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DIESTERWEG'S CONCEPT OF *ANSCHAUUNG* AND ITS IMPLEMENTATION IN HIS *PRAKTISCHES RECHENBUCH*

Summary: The German pedagogue Friedrich Adolph Wilhelm Diesterweg (1790–1866) is well-known for his political and philosophical contributions to the so-called *Volksbildung* (public education). However, his influence on the teaching of mathematics in his time – particularly through his textbook *Praktisches Rechenbuch für Elementar- und höhere Bürgerschulen* [Practical Arithmetic Book for Elementary and Higher Civic Schools], written in collaboration with Peter Heuser – has so far received relatively little attention. In the three exercise books and accompanying teacher commentaries, Diesterweg emphasizes his two key pedagogical principles, derived from the ideas of Pestalozzi and Schopenhauer as well as his own pedagogical philosophy: *Anschauung* and *self-activity*. This article examines Diesterweg's concept of *Anschauung* within the framework of his approach to teaching and learning arithmetic, arguing for his significance as an influential figure in the history of mathematics education in the 19th c. Accordingly, the article elaborates on his concept of *Anschauung* and investigates how it was practically implemented in both his textbooks and the accompanying teacher guidelines.

Keywords: Adolph Diesterweg, *Anschauung*, visualization, intuition, teaching arithmetic in 19th-century Prussia, arithmetic textbook for elementary schools, number-images

Introduction

This article focuses on Friedrich Adolph Wilhelm Diesterweg, one of the leading Prussian educators and teacher trainers of the 19th c. He is particularly known for his commitment to the professionalization of elementary school teachers (*Volksschullehrer*) and for his liberal and emancipatory stance on educational policy in his time. Secondary literature predominantly discusses his general pedagogical and didactic views, often summarized as follows: In the tradition of Rousseau and Pestalozzi, the main principles of his pedagogy are *self-activity* and *Anschauung*[1]. Indeed, these two principles guide not only his systematic reflections on education in general but also his didactic considerations regarding the content and methodology of various school subjects. However, in the scholarly reception of his work, these two principles are often treated merely as conceptual black boxes. What Diesterweg actually meant by *Anschauung* – beyond mere sensory perception – is seldom examined in detail, even though the 19th c. drew on several fundamentally different philosophical traditions to interpret the meaning of *Anschauung*.

Mathematics occupies a special place for Diesterweg, particularly in relation to the principle of *Anschauung*. In his systematic-philosophical writings, he repeatedly used mathematics as a paradigmatic example of a discipline grounded in *Anschauung*[2]. Unlike in other areas of knowledge, Diesterweg posited a specific form of *Anschauung* for mathematics – the *mathematische Anschauung* – to which he assigned a distinctive status. In his articles on mathematics instruction, he emphasized the distinction between mental arithmetic and written arithmetic, underscoring the role of *Anschauung* as a foundation for meaning and understanding. Finally, the primacy of *Anschauung* became the central organizing element in his practical suggestions for mathematics instruction. This applies not only to geometry lessons but also – and especially – to initial arithmetic instruction, which at the time was still often mechanical and repetitive:

> We have thus stated the main principle for elementary arithmetic teaching, as for every branch of elementary teaching: *Anschauung*. It states not only that the first concepts of number should be gained from sensory (inner, induced by external means) perception, but also that all operations should be traced back to original, purely visual cognition [*anschauliche Erkenntnis*], and it rejects all general concepts placed at the top, all positive rules, all given and positive things.[3]

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[1] Since the term *Anschauung* has very different translations depending on the philosophical background theory – intuition, visualization, imagination… – the German term will be retained in the following as *terminus technicus*. However, after a detailed presentation of Diesterweg's understanding, suggestions will be offered for categorizing it within contemporary English-language approaches.

[2] See, e.g., F.A.W. Diesterweg, *Über die Quelle unserer Erkenntnis und über das (einzig richtige) Verfahren bei Erwecken derselben in Andern, nebst einem Anhange über die heuristische Methode in der Raumlehre*, "Rheinische Blätter für Erziehung und Unterricht" 1833, p. 291–324.

[3] F.A.W. Diesterweg, *Wegweiser zur Bildung für Lehrer und die Lehrer werden wollen: und methodisch-praktische Anweisung zur Führung des Lehramtes*, 1st ed., G.D. Bädeker, Essen 1835, p. 613. All translations of Diesterweg in the following article have been prepared by the author. For readability, this is stated only here (and not repeated elsewhere).

The aim of the following discussion is to reconstruct Diesterweg's concept of *Anschauung* as the foundation of these demands. His basic epistemological position and the resulting general conception of *Anschauung* will be examined, along with his philosophical understanding of *mathematische Anschauung*. Special attention will be paid to how these theoretical foundations are interpreted in relation to arithmetic. Finally, the second part of the paper draws on examples from Diesterweg and Heuser's best-selling textbook – vol. 1 of the *Praktisches Rechenbuch für Elementar- und höhere Bürgerschulen* [Practical Arithmetic Book for Elementary and Higher Civic Schools] – to explore the extent to which these philosophical-pedagogical assumptions were incorporated into instructional materials. This serves both to contribute to the reappraisal of *Anschauung* as a general pedagogical principle in Diesterweg's work and to portray him as a knowledgeable and influential didactician of mathematics. A brief biographical sketch will precede the philosophical and didactic analysis.

Teacher educator with a focus on mathematics

Biographical notes

Friedrich Friedrich Adolph Wilhelm Diesterweg was born on 29 October 1790 in Siegen (a small town in the Duchy of Nassau, later part of Prussia), as the seventh child of Carl Friedrich Diesterweg, a lawyer and senior civil servant. His family was characterized by a strong sense of social status and justice. Diesterweg's own schooling at the local Latin school left little positive impression on him; his teachers were mostly authoritarian theologians or former soldiers, and the curriculum consisted primarily of religion, writing, and arithmetic. Much of the instruction relied on rote memorization, which Diesterweg criticized from an early age as unnecessary.

After completing school, he studied philosophy (which at the time included mathematics) at the *Hohe Schule* (a higher education institution) in Herborn and Heidelberg from 1808 to 1810. Later, he enrolled at the Eberhard Karls University of Tübingen to continue studying mathematics. In 1812, Diesterweg became a teacher, although his original ambition was to become a surveying engineer. After seven years in teaching – as a private tutor in Mannheim, at a secondary school in Worms, at the *Musterschule* in Frankfurt, and at the Latin and civic school in Elberfeld – he took over the management of a *Stadtschullehrerseminar* (a teacher training college for urban elementary teachers) in Moers in 1820[4]. In 1832, he moved to Berlin, where he led a teacher training institution until 1847.

[4] For detailed information on Diesterweg's work and his influence on primary school teacher training during his time in Moers, see S. Schmidt, *Rechenunterricht und Rechendidaktik an den Rheinischen Lehrerseminaren im 19. Jahrhundert*, Böhlau, Köln/Wien 1991.

Diesterweg traveled extensively and read widely, which enabled him to develop and articulate his own theoretically and practically grounded views on many pedagogical questions, publishing them in books and journals. His positions were often controversial, especially in conservative circles during the post-1848 period. He was repeatedly reprimanded by the Ministry of Education for promoting a view of church-state relations that was seen as inappropriate and for allegedly inciting elementary school teachers. Despite these official warnings, Diesterweg persisted in publishing his views, and in 1847 he was dismissed from his position without specific charges.

After leaving the teacher training seminar, he continued writing and speaking out against conservative education policy, particularly the Prussian regulations of 1854, which he found especially objectionable. From 1859 until his death in 1866 (of cholera), he also engaged in direct political advocacy for socially responsive education as a member of the Prussian State Parliament representing the liberal party in Berlin[5].

Diesterweg's ideas achieved lasting influence, especially through his founding of the "Rheinische Blätter für Erziehung und Unterricht", a journal for elementary school teachers that he edited from 1827, and to which he contributed numerous articles. In addition, his pedagogical manual *Wegweiser zur Bildung für Lehrer und die Lehrer werden wollen, und methodisch-praktische Anweisung zur Führung des Lehramtes* (first published in 1835; later titled *Wegweiser für deutsche Lehrer* [Guide for German Teachers]) became the standard reference work for (prospective) elementary school teachers[6]. Its wide impact is evident from the fact that four editions were published during Diesterweg's lifetime – each updated with current literature and additional chapters – and that his students continued to release further editions after his death, preserving the core of his theoretical pedagogical insights[7].

[5] See G. Geißler, *Friedrich Adolph Wilhelm Diesterweg (1790–1866) – Was fordert die Zeit?*, [in:] *Klassiker der Pädagogik. Die Bildung der modernen Gesellschaft*, ed. by B. Dollinger, Verlag für Sozialwissenschaften, Wiesbaden 2006, p. 127–149.

[6] The *Wegweiser zur Bildung für Lehrer*... consists of a theoretical first part (The General), in which Diesterweg outlines his didactic and psychological principles for teachers' practical work, and a second, subject-didactic part (The Special), in which these pedagogical principles are applied to the subjects of contemporary elementary schools. In addition to detailed information on the sequence and selection of content, it also includes methodological tips, as well as a discussion of existing textbooks and specialized didactic literature. Diesterweg himself is responsible for the chapters on teaching reading, the German language, natural science and mathematical geography as well as arithmetic and geometry, while for the other subjects he enlisted experienced colleagues from his teacher training seminar as co-authors (See F.A.W. Diesterweg, *Wegweiser zur Bildung für Lehrer*, 1st ed., p. xii).

[7] See S. Schütze, *Diesterwegs Wegweiser zur Bildung für Lehrer – ein Lehrbuch im Spiegel didakitsich-methodischer und politischer Veränderungen*, [in:] *Continuity and Change of Knowledge in Educational Media*, ed. by E. Matthes et. al., Verlag Julius Klinkhardt, Bad Heilbrinn 2024, p. 295–305.

Work on mathematics education

Diesterweg's contributions to mathematics education were no less influential in his own time than his general pedagogical writings, though they are less often acknowledged today[8]. He became known as the author of numerous textbooks across the various subfields of school mathematics. Some of these were designed for teacher training seminars, while others were intended for use in primary and secondary schools:

- *Geometrische Combinationslehre. Zur Beförderung des Elementar-Unterrichts in der Formen- und Größenlehre* [Geometric Combinatorics. On the Advancement of Elementary Teaching in Shape and Size], Büschler, Elberfeld 1820.
- *Leitfaden für den ersten Unterricht in der Formen-, Größen-, und räumlichen Verbindungslehre, oder Vorübungen zur Geometrie* [Guide for the First Lessons in Shape, Size, and Spatial Relations, or Preliminary Exercises in Geometry], 1st ed., Büschler, Elberfeld 1822 (3rd ed., 1836 – separated into an exercise book and a teacher's manual).
- Together with Peter Heuser: *Praktisches Rechenbuch für Elementar- und höhere Bürgerschulen* [Practical Arithmetic Book for Elementary and Higher Civic Schools], 1st ed., 3 vols., Büschler, Elberfeld 1825–1828 (vol. 1: 1825, vol. 2.: 1826, vol. 3: 1828)[9].
- *Elementare Geometrie für Volksschulen und Anfänger überhaupt* [Elementary Geometry for Elementary Schools and Beginners in General], Frankfurt am Main 1860.

The fact that many of these textbooks were frequently reprinted is a clear indicator of Diesterweg's practical influence in schools. Since the focus of this article is on initial arithmetic instruction, the analysis will concentrate on vol. 1 of the *Praktisches Rechenbuch*, co-authored with Peter Heuser, which became a bestseller.

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[8] Exceptions to this are isolated works such as O. Koch, *Diesterweg als Methodiker des Rechen- und Raumlehreunterrichts*, [in:] *Der Pädagoge Adolph Diesterweg*, ed. by H. G. Blotz, Verlag Moritz Diesterweg, Frankfurt 1958, p. 54–70; Th. Jahnke, *Die Regel de Tri. Eine mathematische Reise mit Diesterweg*, [in:] *Adolph Diesterweg. Wissen im Aufbruch*, ed. by Universität-Gesamthochschule-Siegen, Deutscher Studienverlag, Weinheim 1990, p. 210–216; or A. Zeimetz, *Anwendungen und weitere Vernetzungen in Diesterwegs Raumlehre. In Vernetzungen und Anwendungen im Geometrieunterricht*, ed. by A. Filler, M. Ludwig, Franzbecker, Hildesheim 2012, p. 157–176; and with a special focus on Diesterweg's approaches to teaching arithmetic in relation to contemporary teacher training, see S. Schmidt, *Rechenunterricht*, Chapter 4.

[9] During Diesterweg's lifetime, the first exercise book appeared in its 20th edition in 1858, the second exercise book in its 10th edition in 1857, and the third exercise book in its 5th edition in 1854. After Diesterweg's death, all volumes were republished several times, partly revised by Diesterweg's student E. Langenberg (See S. Schmidt, *Zur Rechendidkatik in Deutschland in der Ersten Hälfte des 19. Jahrhunderts*, [in:] *Rechenbücher für den Unterricht in der Elementarschule*, ed. by S. Schmidt, Böhlau, Köln/Weimar/Wien 1993, p. 12–13).

However, these textbooks and exercise collections were not primarily the result of Diesterweg's own classroom teaching experience. Rather, they were grounded in systematically developed pedagogical concepts informed by philosophical study. Diesterweg published his didactic reflections on mathematics – which underpinned these practical materials – in various formats: some as articles in the "Rheinische Blätter für Erziehung und Unterricht", others as subject-specific chapters in his *Wegweiser zur Bildung für Lehrer...*, and some as independent volumes. His core methodological and didactic contributions to arithmetic instruction include:

- *Über den Unterricht in der Geometrie*, "Neue Bibliothek für Pädagogik, Schulwesen und die gesamte pädagogische Literatur Deutschlands" 1815, p. 289–310[10].
- *Bemerkungen über den Rechenunterricht, mit besonderer Beziehung auf das praktische Rechenbuch von Diesterweg und Heuser* [Remarks on Arithmetic Teaching, with Special Reference to the Practical Arithmetic Book by Diesterweg and Heuser], "Rheinische Blätter für Erziehung und Unterricht" 1828, p. 86–101.
- *Über die Quelle unserer Erkenntnis [...] nebst einem Anhange über die heuristische Methode in der Raumlehre* [On the Source of Our Cognition [...] with an Appendix on the Heuristic Method in the Theory of Space], "Rheinische Blätter für Erziehung und Unterricht" 1833, p. 291–324.
- *Der Unterricht in der Zahlenlehre* [Teaching in Arithmetic], [in:] *Wegweiser zur Bildung für Lehrer und die Lehrer werden wollen, und methodisch-praktische Anweisung zur Führung des Lehramtes*, ed. by F.A.W. Diesterweg, G.D. Bädeker, Essen 1835, p. 607–645.
- *Methodisches Handbuch für den Gesammtunterricht im Rechnen* [Methodical Handbook for Comprehensive Arithmetic Instruction], together with P. Heuser (1st vol. written by Diesterweg) (1st ed.), Büschler, Elberfeld 1844 (appeared in 5th ed., in 1850).
- *Das Lehren aus der Anschauung und das Lehren aus dem Gedächtnis* [Teaching Based on Observation and Teaching Based on Memory], "Rheinische Blätter für Erziehung und Unterricht" 1850, p. 291–315.

Diesterweg's concept of *Anschauung*

Diesterweg developed the epistemological foundation of his principle of *Anschauung* primarily in two texts: his 1833 article *Über die Quelle unserer Erkenntnis...* and his most influential book *Wegweiser zur Bildung für Lehrer…*, especially Chapter IV: 'Die Anlage des Menschen und die aus ihrem Wesen entspringenden allgemeinen didaktischen Gesetze und Regeln' [The Human Dispo-

[10] This was Diesterweg's very first scholarly article.

sition and the General Didactic Laws and Rules Arising from Its Nature]. These two works form the basis of the following discussion of Diesterweg's concept of *Anschauung*, as they reflect his systematic-philosophical considerations. Although numerous other writings contain didactic references to *Anschauung*, they largely rely on the conceptual groundwork he established in these two texts.

Basic epistemic assumptions

Diesterweg distinguishes between *knowledge* and *cognition*. Whereas knowledge refers to memorized properties of an object – such as external features, names, or definitions – cognition (*Erkenntnis*) refers to a deeper understanding of the *essence of things*. It involves grasping both the original and derived properties of objects and their interrelations[11]. Diesterweg criticized the tendency in elementary and Latin schools to rely excessively on rote memorization – of rules, Bible verses, or terminology – as a substitute for real understanding.

He insisted that the only valid path to cognition was through *Anschauung* – *vivid perception*. This principle, according to him, applied universally: across all domains of knowledge and all stages of learning[12]. He famously formulated this as a general didactic imperative:

> In the pupil, the most immediate perception, on which a concept, a thought, or a feeling is based, must first be evoked, or, if this is not possible, the new must at least be connected to immediate perceptions by comparison, analogy, or pictorial representation.[13]

To support this pedagogical stance, Diesterweg draws on the anthropological logic of Jakob Friedrich Fries (1773–1843)[14], though without fully embracing Fries's neo-Kantian system. Instead, he adapts key aspects of it to educational practice. One might describe his approach as a *philosophically informed pedagogy* – a pragmatic application of epistemology to teaching.

For Diesterweg, pupils who possess only knowledge but not cognition are unable to work independently or critically. They cannot test what they have learned, nor can they extend it. Their knowledge remains dependent on external authority. In contrast, cognition enables autonomy, critical thinking, and personal development – aims that align with Diesterweg's liberal political and religious convic-

[11] F.A.W. Diesterweg, *Über die Quelle unserer Erkenntnis*, p. 292.

[12] See Ibidem. p. 294.

[13] Ibidem, p. 300.

[14] Diesterweg likely met Fries during his time in Heidelberg, where Fries served as a professor of philosophy and elementary mathematics. Fries formulated his own explicitly neo-Kantian epistemology, to which Diesterweg refers here, and later developed an independent philosophy of mathematics (See G. Schubring, *Philosophie der Mathematik bei Fries*, [in:] *Jakob Friedrich Fries – Philosoph, Naturwissenschaftler und Mathematiker*, ed. by W. Hogrebe, K. Hermann, Peter Lang, Frankfurt am Main 1999, p. 175–193).

tions. As Hohendorf notes, Diesterweg echoes the Protestant principle of the *freedom of individual judgment*, which Paul expresses in Thessalonians 5:21: 'Test everything; hold fast to what is good'[15]. Diesterweg writes:

> Only the spiritually dead-born, the eternally blind person, faithfully repeats what others say to him; but the person striving for spiritual maturity and independence examines and tests whether what he believes to be true accords with the results of his life and the certain views he has directly gained.[16]

Anschauung from external and internal sensations

To illustrate his conception of *Anschauung*, Diesterweg outlines a *map of cognition* that comprises both external and internal sources of perception[17].

He begins with a standard distinction: external *Anschauung* arises from the senses and pertains to the perception of concrete physical objects. Initially, individual properties of these objects are registered as *sensations* (*Empfindungen*), which are passively received. These *sensations* are then actively synthesized by the *figural imagination* (*figürliche Einbildungskraft*) into an *Anschauung* – a holistic mental representation of a specific object with all its features[18]. Thus, while sensations refer to isolated aspects, *Anschauung* represents the integration of these aspects into a single, coherent idea.

These representations, though generated through sense experience, are located in the mind or soul (*Seele*) of the learner. Diesterweg refers to this level of experience as 'empirical consciousness'[19]. He also acknowledges the varying subjectivity of the senses: touch, taste, and smell produce more subjective impressions, while sight and hearing yield more intersubjectively valid perceptions.

Diesterweg then introduces internal *Anschauung*: the perception of inner mental states. These states, like external objects, are composed of sensations – in this case, those of the *soul's own activities*. They are perceived through an inner sense (*innerer Sinn*), which Diesterweg likens to a sensory organ that is receptive to

[15] G. Hohendorf, *Über die Quellen der Pädagogik Diesterwegs*, [in:] *Diesterweg: Pädagogik-Lehrerbildung-Bildungspolitik*, ed. by G. Hohendorf, H.F. Rupp, Deutscher Studienverlag, Weinheim 1990, p. 25.

[16] F.A.W. Diesterweg, *Über die Quelle unserer Erkenntnis*, p. 300.

[17] Many of the terms used by Diesterweg were already in use in the philosophy of *Anschauung* and in the pedagogy of his time that referred to it. However, contemporary approaches differ greatly depending on the philosophical reference context (e.g., in comparisons between sensualists and Kantians). It can be shown, for example, that although Diesterweg at first glance shared Pestalozzi's demand for *Anschauung*-based teaching, the terms used in parallel have completely different meanings (See S. Spies, *Anschauung als Leitprinzip im Rechenunterricht bei Pestalozzi und Diesterweg*, "SieB" 2024, vol. 18, p. 145–174).

[18] F.A.W. Diesterweg, *Über die Quelle unserer Erkenntnis*, p. 302.

[19] Idem, *Wegweiser zur Bildung für Lehrer*, 1st ed., p. 101.

mental states at a given moment. These internal impressions are also synthesized by the imagination into coherent inner *Anschauungen*[20].

Diesterweg identifies imagination (*Vorstellungskraft*) as the faculty that enables both *Anschauung* and abstract thought[21]. Concepts (*Begriffe*) are formed by abstraction, following the operations of the mind (*Verstand*), which acts according to the 'laws of separation and combination'. Abstract concepts are the result of further generalisation: they are simpler in structure but broader in scope. While a *sensation* relates to a specific, concrete phenomenon, an abstract concept refers to a class of phenomena. For Diesterweg, cognition proceeds from the concrete to the abstract – from *sensation* to *Anschauung*, and from *Anschauung* to *concept*.

Anschauung in mathematical cognition

According to Diesterweg, mathematical cognition can be achieved in two fundamentally different ways. The first – and in terms of the learning process, the primary – path begins, as in all fields of knowledge, with external *Anschauung*. He also introduces a second form: *mathematical Anschauung*, which he understands as a kind of inner perception grounded in external experiences. This inner perception arises, for instance, when learners *mentally* process concrete physical objects such as wooden cubes: they may see and feel the sides, edges, and corners, but they can also, with the aid of imagination, 'look into' the cube and mentally visualize the diagonals or internal structures that are not directly visible.

Thus, *mathematische Anschauung* develops from external perception through the activation of the *productive imagination*. Diesterweg's example of the wooden cube illustrates this vividly: while the cube's physical characteristics are accessible to the senses, certain geometric properties must be inferred through internal visualization. In such cases, the imagination supplements the senses, enabling insight into hidden structures. This kind of cognition, he writes, is *vivid* (*anschaulich*) and intuitive. It is also possible for propositions to be directly recognized as true or evident through contemplation[22]. He distinguishes clearly between the physical solid perceived through external *Anschauung* and the mathematical solid, which is an object of internal visualization[23].

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[20] Idem, *Über die Quelle unserer Erkenntnis*, p. 300ff.

[21] Idem, *Wegweiser zur Bildung für Lehrer*, 1st ed., p. 104.

[22] See Idem, *Über die Quelle unserer Erkenntnis*, p. 312.

[23] This is particularly evident in geometry, when, on the one hand, canonical external visualizations in the form of drawings and models are available, but, on the other hand, these are explicitly omitted, at least in the books for the higher grades and teacher education, in order to allow the productive imagination to become directly active. Drawings then take only a subordinate role as means of visualization (cf. J. Lemanski, *Schopenhauers Logikdiagramme in den Mathematiklehrbüchern Adolph Diesterwegs*, "SieB" 2022, vol. 16, p. 16).

In contemporary terms, Diesterweg's account of *mathematische Anschauung* aligns with what is now called *internal visualization*: the mental generation, transformation, and inspection of idealized figures. Through these internal operations, learners form *prototypical images* of mathematical objects, explore their relationships, and draw conclusions. In Diesterweg's examples, particular emphasis is placed on how learners perceive relations between parts of a figure or between different figures – an aspect that resonates with Anna Sierpinska's notion of structural or spatial forms of visualization[24].

However, Diesterweg goes beyond concrete or spatial internal visualization. He also refers to *the awareness of general laws* that govern the soul's activity[25] – including the *basic forms of space and time*[26] – as conditions of possibility for *all* perception. In this sense, his notion of *Anschauung* includes abstract mental entities that reflect spatial properties (shape, position, magnitude) but also possess conceptual qualities such as generality, ideality, and perfection. These are best understood as 'figural concepts', in Ephraim Fischbein's sense: mental constructs that are both visual and conceptual[27].

Crucially, Diesterweg acknowledges that mathematical truths can also be attained analytically, through conceptual cognition. Properties of mathematical objects may be derived logically, without reliance on perception or imagination. For example, in elementary geometry, the number of a cube's edges may be: counted on a physical object (external *Anschauung*), visualized internally (*mathematische Anschauung*), or derived conceptually by analyzing the cube's surfaces and their connections (*conceptual cognition*).

Diesterweg refers to the latter as *analytical cognition* in the tradition of the French, while cognition by *Anschauung* is associated with 'the Greeks'[28]. These two modes differ in their mental operations: vivid cognition [*anschauliche Erkenntnis*]

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[24] See A. Sierpinska, *Visualization is in the mind of the beholder*, "New Zealand Journal of Mathematics" 2003, vol. 32, p.176.

[25] F.A.W. Diesterweg, *Wegweiser zur Bildung für Lehrer*, 1st ed., p. 102.

[26] At this point, Diesterweg does not make any specific reference to Kant or Fries, even though the concept very closely resembles their approaches. These references are only mentioned later (Ibidem, p. 107), in connection with the dissection of judgements, but without mentioning any specific work.

[27] E. Fischbein, *The Theory of Figural Concepts*, ''Educational Studies in Mathematics" 1993, vol. 24, no. 2, p. 143.

[28] The influence of his older brother and patron, Wilhelm, is evident here: as one of the first two professors of mathematics at the University of Bonn, he still emphasized the value of elementary mathematics based on the Greek works of Euclid and Apollonius and preferred these over the 'modern' algebraic-analytical subdisciplines (See G. Schubring, *Wilhelm Diesterweg. Ein Elementarmathematiker in der entstehenden Forschungsuniversität*, [in:] *Adolph Diesterweg. Wissen im Aufbruch*, ed. by Universität-Gesamthochschule-Siegen, Deutscher Studienverlag, Weinheim 1990, p. 75–83).

is supported by imagination, while conceptual cognition is governed by the intellect. This difference also informs the instructional sequence: because vivid cognition is more accessible and intuitive, it should precede abstract reasoning. Hence, mathematics education, particularly at the elementary level, must begin with *Anschauung*[29].

In Diesterweg's view, *Anschauung*-based understanding is not only sufficient for practical applications (e.g., in the trades), but also educationally valuable. Even when a proposition is later proven conceptually, the initial intuitive insight remains important. Indeed, Diesterweg sees great pedagogical merit in re-encountering a mathematical idea from multiple cognitive perspectives: *first* through perception or imagination, *then* through deduction and proof. This multiplicity of perspectives constitutes, for him, true *Bildung*[30]. Thus, for Diesterweg, the goal of mathematics education is not the acquisition of general concepts per se, but rather a comprehensive understanding of mathematical objects and their interrelations. General concepts are the *result* of this process, not its starting point.

Anschauung in elementary arithmetic

In elementary arithmetic, Diesterweg advocates reducing all numbers to the unit (1) and the higher units derived from it through multiplication in the decimal system[31]. This conceptual reduction is to be achieved through vivid instruction (*anschaulicher Unterricht*) using manipulatives. These manipulatives should exceed the number of fingers available for counting and clearly reveal the decimal structure of numbers[32].

Although Diesterweg's concept of *Anschauung* and the instructional methods derived from it differ significantly from those of Pestalozzi[33], he initially (in 1835) recommends Pestalozzi's table of units (*Einheitentafel*) as one possible manipulative. In the first edition of the *Wegweiser zur Bildung für Lehrer…*, he also suggests additional tools that can be used by teachers: a counting board, and marks or dots[34]. In later editions[35], he adds: Denzel's ladder (*Denzel'sche*

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29 See F.A.W. Diesterweg, *Über die Quelle unserer Erkenntnis*, p. 316.

30 Ibidem, p. 317.

31 Ibidem, p. 311.

32 In the following, manipulatives should be understood in the sense of Hartshorn and Boren as 'objects that can be touched and moved by students to introduce or reinforce a mathematical concept' (R. Hartshorn, S. Boren, *Experiential Learning of Mathematics: Using Manipulatives*, "ERIC Digest" 1990, ED321967, p. 2).

33 See S. Spies *Anschauung als Leitrprinzip*.

34 F.A.W. Diesterweg, *Wegweiser zur Bildung für Lehrer*, p. 613.

35 See Idem, *Wegweiser zur Bildung für Lehrer*, 4th ed., p. 357.

Leiter)[36], the so-called 'Russian calculating machine'[37], and Friedrich Fröbel's Third Gift[38].

Diesterweg takes the view that arithmetic instruction *requires* concrete, visual materials, but the choice of material is part of the teacher's *method*, and may be selected by the teacher personally. This pragmatic attitude reflects Diesterweg's overarching goal: to teach arithmetic vividly and to foster inner number concepts, i.e., *mathematische Anschauung*. The manipulative serves merely as an empirical representative of the unit, not as *Anschauung* itself. In fact, in his detailed teacher guidelines (1844)[39], Diesterweg prefers marks on the blackboard to introduce counting and the first symbolic representations of numbers. This may be due to the simplicity of printing linear marks compared to images of three-dimensional objects, and to the universal availability of blackboards and slates in the classroom, which made separate purchases of materials unnecessary.

Vol. 1 of the *Praktisches Rechenbuch für Elementar- und höhere Bürgerschulen* similarly begins with exercises using marks written on slates. All four basic operations are introduced using these representations. Arithmetic is initially reduced to counting marks, and operations are recorded by drawing strokes to show the process and the result. A noteworthy feature is the use of Roman numerals to represent bundles of ten before introducing Indian-Arabic numerals and the decimal place-value system. This allows pupils to extend the number range beyond

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[36] This is a decimal structured material developed by Bernhard Gottlieb Denzel (1773–1838), director of the Württemberg seminary and a Pestalozzi expert. In it, rods representing one ('rungs') are bundled into tens ('ladders'), which can in turn be bundled on a large 'ladder' to expand the number range (see the detailed description in K.C.G. Zerrenner, *Mittheilungen und Winke die Einführung der wechselseitigen Schuleinrichtung betreffend*, Heinrichshofe, Magdeburg 1834, p. 37f).

[37] The manipulative is a decimal structured arithmetic frame with several bars, each bearing ten movable beads that are colour-coded and divided into groups of five. Each bar represents a power of ten. Unlike Denzel's ladder, this material was designed as an abacus, i.e., as a calculating aid and not primarily as a visualization aid (See H-J. Vollrath, *Verborgene Ideen: Historische mathematische Instrumente*, Springer-Spectrum, Wiesbaden 2013, p. 111).

[38] This consists of a larger wooden cube made up of eight small cubes, which, according to Diesterweg, can be used 'to visualize the first arithmetic operations' (F.A.W. Diesterweg, *Wegweiser zur Bildung für Lehrer*, 4th ed., p. 357). It should be noted that Diesterweg only met Fröbel personally shortly before the publication of the fourth edition of the *Wegweiser* and it was only then that he became acquainted in detail with the content of Fröbel's ideas and manipulatives. Diesterweg was so impressed by this encounter that he dedicated the 4th edition to Fröbel (see the preface). It remains unclear exactly why Diesterweg mentions the Third Gift here, although Fröbel himself envisaged the laying of rods for arithmetic lessons (See A. Rahn, S. Spies, *Arithmetic by manipulatives: a view through historical examples*, [in:] *"Dig where you stand" 7. Proceedings of the seventh International Conference on the History of Mathematics Education*, ed. by K. Bjarnadóttir et. al., WTM-Verlag, Münster 2023, p. 213–226).

[39] F.A.W. Diesterweg, P. Heuser, *Methodisches Handbuch für den Gesamt-Unterricht im Rechnen: Als Leitfaden beim Rechenunterrichte und zur Selbstbelehrung*, Büschler, Elberfeld 1844.

ten or twenty in a vivid way and to solve operations with large numbers by simply counting signs. In an article promoting the textbook, Diesterweg writes:

> Since these signs [the Roman numerals] represent the ideas of numbers much more vividly and sensually than our Arabic numerals, these signs can be called number-images [Zahlenbilder], and arithmetic with them can be called arithmetic by number-images.[40]

Implementation in the successful standard textbook

In the following, the concept of the *Praktisches Rechenbuch* will first be presented on the basis of the detailed 1828 description[41]. The aspect of *Anschauung* in arithmetic lessons will then be illustrated using selected examples from vol. 1[42]. For reasons of space, and because Diesterweg's primacy of *Anschauung* becomes particularly clear there, the focus is on the first section.

Collection of exercises in three volumes

Together with Peter Heuser, Diesterweg published a three-volume *Praktisches Rechenbuch*. These books were designed as collections of exercises for pupils, with a separate solution manual for teachers. First published in 1825, vol. 1 alone appeared in twenty editions during Diesterweg's lifetime, indicating its wide acceptance and success in school practice.

Diesterweg and Heuser intended the books to respond to the real conditions of contemporary elementary schools, which were often overcrowded and included multiple grade levels in a single class. To this end, they developed exercises that could be worked on by pupils independently (*self-study*), allowing the teacher to attend to others. The books were thus meant to be both practical and educational, focusing not only on acquiring everyday arithmetic skills but also on promoting reflection and intellectual development.

The volumes differ in terms of target group and content: vol. 1 is designed for elementary schools. It introduces number representation, the four basic operations, fractions, the Rule of Three, and elementary geometry (including area calculation and compass-and-straightedge constructions, but omitting formal proofs). Vol. 2 addresses the needs of more advanced students, such as merchants and craftsmen. It focuses on principles and applications of commercial arith-

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[40] F.A.W. Diesterweg, *Bemerkungen über den Rechenunterricht, mit besonderer Beziehung auf das Rechenbuch von Diesterweg und Heuser*, "Rheinische Blätter für Erziehung und Unterricht" 1828, p. 95.

[41] Ibidem.

[42] All examples of the following analysis are taken from F.A.W. Diesterweg, P. Heuser, *Praktisches Rechenbuch für Elementar-und höhere Bürgerschulen: Erstes Uebungsbuch*, 16th ed., Büschler, Elberfeld 1846.

metic. Vol. 3 targets students with special interests in mathematics. It includes content that goes beyond the necessary, such as negative numbers, progressions, square roots, and probability – material that is ‘useful, pleasant, interesting, and intellectually stimulating’[43].

Although the *Praktisches Rechenbuch* is ‘only’ a collection of exercises for independent practice, it is not just a matter of syntactically practicing what may have previously been presented or worked on vividly in class together with the teacher. Rather, the design also follows the central teaching principle of *Anschauung*. This will be illustrated below using a few examples from vol. 1, section 1.

Examples of Anschauung-based exercises

Introduction to numbers using marks: The book begins by assuming that children either already know the number words up to a certain point or are introduced to them orally by the teacher. The exercises then ask pupils to: name quantities represented by marks, write out numbers in words, and relate numbers to the unit (1) as a counting base. In Tasks 1 and 2: pupils respond to the question, ‘How many times one is…?’, identifying numbers as multiples of the unit. The number range exceeds finger-counting, and marks are already grouped into bundles (e.g., dashes), thus preparing both additive and multiplicative number constructions.

This is already used in Task 3 (§2): pupils are asked to write groupings such as ‘five times two’ using marks[44]. This introduces multiplication *without* requiring formal instruction by anchoring it in visual representations and familiar language.

In Task 1 (§2): the successor principle is taught as pupils write numbers 1 to 10 in sequence using marks, vividly experiencing that each new number adds one unit to the previous one. In Task 4 (§2), pupils are given prompts like ‘eight and thirty’, which simulate German additive number words (*achtunddreißig*). Sometimes the order is reversed to prevent automatic counting and to promote the mental structuring of numbers. These tasks prepare addition and multiplication, but do not yet ask for results – they train structural understanding first.

Representing decimal units using Roman numerals: In §5, large numbers are introduced using Roman numerals as ‘number-images’. After introducing X as the symbol for ten, pupils: read and interpret groupings, are presented with examples in random order to discourage pattern-based guessing, and discover through variation (for example, the representation of 25 as XII X III) that order does not affect value in this additive system.

Diesterweg uses Roman numerals not as an alternative numeral system, but as a visual manipulative that foregrounds bundled tens, without including subtrac-

[43] See Ibidem.

[44] This indicates that Diesterweg did not want manipulatives to remain solely in the hands of teachers, which Schmidt described as common practice at that time (See S. Schmidt, *Zur Rechendidaktik*, p. 32).

tive elements (like IV or IX)[45]. This is meant to foster a robust *Anschauung* of decimal units and help children differentiate between the *number concept* and the *numerical sign*[46].

Basic arithmetic operations: Section §3 is titled 'Adding up the number-images' – a phrasing that signals two things: a process-oriented understanding of addition (not static equality), and an explicit reference to visual groupings or number-images (*Zahlbilder*).

The first exercises begin with grouped strokes or marks, and pupils are asked: 'How many times 'one' are presented by the following marks together?' This explicitly frames addition as counting bundled units, grounded in *Anschauung*. In subsequent tasks, this is shortened to prompts such as 'Count together:…' – again reinforcing the processual character of addition.

Especially notable is Task 4 (§3), where Diesterweg combines addition with earlier multiplicative formulations: 'How many times one make four times three and seven times five?'. This encourages pupils to reflect on mixed operations, with an intuitive basis in grouped visual elements. These preparatory tasks precede formal symbolic work but already train mental arithmetic.

In §4, subtraction is introduced through the idea of removal (*Wegnehmen)*, again based on *Anschauung*. The guiding question is: 'How many times one remain if you take away […] from […]?' Initially, quantities are represented with marks. Later, number words (both additive and multiplicative) are used. This progression from visual to verbal to symbolic gradually abstracts the concept of subtraction without detaching it from meaning.

Multiplication builds directly on the earlier tasks that already used formulations like 'five times six'. Now, however, pupils are explicitly asked to determine the total: 'How many ones are there in five times six?'. This encourages pupils to mentally construct a set of bundles of equal size and count the total either by grouping or unpacking the bundles into individual marks. Whether pupils adopt a cardinal or ordinal number concept depends on prior instruction and the classroom culture. The didactic design accommodates both perspectives.

A first symbolic transformation of multiplication into addition (e.g., $5 \times 6 = 6 + 6 + 6 + 6 + 6 = 30$) appears only in §14, after the decimal place-value system has been introduced. There, pupils also construct a multiplication table (*1×1-Tafel*), and Task 29 asks for memorized products: 'What is the product of the numbers from one to 20?'.

Also of note is Task 2 in §6, where very large numbers are introduced using Roman numerals as *number-images*. Without formal multiplication tables,

45 The ordinary use of Roman numerals is introduced only in §11 under the heading 'The meaning of Roman numerals', and is paired with some exercises in reading and writing Roman numerals.

46 F.A.W. Diesterweg, *Bemerkungen über den Rechenunterricht*, p. 96.

pupils must use rearrangement and grouping strategies to simplify: 'What is four times (one hundred and twenty)?'. Pupils are expected to reorganize CXX CXX CXX CXX into CCCC XXXXXXXX, which can be directly interpreted as 'four hundred and sixty'. This exercise implicitly applies the distributive law – *without naming or proving it*. The visual character of the number-images validates the operation by *Anschaulichkeit* alone. This is a clear example of Diesterweg's preference for intuitive justification over formal deduction at the elementary level.

Division is introduced through the idea of sharing (*Aufteilen*). Pupils are shown number-images (marks and Roman numerals) and instructed to divide them into a given number of equal parts. The first goal is to visualize partitioning: splitting a set into halves, thirds, fourths, etc. Once this is understood, the tasks shift to naming the resulting part (e.g., 'What is one third of...?'). This prepares pupils linguistically and conceptually for unit fractions. The idea is taken up again in §23 when fractions are introduced: A square is divided into equal parts, and pupils name the fraction (e.g., 1/4, 3/5)[47].

Diesterweg and Heuser also include reflective prompts such as: 'What do you think about 4/5, 3/6,…?', which perhaps should be phrased as: 'What do you see in your mind's eye?'. The following tasks also encourage the children to use their imagination to obtain certain fractions of an (arbitrary) whole or to represent natural numbers by fractions with a given denominator, which leads to an *Anschauung* of n/m with $n > m$.

Word problems: The first section of vol. 1 concludes with a series of word problems designed to be solved by 'writing in number-images'. These exercises draw on scenarios from children's everyday lives and include both simple and compound problems that help to consolidate mathematical terminology and operations. To solve these tasks, pupils must perform a transfer: they must recognize which elements of the story correspond to numerical quantities and then represent these abstractly, using their internalized *Anschauung*. For instance, instead of using a group of marks, a pupil may now think of a set of pennies being added, multiplied, or divided. The number of children in a problem might determine how many parts a quantity is to be divided into – i.e., the divisor.

Importantly, these applications are not used as motivational entry points. Diesterweg does not introduce arithmetic through concrete everyday situations, nor are these real-world contexts used to derive mathematical operations. Instead, they serve a secondary, illustrative function. In Diesterweg's approach, number-images – whether marks or Roman numerals – remain the primary medium through which mathematical meaning is constructed. Everyday objects (apples, coins, etc.) are not meant to serve as the focus of *Anschauung* itself, but merely

[47] For a detailed study on the teaching of fractions by Diesterweg and his contemporaries, see S. Schmidt, *Zur Rechendidaktik*, p. 59ff.

as narrative anchors that support abstraction. Diesterweg explicitly warns against overusing artificial applications:

> Anyone who seeks the love of the matter in the application of numbers to apples and nuts, or in invented stories in which arithmetical problems are hidden, has not grasped the essence of the matter. Variation is good, indeed necessary; but the main thing remains the pure working through of the material.[48]

Concluding remarks

This paper has examined Diesterweg's principle of *Anschauung* with particular focus on its role in arithmetic instruction and its implementation in the *Praktisches Rechenbuch*. The analysis was guided by the question: How did Diesterweg translate his epistemological and pedagogical ideas into concrete teaching materials?

Section 3 demonstrated that Diesterweg's notion of *Anschauung* is rooted in a unique adaptation of Jakob Friedrich Fries's neo-Kantian philosophy. He develops *Anschauung* not merely as sensory perception, but as a layered concept encompassing external and internal perception, imagination, and mental structuring. Mathematics serves as his prime example of a discipline in which *Anschauung* plays a foundational role. In particular, *mathematische Anschauung* emerges in two forms: as a structural or figural form of internal visualization, enabling vivid insight into objects and relations, and as a figural concept (in Fischbein's sense), reflecting abstract, idealized properties such as generality and perfection.

Section 4 analyzed how these theoretical foundations are realized in practice, focusing on vol. 1 of the *Praktisches Rechenbuch*. The textbook systematically implements Diesterweg's ideas through the consistent use of number-images (marks, Roman numerals), the visual introduction of all four basic operations, tasks that encourage learners to internalize numerical structures, and a pedagogical progression from vivid to conceptual cognition. Crucially, the book avoids premature formalization or definition. Instead, it prioritizes vivid understanding and mental visualization over rule-based learning. In this way, Diesterweg maintains a coherent alignment between philosophical principle and pedagogical practice.

However, this study has focused primarily on elementary arithmetic as presented in vol. 1. A fuller picture of Diesterweg's mathematical pedagogy requires further investigation of vol. 3 of the *Praktisches Rechenbuch*, which covers more abstract topics such as negative numbers, progressions, and probability, and may place greater emphasis on conceptual cognition, Diesterweg's geometry textbooks, which may reveal how he implemented *Anschauung* in visual-spatial reasoning beyond arithmetic, and actual classroom usage: how teachers employed

[48] Idem, *Wegweiser zur Bildung für Lehrer*, 1st ed., p. 626.

the textbook in different instructional settings, and whether the vivid, heuristic method was truly practiced as intended.

It remains an open question whether Diesterweg's heuristic method – a form of Socratic, discovery-based learning in large-group settings – is necessarily tied to his concept of *Anschauung*, or whether it can function independently as a pedagogical tool. Moreover, since both the *Wegweiser zur Bildung für Lehrer…* and the *Praktisches Rechenbuch* continued to be published after Diesterweg's death, it would be worth exploring their influence on later didactical developments. Diesterweg's ideas, in many ways, anticipate modern concepts of visualization, discovery learning, and learner autonomy. A detailed reception history could help trace the trajectory of his thought into the 20th and 21st c.

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