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Alexandra Sofia Rodrigues
ORCID 0000-0002-4688-7159
EDUNOVA.ISPA – Centro de Investigação Interdisciplinar em Educação
Faculdade de Ciências e Tecnologia da Universidade NOVA de Lisboa
CICS.NOVA, UIED, Portugal

TEACHING GEOMETRY IN THE PREPARATORY CYCLE OF TECHNICAL EDUCATION IN PORTUGAL (1947–1967)

Summary: Between 1947 and 1967, students in Portugal received technical training through the *Ciclo Preparatório do Ensino Técnico*. This research undertakes a documentary analysis of two curricular dimensions: the 'prescribed' and 'presented' mathematics curricula during this period. To achieve this, two methodological frameworks were combined – one proposed by Okeeffe and the other by Krüger. Almeida and Rodrigues previously employed this combination to analyse chapters on integers in textbooks from secondary schools and technical schools in Portugal during the Modern Mathematics Movement, and Rodrigues used it to analyse a chapter on integers from an industrial school's textbook. The mathematics programme remained stable throughout this period. This text examines the textbook *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, for the second year, published in 1954, focusing on the geometry chapters. The information gathered from the textbook analysis enables us to reflect on the importance of the textbook in student learning at a time when mathematics teaching underwent significant transformations. The textbook's author endeavoured to incorporate his teaching experience into its preparation, focusing on the importance of preparatory training for life in society.

Keywords: technical education, geometry, textbooks, preparatory cycle, history of mathematics education

Introduction

The *Ciclo Preparatório do Ensino Técnico* [Preparatory Cycle of Technical Education] lasted for 20 years. It was established in 1947[1] and continued until 1967[2] when the Preparatory Cycle of Secondary Education was created, merging the Preparatory Cycle of Technical Education with the 1st Cycle of Secondary Schools. The rationale behind this union was to postpone the decision-making process for children (aged 10 or 11) about their choice between technical and secondary education.

The constitution of the Preparatory Cycle of Technical Education, during the period of this study, comprised two degrees. The 1st degree consisted of an elementary preparatory cycle of general education, lasting two years, designed to provide the necessary qualifications for admission to various technical courses. Admission to this cycle was contingent upon passing the 4th class of primary education or the examination scheduled at the end of primary education. The 2nd degree encompasses six different types of courses: i) industrial and commercial courses, supplementary learning; ii) industrial and commercial vocational training courses; iii) industrial and commercial professional development courses; iv) industrial master's programmes; v) preparatory sections for enrolment in secondary institutes and schools of fine arts; and vi) other courses organised following the preparatory cycle. These courses had variable durations depending on the nature of the profession, but could not exceed four years. This study focuses on the geometry taught during the 1st degree.

It was in 1952[3] that the programmes for all subjects in the Preparatory Cycle of Technical Education were published, including mathematics. In this programme, the purposes of studying mathematics are related to:

> The educational purpose of mathematics is to form spontaneous judgment and mathematical reasoning; its social purpose also requires that we consider the discipline's material value in solving practical life problems.
>
> This criterion of material value could move us quite far from the usual aspect of arithmetic and geometry programmes formerly taught in secondary schools. With this separation, we would do nothing more than follow the principles of modern pedagogy, above all concerned with the practical usefulness of acquired knowledge and its retention in students' minds.[4]

Geometry should lead students to develop an intuition of space and enhance their ability to comprehend, represent, estimate, and calculate spatial quantities. The programme encourages interdisciplinarity with other subjects, such as drawing

[1] Ordinance No. 2.025, *Diário do Governo*, I Series, no. 139, 19.06.1947, p. 571–576.
[2] Ordinance No. 47.480, *Diário do Governo*, I Series, no. 1, 02.01.1967, p. 01–04.
[3] Ordinance No. 13.800, *Diário do Governo*, I Series, no. 8, 12.01.1952, p. 17–236.
[4] Ibidem, p. 28. All translations in this article are by the author.

and manual work. Connections were also established within mathematics, linking to arithmetic knowledge.

During the 1st year, the programme does not recommend using a textbook in the mathematics course. However, adopting a manual in the 2nd year is expected for two reasons: to monitor the study of mathematics in classes and to provide a future reference for the subjects studied, especially for students who do not continue their studies[5].

During this period, textbooks supporting mathematics education were published. In Portugal, a law established the existence of a single book called *Livro único* (Fig. 1), approved by the central administration to support the teaching of the subject. The Ministry of National Education numbered and authenticated all these textbooks[6].

Fig. 1. Stamp in the unique book analysed, numbered 18618.

Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, Livraria Popular de Francisco Franco, Lisboa 1954, no page.

Matemática: ensino técnico profissional: segundo ano do ciclo preparatório, by António Oleiro dos Santos Heitor, published in 1954, was the approved textbook for the 2nd year of mathematics in the Preparatory Cycle of Technical Education, and it is analysed in this article.

In 1967, the Preparatory Cycle of Technical Education, which served as an exclusive training pathway for technical education, was abolished, with the intention of delaying young people's choice between technical and secondary schools. Following the promulgation of a decree-law, the preparatory training cycle in both secondary and technical schools was unified. This educational cycle, for children aged 10 to 11, aims to provide a common learning pathway for all students, whether they wish to continue in technical schools or pursue academic studies in secondary schools.

By studying the mathematics textbook used during the Preparatory Cycle of Technical Education, the aim is to answer the following research questions: i) What is the relationship between the curriculum prescribed, through an official ordinance in the *Diário do Governo*, and the curriculum presented to teach-

[5] Ordinance No. 13.800.

[6] A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, Livraria Popular de Francisco Franco, Lisboa 1954; Ordinance No. 13.800, 12.01.1952.

ers, through the school textbook?; ii) How does the school textbook relate to the teaching methodologies proposed in the official curriculum?; and iii) How was the geometry taught aligned with the initial training needed to pursue a path in technical education?

The prescribed curriculum

The mathematics programme for the Preparatory Cycle of Technical Education, among other disciplines, was published in 1952 through Ordinance No. 13.800 and was signed by the Minister of Education, Fernando Andrade Pires de Lima. This programme aims to align the mathematics class with the other subjects of the preparatory cycle, described as 'a series of experiences lived by the student, within their centres of real interest'[7]. During the two-year course, it was intended that students use daily notebooks to record the exercises they completed, along with one or two succinct teacher comments on the concepts addressed. Formalism would only be necessary until 'the acquisition of a good numerical calculation technique, but always based on the resolution of concrete problems'[8].

In the first year, the programme is divided into five parts: i) Measurements and counting, ii) Arithmetic operations, iii) Fractional numbers, iv) Intuitive geometry, and v) Everyday life problems. Within the scope of geometry, concepts related to angles and the properties of angles in circles are covered.

The mathematics programme for the second year of the Preparatory Cycle of Technical Education is structured into three parts: i) Review of the first year – recalling the essential topics of the subjects covered (this does not have to be done at the beginning; it can be completed before delving into geometry and algebra content); ii) Intuitive geometry – methodization and expanding upon previous knowledge based on experimental verification; and iii) Practical arithmetic – an elementary study based on everyday problems.

Regarding geometry, the programme prescribes the methodization and expansion of previous knowledge through experimental verification (not through logical 'demonstration'). The topics covered include the teaching of: i) Triangles and relationships between their elements; ii) Cases of equality of triangles; iii) Comparison of perpendicular and oblique segments taken from the same point to the same straight line, distance between two parallel lines; iv) Quadrilaterals: parallelogram, rhombus, rectangle, square, and trapezoid. Most important properties; v) Circumference. Arc, radius, chord, diameter, secant, and tangent. Relative position of two circles; vi) Circle. Circle segment, circular sector, circular ring; vii) Circumference inscribed and circumscribed about a triangle. Other inscribed

[7] Ordinance No. 13.800, p. 28.

[8] Ibidem.

polygons; viii) Circumference perimeter. Experimental determination of the value of π; ix) Equivalent figures. Equivalences of parallelograms and trapezoids to rectangles. Equivalence of the triangle to the parallelogram. Areas of these figures: Areas of flat, regular, and irregular polygons. Areas of the circle, circular sector, circle segment, and circular ring crown. Areas of the surfaces of the right prism, the regular pyramid, the cylinder, and the cone of revolution; x) Volumes of indicated solids[9].

Santos Heitor: professor, methodologist, and textbook author

António Oleiro dos Santos Heitor was born in Abrantes on 7 October 1904. His parents were Maria José da Silva Oleiro and the republican doctor Eduardo dos Santos Heitor, who was born in Coimbra and served as a municipal doctor in Abrantes (a small Portuguese town in the district of Santarém, in the centre of the country). He completed his primary education in his hometown and later attended the *Liceu* in Coimbra[10]. In 1921, after finishing his studies at the *Liceu* at the age of 17, he moved to Lisbon to enrol at the *Instituto Superior Técnico*, where he studied Mechanical Engineering. During his studies, he was influenced by Mira Fernandes, a teacher whom Santos Heitor described as 'exceptional and unforgettable'[11].

Regarding his professional career, after completing the course, Santos Heitor returned to Abrantes, where he worked at *Metalúrgica Duarte Ferreira* in Tramagal. During the 1930s, he relocated to Lisbon and started working as an engineer at the General Directorate of Industrial Services. Simultaneously, he began his career in professional technical education, teaching at night in schools such as *Fonseca Benevides*, *Afonso Domingues*, and *Machado de Castro*. He taught technical subjects at these institutions, including mathematics, mechanics, and machine design. By the mid-1950s, he was invited to become a methodology professor at the Industrial School *Marquês de Pombal*, where he remained until the reform[12].

Santos Heitor made significant contributions to the teaching of mathematics. During the reform of Modern Mathematics during the 1960s, he directed the Study Commission for the Reorganisation of Mathematics Teaching in industrial training courses[13]. He advocated for a more comprehensive approach to math-

[9] Ordinance No. 13.800, p. 28.

[10] M.J. Cortesão, M.C. Serra, "O percurso de Santos Heitor", interview conducted by A.S. Rodrigues and J.M. Matos, 8.05.2021, audio recording, unpublished.

[11] Ibidem.

[12] Ibidem.

[13] A.S. Rodrigues, J.M. Matos, M.C. Almeida, *A visão da aprendizagem da matemática no ensino técnico do metodólogo Santos Heitor*, [in:] *Anais/actas do 8.º encontro Luso-brasileiro de História da Matemática*, ed. by M. Lübeck, S.R. Nobre, Sociedade Brasileira de História

ematics, tailored to the technological needs of the future, believing that 'Modern Mathematics can be a means to provide students in technical schools with more thorough training'[14]. Throughout his career as a methodologist, he presided over and served as a member of State Examination juries for permanent teachers in the 1st group of Professional Education[15].

Since the late 1930s, Santos Heitor has produced mathematics textbooks independently or in collaboration with his son Raul and some former interns[16]. These textbooks, which he authored throughout his life, encompassed topics ranging from pre-primary education to technical education[17]. In early childhood education, he employed the principles of Jean Piaget's constructivism, tailoring the content to align with children's developmental stages. His children, Raul and Maria José, actively contributed to the creation of the textbook illustrations, in the words of his daughter, Maria José: 'We even made drawings for him. Look! This is mine! And adding: Raul did more than I!'[18].

In addition to being a teacher, Santos Heitor was an engineer at a screw factory in Lisbon, the *Fábrica Florescente*, where he collaborated with the owner, Dr Restany, and undertook several work trips across Europe[19]. He also organised and attended numerous professional development courses for teachers in Portugal and abroad, often in partnership with prominent educational figures, such as Professor Sebastião e Silva from the University of Lisbon[20]. In the 1970s, he received a scholarship from the Ministry of Education to attend a mathematics teacher training course at Columbia University in New York.

His granddaughter Marta described Santos Heitor as 'a provocative person who liked to challenge others'. He stood out for his work ethic and dedication to education, and he was always working[21]. In October 1974, shortly before retiring at 70, he was elected to the first board of directors of the *Marquês de Pombal* Industrial School and presided over it during the period of revolution.

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da Matemática (SBHMat), Sociedade Portuguesa de Matemática/Seminário Nacional de História da Matemática (SPM/SNHM), Universidade Estadual do Oeste do Paraná (Unioeste), Foz do Iguaçu 2021, p. 88.

[14] A.S. Rodrigues, B.W. D. Novaes, J.M. Matos, *A cultura escolar em conflito: ensino técnico e matemática moderna em Portugal* [School culture in conflict: technical education and modern mathematics in Portugal], "Diálogo Educacional" 2016, vol. 16, no. 48, p. 387.

[15] A.S. Rodrigues, J.M. Matos, M.C. Almeida, *A visão da aprendizagem da matemática no ensino técnico do metodólogo Santos Heitor*, p. 88.

[16] M.J. Cortesão, M.C. Serra, "O percurso de Santos Heitor", interview.

[17] A.S. Rodrigues, J.M. Matos, M.C. Almeida, *A visão da aprendizagem da matemática no ensino técnico do metodólogo Santos Heitor*, p. 88.

[18] M.J. Cortesão, M.C. Serra, "O percurso de Santos Heitor", interview.

[19] Ibidem.

[20] A.S. Rodrigues, B.W. D. Novaes, J.M. Matos, *A cultura escolar em conflito: ensino técnico e matemática moderna em Portugal*, p. 390.

[21] M.J. Cortesão, M.C. Serra, "O percurso de Santos Heitor", interview.

António Oleiro dos Santos Heitor passed away in July 1993, succumbing to cancer at his home in Lisbon. He enjoyed listening to classical music throughout his life, reflecting his appreciation for culture and art[22].

Methods and theoretical framework

This study employs a documentary analysis approach[23] to investigate the 'prescribed' and 'presented' mathematics curriculum during the Preparatory Cycle of Technical Education in Portugal. Bardin[24] defines document analysis as a systematic process of classifying, indexing, and transforming documentary sources to represent their content in a condensed form. This approach facilitates the creation of secondary documents containing relevant information about the topic under study. The materials under study include official curriculum documents, textbooks, and supplementary curricular documents meant to present the curriculum to educators.

Textbooks are central to this analysis. While Julia[25] defines textbooks only by their usage, Choppin[26] highlights that textbooks are more than just books used in schools. They are intentionally designed and organised to meet instructional objectives from both the students' and the teacher's perspectives, playing a crucial role in the educational process. This definition serves as the basis for the approach adopted in this article.

Furthermore, historical curriculum analysis seeks to maintain a dual perspective on the 'prescribed' and 'presented' curriculum, as emphasised by Choppin[27]. The textbook is regarded as a complex object that serves a didactic function, transmitting knowledge, skills, and cultural or social perceptions reflective of a specific area and time. Textbooks are shaped by the technical and economic constraints of their production era, and they participate in a broader economic system that affects their design and material realisation. As historical research sources, textbooks do not exist in isolation; rather, they are supplemented by regulations, programmes, school instructions, public debates, professional journals, and other materials such as notebooks and posters, offering a more comprehensive view of curricular frameworks[28]. Given their significance in teaching

[22] Ibidem.

[23] G. McCulloch, *Documentary research: in education, history and the social sciences*, Routledge Falmer, London 2004, p. 25.

[24] L. Bardin, *Análise de Conteúdo*, Edições 70, Lisboa 1977, p. 9.

[25] D. Julia, *A cultura escolar como objeto histórico*, transl. by G. de Sousa, "Revista Brasileira de História da Educação" 2001, vol. 1, p. 34.

[26] A. Choppin, *O manual escolar: uma falsa evidência histórica*, transl. by M.H.C. Bastos, "História da Educação" 2009, vol. 13, no. 27, p. 27.

[27] Ibidem, p. 65.

[28] M.C. Almeida, J.M. Matos, A.S. Rodrigues, *Inovação curricular em livros de texto de matemática*, [in:] *Memorias del VI Congreso Iberoamericano de Historia de la Educación*

and learning contexts, textbooks play a crucial role in shaping curricular visions through their content and structure[29].

A content analysis method[30] was employed to ensure rigour, focusing on both the official programme and the mathematics textbook. Content analysis is a preferred technique for examining curricula, as it enables replicable and valid inferences regarding the representation and communication of content. This methodological framework is particularly suited to assessing the interplay between 'prescribed' curricula (official programmes) and 'presented' curricula (textbooks and supplementary materials). The analysis is supported by two complementary theoretical frameworks: Lisa Okeeffe's[31] model, which offers a structured method for examining textbooks, and Jenneke Krüger's[32] categories, which investigate historical coherence, visualisation, and contextual application. Together, these frameworks facilitate a multifaceted evaluation of curriculum materials, ensuring a balanced approach to both structural and contextual aspects. Rodrigues[33] previously used these combined frameworks to analyse a chapter on integers in an industrial school's textbooks, and Almeida and Rodrigues[34] used them to compare the approach to teaching integers in technical and general secondary school settings.

Okeeffe's[35] framework encompasses four primary dimensions of analysis: content, structure, expectation, and language.

Content: Examines aspects such as value formation, motivational elements, accessibility, illustrations, and study guides.

Structure: Focuses on the organisation of the textbook, the distribution of content, interconnections between subjects, and the overall process of information presentation.

Matemática, ed. by E. Gutiérrez, J.L. Prieto, Asociación Aprender en Red, Maracaibo 2022, p. 817; C.S. Lopes de Sousa, *O Ensino de Matemática no CPES. Análise de Manuais*, master thesis, Universidade Nova de Lisboa 2012, p. 3, http://hdl.handle.net/10362/9071 [accessed 1.01.2026].

[29] L. Okeeffe, *A Framework to Textbook Analysis*, "International Review of Contemporary Learning Research" 2013, vol. 2, no. 1, p. 2.

[30] L. Cohen, L. Manion, K. Morrison, *Research methods in education*, 6th ed., Routledge, London 2007, p. 475.

[31] L. Okeeffe, *A Framework to Textbook Analysis*, p. 11.

[32] J. Krüger, *Lessons from the early seventeenth century for mathematics curriculum design*, "BSHM Bulletin" 2010, vol. 25, p. 153–156.

[33] A.S. Rodrigues, *Los números enteros durante la matemática moderna en las escuelas técnicas de Portugal*, "PNA. Revista em Didática de la Matemática" 2025, vol. 19, no. 5, p. 419–437.

[34] M.C. Almeida, A.S. Rodrigues, *Distinct approaches to integers in technical schools and in liceus, during modern mathematics in Portugal*, [in:] *Current Trends, Practices, and Ideas in the History and Epistemology in Mathematics Education: Reflections based on ESU-9*, *Salerno, July 2022*, ed. by E. Barbin, M.N. Fried, M. Menghini, F.S. Tortoriello, Springer, Cham 2025, p. 513–526.

[35] L. Okeeffe, *A Framework to Textbook Analysis*, p. 6–10.

Expectation: Considers implicit expectations in textbooks that influence student approaches to learning. For instance, an emphasis on repetition and practice may limit the use of problem-solving skills.

Language: Analyses the type of discourse (e.g., narrative or descriptive), sentence connectors, semantic structures, and specific elements such as notational and graphic signs.

Krüger[36] emphasises three categories relevant to historical and contextual analyses:

Coherence: Assesses the consistent application of previously taught material to reinforce skills and facilitate their use in complex or diverse contexts.

Visualisation: Evaluates the role of illustrations and visual elements, including their artistic quality and effectiveness in enhancing content comprehension.

Context: Considers the integration of practical or professional situations, connecting mathematical content to real-world applications relevant to students' environments.

By integrating Okeeffe's systematic descriptions with Krüger's contextual focus, the study conducts a meta-analysis of the curriculum. The textbook organisation, its suitability for the prescribed curriculum, and its alignment with instructional goals are examined through these perspectives. Furthermore, this dual framework facilitates a comprehensive understanding of the pedagogical, historical, and cultural dimensions embedded in the curriculum materials. The analysis also considers the socio-political and economic context of the Preparatory Cycle of Technical Education, ensuring a nuanced interpretation of the findings. This dual focus on 'prescribed' and 'presented' curricula emphasises the evolution of mathematical education in technical schools and its implications for teaching practices during this transformative period.

Analysis of the textbook

In countries with centralised educational systems, such as Portugal, in the decades under study, the prescribed curriculum (the programme) is determined by government entities. Once established, the programmes are presented to teachers through curricular materials, mainly through textbooks. The Ministry of Education approves a single textbook.

The mathematics programmes remained stable throughout the Preparatory Cycle of Technical Education reform. In this article, we analyse the textbook *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, published in 1954, for the second year[37], written in Portuguese.

[36] J. Krüger, *Lessons from the early seventeenth century for mathematics curriculum design*, p. 153–156.

[37] A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*.

Structurally, it is a hardcover book with 299 pages printed in black-and-white, featuring figures that illustrate the context or facilitate mathematical explanations through geometric visualisation. For example, in Fig. 2, we see an illustration that connects geometric forms to real-life objects.

Fig. 2. Illustration of geometric forms.
Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, p. 16.

After a chapter with revisions of concepts taught in the 1st year, the textbook is organised into six chapters: i) Measurements and counts; ii) Arithmetic operations; iii) Fractional numbers; iv) Intuitive geometry – review and generalisation of concepts; v) Intuitive geometry; and vi) Practical arithmetic. The article focuses on the analysis of the geometry chapters. Chapters I-IV are referred to as revision chapters for the 1st year, and the last two chapters are for the 2nd year.

At the beginning of the textbook, in a section entitled 'Presentation', the author notes the presence of mathematics in students' and their parents' lives. In this section, he introduces four characters from his book: Mr. Silva (father), D. Helena (mother), their son José, and their daughter Maria Amélia. Another character, Captain Silva, José's and Maria Amélia's uncle, also appears to clarify more difficult and curious questions. According to the author, real-life problems are contextualised through the Silva family to facilitate understanding of the concepts.

Although the textbook includes an index at the end that lists the mathematical content – for instance, the first section of the first chapter, titled 'Length Measurements', indicates that the topic can be found on pages 9, 92, and 103 – the titles of the sections differ significantly. On p. 9, the title is 'Measuring and Counting, Bases of Calculation. Numbering, Concrete Numbers and Abstract Numbers. Mr. Silva Takes Measurements in the Backyard!'; on p. 92, it reads 'Revisions of 1st Year. Measurements. Reasons for New Interests'; and on p. 103 it states 'Measurement Technique. Esteem and Measure'.

Regarding content, we will focus on chapters: i) Measurements and counts; iv) Intuitive geometry – review and generalisation of concepts; and v) Intuitive geometry. The introduction to geometric concepts is based on a progression from concrete to abstract approaches. First, the author introduces planimetric objects, and at the end of the discussion, stereometric objects are introduced, including areas and volumes of solids.

Chapter I, 'Measurements and counts', is organised into seven parts: a) Length measurements, b) Surface measurements, c) Volume and capacity measurements, d) Relationships between weight measurements and volume and capacity measurements, e) Criticism of some incorrect results, f) Graphical representation of counts and measurements, and g) A simple summary of the study conducted. This chapter spans up to p. 279, where the topic 'Graphic representation of measurements and counts' is explored – even though the second chapter starts on p. 153.

One example in this chapter, featuring beautiful illustrations that involve a member of the Silva family, introduces the concepts of parallel and perpendicular lines, illustrated through the architectural evolution of houses (Fig. 3).

> – Wherever man appears, Captain Silva observed to his nephews – *parallel* and *perpendicular* lines appear, or *parallelism* and *perpendicularism*, [...] And, as civilisation increases and man builds more and more, the parallel and perpendicular lines defined around him also increase. See, through the engravings, how the construction of the house evolved from the most ancient times until today.[38]

Fig. 3. Illustration of construction evolution.

Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, p. 42–43.

Illustrations are also used to simplify the learning of mathematical concepts. Refer to the following example in Fig. 4, after exploring the concepts of direction and sense associated with a straight line

> Let's look at the figure [Fig. 4 – A.S.R.]:
>
> The feet of the landmark and the tree trunk (considered as points) define a straight line. On this straight line (or in this direction), we can move in two ways:
>
> From the landmark to the tree, which is indicated by arrow A;
>
> In the *opposite direction*, from the tree to the landmark, which we indicate by arrow B.[39]

[38] Ibidem, p. 42.

[39] Ibidem, p. 22.

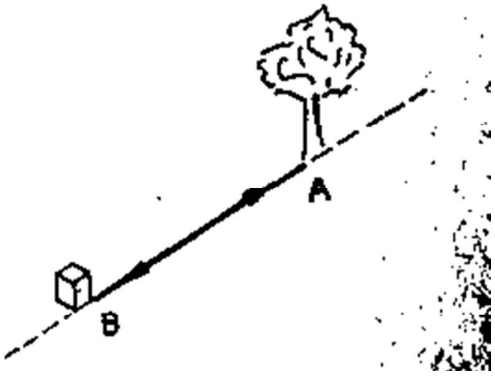

Fig. 4. Illustration to help students understand mathematics topics.
Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, p. 22.

After the mathematical concepts are taught, the author is almost always concerned with relating them to real-life problems. Observe the example in Fig. 5, which connects the concepts of direction and orientation associated with a straight line to traffic signs and the movement of a car.

Exercises
1. a) What adjective can be used to classify the two orientations we have learned, relative to the other?
b) The car will enter this street, as the figure indicates. However, a "no-entry" traffic sign is posted at the entrance. The driver risks being fined. Would the driver take that risk if the sign indicated only a prohibited direction?[40]

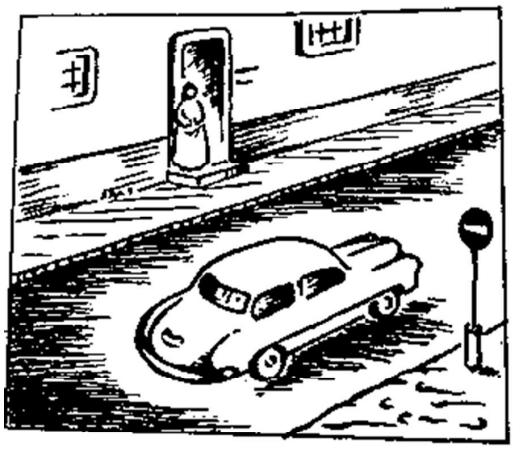

Fig. 5. Illustration of question 1b).
Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, p. 22.

[40] Ibidem.

The author provides all the real-world problems in the book. In the textbook, there are no suggestions that encourage teachers to take students outside the classroom to solve real-world problems, nor are there methodological suggestions that take into account students' previous experience.

In turn, Chapter IV, 'Intuitive Geometry – review and Generalisation of Concepts', consists of four parts: a) Straight-line angles; b) Circumference and angle at the centre; c) Relative position of two straight lines in a plane; and d) Internal angle and external angle of a triangle and any convex polygon. Please note that these chapters (I and IV) are distributed across overlapping page ranges. The contents of Chapter I appear on pages 1 to 153, and Chapter IV on pages 15, 26, 29, 38 to 48, 50, 82, and 83.

The textbook also proposes experimental activities, such as the one suggested to students to verify that 'the sum of the external angles of a convex polygon is equal to 360°'[41].

> Experience:
> Let us draw an irregular [convex] polygon on paper. Let's extend the sides, always in the same direction (for example, straight), as the 1st figure shows.
> With a pocket knife, let's cut out the external angles that were thus obtained.
> Let's add these external angles, as shown in the 2nd figure. [...]
> We can repeat the previous experiment with any convex polygon. We always get the same result.[42]

The experimental activity is accompanied by Fig. 6.

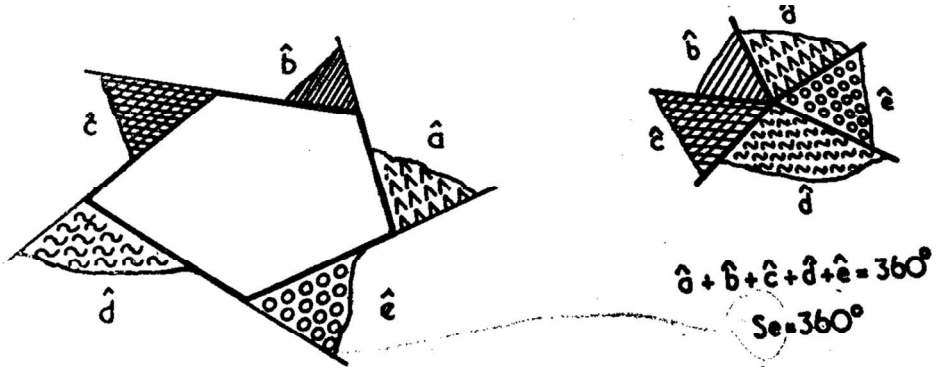

Fig. 6. The sum of the external angles of a convex polygon.
Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, p. 82.

This type of activity is frequent, reflecting the author's strict interpretation of the programme and his emphasis on experimental activities. All the tasks pro-

[41] Ibidem, p. 82.
[42] Ibidem.

posed are very simple. The requested geometric constructions allow an approach from the concrete to the abstract. The manual presents its contents using concrete examples. For example, to deduce the formula for calculating the area of a parallelogram, students are asked to construct an oblique parallelogram on a sheet of paper and are told that, with two cuts, it will be possible to transform the parallelogram into an equivalent rectangle, and only after the experimental activity is it concluded that the area of a parallelogram is equal to the product of its base and height[43].

Chapter V, 'Intuitive Geometry', referring to the 2nd year, is divided into 7 parts as follows: a) Triangles; b) Comparison of perpendicular and oblique segments; c) Quadrilaterals; d) Circumference; e) Circumference inscribed and circumscribed about a triangle. Other inscribed polygons. Circumference perimeter; f) Equivalent figures. Areas; and g) Areas of solid surfaces. Volumes. Note that the topics in this chapter are found between pages 29 and 130, complementing chapters I and IV.

Reviews of concepts stand out throughout the textbook, such as the definition of circumference and some of its elements, as we can see in the following excerpt:

> The circumference is the curved, flat, closed line with all points equidistant from a central point, known as the centre.
>
> An arc of the circumference is a section defined by two extreme points.[44]

The textbook has reminders and observations to guide the student's study, such as '$3cm^2$ is not the same as $(3cm)^2$. In fact, $3cm^2$ represents 3 times the area of $1cm^2$'[45].

There are also worked-out problems. Let's examine the configuration of Mr Silva's backyard, which offers two possible solutions to address the issue.

> Mr. Silva changes his backyard configuration
>
> But, after all, Mr. Silva hasn't begun cultivating the garden yet [Fig. 7 – A.S.R.]! He hasn't even started digging it, nor has he bought a net to make the fence. It's just that D. Helena and her daughter don't like the backyard's layout.
>
> They wanted it to have a regular shape, which could be achieved only by exchanging land with the neighbours.[46]

[43] Ibidem, p. 115.
[44] Ibidem, p. 77.
[45] Ibidem, p. 114.
[46] Ibidem, p. 158.

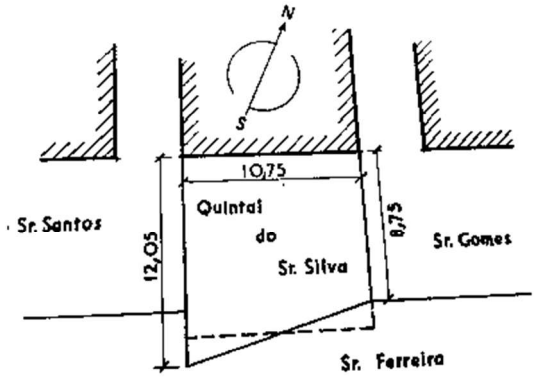

Fig. 7. Silva's family backyard.

Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, p. 158.

The first solution, which seemed to be the most practical, was to exchange land with Mr. Ferreira. Note that the lower triangle of Mr. Silva's land has an area equivalent to the upper triangle of Mr. Ferreira's land, so a simple exchange would suffice. 'The yard would take the form of a rectangle with a base of 10.75 m and a height of 10.40 m'[47]. The value of 10.40 comes from calculating the median of 12.05 and 8.75.

A second solution would also be to negotiate with Mr. Gomes and Mr. Santos, purchasing the land marked by III in the figure (Fig. 8). At the same time, the triangle marked II in the figure would be sold to Mr. Ferreira.

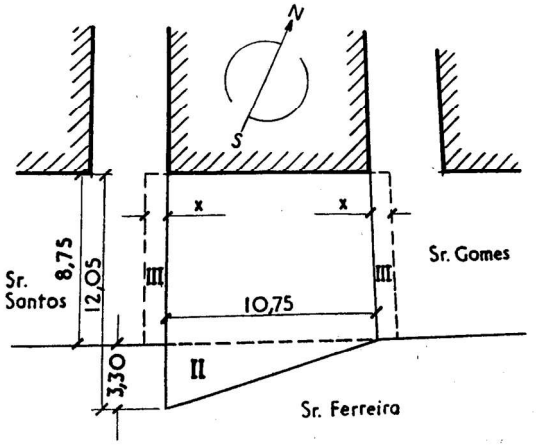

Fig. 8. Scheme to solve the backyard problem.

Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, p. 159.

[47] Ibidem, p. 159.

Considering that the land would have the same price per square meter, Maria Amélia calculated the land area, concluding that the area of triangle II equals 17.74 square meters. Given this value and in order for the areas to be, she determined the height of the rectangle and realised that the height of rectangle II would be 1 meter.

The textbook's language is narrative and aimed at students. It is written in the third person. Exercises, notes, and intriguing facts are included to motivate learners. Illustrations help students grasp mathematical concepts and connect the discipline to real-life problems. The approach to the theme of geometry often contextualises situations from the daily life of the Silva family and visualises them in relation to the figures being examined. Highlights include experimental activities designed for students and exercises linked to real-life problems that appear throughout the textbook whenever a new mathematical concept is introduced, along with observations and engaging facts, as demonstrated in the following excerpt regarding the effect of diameter variation on the perimeter.

Curiosity:

> Let's imagine that an aerial wire goes around the Earth completely, along a circle 50 cm away from the equator, and concentric with it.
>
> What is the difference between the perimeter of the wire's circumference and that of the equator?
> Do you think it's huge? It is not. Now let's see:
>
> The increase in diameter is 2×50 cm = 100 cm.
>
> Therefore, the increase in the perimeter is: 3.14×100 cm = 314 cm = 3.14 m.
>
> Just over 3 meters![48]

Analysing students' *expectations* about the textbook reveals that it is written in a way that allows students to follow and understand the content. Practice is systematically encouraged through exercises. The Silva family highlights many practical everyday situations, and the students become familiar with them. There are problems and exercises in the textbook for the students to practice. Some of those exercises include some constructions or drawings, such as the one that follows.

> Draw two equal triangles, as shown in the figure [Fig. 9 – A.S.R.]. In the 1st triangle, draw a height. This thus divides the triangle into two right-angled triangles (represented in dashed lines and white). Join these two right-angled triangles to the 2nd triangle, as indicated by the arrows. What figure do you obtain? How tall is it? What is the base of this figure?[49]

[48] Ibidem, p. 110.

[49] Ibidem, p. 119.

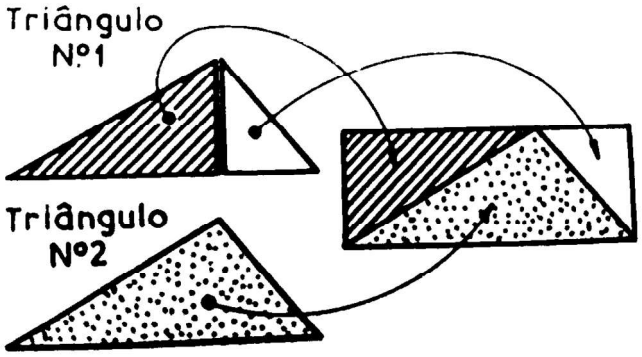

Fig. 9. Two triangles equivalent to one rectangle.
Source: A.O.S. Heitor, *Matemática: ensino técnico profissional: segundo ano do ciclo preparatório*, p. 119.

Throughout this section, examples were selected to illustrate the textbook's versatility, ranging from illustrative images related to real-life contexts to supporting illustrations for mathematical content, curiosities, exercises, experimental activities, and notes that remind students of common errors. This choice was made to facilitate a more comprehensive analysis of the school textbook, highlighting the various approaches the author employed to engage readers.

Final considerations

The theoretical framework proposed by Krüger (2017) allows us to carry out a meta-analysis of the textbook and relate it to the prescribed curriculum, while acknowledging that the author was an experienced teacher who, during this reform, served as a methodology teacher at the *Marquês de Pombal* Industrial School in Lisbon.

The textbook is coherent horizontally and vertically and has the potential to reinforce skills through practice. It follows the programme's prescribed sequence, yet its organisation reads more like a storybook than a schoolbook. The school textbook follows the prescribed curriculum, adopting an innovative approach that tells a story and presents the content in a narrative that reflects the author's perspective, perhaps due to his professional experience as a teacher and methodologist in technical education. The prescribed programme advocates experimental teaching. In the school textbook, geometric constructions enable students to make conjectures and apply their learning in practice. However, there are no suggestions based on students' prior knowledge or encouraging field classes.

The connection between students' future professional practice and the completion of technical education is unclear. The problematic situations that appear

throughout the book are largely intended to address everyday family problems, particularly those involving the Silva family.

Despite being printed in full colour, the textbook is pleasant to look at. Several illustrations enhance students' understanding of the content, both in terms of mathematical concepts and in visualising the problem's context. The textbook includes exercises that apply mathematics to various contexts, connecting it to students' reality. Additionally, it features notes and curiosities that foster motivation. The Silva family adds practicality to the school textbook, whose organisation differs significantly from the usual: instead of being arranged by year or theme, it is organised around mathematical topics, often contextualised through real-life problems. The Silva family serves as an example of how mathematics can be applied to various contexts in their daily lives. The author concludes the book with a final note, where he says, 'Always Mathematics? Yes, mathematics, which is always at the service of Life, Science, and calculated adventure, today allows us to fly over the poles'[50].

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Alexandra Sofia Rodrigues holds a PhD in Mathematics Didactics awarded by Beira Interior University. She is an invited professor at the Faculty of Science and Technology at NOVA University of Lisbon, where she teaches pre-service mathematics teachers. Her research interests focus on the history of mathematics education and the teaching and learning of mathematics in professional education.
e-mail: alexsofiarod@gmail.com