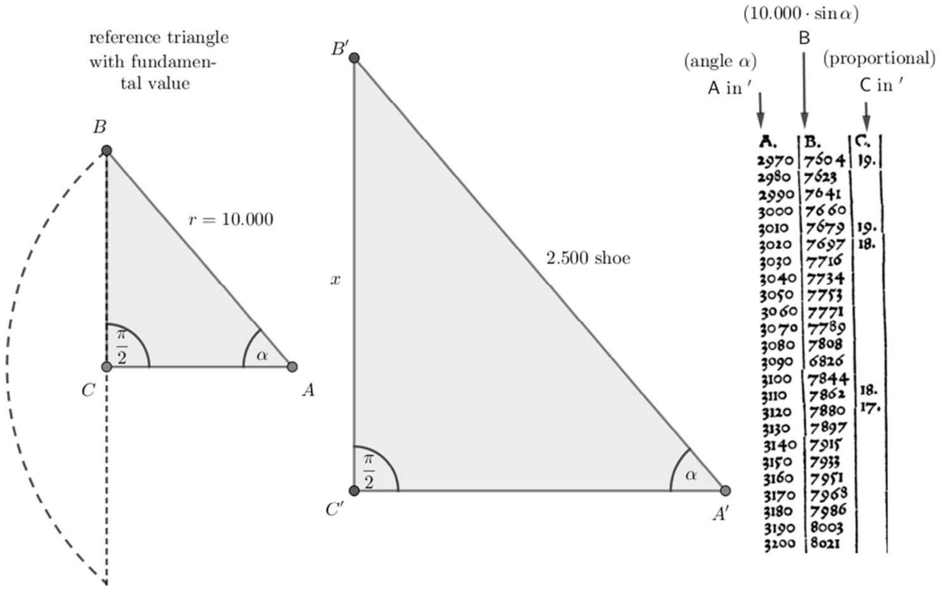
[37] E. Reinhold, *Gründlicher vnd Warer Bericht. Vom Feldmessen [...] Desgleichen, vom Marscheiden, kurtzer vnd gründlicher vnterricht*, Georgius Bawman (Georg Baumann), Erfurt 1574.

[38] For a more detailed analysis of the history of mathematics, see T. Reimers, *Wurzeln des Markscheidewesens im Spiegel gelehrter Schriften: Eine mathematikhistorisch-bibliographische Analyse*, [in:] *SieB. Siegener Beiträge zur Geschichte und Philosophie der Mathematik*, p. 93–127.

[39] M. Fuhrmann, *Columella*, [in:] *Der Kleine Pauly. Lexikon der Antike*, ed. by K. Ziegler, W. Sontheimer, vol. 1, Alfred Druckenmüller Verlag, Stuttgart 1964, col. 1251–1252.

[40] A. Kühne, *Reinhold, Erasmus*.

[41] Idem, *Rheticus, Georg Joachim*, [in:] *Neue Deutsche Biographie (NDB)*, vol. 21, ed. by H.G. Hockerts, Duncker & Humblot, Berlin 2003, p. 496–497.



A.	B.	C.
2970	7604	19.
2980	7623	
2990	7641	
3000	7660	
3010	7679	19.
3020	7697	18.
3030	7716	
3040	7734	
3050	7753	
3060	7771	
3070	7789	
3080	7808	
3090	6826	
3100	7844	
3110	7862	18.
3120	7880	17.
3130	7897	
3140	7915	
3150	7933	
3160	7951	
3170	7968	
3180	7986	
3190	8003	
3200	8021	

Fig. 2. Modern interpretation of Reinholdus' AB tables.

Note: Author's visualisation, inspired by H. Wunderlich, *Kursächsische Feldmeßkunst, artilleristische Richtverfahren und Ballistik im 16. und 17. Jahrhundert*, Deutscher Verlag der Wissenschaften, Berlin 1977, p. 27.

field measurement[42], '[t]he it is one of the most scientifically sound works of this genre [...]'[43]. The textbook therefore shows not only that the author – in contrast to Agricola – understood his discipline, but also that a didactic goal was pursued. In doing so, this evaluation is based (presumably decisively)[44] on the reception of mathematics history by Abraham Gotthelf Kästner (1716–1800)[45] and the analy-

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[42] Weißflog received Wunderlich, See E. Weißflog, *Erasmus Reinhold: "Von dem Marscheiden"*, [in:] *Kaufmanns-Rechenbücher und mathematische Schriften der frühen Neuzeit*, ed. by R. Gebhardt, Adam-Ries-Bund, Annaberg-Buchholz 2011 (Schriften des Adam-Ries-Bundes Annaberg-Buchholz, vol. 22), p. 261–276.

[43] '[D]a es zu den wissenschaftlich am besten fundierten Werken dieser Gattung gehört', see H. Wunderlich, *Kursächsische Feldmeßkunst, artilleristische Richtverfahren und Ballistik im 16. und 17. Jahrhundert*, Deutscher Verlag der Wissenschaften, Berlin 1977 (Veröffentlichungen des Staatlichen Mathematisch-Physikalischen Salons, vol. 7), p. 25.

[44] A.G. Kästner, *Die Geschichte der Mathematik seit der Wiederherstellung der Wissenschaften bis an das Ende des achtzehnten Jahrhunderts*, vol. 1, Johann Georg Rosenbusch, Göttingen 1796, p. 699 sqq.

[45] J.E. Hofmann, F. Menges, *Kästner, Abraham Gotthelf*, [in:] *Neue Deutsche Biographie (NDB)*, ed. by Historische Kommission bei der Bayerischen Akademie der Wissenschaften, vol. 10, Duncker & Humblot, Berlin 1974, p. 734–736.

sis of the West German mining engineer Wilhelm Wilkening (b. 1924)[46]. Another didactic argument is that the book begins with elementary measuring and – even if addition and subtraction are not explicitly treated – the handling of multiplication and division is introduced in a textbook manner.

In addition to other aspects of mathematics history, the introduction of trigonometric tables for mine surveying applications is a merit of Reinholdus (the Younger). To measure triangles, he recommends measuring rods or wire to generate right angles, right-angled triangles or ropes knotted on the basis of the Pythagorean triple number (3, 4, 5). In the place of the calculation of circles and arc length determination, Reinholdus' most important achievement appears: his sine tables[47], which he only calls A and B tables, whereby, according to Wunderlich, A is angular-valued and B sinusoidal[48]. In the right-angled triangle *ABC* with constant hypotenuse length r, a certain sine segment *BC* belongs to a certain angle α. This distance is measured in r-units and is a measure of the corresponding angle. Reinholdus chose $r = 10{,}000$ as the fundamental quantity. In the A column, the angles are given in minutes from 0 to 5,400, i.e. $\frac{\pi}{2}$ or 90°, with increases of 10' each. For example, for A = 3,180 or $\alpha = 53°$ according to the B column, the sine distance is 7,986 on r or $\sin\alpha \approx 0.7986$. The C column can also be used to determine the B values for individual minutes: Let A = 3,187, i.e. 3,180 + 7, then the surcharge x to the B value $7{,}986 + x$ results from the proportion

$$\frac{x}{17} = \frac{7}{10}$$

based on the C-value 17, i.e.

$$x = \frac{17 \cdot 7}{10} \approx 12,$$

ergo B = 7,998[49]. Fig. 2 shows a modern interpretation of an excerpt from Reinholdus' AB tables. The AB tables are sine tables adapted for field measurement or mine cutting, as they can be found, for example, as early as 1554 in the astronomical book *Primus Liber Tabularum Directionum* by Reinholdus the Elder[50]. In 1584, his son, together with Andreas Schato, professor of *inferior mathematics* in Wittenberg, republished[51] the trigonometric tables of Johann Regiomontanus

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[46] W. Wilkening, *Erasmus Reinholdt – Der Verfasser der ersten deutschen Markscheidekunde*, "Mitteilungen aus dem Markscheidewesen" 1960, vol. 67, p. 13–15, 58–74.

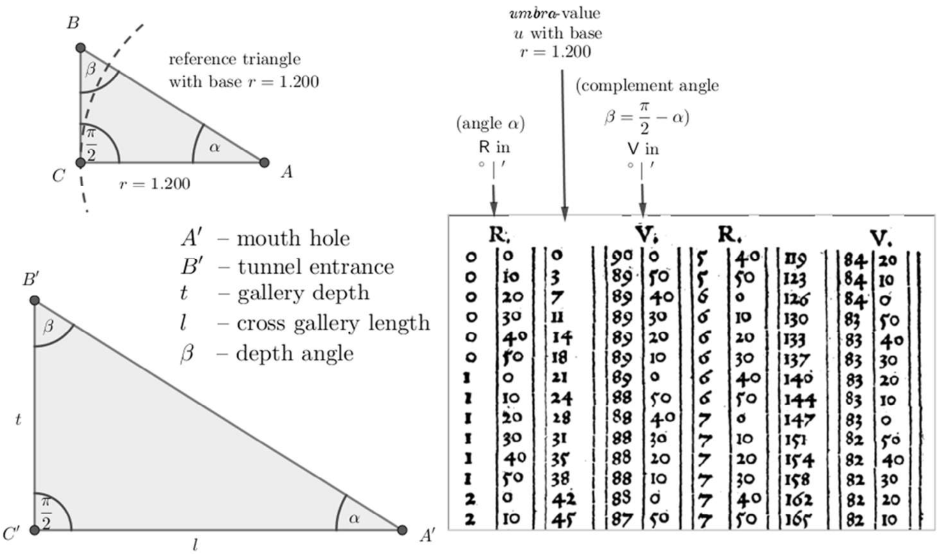
[47] E. Reinhold, *Gründlicher vnd Warer Bericht. Vom Feldmessen [...] Desgleichen, vom Marscheiden, kurtzer vnd gründlicher vnterricht*, chapter 19, s.p.

[48] H. Wunderlich, *Kursächsische Feldmeßkunst, artilleristische Richtverfahren und Ballistik im 16. und 17. Jahrhundert*, p. 26.

[49] Ibidem, p. 26 sq.

[50] E. Reinhold, *Primus liber tabularum directionum. Discentibus prima elementa astronomiae necessarius & utilissimus*, heirs of Ulrich Morhard, Tübingen 1554, foll. 17 sqq.

[51] K. Heinz, *Die Wittenberger Philosophische Fakultät 1502–1817*, Böhlau, Köln / Weimar / Wien 2002 (Mitteldeutsche Forschungen, vol. 117), p. 455.



R.			V.		R.			V.	
0	0	0	90	0	5	40	119	84	20
0	10	3	89	50	5	50	123	84	10
0	20	7	89	40	6	0	126	84	0
0	30	11	89	30	6	10	130	83	50
0	40	14	89	20	6	20	133	83	40
0	50	18	89	10	6	30	137	83	30
1	0	21	89	0	6	40	140	83	20
1	10	24	88	50	6	50	144	83	10
1	20	28	88	40	7	0	147	83	0
1	30	31	88	30	7	10	151	82	50
1	40	35	88	20	7	20	154	82	40
1	50	38	88	10	7	30	158	82	30
2	0	42	88	0	7	40	162	82	20
2	10	45	87	50	7	50	165	82	10

Fig. 3. Modern interpretation of Reinholdus' RV tables.

Note: Author's visualisation, inspired by H. Wunderlich, *Kursächsische Feldmeßkunst, artilleristische Richtverfahren und Ballistik im 16. und 17. Jahrhundert*, p. 30.

(1436–1476) from his[52] estate[53]. With the help of the AB tables, the main tasks on the right-angled triangle can now be solved.

Fig. 3 shows a modern interpretation according to Wunderlich's reception of Kästner of an excerpt from Reinholdus' RV tables[54]. A simpler version, which only returns the tangent to the full degree – but at the fundamental variable 10,000 – can also be found in Reinholdus' Regiomontan tables[55]. According to Kästner, the angles α in degrees and minutes are given in the two R-columns and are assigned to both the *umbra* value u – i.e. $u = 1{,}200 \cdot \tan \alpha$ – and the V-value – i.e. the complement angle[56] $\beta = 90° - \alpha$. The use of the RV tables will be illustrated by a short example:

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[52] M. Folkerts, A. Kühne, *Regiomontanus, Johannes*, [in:] *Neue Deutsche Biographie (NDB)*, vol. 21, ed. by H.G. Hockerts, Duncker & Humblot, Berlin 2003, p. 270–271.

[53] E. Reinholdus, *Ioannis de Monteregio Mathematici Clarissimi, Tabulae Directionum Profectionumque, non tam Astrologiae Iudiciariae, quam Tabulis Instrumentisque Innumeris Fabricandis Utiles ac Necessariae*, Matthäus Welack, Wittenberg, 1584, foll. 31 sqq.

[54] A.G. Kästner, *Die Geschichte der Mathematik seit der Wiederherstellung der Wissenschaften bis an das Ende des achtzehnten Jahrhunderts*, p. 704.

[55] E. Reinholdus, *Ioannis de Monteregio Mathematici Clarissimi, Tabulae Directionum Profectionumque, non tam Astrologiae Iudiciariae, quam Tabulis Instrumentisque Innumeris Fabricandis Utiles ac Necessariae*.

[56] A.G. Kästner, *Die Geschichte der Mathematik seit der Wiederherstellung der Wissenschaften bis an das Ende des achtzehnten Jahrhunderts*, p. 703.

Wanted is the tunnel depth $t = |B'C'|$ given transverse tunnel length $l = |A'C'| =$ 2,400 shoes (≈ 672 metres) and the depth angle measured, $\beta = 83°50'$, at the tunnel entrance B'. From the RV table, the complement angle is $\alpha = 6°10'$ and an *umbra* value $u = 130$.

$$\frac{t}{l} = \frac{u}{1{,}200} \Leftrightarrow t = \frac{l \cdot u}{1{,}200} \rightarrow t \approx 260 \text{ Schuh } (\approx 73 \text{ metres}).$$

It was therefore only necessary to divide the product of the transverse tunnel length l and the *umbra* value u from the table by 1,200 in order to determine the tunnel depth t.

Therefore, Reinholdus' intention was less to write a textbook for mine surveyors than to give the trades the option of carrying out measurements on their own, so that they 'may also have their *proba*, in addition to the common marking'[57]. Reinholdus is more likely – similar to the Saxonian metallurgical and mint master Lazarus Ercker (1530–1594)[58] – to have focused on administrative and political authorities and – similar to Rülein – potential investors or the economic management of a trade as a presumed readership. However, his explanations – such as the sighting compass, the trigonometric method, the documentation forms and the German language – suggest that he wanted to make a contribution to the improvement of mine surveying practice, or at least to give an appearance that justified his promotion to *Bergvogt*.

It is not surprising that an outspoken *mathematicus* chooses an aspect of treatment that corresponds to his subject. In a sense, Reinholdus is to the learned art of mine surveying what Georg Peuerbach (1423–1461)[59] is to astrometry: Both describe innovative measuring instruments – Reinholdus his visor*compast*, Peuerbach his *quadratum geometricum* – and both introduce trigonometric tables to their respective disciplines. The flaw of the practical distance that Morel demonstrates for the application of *geometria subterranea* in Agricola[60] may not cling to

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[57] '[A]uch ihre Proba mögen neben dem gemeinen Mar(k)scheiden haben', see E. Reinhold, *Gründlicher vnd Warer Bericht. Vom Feldmessen [...] Desgleichen, vom Marscheiden, kurtzer vnd gründlicher vnterricht*, cf. *An den Leser*, s.p.

[58] P.O. Long, *Artisan/Practitioners and the Rise of the New Sciences, 1400–1600*, p. 346qq., on Ercker see P.R. Beierlein, *Lazarus Ercker: Bergmann, Hüttenmann und Münzmeister im 16. Jahrhundert*, Akademie-Verl., Berlin 1955 (Freiberger Forschungshefte, Series D, no. 12), or L. Kubátová, H. Prescher, W. Weisbach, *Lazarus Ercker (1528/30–1594): Probierer, Berg- und Münzmeister in Sachsen, Braunschweig und Böhmen*, Deutscher Verlag für Grundstoffindustrie, Leipzig 1994.

[59] H.F. Haupt, *Peu(e)rbach (auch Purbach), Georg von*, [in:] *Neue Deutsche Biographie (NDB)*, ed. by Historische Kommission bei der Bayerischen Akademie der Wissenschaften, vol. 20, Duncker & Humblot, Berlin 2001, p. 281–282.

[60] T.T. Morel, *De Re Geometrica*: *Writing, Drawing, and Preaching Mathematics in Early Modern Mines*, "Isis" 2020, vol. 111, no. 1, p. 32.

Reinholdus to the same extent against the background of his intention(s). It would therefore be somewhat hasty to measure these two scholars with the same yardstick from the point of view of the execution of the *geometria subterranea*[61].

Johann Friedrich Weidler and *mathematisation* in the 18th c.

After Reinholdus, less extensive manuscripts dominated the mine surveying literature[62]: a development that this discipline shares with many others during the Thirty Years' War[63]. With the exception of Nikolaus Voigtels[64] (1658–1714)[65] *Geometria Subterranea oder Markscheide-Kunst*[66] (and by Balthasar Rößler's (1605–1673)[67] *Speculum Metallurgiæ Politissimum*[68], which only scratched the surface of the topic), a book dedicated to the art of mine surveying (mathematically) did not go into print until 1726: The *Institutiones Geometrieae Subterraneae*[69], which recently have been explored historically and analysed in detail[70]. It is again a Leucorean mathematics professor – Johann Friedrich Weidler (1691–1755)[71], as one of the outstanding mathematics professors of the Central German Enlightenment – who actively contributed to the *mathematisation* of his epoch through his numerous publications[72].

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[61] T. Reimers, *Wurzeln des Markscheidewesens im Spiegel gelehrter Schriften: Eine mathematikhistorisch-bibliographische Analyse*, p. 120.

[62] See T.T. Morel, *De Re Geometrica*: *Writing, Drawing, and Preaching Mathematics in Early Modern Mines*, or Idem, *Five Lives of a Geometria subterranea (1708–1785). Authorship and Knowledge Circulation in Practical Mathematics*, "Revue d'histoire des mathématiques" 2018, vol. 24, no. 2, p. 207–258.

[63] M. Koch, *Geschichte und Entwicklung des bergmännischen Schrifttums*, Diss. Bergakademie Clausthal, Clausthal-Zellerfeld 1960, p. 67 sqq.

[64] Surveyor and electoral Saxon tithe collector.

[65] A.W.S. Günther, *Voigtel, Nikolaus*, [in:] *Allgemeine Deutsche Biographie (ADB)*, ed. by Historische Kommission bei der Bayerischen Akademie der Wissenschaften, vol. 40, Duncker & Humblot, Leipzig 1896, p. 212.

[66] N. Voigtel, *Geometria subterranea, oder Marckscheide-Kunst*, Johann Dietzel, Eisleben 1686.

[67] H. Meixner, W. Schellhas, P. Schmidt, *Balthasar Rösler: Persönlichkeit und Wirken für den Bergbau des 17. Jahrhunderts*, Deutscher Verlag für Grundstoffindustrie, Leipzig 1980.

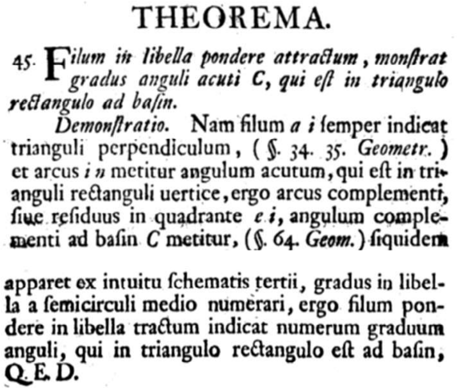
[68] B. Rößler, *Speculum Metallurgiae Politissimum oder: Hell-polierter Berg-Bau-Spiegel*, Johann Jacob Winckler, Dresden 1700.

[69] J.F. Weidler, *Institutiones geometriae subterraneae*, apud viduam Gerdesiam (Gerdes' Witwe), Wittenberg 1726.

[70] T. Reimers, *Geometria subterranea im Lehrbuchkanon der Universität Wittenberg im 18. Jahrhundert: Zur Herausbildung der Wissenschaft des Markscheidens*, p. 127–172.

[71] A.W.S. Grünther, *Weidler, Johann Friedrich*, [in:] *Allgemeine Deutsche Biographie (ADB)*, ed. by Historische Kommission bei der Bayerischen Akademie der Wissenschaften, vol. 41, Duncker & Humblot, Leipzig 1896, p. 453–455.

[72] T. Reimers, *Der Beitrag des Wittenberger Mathematikers Johann Friedrich Weidler zur Begriffsgenese der Angewandten Mathematik.*



THEOREMA.

45. *Filum in libella pondere attractum, monſtrat gradus anguli acuti C, qui eſt in triangulo rectangulo ad baſin.*

Demonſtratio. Nam filum *a i* ſemper indicat trianguli perpendiculum, (§. 34. 35. *Geometr.*) et arcus *i n* metitur angulum acutum, qui eſt in trianguli rectanguli uertice, ergo arcus complementi, ſiue reſiduus in quadrante *e i*, angulum complementi ad baſin *C* metitur, (§. 64. *Geom.*) ſiquidem apparet ex intuitu ſchematis tertii, gradus in libella a ſemicirculi medio numerari, ergo filum pondere in libella tractum indicat numerum graduum anguli, qui in triangulo rectangulo eſt ad baſin, Q. E. D.

The tightened string in the applied scale shows the acute angle *C*, that is at the base of the right triangle.

Proof:

For the cord *a i* always shows the perpendicular of the triangle and the arc *i n* measures the acute angle, which is in the vertex of the right triangle, so the complementary arc or the rest in the quadrant *e i*, measures the complementary angle to the base *C*,

mathematical analysis

For it appears from the third figure that the degrees of the protractor are counted from the centre of the semicircle, so the cord stretched by the weight in the protractor indexes the degree of the angle, that is at the base of the right triangle.

practical sythesis

Fig. 4. Example of Weidler's mathematical-analytical and practical-synthetic approach.

Note: Author's deconstruction of Weidler's proof, § 45, J.F. Weidler, *Institutiones geometriae subterraneae*, apud viduam Gerdesiam (Gerdes' Witwe), Wittenberg 1726, p. 24.

Weidler sees the core of his *Institutiones Geometrieae Subterraneae*, which comprises only 88 pages with 105 paragraphs, as follows:

> This art, even if it flourishes excellently among our miners, is nevertheless, due to the discomforts and dangers with which it is surrounded, for the most part handled only by practical men, **less knowledgeable in the mathematical sciences**, and therefore treated by them in a more obscure and not sufficiently solid manner. I have thus made an attempt to shape it somewhat more precisely and to demonstrate it from the principles of pure mathematics.[73]

In these, he set new standards and introduced the innovations which are typical of the Enlightenment: For example, the algebraization of geometrical facts – with the help of variables, terms, equations – mathematical conceptualisation and the *methodus mathematica*, as he also used it as the basis for his *Institutiones Matheseos* (1718)[74] and which was an essential characteristic of the *Anfangsgründe* textbook tradition[75], to whose first generation of authors Weidler must certainly be counted – even if he always published in Latin[76]. Although his *Institutiones*

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[73] J.F. Weidler, *Institutiones geometriae subterraneae*, p. 3.

[74] The first edition was still published under: J.F. Weidler, *Institutiones mathematicae decem et sex purae mixtaeque matheseos disciplinas complexae*, sumptibus Samuelis Hannauer(i), Wittenberg 1718.

[75] D. Kröger, *Abraham Gotthelf Kästner als Lehrbuchautor: Unter Berücksichtigung weiterer deutschsprachiger mathematischer Lehrbücher für den universitären Unterricht*, Diss. Bergische Universität Wuppertal, Wuppertal 2014.

[76] T. Reimers, *Traces of the Impact of the Works of the Wittenbergian Mathematician Johann Friedrich Weidler on Textbooks and Academic Teaching of the 18th Century*, [in:] *"Dig where you stand" 7: Proceedings of the Seventh International Conference on the History of Mathematics Education, September 19–23, 2022, Mainz, Germany*, ed. by K. Bjarnadóttir, F. Furinghetti, A. Karp, J. Prytz, G. Schubring, Y. Weiss, J. Zender, WTM-Verlag, Münster 2023, p. 227–238.

Geometriae Subterraneae are also in Latin, Fig. 4 shows his practical demand by way of proving and problem solving.

Each paragraph is clearly titled *definitio*, *theorema*, *demonstratio*, *scholion*, *consectarium*, *problema*, *resolutio*, and these structures the surveying writings – which were previously written in continuous text form – textually, epistemically, and didactically. In the case of *purely* mathematical references, he always refers to corresponding paragraphs of his *Institutiones Matheseos* and embeds the *geometria subterranea* in the *mathesis applicata*. The art of mine surveying is now seen as part of applied mathematics at least until the end of the 18th c.: One example is *Neue Beyträge zur Mathesi applicata* (1786) by the science and mathematics' professor Johann Peter Eberhard (1727–1779) in Halle[77]. The title shows that the term *mathesis applicata*, Weidler disseminated with the (six) editions of his *Institutiones Matheseos*, is established – also as *angewandte Mathematik* in Abraham Gotthelf Kästner's (1719–1800) *Anfangsgründe der Angewandten Mathematik* (1781)[78]. Eberhard's *Neue Beyträge* includes – next to mine surveying – mill building art and hydrotechnology, which shows exemplarily the diversity of 18th c. *mathesis applicata*.

In Kästner's *Abhandlungen über die Markscheidekunst*, he wrote in 1775:

> I used Weidler's *Institutiones* [*G*]*eometriae* [*S*]*ubterraneae* for the lectures, which have also been made even more accessible to the public by Mr. P[ater] Fuchsthaler's German translation. More complete instructions, such as those of Mr. v. Oppel and Beyers, are not for academic lectures. Among those that are written for this purpose, Weidler's is the only one that can be seen as viable [...].[79]

In fact, however, the two practitioners – the surveyor August Beyer (1677–1753)[80] and the mining official Friedrich Wilhelm von Oppel (1720–1769)[81] – also take up Weidler's innovations in mathematical treatment in the (textbook) literature of surveyors[82]. Von Oppel's academic background is certainly one of the

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[77] J.P. Eberhard, *Neue Beyträge zur Mathesi applicata. Worin die ersten Gründe der Mühlenbaukunst, Hydrotechnik und Bergwerkswissenschaft erklärt werden. Nebst einigen Zusätzen zur Mechanik, Optik und Gnomonik*, Rengerische Buchhandlung, Halle 1773.

[78] A.G. Kästner, *Anfangsgründe der angewandten Mathematik*, im Verlag der Witwe Vandenhoeck, Göttingen 1781.

[79] Idem, *Anmerkungen über die Markscheidekunst: Nebst einer Abhandlung von Höhenmessungen durch das Barometer*, im Verlag der Wittwe Vandenhoeck, Göttingen 1775, Vorrede.

[80] W.L. Pieper, *Beyer, August der Ältere*, [in:] *Neue Deutsche Biographie (NDB)*, vol. 2, ed. by Historische Kommission bei der Bayerischen Akademie der Wissenschaften, Duncker & Humblot, Berlin 1955, p. 203.

[81] J. Matzerath, *Oppel, Friedrich Wilhelm von*, [in:] *Neue Deutsche Biographie (NDB)*, vol. 19, ed. by Historische Kommission bei der Bayerischen Akademie der Wissenschaften, Duncker & Humblot, Berlin 1999, p. 557–558.

[82] F.W. von Oppel, *Anleitung zur Markscheidekunst nach ihren Anfangsgründen und Ausübungen kürtzlich entworfen*, Georg Conrad Walther, Dresden 1749; A. Beyer, *Gründlicher Unterricht von Berg-Bau, nach Anleitung der Marckscheider-Kunst*, Carl Wilhelm Fulde, Schneeberg 1749.

main factors why he was open to Weidler's mathematical style. Despite his short work, von Oppel laid the first foundations for the importance of mathematics at the Freiberg Mining Academy, which he co-founded in 1765[83].

Weidler's *Inſtitutiones Geometriae Subterraneae* – written in Latin as all Weidlerian publications and especially his textbooks, because Weidler was an opponent of German language in science and philosophy, and indeed this choice provided his book a wider spread of international circulation[84] – have been translated into German and Russian and served as textbooks in Vienna and St. Petersburg[85]. How exactly and to what extent can no longer be completely reconstructed, but due to the brief treatment of general mathematical content – such as arithmetic and geometric basics – it can be assumed that other textbooks were used to supplement it. At the Imperial University of Moscow, Weidler's mathematical textbooks had a demonstrably high reputation: In the second half of the 18th c., all pure and applied mathematical disciplines were largely taught according to Weidler's publications[86]:

> R[ost] read from Weidler's work: *Institutiones Matheseos selectis observationibus illustratae in usum academicarum*, which consists of separate sections devoted to both pure and applied mathematics. The latter was represented in it by the following disciplines: *Optica proprie dicta, Ars perspectiva, Catoptrica, Dioptrica, Aſtronomia sphaerica, Astronomia theorica, Geographia generalis, Chronologia, Gnomonica, Mechanica, Hydrostatica, Aërometria, Hydraulica, Architectura civilis, Architectura militaris.* All these sciences were not observed in any particular order in their succession one after another.[87]

This quote reflects ones more the diversity also of *prikladnaia matematika*, i.e. applied mathematics.

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[83] T.T. Morel, *Mathématiques et politiques scientifiques en Saxe (1765–1851): Institutions, acteurs et enseignements*, Diss. Université Bordeaux 1 / Université de Bordeaux, Bordeaux 2013, p. 141 sqq.

[84] T. Reimers, *Geometria subterranea im Lehrbuchkanon der Universität Wittenberg im 18. Jahrhundert: Zur Herausbildung der Wissenschaft des Markscheidens*, p. 117, 158–90.

[85] J.F. Weidler, *Anleitung zur unterirdischen Meß- oder Markscheidekunst*, transl. and ed. by N. Fuchsthaler, Trattner, Wien 1765; Idem, *Vejdlera nastavleniâ k" podzemnoj geometrii ili markšejderskoj naukě*, transl. by A. Martov, pečatany pri Gornom učilišče, Sankt-Peterburg 1777.

[86] T. Reimers, *Traces of the Impact of the Works of the Wittenbergian Mathematician Johann Friedrich Weidler on Textbooks and Academic Teaching of the 18th Century.*

[87] V.V. Bobynin, *Rost, Ivan Akimovič*, [in:] *Ènciklopedičeskij slovar' Brokgauza i Efrona: v 86 t. (82 t. i 4 dop.)*, vol. 27, ed. by K.K. Arsen'ev, F.F. Petruševskij, Akcionernoe izdatel'skoe obščestvo 'F.A. Brokgauz – I.A. Efron', Sankt-Peterburg 1899, p. 142–143.

Conclusion and discussion

The essential contributions of the two Wittenbergian scholars Reinholdus (the Younger) and Weidler have been spotlighted and examined for their mine-surveying mathematical content: Based on his father's mathematical manuscripts, Reinholdus introduced trigonometric tables and, as a result, introduced new methods as a scholarly suggestion in the interpretative approach to mine-surveying measurements. Weidler combined the measurement and calculation problems of mine surveying with the mathematical means of his time by algebraizing geometric content and using the *mathematical method* as the basis for his *Inſtitutiones Geometriae Subterraneae*.

While Heron's account can be regarded as a purely scholarly reconstruction of method[88], the evaluation of Reinholdus' contribution is less straightforward: Although he too belongs to a scholarly *Denkkolektiv*, he is significantly closer to practical application – after all, he was presumably appointed as a mining official following or as a result of his publication and introduced innovations in mine surveying. However, according to Morel, these were only incorporated into surveying practice more than half a century after the publication of *Vom Marckscheiden*. Whether this development can be traced back to Reinholdus or whether, as Morel argues, it evolved independently of scholarly literature remains speculative: Morel, at any rate, asserts that authority among surveyors outweighed scholarly *expertise*[89]. It is certainly not merely due to the reception of Reinholdus' publication that trigonometric methods permeated surveying practice – after all, as Fleck points out, knowledge must first be stabilised and transmitted[90] – but it is conceivable that Reinholdus' later (in)direct influence as a mining administrator fostered this knowledge through interaction with the *superior artisans* – the surveyors – in the context of the *contact* or *trading zone*. In any case, it appears that this was one of Reinholdus' intended aims.

It is not Weidler's actions, but rather the incipient receptions of his *Institutiones Geometriae Subterraneae*, that soften the independent traditions of mine surveying: On the one hand, the mathematisation applied by Weidler to mine surveying also prevails in the (textbook) literature of mine surveyors, and on the other hand, his *Institutiones* introduced this discipline, which was dominated primarily by practitioners, into academic audiences and thus opened up a first scholarly discourse, which ultimately also prepared the ground for its scientification; lead to the categorisation of mine surveying as *mathesis applicata* – and later, by

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[88] T. Reimers, *Wurzeln des Markscheidewesens im Spiegel gelehrter Schriften: Eine mathematikhistorisch-bibliographische Analyse.*

[89] T.T. Morel, *Underground Mathematics. Craft Culture and Knowledge Production in Early Modern Europe*, Cambridge University Press, Cambridge 2023, p. 140 sqq.

[90] L. Fleck, *Entstehung und Entwicklung einer wissenschaftlichen Tatsache. Einführung in die Lehre vom Denkstil und Denkkollektiv.*

emancipation, to be an own engineering discipline, which is part of another later chapter in the history of applied mathematics and its teaching[91].

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[91] T. Reimers, *Geometria subterranea im Lehrbuchkanon der Universität Wittenberg im 18. Jahrhundert: Zur Herausbildung der Wissenschaft des Markscheidens*, p. 164 sqq., 191 sqq.

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