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**SEFER OVER LA-SOHER – A COMMERCIAL ARITHMETIC BOOK  
IN HEBREW PRINTED IN VENICE AT THE BEGINNING  
OF THE 17TH CENTURY**

**Summary:** In 1627, an arithmetic book in Hebrew was published in Venice. The name of the book was *Sefer Over la-Soher* [Money current among merchants], and it was printed by one of the students of the mathematician-astronomer Menahem Zion (Emanuel) Port (Porto) Kohen Rappa. Printing books in Hebrew in northern Italy, and particularly in Venice, was not unusual at that time. Beginning in the mid-15th c., many Hebrew printing houses were established in Italy in various places, and even Christian-owned printing houses printed books in Hebrew. However, the number of Hebrew books dealing with mathematics was minimal (a few percent).

Menachem Zion's book is the first arithmetic book printed in Italy and the second overall. It was preceded only by Eliyahu Mizrahi's books, which were printed in Constantinople and Basel at the beginning of the 16th c.

Currently, this is the only known arithmetic book printed in northern Italy during the 16th and 17th c., making it a rare piece of evidence of the Jewish community's interest in this subject in the region. Consequently, research on this topic is essential.

This paper will present the author's goals for writing the book, its mathematical content, its sources, and comparisons with other texts on the same subject in Hebrew and other languages. Moreover, it will attempt to answer why Hebrew arithmetic books were so rare in this period while describing the cultural environment of the Jews of Northern Italy.

**Keywords:** history of mathematics, commercial math books, 17th century, Italy, mathematics education, Hebrew mathematical book

## Introduction<sup>1</sup>

From the 12th c. until the beginning of the 16th c., comprehensive Hebrew literature on mathematical subjects flourished. The beginnings were in the communities of Spain and southern France, and later in Italy and Byzantium. The mathematical texts were manuscripts (translations, commentaries, and original works) that dealt with various subjects: arithmetic, algebra, geometry (Euclidean and spherical), number theory, and combinatorics<sup>2</sup>.

The arithmetic manuscripts known today (several dozen) present the general principles for performing arithmetic operations. They are structured in various models, most of which are based on models from Arabic arithmetic books. Some of these texts include extensive explanations and proofs of the proposed algorithms, while others are content with presenting the various calculation methods without further explanation<sup>3</sup>.

The first printed mathematical book in Hebrew was Elijah Mizrahi's *The Book of Number*, published in Constantinople in 1534. It is an example of a text that thoroughly explains and proves all theorems and algorithms. It is possible that Mizrahi's book also marked the end of an era. After him, new mathematical books in Hebrew were rarely published. Menachem Zion's book, the focus of this article, is probably the second printed book to discuss arithmetic in Hebrew, appearing nearly a century after the first. Research on this book can deepen the understanding of Hebrew mathematical texts from the 16th c. onward, as Levy states:

In particular, the development of Hebrew mathematics in Italy in the fifteenth and sixteenth centuries, whose sources are no longer merely Arabic but also Latin and vernacular, is still poorly appreciated. Moreover, the effects of the cultural context and of the evolution of the Jewish communities on the works of these mathematicians cannot be evaluated until those texts have been properly edited and analyzed.<sup>4</sup>

## Research objectives

Although teaching arithmetic during this period was widely studied<sup>5</sup>, references to Jewish communities are absent. This study tries to enrich the research

<sup>1</sup> I would like to thank Alfred Holl, who first introduced me to the book *Sefer Over la-Soher*.

<sup>2</sup> T. Lévy, *The Hebrew Mathematics Culture (Twelfth–Sixteenth Centuries)*, [in:] *Science in Medieval Jewish Cultures*, ed. by G. Freudenthal, Cambridge University Press, Cambridge 2012, p. 155–171.

<sup>3</sup> N. Aradi, *The Arithmetic of the Jews in the Middle Ages: Introduction and Selected Samples*, PhD thesis, Hebrew University of Jerusalem 2015, p. 17–26.

<sup>4</sup> T. Lévy, *The Hebrew Mathematics Culture (Twelfth–Sixteenth Centuries)*, p. 160.

<sup>5</sup> K. Bjarnadóttir, *History of Teaching Arithmetic*, [in:] *Handbook on the History of Mathematics Education*, ed. by A. Karp, G. Schubring, Springer, New York 2014, p. 431–457.

by presenting an arithmetic book in Hebrew. In the following, I will try not only to characterize Menachem Zion's book of arithmetic but also to give a more general picture of the nature of the general studies, and in particular the studies of mathematics, in the Jewish community of Venice at the beginning of the 17th c.:

- What are the characteristics of the book, and what is its content compared to other mathematical works in Hebrew and other languages at the beginning of the 17th c.?
- How do we explain the small number of printed books on arithmetic after 1534, the publication date of Elijah Mizrahi's book?
- For whom was the book written?

### **Learning mathematics and arithmetic in the Jewish communities in Northern Italy at the beginning of the 17th c.**

In the 16th c., Jewish communities existed in several cities in Northern Italy, such as Venice, Padua, Mantua, Cremona, etc. These communities were made up of Italian Jews and also refugees from the expulsion from Spain and Portugal (Sephardic Jews<sup>6</sup>, and Ex-converts<sup>7</sup>) or from northern countries in central Europe (Ashkenazi Jews<sup>8</sup>).

These communities were influenced by the Renaissance cultural upheaval, during which notable personalities such as the scholar, rabbi, and poet Leon Modena grew up.

Education in these communities primarily took place at home, with fathers hiring teachers for their children. The first phase was primarily intended to impart basic knowledge of Hebrew, enabling the boys to pray and read religious texts. However, the children later learned to read and write in Italian, as well as other subjects, including Latin, logic, music, and more. At an older age, some boys studied in a yeshiva (religious high school) to deepen their knowledge of religious issues, and others went to Christian universities (like Padua

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<sup>6</sup> Sephardic Jews are descendants of Jewish communities that lived in Spain and Portugal until their expulsion in the late 15th c. Afterwards, they settled in regions like North Africa, the Ottoman Empire, and parts of Europe. They developed distinct religious customs, spoke Ladino (a Judeo-Spanish language), and have unique cultural traditions.

<sup>7</sup> Jews who converted to Christianity to escape persecution but continued to practice Judaism secretly.

<sup>8</sup> Ashkenazi Jews are descendants of Jewish communities that settled in Central and Eastern Europe, particularly in areas such as Germany, Poland, and Russia, from the early Middle Ages onward. They developed distinct religious customs and cultural traditions and spoke Yiddish – a language that combines elements of German, Hebrew, and Slavic.

University)<sup>9</sup>, where they mainly studied medicine. With a few exceptions, the girls did not learn to read and write.

Mathematics, in general, and arithmetic, in particular, were not very important subjects. We can assume that small children learned basic arithmetic, but not more, until later in their education. However, certain mathematical knowledge was necessary for both medical studies and those engaged in commerce. Physicians sometimes used astrology to determine the prognosis of their patients. For that, they had to possess a considerable amount of astronomical knowledge, which in turn required a solid understanding of mathematics. Where could they get this mathematical knowledge?

Those who attended universities could take courses in mathematics, even if their main subject was different. The medicine program included some mathematics (even compulsory in some universities, such as the University of Bologna)<sup>10</sup>. We will note that Galileo taught mathematics at the University of Padua during this period. We have at least one testimony of a Jewish student who mentions Galileo as his teacher: the rabbi, physician, mathematician, and astronomer Joseph Solomon Delmedigo (1591–1655), known in Hebrew as ‘Yashar mi-Candia’, studied medicine at the University of Padua<sup>11</sup>.

It was also possible to study mathematics at the Abbacus schools. These institutions were primarily designed for clerks, merchants, and others who needed accounting knowledge for their jobs, so they primarily taught arithmetic and practical geometry. Did Jews study in these schools? We have no proof of this, but we can assume that at least some did. But there was also a third possibility: learning mathematics and arithmetic by reading books. During this period, many arithmetic books were printed in Italian.

Thus, Italian-speaking Jews had a variety of options for learning arithmetic. In contrast, those who did not know Italian well enough or who did not want to study in an Italian-speaking Christian environment faced a problem. We can hypothesize that this was the target population for Menachem Zion’s book. It is also possible that the book could have been used as a teacher’s aid for groups of Jewish students who wanted to learn in Hebrew, as abbasus texts were manuals for training teachers and merchants<sup>12</sup>.

<sup>9</sup> There are records of eighty Jews who graduated in Padua between 1517 and 1619 (C. Roth, *The Jews in the Renaissance*, Jewish Publication Society, Philadelphia 1959, p. 38).

<sup>10</sup> J. Høytrup, *Mathematics Education in the European Middle Ages*, [in:] *Handbook on the History of Mathematics Education*, ed. by A. Karp, G. Schubring, Springer, New York, 2014, p. 109–124.

<sup>11</sup> J. Adler, *Joseph Solomon Delmedigo: Student of Galileo, Teacher of Spinoza*, “Intellectual History Review” 2013, vol. 23, no. 1, p. 141–157.

<sup>12</sup> A. Petrocchi, *Mercantile arithmetic in Renaissance Italy: A translation and study of selected passages from a vernacular Abbaco work*, “Cerae” 2016, vol. 3, p. 2; K. Bjarnadóttir, *History of Teaching Arithmetic*, p. 435.

### Hebrew books in Northern Italy (16th–17th c.)

Since the first half of the 16th c., Venice has been the main center of Hebrew printed book production. Jewish or Christian-owned printing houses produced many books, most of which were on religious subjects. However, it is essential to emphasize that the appearance of the printed book did not eliminate the use and production of manuscripts<sup>13</sup>. So, if we want to know what mathematical books were available to the Jewish community in Northern Italy during this period, we have to check:

- What Hebrew mathematics books were printed?
- What mathematics books or manuscripts in Hebrew or other languages were owned by the Jews?

Of all the books in Hebrew with scientific content that were printed between the 16th and 18th c., only 3% had mathematical subjects, while in the 16th–17th c., fewer than ten books were printed<sup>14</sup>.

Baruchson's research<sup>15</sup> provides detailed information about the mathematics books and manuscripts owned by Jews in Mantua at the end of the 16th c. Using statistical considerations, Baruchson demonstrates that we can assume the contents of other Jewish libraries in Northern Italy at the end of the 16th century were very similar to those in Mantua.

The number of Hebrew mathematics books possessed by the Jews in Mantua at the end of the 16th c. was 14, four of which were printed copies of Elijah Mizrahi's *Sefer ha-Mispar le-ha-Hakam ha-Elohi Eliya ha-Mizrahi* [The Book of Number]<sup>16</sup>. Among the manuscripts are five whose subject is arithmetic (including Ibn Ezra's *The Book of Number*).

The Jews owned a few books in foreign languages, with about 5% being science books. The Mantua Jewish libraries owned 33 mathematics books in Italian, including 18 arithmetic books, six dealing with measurements, weights, accounting, and currency exchange, and the rest being general books (comprising arithmetic and applied geometry together)<sup>17</sup>. Table 1 summarizes these data.

<sup>13</sup> E.G.L. Schrijver, *Jewish Book Culture Since the Invention of Printing (1469 – c. 1815)*, [in:] *The Cambridge History of Judaism*, ed. by J. Karp, A. Sutcliffe, Cambridge University Press, Cambridge 2017, p. 295–300.

<sup>14</sup> E. Lapon-Kandelshein, S. Baruchson-Arbib, *Hebrew Scientific Publications from the 15th to the 18th Centuries: Social and Cultural Aspects*, "La Bibliofilia" 2002, vol. 104, no. 2, p. 167–188.

<sup>15</sup> Baruchson's book (1993) describes a study based on the lists of books and manuscripts owned by the Jews of Mantua that were handed over to the censors on behalf of the church in 1595.

<sup>16</sup> E. Mizrahi, *Sefer ha-Mispar le-ha-Hakam ha-Elohi Eliya ha-Mizrahi*, Soncino, Constantinople 1534.

<sup>17</sup> S. Baruchson, *Books and Readers: The reading Interests of Italian Jews at the Close of the Renaissance*, Bar-Ilan University Press, Ramat-Gan 1993, p. 91–93, 167, 187.

Table 1. Hebrew books and manuscripts owned by Jews of Mantua in 1595.

	total	Math	Arithmetic	Some of the titles
Hebrew books/ manuscripts	20,381 96.4%	14	9	<i>The Book of Number</i> – E. Mizrahi (4 copies) <i>The Book of Number</i> – Ibn Ezra
Other languages	761 3.6%	33	18	<i>Abacho</i> (15 copies) <i>Trattato di numeri et misure</i> – N. Tartaglia <i>Summa de aritmetica, geometria</i> – Juan de Ortega
	21,142	47 (0.2%)	27 (0.1%)	

Source: S. Baruchson, *Books and Readers: The reading Interests of Italian Jews at the Close of the Renaissance*, Bar-Ilan University Press, Ramat-Gan 1993, p. 91–93, 167, 187.

### Menahem Zion Cohen Port Rapha (Rappa) – the author

Menahem Zion Port Cohen, also known as Emanuel<sup>18</sup>, was a member of the well-known Jewish Rappaport (or Rappoport) family, originally from Lublin. They arrived in Italy, passing through Germany, and settled in the small village of Porto in the province of Mantua. The name Rapha (Rappa) has several explanations. Some believe it originates from the German word ‘Rappe’, which means ‘raven’<sup>19</sup>. Moreover, we can see a raven in Fig. 1<sup>20</sup>. Others believe Rapha is connected to the Hebrew word ‘roffe’ (רוּפֵּא), meaning physician. Anyway, after arriving in Italy, the family added to the name Rapha the name of the place where they settled, Porto, to distinguish itself from other branches of the Rappa family, so we later got the name Rappaport.

We know very little about Menachem Zion’s life. He was born in Trieste toward the end of the 16th c. and was the town’s chief rabbi. He also served as the chief rabbi in Padua, where he died in 1660. In addition to the arithmetic book discussed here, he wrote other books on various subjects in Hebrew and Italian:

- *Porto Astronomico* (Padua 1636).
- *Breve Istituzione della Geographia* (Padua 1636).
- *Diplomologia, Qua Duo Scripturæ Miracula de Regressu Solis Tempore Hiskiaë et Ejus Immobilitate Tempore Josuæ Declarantur* (Padua 1643).

<sup>18</sup> I use the name Emanuel (with only one ‘m’), which is a direct transliteration from Hebrew, as other scholars also do (see M.J. Heller, *Further Essays on the Making of the Early Hebrew Book*, Brill, Leiden, Boston 2024 (Brill’s Series in Jewish Studies, vol. 78)).

<sup>19</sup> M.J. Heller, *Further Essays on the Making of the Early Hebrew Book*, p. 83.

<sup>20</sup> In this coat of arms, we can read the name of ‘Avraham Menachem ben Yaakov HaCohen’ (the upper part) and ‘Rafa [may his memory be blessed] from Porto’ (at the bottom). The meaning of the name Cohen is ‘priest’. The hands symbolize the Blessing of priests (‘birkat kohanim’). During the blessing, the hands of the Kohanim are spread out over the congregation.



Fig. 1. Coat of arms of the Rappaport family, probably from the 1600s, Italy.

Source: Wikimedia Commons.



Fig. 2. The cover page of the book *Over la-Soher*.

Source: <https://www.hebrewbooks.org/11814> [accessed 1.01.2026].

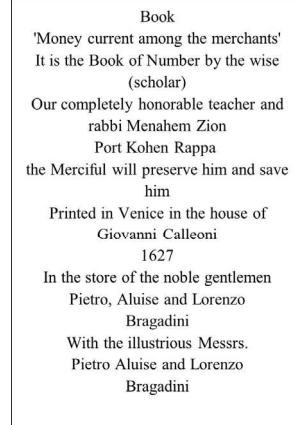


Fig. 3. English translation of the cover page of *Over la-Soher*.

The last book, originally written in Italian, was dedicated to Emperor Ferdinand III. Menahem Zion translated it into Hebrew, and Lorenzo Dalnaki of Transylvania translated it into Latin.

According to Gottheil and others<sup>21</sup>, Menachem Zion was highly regarded by personalities of his time, such as the mathematician and astronomer Andrea Argoli. Menachem Zion also maintained an active correspondence with the Protestant Hebraist Johannes Buxtorf (the younger).

### *Sefer Over la-Soher*

The book *Sefer Over la-Soher* was printed in Venice in 1627 in the printing house of the brothers Pietro, Aluise, and Lorenzo Bragadini, managed by Giovanni Calleoni, as the title page shows (Fig. 2, Fig. 3). Pietro, Aluise, and Lorenzo belonged to the well-known Bragadini family, which printed Hebrew books in Venice at the beginning of the 16th c.<sup>22</sup>

The title ‘*Sefer Over la-Soher*’ means that the book deals with currency trading between merchants and is taken from Abraham’s purchase of the Makhpelah from Ephron the Hittite:

<sup>21</sup> R. Gottheil, I. Broyde, I. I. Elbogen, *Emanuel Porto or Menahem Zion Porto Cohen*, <https://jewishencyclopedia.com/articles/12295-porto#anchor> [accessed 4.12.2024].

<sup>22</sup> Bragadini’s printing house was one of several that were active in printing Hebrew books in Venice in the 16–17th c., like Daniel Bomberg, Marco Antonio Giustiniani (Justinian), Giovanni (Jean) di Gara, and more (S. Baruchson, *Books and Readers: The reading Interests of Italian Jews at the Close of the Renaissance*, p. 29–31).

And Abraham hearkened unto Ephron, and Abraham weighed the silver for Ephron, which he had named in the audience of the sons of Heth: four hundred shekels of silver, “current money with the merchant”. (Genesis 23:16)

### *The introduction*

Before the introduction, there is a little poem in rhymes that encourages people to buy the book<sup>23</sup>:

Hasten to buy The Book of Number,  
 Look at it, good seeker!  
 He who listens to him will surely live  
 in all wisdom if it is not too late.  
 A wise man will listen; he will add a lesson.  
 Anyone who is fed up with him has terrible days  
 and snorts in pain.  
 The object will give its value,  
 It will give you tools.  
 Four hundred silver shekels  
 It is Money current among merchants.<sup>24</sup>

In the introduction, Menachem Zion discusses the significant benefits of learning the wisdom of numbers, particularly for merchants and others involved in financial transactions, as well as the negative consequences that can result from a lack of understanding of this wisdom, including the potential collapse of institutions. He also describes requests from his students for help on this subject. Therefore, he responded to his students' requests and wrote a book on the subject, choosing to write a concise and clear book so that it would be easy to understand and study. He mentions Rabbi Elijah Mizrahi's extensive and in-depth book. He speaks of it with appreciation, but he chooses the short route because of his target audience, merchants, traveling merchants, and 'donkey riders'. Therefore, his book does not include introductions or in-depth explanations; instead, he provides brief methods. He further praises his patron, R. Abraham ben Mordecai Ittiniger<sup>25</sup>.

In the second introduction, written by one of Menachem Zion's students, R. Gershom ben Kalonymous Hefez, who helped publish the book, there is an-

<sup>23</sup> This poem, like other excerpts from Menachem Zion's book that appear in the article, was translated by the author of this article.

<sup>24</sup> M.Z.K. Rappa, *Sefer Over la-Soher*, Pietro, Aluise and Lorenzo Bragadini, Venice 1627, p. 2, verso of the title page.

<sup>25</sup> This name is associated with Jewish names like Öttingen, Öttinger, or Ettinger, which are habitational names for Ashkenazi Jews from various locations in southern Germany, France, or Switzerland.

other encouragement to buy the book as an investment that will pay back the purchase due to its usefulness.

### **The mathematical content of *Sefer Over la-Soher***

The book introduces the symbols needed to write numbers using the positional decimal system. The author presents the first nine digits using the first nine Hebrew letters and the zero sign, and then the 10 Hindu-Arabic digits. Later, he will only use the Hebrew numerals.

Zero is called ‘round’, unlike earlier Hebrew texts that used different terms. However, like its predecessors (Ibn Ezra, Comtino, Mizrahi), zero is not considered a number but a placeholder.

The book contains 12 chapters (see Table 2), including explanations and examples of basic arithmetic operations with integers and fractions, an explanation of the rule of three<sup>26</sup>, word problems, problems involving money conversions, and examples illustrating the use of units of length and weight.

One or two algorithms are usually presented for each calculation operation, accompanied by several examples and methods for checking the calculation. The ‘check by nines’<sup>27</sup> method for checking a calculation’s result appears in several places.

Commercial calculations (currency exchange, measurements) already appear in the chapters on operations with fractions.

Before the word problems chapters, there is a brief explanation of The Rule of Three and its Italian name, ‘rigula dil tri’ (ריגולה דיל טרי), including a warning about using appropriate units.

Although the book was written to teach applied arithmetic, chapters 11–12, devoted to word problems, contain only 34 word problems. Some of the problems in this chapter also appear in Mizrahi, but there are problems of types that do not usually appear in Hebrew books, such as calculating interest rates. The selection of the problems was probably influenced by the commercial arithmetic books in Italian that were in use at the time. The main topics that appear in the word problems are:

- problems with monetary exchange and merchandise,
- partnership/company problems,
- calculations on barter and exchange of goods,
- calculations of simple and compound interest,
- problems with alloying/mixtures.

<sup>26</sup> The Rule of Three allows you to solve problems based on proportions where three values are known and the fourth is unknown.

<sup>27</sup> The ‘check by nines’ method is explained in detail later.

Table 2. The content of *Sefer Over la-Soher*.

Content	Chapter
Poem, author's introduction, student introduction, explanation about the number system, and presentation of the book's chapters	Introduction
Addition of integers: algorithm and several checking methods	Chapter 1
Subtraction of integers	Chapter 2
Multiplication of integers	Chapter 3
Division of integers: division with/without remainder, division by ten	Chapter 4
Fractions: definitions, notations, and reduction of fractions	Chapter 5
Addition of fractions: algorithms, applications in currencies and weights	Chapter 6
Subtraction of fractions: algorithms, applications	Chapter 7
Multiplication of fractions: algorithms, applications	Chapter 8
Division of fractions: algorithms, applications	Chapter 9
The Rule of Three: general explanation and example	Chapter 10
21 word problems about monetary exchange, partnership, simple, and compound interest	Chapter 11
13 Money problems (alloying)	Chapter 12

Not all problems have a detailed solution. Sometimes Menachem Zion presents the beginning of a solution, followed by a final answer; in other cases, he only comments that the problem is similar to a previous one. The methods for solving these problems usually involve the rule of three, and sometimes the rule of false position (*regula falsi*)<sup>28</sup>.

In formulating the problems, one sees adaptations of the stories to the reality of the target audience for whom the book is intended. For example, when the names of cities appear, if in Mizrahi's book the cities mentioned are mainly from the Ottoman Empire, such as Constantinople, Bursa, and Adrianople, in Menachem Zion, we find cities from Northern and Eastern Europe, such as Lublin, Poznan, Prague, and Krakow. Menachem Zion also uses names of cities from the Ottoman Empire, such as Hebron and Damascus, as well as from ancient Babylon, like Pumbedita and Nehardea. We can also observe the adaptation of coin types and the measurement units for weights and lengths. He explains the coins and weight units used in Italy and those in Ashkenaz (Germany and northern Europe)<sup>29</sup>.

### Some examples

Most of the topics in the book are presented similarly to previous Hebrew texts on the subject or in *Abacus* books. However, I will present several less common examples below.

<sup>28</sup> The false position method is a strategy of guessing a solution, checking the result, and adjusting the guess proportionally to reach the correct answer.

<sup>29</sup> M.Z.K. Rappa, *Sefer Over la-Soher*, p. 24–25.

### *Checking arithmetic operations*

Despite the book's small scope, the author sometimes presents multiple methods for verifying the calculations. For example, six methods are presented for adding integers, and he states that the last three are 'excellent'<sup>30</sup>.

The first method presented is the 'check by nines' method, already known in previous arithmetic texts. In this method, we check whether the sum of remainders (when dividing by 9) of the added numbers equals the remainder of their sum. The remainders of dividing by nine are calculated by adding the digits of the numbers as units. The author states in advance that this method is not reliable. For example, in the following incorrect calculation,  $125 + 356 = 418$  (the correct result is 481), the check of the remainders is correct ( $8 + 5 \equiv 4 \pmod{9}$ ), but the checked calculation is incorrect. On the other hand, if the check fails, we can be sure that the calculation is wrong.

He also presents methods in which the remainder of the division of the numbers is checked by some other number, such as 6 or 8, emphasizing that, in this case, it is impossible to calculate the remainder by adding the digits of the numbers as units.

The last checking method is particularly interesting. It is an extended check by (modulo)  $k$  that uses both the quotients and the remainders. In this method, the numbers to be added and their sum must be divided by  $k$ . The quotients from dividing by  $k$  and the remainders must be recorded. Then, we will verify whether the sum of the multiples of  $k$  and the sum of the remainders of the added numbers equal the result. If they do, the calculation is correct.

Below (Fig. 4) are the diagrams of the calculation itself (on the left), and next to them are the checking calculations for  $k = 8$ , written in Hindu-Arabic numerals (in the book, they are written in Hebrew letters):

For example, when we divide 2321 (the first number in the calculation) by eight, the quotient is 290, and the remainder is one. These results are written in the first row of the right-hand side diagram. The remainders in the right-hand side diagram are added, and if their sum exceeds  $k = 8$ , then the sum of the quotients is increased. For example, in the rightmost column,  $1 + 1 + 6 + 6 = 14 \equiv 6 \pmod{8}$ . Therefore, we write six and carry 1 for the sum of the quotients.

$$\begin{array}{r}
 2321 \\
 5625 \\
 1214 \\
 \hline
 6926 \\
 \hline
 16086
 \end{array}
 \qquad
 \begin{array}{r}
 290 : 1 \\
 703 : 1 \\
 151 : 6 \\
 865 : 6 \\
 \hline
 2010 : 6
 \end{array}$$

Fig. 4. Checking addition.

<sup>30</sup> Ibidem, p. 7.

We can assume that Menachem Zion knew ‘the improved check by nines’ for addition from Mizrahi’s book<sup>31</sup> and adapted it here for checking by eighths.

Other mathematicians, such as Planudes or Pacioli, also customarily wrote the remainders next to the numbers. As we saw, Menachem Zion added a special diagram showing the number of multiples and the remainders.

### *The arrangement of the calculations in integer division*

Unlike his predecessors, Menachem Zion presents a form of writing for long division that is economical and closely resembles the method used today. We can see this arrangement for dividing 1729 by 7 in Fig. 5. When a number smaller than the divisor appears in the last row, it will be the remainder.

This calculation arrangement is different from other arrangements in arithmetic books of the period.

7	:	1	7	2	9	:	2	4	7
			3	2					
				4	9				

Fig. 5. The arrangement of the calculations in integer division.

### *Reducing fractions*

At the beginning of the chapter on fractions, a method for reducing fractions is presented to arrive at a fraction in the lowest terms. Menachem Zion uses a variation of Euclid’s algorithm to calculate the greatest common divisor (gcd) of the numerator and denominator (without mentioning Euclid). In this version, repeated subtractions are used instead of repeated divisions to find the gcd of the numerator and denominator.

In the example of  $\frac{932}{1344}$  he uses Euclid’s algorithm to calculate  $\text{gcd}(1344, 932) = 4$  and so reduces the fraction to:  $\frac{233}{336}$ . Curiously, he is unsatisfied with this and re-runs Euclid’s algorithm to prove that  $\text{gcd}(233, 336) = 1$ .

Mizrahi did not use this method to simplify fractions. Italian books starting with Pacioli used this method, usually called a method for ‘abbreviating’<sup>32</sup>. Menachem Zion may have found the method in Italian arithmetic books.

Fig. 6 shows the calculation arrangement to get  $\text{gcd}(1344, 932) = 4$  and  $\text{gcd}(233, 336) = 1$ . (In the original Hebrew diagram, the left digit of the last line is missing. There should have been the letter *gimel*, which stands for 3.)

<sup>31</sup> E. Mizrahi, *Sefer ha-Mispar le-ha-Hakam ha-Elohi Eliya ha-Mizrahi*, p. 11; S. Segev, *The Book of the Number by Elijah Mizrahi: A Textbook from the 15th century*, PhD thesis, The Hebrew University of Jerusalem 2010, p. 44–45.

<sup>32</sup> D.E. Smith, *History of Mathematics*, vol. 2, Dover Publications, Inc., New York 1953, p. 222.

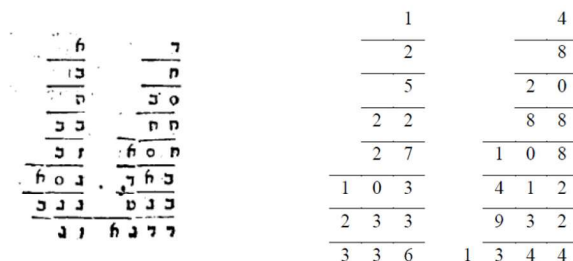


Fig. 6. Calculating  $\text{gcd}(1344, 932) = 4$  and  $\text{gcd}(233, 336) = 1$ . On the right-hand side, the calculation in Hindu-Arabic numerals.

Source: M.Z.K. Rappa, *Sefer Over la-Soher*, Pietro, Aluise, and Lorenzo Bragadini, Venice 1627, p. 21.

**Example of a calculation with money/currency**

Section 10 of Chapter 6, on adding simple fractions, provides an example of money calculations (compound denominate numbers). Menachem Zion explains the coins used in Italy: Literin, Simple (*pshutim*), and Pennies (*prutot*)<sup>33</sup>, as well as the conversion relationship between them: 1 Literin = 20 Simples, 1 Simple = 12 Pennies. When writing, he separates the types of coins with colons.

In Fig. 7 and Fig. 8, we can see an example<sup>34</sup>. We add here three amounts of money:  
 a. 12 Literins, 3 Simples, and 4 Pennies,  
 b. 24 Literins, 16 Simples, and 8 Pennies,  
 c. 19 Literins, 17 Simples and 6 Pennies.

We start by adding the Pennies column (the rightmost column):  $4 + 8 + 6 = 18$ , which is one Simple and 6 Pennies. We write 6 in the Pennies column, carry one Simple to the next column, continue to the Simple column, and so on.

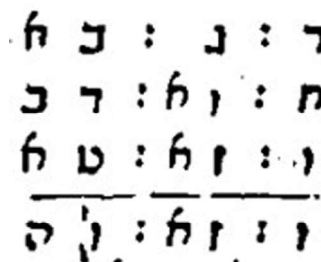


Fig. 7. Money calculation.

Source: M.Z.K. Rappa, *Sefer Over la-Soher*, p. 24.

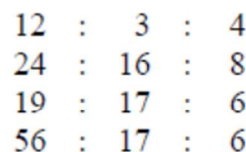


Fig. 8. Money calculation in Hindu-Arabic numerals.

<sup>33</sup> It is possible that these are the currencies: Lira, Soldi, and Denari that were used in his time in Northern Italy and whose conversion rate is the same (A. Petrocchi, *Mercantile arithmetic in Renaissance Italy: A translation and study of selected passages from a vernacular Abbaco work*, p. 16)

<sup>34</sup> M.Z.K. Rappa, *Sefer Over la-Soher*, p. 24.

Several examples of money calculation can be found in the other chapters that deal with fractions.

### *A partnership problem*

The second problem in Chapter 11 is a partnership problem:

Three partners agreed that the first would take half the profit, the second, a third, and the third a quarter. They earned 70 *zehuvim* [gold coins]. How much profit will each one make?<sup>35</sup>

The same problem appears in Mizrahi's book<sup>36</sup>. Only the profit is different, 12 coins (instead of 70). According to the given data, Menachem Zion explains that the second person will earn two-thirds of the first person's profit, and the third person half of the first person's profit. Then, he uses the 'regula falsi' method and supposes that the first earned 30, the second 20, and the third 15, meaning a total of 65. Using the rule of three, he then calculates the profit of each partner:  $32\frac{4}{13}$ ,  $21\frac{7}{13}$ ,  $16\frac{2}{13}$ .

Mizrahi's solution is similar. However, this is where the similarity between the solution methods of the two mathematicians ends. Menachem Zion is satisfied with this solution and moves on to the next question. Mizrahi, however, notices a fundamental difficulty with this question. In the problem's formulation, we are not discussing the relationship between the different profits, but rather the components that make up the total profit. If you add all the parts, you get a result greater than one!  $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{26}{24}$ . Therefore, this is a question with no solution<sup>37</sup>.

Did Menachem Zion notice this fact? Why did he choose to take from Mizrahi precisely this question, in which Mizrahi himself writes that it is a question that 'causes embarrassment'?

## Conclusions

Freudenthal<sup>38</sup> and other scholars present an approach in which research in the history of mathematics should focus not only on the development of mathematical knowledge, but also on the sociological contexts and the transfer of knowledge between different cultures.

<sup>35</sup> Ibidem, p. 36–37.

<sup>36</sup> E. Mizrahi, *Sefer ha-Mispar le-ha-Hakam ha-Elohi Eliya ha-Mizrahi*, p. 191.

<sup>37</sup> Ibidem.

<sup>38</sup> G. Freudenthal, *Introduction: The History of Science in Medieval Jewish Cultures: Toward a Definition of the Agenda*, [in:] *Science in Medieval Jewish Cultures*, ed. by G. Freudenthal, Cambridge University Press, Cambridge 2011, p. 2–5.

In this perspective, Menachem Zion's book is of interest not only because of its content but also because of what can be learned about the Jewish community in Northern Italy during this period and its mathematical interest.

We can summarize some insights from reading the *Sefer Over la-Soher*:

- a. The paucity of mathematics books printed in Hebrew in northern Italy in the 16th-17th c. does not necessarily indicate a lack of interest in this subject. As I have shown before, it is reasonable to assume that Jews interested in mathematics found what they were looking for in studies at non-Jewish institutions or in books written in languages other than Hebrew.
- b. Menachem Zion's book was intended for a very specific Jewish population that was primarily interested in applied mathematics, the kind that could assist travelers and those engaged in commerce.
- c. The choice of names for places in the word problems chapter can hint at the book's target audience. It is possible that among them were immigrants and travelers from Central and Northern Europe, as well as the Ottoman Empire. The mention of places from ancient Babylon can indicate people who studied the Talmud (the central text of Judaism after the Hebrew Bible).
- d. Mathematically, Menachem Zion's book integrates Jewish sources (mostly Mizrahi's book) with local sources (Italian abacus books) and covers various topics in a concise manner, avoiding proofs and lengthy explanations.

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