DOI: 10.24425/asmdn.2025.158168

Peter Ullrich
ORCID 0000-0003-0913-0971
Fachbereich 3, Mathematisches Institut
Universität Koblenz
Koblenz, Germany

ABOUT THE TIME WHEN CALCULUS WAS BANNED IN PRUSSIAN GYMNASIA

Summary: The Meran Reform of 1905 is often credited with bringing infinitesimal calculus to the Prussian gymnasia and also to the secondary schools preparing for technical studies. Evidence for this comes from comparing the document on the final examination of the gymnasium from 1788 and the description of the curricula for secondary schools from 1882. A closer study shows, however, that the story is not that simple and that interesting things happened during the almost century between these years: The Humboldtian reform in the first decades of the 19th c. raised mathematics to the status of a main subject of the Prussian gymnasia. During the following years, however, other school subjects like Latin claimed their supremacy. In particular, in 1829 the Prussian authorities issued an order banning infinitesimal calculus from the gymnasia since it was considered too difficult for the pupils – which implies that it had been taught at some schools before! Furthermore, many teachers did not want to abandon the tasks dealing with the determination of local extrema and tried to get around the ban. The most prominent example was the Berlin mathematics educator Karl Heinrich Schellbach (1804–1892) who published a method that avoided the open use of infinitesimal ideas. Both Schellbach's and others' texts, even official documents, from this time also show that the term 'function' was standard for Prussian mathematics teachers in the mid-19th c.

Keywords: Prussian gymnasium, Humboldtian reform, infinitesimal calculus at secondary schools, Karl Heinrich Schellbach, schools preparing for polytechnical schools

Introductory remarks

The present article does not study infinitesimal calculus as a mathematical field for its own sake but uses the story of the appearing, disappearing and reappearing of calculus in Prussian gymnasia as a case study of general phenomena:

On the one hand, this story gives further evidence for the fact that history, in this case: history of mathematics education, often enough meanders rather than proceeds in a straightforward way.

On the other hand, one can identify different groups of 'stakeholders' within the events who became active and interfered at the stage of the development of mathematics education:

- firstly, the people with the 'big ideas': reformers – the heroes whom one likes to talk about and sometimes even worships,
- secondly, the state administrators who are supposed to feed the 'big ideas' into the educational system, but who are also responsible for keeping the whole system running,
- thirdly, the teachers in the classroom who have their own ideas and experiences about what should happen there, in particular, with respect to their own scientific discipline and its role within the educational system, and
- fourthly, groups of people, mainly from the economy, who look at the educational system as a means to train people with specific professional abilities[1].

A reason for choosing calculus as a case study

Such a case study can be undertaken with numerous other fields within mathematics education as a basis, for example, analytic geometry, conic sections, and geometry in three-dimensional space. But there is one specific reason for the choice of infinitesimal calculus on this occasion, namely the anniversaries of Felix Klein in 2024 (born: 25 April 1849) and in 2025 (died: 22 June 1925).

As for mathematics education in Germany, Klein's name is associated with the so-called Meran Reform or, more precisely, Meran plans for the German, in particular, Prussian, gymnasium which were initiated by him and approved by the general assembly of the German Mathematical Society (*Deutsche Mathematiker-Vereinigung*) at its meeting in Meran in 1905.

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[1] The analysis of a historical curriculum reform in the present article is structured with respect to these groups. Ian Westbury, on the contrary, has used the categories 'intended curriculum', 'enacted curriculum', 'implemented curriculum' when analyzing the curriculum reforms of the last decades. For a recent exposition of his approach see S. Rezat, I. Westbury, *Textbooks and Curriculum from a Governance Perspective*, [in:] *Fourth International Handbook of Mathematics Education*, ed. by M.A. (Ken) Clements, B. Kaur, T. Lowrie, V. Mesa, J. Prytz, Springer Nature, Cham 2024, p. 601–631.

Gert Schubring comments on the reception of this reform as follows:

> The Meran Reform represents an almost mythical reference point for the German didactics of mathematics, but its historical research has so far concentrated on the year 1905 and its immediate environment, as well as on the person of Felix Klein (1849–1925).[2]

An analysis of the Meran Reform with special emphasis on kinematic functional thinking has been undertaken by Katja Krüger. With respect to the consequences of this reform, she comes to the conclusion:

> The Meran Reform is credited with introducing the theory of functions and differential and integral calculus into higher mathematics teaching, and thus having a profound impact on gymnasium curricula in the 20th century. From this point of view, it appears that the ideas of the reformers around Felix Klein have been successfully incorporated into school practice since the beginning of the 20th century. The concept of function is at the center of lower-secondary education (Sekundarstufe I), and analysis lessons are now an essential part of upper-secondary mathematics.
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> However, if one measures the success of the Meran Reform against its original main goal of "educating toward the habit of functional thinking", a different picture emerges. In the following, it will be shown that against this background, the Meran Reform can be viewed as a failure. In order to find evidence and causes for failure, it is necessary first to explain the complexity of the term "functional thinking".[3]

The last quote indirectly claims that Klein brought both the notion of function and infinitesimal calculus into the secondary mathematics education in Germany. This claim is a frequent narrative in expositions on the history of mathematics education, but it is only true in the sense that the Meran Reform brought calculus *back* to the classroom from which it had been expelled during the 19th c. The introduction of the notion of function into secondary education was a complicated, long-lasting process[4]. Indeed, there exist so-called *Lehrpläne*, formal listings by the ministry of what should be taught, that do not contain the word 'function'[5].

[2] G. Schubring, *Der Aufbruch zum "funktionalen Denken": Geschichte des Mathematikunterrichts im Kaiserreich*, "NTM Zeitschrift für Geschichte der Naturwissenschaften, Technik und Medizin" 2007, vol. 15, p. 1.

[3] K. Krüger, *Kinematisch-funktionales Denken als Ziel des höheren Mathematikunterrichts – das Scheitern der Meraner Reform*, "Mathematische Semesterberichte" 2000, vol. 47, p. 221. See also eadem, *Erziehung zum funktionalen Denken. Zur Begriffsgeschichte eines didaktischen Prinzips*, PhD thesis, Universität Frankfurt (Main) 1999, also Logos Verlag, Berlin 2000.

[4] For Bavaria, the process of introducing the notion of function into the lecture halls and classrooms during the 19th c. was analyzed in the doctoral thesis: H. Säckl, *Die Rezeption des Funktionsbegriffs in der wissenschaftlichen Basis an Hochschule und Schule im neunzehnten Jahrhundert: eine Fallstudie zur Sozialgeschichte der Mathematik mit besonderem Blick auf Bayern*, PhD thesis, Universität Regensburg 1984.

[5] M. Mattheis, *Wie der Funktionsbegriff in die Schule kam*, "Siegener Beiträge zur Geschichte und Philosophie der Mathematik" 2020, vol. 13, Section 4.

But these curricula do not faithfully represent the reality in the classrooms: Many of the following quotes, even from official school documents, give evidence that the word ‘function’ was standard in texts for secondary mathematics education as early as the first decades of the 19th c.

Intentions and limitations

Therefore, the present article will give the proof that the notion of ‘function’ had entered mathematics classrooms in Prussia long before Klein only as a by-product. Instead, this article will concentrate on the change in the status of infinitesimal calculus in secondary mathematics education. Here, the timeline will run in the direction opposite to Krüger’s studies[6], not starting with the Meran Reform, but ending just before it and beginning instead with the reform of the Prussian educational system under Wilhelm von Humboldt (1767–1835) in the decade following 1809, which shaped the neohumanistic Prussian gymnasium.

It would be rather surprising if nothing had happened with this type of secondary school during the almost one hundred years until the Meran Reform of 1905. In fact, the status of infinitesimal calculus in mathematics education at Prussian gymnasia changed in the time from 1809 to 1905, but not in the order that one would expect from the rising economic importance of the natural and engineering sciences during this period, sciences that use infinitesimal calculus as a tool. Instead, the chronological order was as follows: from ‘should be obligatory’ to ‘accepted’ or at least ‘tolerated’, then to ‘not admitted as a topic of the final examination’ and, finally, to an explicit ‘forbidden’, even at those gymnasia that had an emphasis on mathematics and the natural sciences.

Surely, a complete study of the position of calculus in secondary education in Germany would have to take into account both the years before and after the Meran Reform. The restriction to the years between 1809 and 1905 in the present article, however, is not only due to questions of space but also to the fact that studies on the consequences of the Meran Reform already exist, such as Krüger’s analysis[7].

Furthermore, the study is restricted to the political structure to which the Meran Reform mainly aimed, namely the Kingdom of Prussia, even though numerous other states had established gymnasia as a type of school.

Only those types of secondary schools will be considered that could grant their graduates the right to study, whether at a university, at a polytechnic or one of its successor institutions such as a *Technische Hochschule* (an institute of technol-

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[6] K. Krüger, *Erziehung zum funktionalen Denken. Zur Begriffsgeschichte eines didaktischen Prinzips*. And eadem, *Kinematisch-funktionales Denken als Ziel des höheren Mathematikunterrichts – das Scheitern der Meraner Reform*.

[7] Ibidem.

ogy), even if the regulations of admission to such institutions of higher education were formalized only in 1834. Consequently, the gymnasium is at the center; the *Volksschule* (that granted the minimal schooling) and the *Bürgerschule* (a secondary school preparing students for practical professions, particularly in administration and business) will not be discussed, and the *Realschule* (a secondary school with an emphasis on mathematics, the sciences, and the modern languages) will appear only at the end. This also implies that only schools for boys will be under consideration[8].

The Humboldtian Reform of the gymnasium in Prussia

The reform of the educational system in Prussia from 1809 onwards was also a consequence of the disasters suffered by the Prussian army and state during the early Napoleonic Wars. In its beginning, the reform was directed by Wilhelm von Humboldt and is therefore associated with his name. Here, we are mainly interested in the part of this reform that referred to the gymnasium, a type of higher secondary school that had already been in existence before but now got a specific neohumanistic profile centered on three core school subjects: the classical languages (where the prefix ‘neo’ of ‘humanistic’ indicated that Greek was valued higher than Latin, at least in the beginning of the reform), history, and mathematics.

The story of the Humboldtian Reform has already been documented in the literature[9]. Thus, one can refer to those and other sources for an exposition of the general reform process and restrict the present study to those aspects that concern calculus itself.

One general aspect, however, should be stressed at the outset: A ministry or any other institution that wants to introduce a new kind of secondary education can use at least three kinds of administrative regulations to steer the reform:

- firstly, regulations concerning the examination of teachers for the type (or types) of school under consideration,
- secondly, regulations on what has to be taught at that type(s) of school: curricula decreed by the school authorities (*Lehrpläne*), and
- thirdly, regulations on what was going to be tested in the final examination at that type of school[10].

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[8] See K. Krüger, *The dawn of mathematics education for girls at the Höhere Mädchenschule in Prussia in the early 20th century*, this volume of “Analecta”.

[9] Specific for mathematics as a school subject is G. Schubring, *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870)*, 2nd, corrected and enlarged edition, Deutscher Studienverlag, Weinheim 1991, in particular Chapters 5 and 6.

[10] Furthermore, there were ‘Schulkonferenzen’ for the gymnasia in Prussia, meetings of large groups of teachers, teacher trainers and officials from the educational administration in 1873, 1890,

Obviously, it is most efficient if all these kinds of regulations are in accord. But for the Humboldtian reform in Prussia they differed both in the point of time of their publication and in their legal compulsoriness. This gave rise to frictions and tensions, not only in general but especially in mathematics[11].

The regulations for the examination of teachers for the gymnasium strengthened the position of mathematics as a school subject increasingly: The decree that first introduced these examinations (*Edikt, betreffend die Einführung einer allgemeinen Prüfung von Schulamtskandidaten*) on 12 July 1810 only prescribed that the candidates had to prove certain knowledge concerning philology, history, and mathematics in the examinations at the end of their university studies. The revision of this edict in 1831 then explicitly declared mathematics as one of the three main school subjects (besides history and old languages), of which one had to be studied in order to qualify as a teacher at a gymnasium.

The Süvernsche Lehrplan, in particular Tralles' curriculum for mathematics

The situation concerning both the curricula (*Lehrpläne*) and the regulations for graduation from the gymnasium (be it called *Abitur*, *Matura* or *Reifeprüfung*) were more complicated: Indeed, von Humboldt personally raised mathematics to the prominent status described above[12]. But he himself was trained in the humanities and did not have enough knowledge of mathematics to give guidance as to which of its fields and to what extent they should be included in secondary education. Concerning the competence to evaluate the different fields of mathematics, the same statement held true for the Prussian teacher and politician Johann Wilhelm Süvern (1775–1829), who was responsible for the gymnasia in the Prussian Ministry of Interior during the reform years from 1810 to 1816 and led the formulation of the first curriculum for the Prussian gymnasia, which is therefore called *Süvernscher Lehrplan*. (Such a *Lehrplan* had become necessary because

and 1900. These bidirectional elements in the reform process attracted considerable interest in the secondary literature but did not lead to significant changes in classrooms practice, partly because the participating teachers disagreed in their views.

[11] G. Schubring, *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870)*, Sect. 5.5., in particular p. 62–64.

[12] One of the reasons for this was that mathematics was related to the Greek antiquity. Another reason was the success of the French army which made the *École polytechnique* in Paris an interesting model for reformers with its fundamental training in mathematics. But already during the period of enlightened absolutism, princes had founded higher schools for their officers, both military and civil, that stressed the *Realien*: mathematics and the natural sciences but also modern languages. Seen from the point of view of mathematics, the most important of these was the *Collegium Carolinum* in Braunschweig which had been founded in 1745 and which Carl Friedrich Gauß (1777–1855) attended from 1792 to 1795.

the starting point of the reform of the educational system was that the Prussian state was responsible for this system and not the local authorities or the churches.) This curriculum was published in 1816 and sent to the regional authorities in all provinces of the Prussian kingdom (the so-called *Provinzial-Konsistorien*) as a guideline, but it was never enforced by law in that form[13]. Only in 1837 was a curriculum for the gymnasium published that had a legal status; it allotted three to four hours for mathematics per week.

As mentioned above, neither von Humboldt nor Süvern had any specific knowledge in mathematics. Therefore, the contents of the *Lehrplan* were discussed among the professional mathematicians who were involved as experts. The most important of them was Johann Georg Tralles (1763–1822), an academic and university scholar[14] who was chiefly responsible for formulating the mathematics part of the *Süvernsche Lehrplan*[15]. In his concept for the mathematics curriculum of the gymnasia one finds the following elements that point specifically towards calculus[16]. (The gymnasium started with the class *Sexta* and ended with the class *Prima*; the *Secunda* consisted of two and the *Prima* of even three school years.)

- *Sekunda*, 2nd year: fundamentals of the theory of series, theory of indeterminate coefficients and applications to some series, use of the theory of combinations for the multiplication and division of series, including their powers,

[13] For a study on its introduction see G. Schubring, *Der Süvernsche Lehrplan. "Ideales Muster" oder staatlicher Zugriff?*, "Zeitschrift für Pädagogik" 1990, vol. 36, no. 3, p. 393–418.

[14] Tralles was born in Hamburg, and studied at Göttingen with Abraham Gotthelf Kästner (1719–1800) and Georg Christoph Lichtenberg (1742–1799). Kästner recommended him for the 'Hohe Schule' at Bern, where Tralles was a professor of mathematics and physics from 1785. In 1804 he was called to the Berlin Academy as an ordinary member and from 1810 onwards he was secretary of the mathematical section. Additionally, he was the first full professor of mathematics (and physics) at the newly founded Friedrich Wilhelms University (today: Humboldt University) in Berlin. He was respected as an applied mathematician, geodesist, and physicist, and also wrote a *Lehrbuch der reinen Mathematik* of more than 300 printed pages in 1788. For a detailed exposition on Tralles see G. Schubring, *Zur Bedeutung von Johann Georg Tralles als erstem Mathematikprofessor der Universität Berlin*, [in:] *Natur, Mathematik und Geschichte. Beiträge zur Alexander-von-Humboldt-Forschung und zur Mathematikhistoriographie*, ed. by H. Beck, R. Siegmund-Schultze, C. Suckow, M. Folkerts, Deutsche Akademie der Naturforscher Leopoldina, Halle (Saale) 1997 (Acta Historica Leopoldina, vol. 27), p. 325–338.

[15] See G. Schubring, *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870)*, in particular Section 5.2 and Anhang: Dokument 1. Even if the 2nd edition of this book is used here, the analysis of the documents is already given in the 1st edition of this book in 1983. A later presentation of the topic is H.N. Jahnke, *Mathematik und Bildung in der Humboldtschen Reform*, Vandenhoeck & Ruprecht, Göttingen 1990 (Studien zur Wissenschafts-, Sozial- und Bildungsgeschichte der Mathematik, vol. 8), Section C.V.3.

[16] See G. Schubring, *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870)*, p. 44. Also H.N. Jahnke, *Mathematik und Bildung in der Humboldtschen Reform*, p. 347.

- *Prima*, 2nd year: arithmetic series, Taylor's theorem, series expansions using this theorem, and the connection with previous knowledge of series.

Here, however, Taylor's theorem was not considered a part of infinitesimal calculus but rather a method for developing a given power series at other points of expansion. Therefore, the above program could be handled without reference to differential and integral calculus[17].

Another member of the mathematics commission for the *Süvernsche Lehrplan*, however, wanted true infinitesimal calculus in the curriculum. Somewhat surprisingly, it was the person from the classroom, the teacher and school inspector Georg Wilhelm Bartholdy (1765–1815)[18]. He called for the following additions to the curriculum:

> In addition to what has hitherto been the norm, we must also teach what the French call géométrie descriptive and which is dealt with in Lacroix's more advanced expositions of geometry, furthermore, the elements of combinatorial analysis with the first principles of differential and integral calculus. The first of these is a not insignificant training and further development of constructive abilities, as it teaches how to design the constructions for the entire space according to all its three dimensions, which are constructed in elementary geometry only on a plane, and for space without depth: the other two contain such distinctive modes of investigation and methods of invention that a clear insight into these cannot be dispensed with for anyone who is interested in researching all of human knowledge, although I will conveniently ignore the material benefits here.[19]

Even though Bartholdy praised infinitesimal calculus with such impressive words, his suggestion with respect to this mathematical subject was unsuccessful: Calculus was not mentioned in the *Süvernsche Lehrplan*. However, this curriculum was intended by its authors as a minimal program that each gymnasium had to meet. Interested teachers were allowed to teach calculus in the first years of the Humboldtian reform. (For comparison: This was the same legal status for

[17] To describe this kind of approach to analysis in schools, Hans Niels Jahnke has used the term *algebraische Analysis* (algebraic analysis) in H.N. Jahnke, *Mathematik und Bildung in der Humboldtschen Reform*, e.g., p. 381. In G. Schubring, *Die Debatten um einen Mathematiklehrplan in Westfalen 1834. Eine regionale Sozialgeschichte der Einführung von Mathematik als Hauptfach*, WTM Verlag, Münster 2010, one finds a critical discussion of this wording, at least for the Prussian province of Westphalia. For a more recent exposition by Jahnke see H.N. Jahnke, *Die Algebraische Analysis in Felix Kleins "Elementarmathematik vom höheren Standpunkte aus"*, "Mathematische Semesterberichte" 2018, vol. 65, p. 211–251.

[18] Bartholdy went to school in Kolberg (today: Kołobrzeg, Poland) and Stettin (today: Szczecin, Poland) and studied at Halle. From 1787 to 1790 he worked at the Gedikesche Seminar for teacher training in Berlin and later as a gymnasium teacher there. On 30.06.1797 he became a teacher of mathematics and physics at the (later: *Vereinigte Königliche und Stadt-*) gymnasium in Stettin. Finally, he was a *Schulrat* (school inspector) and director of the pedagogical seminar.

[19] G. Schubring, *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870)*, p. 256–257.

calculus as in the years after the Meran Reform in 1905 until the *Richertscher Lehrplan* of 1924 which made it compulsory.)

And, in fact, there were not only individual teachers but even entire gymnasia that took advantage of the opportunity to extend the minimal program. For example, the gymnasium in Stralsund announced in its widely published program in autumn 1822 that the *Anfangsgründe der Functionenlehre* [Beginnings of the theory of functions] would be taught at this school[20].

The regulations for the graduation from the gymnasium

Contemporaneous with the curricula, but somewhat orthogonal in content, regulations were set up for the final examination of the pupils leaving the gymnasium for university. To be sure, there had already been such an edict in 1788, but this was only used to compare students from different schools when stipends and other benefits were granted at university. On 25 June 1812, however, a decree on the examination of pupils leaving for a university (*Edikt wegen Prüfung der zu den Universitäten übergehenden Schüler*) was issued, which made the final examination obligatory, at least the attempt to pass it. Again, failing the exam had negative consequences for stipends, but did not hinder admission to university. This examination consisted of six written tests: an essay each in German, Latin, and French, a written translation from Greek and one into Greek and a written exam in mathematics.

The decisive and fatal point, however, was that this regulation restricted the contents of the examination in mathematics to the first four classes of the gymnasium so that the topics listed by Tralles for *Secunda* and *Prima* were not relevant for the graduation from the gymnasium. This became even worse in the following years concerning calculus (and other mathematical subjects like analytic geometry, conic sections and spherical trigonometry) as the final examination at the gymnasium gained more importance, ending on 4 June 1834, with the regulation that it had to be successfully passed before one could enter into the study at a university.

The heaviest blow against additional mathematical subjects in the classroom then was a decree on December 14 of the same year that determined that the *Süvernsche Lehrplan*, originally intended as a minimal list, was now also to be considered a maximal list, which excluded the topics not explicitly mentioned in it[21]. This decree also gave a compilation of the mathematical topics relevant to the final examination at the gymnasium.

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[20] J.F. Neigebaur, *Die Preußischen Gymnasien und höheren Bürgerschulen: Eine Zusammenstellung der Verordnungen, welche den höheren Unterricht in diesen Anstalten umfassen*, Mittler, Berlin, Posen, Bromberg 1835, p. 177.

[21] See M. Folkerts, G. Schubring, *Adolph Tellkampf (1798–1869). Elementarmathematik und ihre Grenzen*, ed. by M. Folkerts, Dr. Erwin Rauner Verlag, Augsburg 2020 (Algorismus, vol. 86), p. 32.

Pushing back infinitesimal calculus

In the 1830s the influence of conservative, not to mention restorative, forces increased, partly due to the revolutions of 1830/1831. With respect to the Humboldtian reform of the educational system this could be perceived even within the old languages alone: As already mentioned, the specific neohumanistic approach emphasized Greek over Latin. But this was driven back by the 1834 regulations for the graduation from the gymnasium which increased the number of written tests in Latin at the expense of those in Greek.

Therefore, it is not surprising that mathematics was also hit by these tendencies, beginning even eight years earlier. The formal gateway was the inconsistency that the *Süvernsche Lehrplan* was completed only in 1816 and had no explicit legal status other than that of a guideline, whereas the regulations for the final graduation had already been published in 1812 with a much shorter catalogue of mathematical fields.[22]

The first explicit attack on calculus and also other mathematical fields as not adequate for the classroom of the gymnasium was a *Circular-Rescript* (circular decree) of the *Provinzial-Schulkollegium* (the regional school authority) in Magdeburg for the Province of Saxony of the Kingdom of Prussia, dated 11 October 1826. (The Province of Saxony plus the former Duchy of Anhalt now constitute the German federal state of Sachsen-Anhalt[23].) It contained the following text:

> If in modern times the teaching of the mathematical sciences has taken on the status and position it deserves in the curricula of our gymnasia, it must remain the guiding principle that it is always in a correct relationship to the classical studies, the pursuit of which gives each gymnasium its character, and not to impair classical studies either by the extent to which one assigns it, nor by the number of hours one allocates to it, nor by the work that one assigns to the private diligence of the students. This principle in particular must first decide on the scope of mathematical studies. And so, as lying outside its essential limits, are not included: the purely analytical treatment of conic sections, higher-order curves, the theory of higher-degree equations, the polynomial theorem in its generality, the development of logarithms and trigonometric functions in series, even if the representation is elementary, differential and integral calculus, applied mathematics in all its detail.[24]

[22] For a detailed analysis see G. Schubring, *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870)*, Section 5.5. Also M. Folkerts, G. Schubring, *Adolph Tellkampf (1798–1869). Elementarmathematik und ihre Grenzen*, p. 31.

[23] The story how it came to this decree and why this took place specifically in the Province of Saxony is told in M. Folkerts, G. Schubring, *Adolph Tellkampf (1798–1869). Elementarmathematik und ihre Grenzen*, p. 31–32.

[24] L. von Rönne, *Die Verfassung und Verwaltung des Preußischen Staates, Achter Theil: Die kirchlichen und Unterrichts-Verhältnisse, Zweiter Band: Das Unterrichts=Wesen. Höhere Schulen. Universitäten. Sonstige Kultur-Anstalten*, Veit & Comp., Berlin 1855, p. 224.

After this text comes a list of the items that were to be done in mathematics education instead, in particular it refers to a decree of 7 April of the same year, stressing the importance of practical arithmetic in the first two classes of the gymnasium. Notably, this stress on arithmetic seems not to have been due to economic reasons, for example bad experiences with everyday calculations, but resulted from the personal initiative of Martin Ohm (1792–1872)[25].

The line of argument in the above quote is obvious: The classical languages are seen as the essential core of the education at the gymnasium, whereas mathematics is considered only of secondary importance. Even if the decree was valid only for the Province of Saxony, it became detrimental for the whole of Prussia when another problem came into play.

On 29 March 1829, the ministry in Berlin issued a decree aimed at reducing the workload of pupils. Parents in the Province of Brandenburg (next to Berlin) had complained about having 38 teaching hours per week instead of the prescribed 32, as well as too much homework. The text of the decree was formulated against the teaching load in Greek rather than that in mathematics, but the urge of the ministry to reduce the teaching load in combination with the decree for the Province of Saxony was widely perceived as forbidding 'higher mathematics', in particular infinitesimal calculus at the gymnasium.

Reactions to the ban

A reaction from the far east of Prussia as early as 1830

Contrary to the usual picture of Prussian officials, including teachers, as obedient to authority, this interpretation caused contradiction, even far away from the central authorities of Prussia. Gumbinnen (today: Gussew, Russia) was the capital of the most eastern Prussian *Regierungsbezirk* (government district)[26]. The *Königliche Friedrichs-Gymnasium* there issued in 1830, just one year after the decree of the ministry mentioned above, a school program, a booklet that was directed to the general public and sent to other gymnasia as well as the adminis-

[25] B. Bekemeier, *Martin Ohm (1792–1872): Universitäts- und Schulmathematik in der neuhumanistischen Bildungsreform*, Vandenhoeck & Ruprecht, Göttingen 1987 (Studien zur Wissenschafts-, Sozial- und Bildungsgeschichte der Mathematik, vol. 4), Subsection I.2.a. And G. Schubring, *Die Entstehung des Mathematiklehrerberufs im 19. Jahrhundert. Studien und Materialien zum Prozeß der Professionalisierung in Preußen (1810–1870)*, Subsection 5.6. Getting back to the question whether the notion of 'function' was in use in mathematics education in the 19th c., on the one hand, the above quote gives explicit evidence for this. On the other hand, also Martin Ohm, more or less a mathematical autodidact, used this term, even though he restricted it to polynomials (including power series) and rational functions in M. Ohm, *Versuch eines vollkommen consequenten Systems der Mathematik, 1. Theil*, T. H. Riemann, Berlin 1822, p. XIV.

[26] For a discussion of the introduction of the *Süvernsche Lehrplan* in this district see G. Schubring, *Der Süvernsche Lehrplan. "Ideales Muster" oder staatlicher Zugriff?*, p. 407–409.

trative authorities. Such a program had to contain a scientific article written by one of the gymnasium teachers. In this case, it was J.G.A. Sperling who presented *Eine neue Methode das Maximum und Minimum zu finden* [A new method to find the maximum and minimum]. He introduced his mathematical presentation with the following text:

> According to the wise discretion of our high ministry of spiritual culture, in order to avoid excess in the mathematical disciplines in the gymnasia, the reasonable requirement was brought to mind that, among other violations of school teaching, the presentation of the differential calculus should be discontinued. This, of course, deprives the teachers who enjoyed having formed esoteric students of some of their pleasure, but does not entirely destroy the joy; because many individual things, e.g. what Taylor's theorem carries within itself, can also be achieved and found in an elementary way. I do not need to mention that this includes various series expansions; however, it would be pleasant to see the theory of maxima and minima, one of the most interesting chapters of differential calculus, treated without their help and based in such a way that the mathematical elements that our current school education is supposed to present in accordance with the regulations would already be sufficient to solve many problems about the maxima and minima. – The rescue of this so diverse and beautiful object, if one wanted to incorporate it – where it actually belongs – into the theory of functions, or to place this theory in isolation just for the purpose of use in homework exercises, would – if it could be made clear, safe and independent of higher calculus – certainly not be unpleasant also to those who do not allow themselves to cross over into the field of analysis of the infinite in their school lessons.[27]

One should note that the name used for the ministry in charge, in German: 'Ministerium des Geistlichen Cultus' was not ironic but the official short title. While a section in the Ministry of Interior was responsible for educational affairs, in particular the gymnasia, until November 1817, from that time onwards the responsibility passed to a newly founded *Ministerium der Geistlichen, Unterrichts- und Medizinal-Angelegenheiten* [Ministry of Spiritual, Educational and Medical Affairs].

But it becomes clear from the above text that the mathematics teacher Sperling was not willing to set aside the determination of maxima and minima in favor of other school subjects. His reasons may have included applications, particularly in physics, that can be treated with it and even lead to metaphysical ideas such as the principle of least action, but it may also have been that he preferred using the collected exercises on the determination of maxima and minima.

[27] J.G.A. Sperling, *Eine neue Methode das Maximum und Minimum zu finden*, [in:] *Zu der öffentlichen Prüfung im hiesigen Königlichen Friedrichs-Gymnasium am 30sten September und 1sten Oktober d. J. (Schulprogramm)*, Gedruckt in der Meltzerschen Buchdruckerei, Gumbinnen 1831, p. 1.

A systematic approach in a teachers' training institution in Berlin

As mentioned above, Gumbinnen was far away from Berlin. But the ban on infinitesimal calculus was also circumvented at Berlin itself, and this was done systematically by a very prominent mathematics teacher and teacher trainer. Karl Heinrich Schellbach (1804–1892) was a member of the commission for the examination of gymnasium teachers, the founding director of the *Mathematisch-pädagogisches Seminar* (mathematical-pedagogical seminar) for the practical training of gymnasium teachers in mathematics at the Friedrich Wilhelm-Gymnasium, and one of the co-editors of the "Journal für die reine und angewandte Mathematik" from 1857 to 1881. Last but not least, he was the personal mathematics tutor of the Prussian (Crown) Prince Friedrich (Wilhelm) (1831–1888), later Kaiser Friedrich III[28].

Schellbach's method for determining local extrema of functions can, in short and from a historical perspective, be described as the second method presented in Pierre (de) Fermat's (1607–1665) *De maxima et minima* [On maxima and minima][29]. As an algorithm in modern terms, it can be described as follows:

In order to find a local maximum or minimum x_m of a function f,

- take x_1, x_2 such that $x_1 < x_m < x_2$ will surely hold,
- factor from the difference $f(x_2) - f(x_1)$ as many linear factors $x_2 - x_1$ as one can:

$$f(x_2) - f(x_1) = (x_2 - x_1)^n \cdot g(x_1, x_2),$$

with $g(x_1, x_2)$ not divisible by $x_2 - x_1$ and
- then solve the equation

$$g(x_m, x_m) = 0$$

for the local maximum or minimum x_m.

Later authors have interpreted this and other methods connected with Schellbach's name in different ways. Klein, for example, considered it as a reformulation of the differential quotient as the limit of the slopes of secants and assumed that Schellbach only used this 'clothing' (Ger. *Einkleidung*) because differential calculus was forbidden at school and he did not want to miss out on these ideas ('Schellbach selbst hat diese Einkleidung gewiß nur gebraucht, weil die Differentialrechnung auf der Schule verboten war und er diese Ideen doch nicht

[28] For further details on Schellbach's life, work, and influence, see P. Ullrich, *Karl Schellbach (1804–1892) und seine Beiträge zu Mathematik, Lehrerbildung und Wissenschaftspolitik*, [in:] *Exkursionen in die Geschichte der Mathematik und ihres Unterrichts*, ed. by H. Fischer, T. Sauer, Y. Weiss, WTM Verlag, Münster 2021, p. 223–234.

[29] P. (de) Fermat, *De maxima et minima*, [in:] *Oeuvres de Fermat*, vol. 1, ed. by P. Tannery, C. Henry, Gauthier-Villars et fils, Paris 1891, Section IV. French translation in: *Oeuvres de Fermat*, vol. 3, Gauthier-Villars et fils, Paris 1896, p. 121–156.

missen wollte')[30]. Hans Niels Jahnke[31] and Heike Renate Biermann[32] question Klein's interpretation and assume that Schellbach used this method for reasons of content, namely in the tradition of *algebraische Analysis*[33].

Before disputing about Schellbach's intention, however, one should take into consideration that the source[34] that is quoted for it was not written by Schellbach himself but by two teachers, A. Bode and Eduard Fischer, who had attended his *Mathematisch-pädagogische Seminar*. Therefore, one cannot say whether Schellbach gave more detailed explanations and justifications for his method than the short geometrical argument found in *Mathematische Lehrstunden*[35]. In particular, one does not know whether Schellbach himself mentioned Fermat's treatise[36] and gave credit to him. The brevity of the exposition in *Mathematische Lehrstunden*[37] is all the more surprising since in an article on the calculus of variations[38] Schellbach explicitly stressed that one should not only apply algorithms but also understand them.

An example from a final examination at a gymnasium in the year 1904

Hermann Weyl (1885–1955) took his PhD with David Hilbert (1861–1943) in 1908, became a professor at the *Eidgenössische Technische Hochschule* in Zurich in 1913 and Hilbert's successor in Göttingen in 1930. Furthermore, he was one of the founding faculty members of the Institute for Advanced Study in Princeton in 1933. At Easter 1904, one year before the approval of the Meran plans by the German Mathematical Society, Weyl passed his graduation at the gymnasium *Königliches Christianeum in Altona*. At that time, Altona belonged to Prussia (and became incorporated into Hamburg only in 1937/1938). So, Weyl's solution to his final mathematics test gives an example of how an excellent schoolboy would deal with the problems posed.

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[30] F. Klein, *Elementarmathematik vom höheren Standpunkte aus*, vol. 1, 4th ed. (reprint of the 4th ed., Berlin 1933), Springer, Berlin/Heidelberg 1968 (Die Grundlehren der mathematischen Wissenschaften in Einzeldarstellungen, vol. 14), p. 240.

[31] H.N. Jahnke, *Mathematik und Bildung in der Humboldtschen Reform*, p. 327.

[32] H.R. Biermann, *Praxis des Mathematikunterrichts 1750–1930. Längsschnittstudie zur Implementation und geschichtlichen Entwicklung des Mathematikunterrichts am Ratsgymnasium Bielefeld*, PhD thesis, Universität Duisburg Essen 2010, also Logos Verlag, Berlin 2010, p. 319–321.

[33] For a discussion of the term *algebraische Analysis* see footnote 17.

[34] K.H. Schellbach, *Mathematische Lehrstunden, Aufgaben aus der Lehre vom Größten und Kleinsten. Bearbeitet und herausgegeben von A. Bode und Eduard Fischer*, G. Reimer, Berlin 1860.

[35] Ibidem, p. 17.

[36] P. (de) Fermat: *De maxima et minima*.

[37] K.H. Schellbach, *Mathematische Lehrstunden*, p. 16–19.

[38] Idem, *Probleme der Variationsrechnung*, "Journal für die reine und angewandte Mathematik" 1851, vol. 41, p. 293–363.

One of the problems was to circumscribe a triangle of smallest area to a given parallelogram[39]. Denoting by h the height of the parallelogram and by y the height of the triangle, Weyl transferred this to the task to minimize

$$f = \frac{y^2}{y-h}$$

as a function of y. For this he set up the equation

$$f(y+\varepsilon) = f(y), \text{ i.e., } \frac{(y+\varepsilon)^2}{y+\varepsilon-h} = \frac{y^2}{y-h}$$

(without mentioning that ε should be infinitely small), and transformed it to

$$(y+\varepsilon)^2(y-h) = y^2(y-h+\varepsilon).$$

Suppressing the infinitely small terms of higher degree ('mit Und[!]erdrückung der von höherem Grade unendlich kleinen Glieder')[40], Weyl got the equation:

$$y^2(y-h) + 2y\varepsilon(y-h) = y^2(y-h) + y^2\varepsilon,$$

which he resolved as $2(y-h) = y$, i.e. $y = 2h$.

Besides this solution, Weyl also gave two further ones. On the one hand, he applied the discriminant method to the equation $y^2 - fy + fh = 0$. On the other hand, he stated that the easiest way to achieve the result is to use differential calculus ('Am einfachsten führt die Anwendung der Differentialrechnung zum Ziel')[41] and calculated from $f = \frac{y^2}{y-h}$ the condition

$$\frac{df}{dy} = \frac{2y(y-h) - y^2}{(y-h)^2} = 0.$$

Notably, according to the annual report of his school, differential calculus had not been treated in the school lessons[42].

Schools preparing for (poly)technical schools

From its foundation in 1794 onwards, the *École polytechnique* in Paris trained its students on the basis of a strong common foundation in higher mathematics like calculus but also conic sections, descriptive geometry, etc. This was the blueprint for numerous polytechnic schools in the German states in the first half of

[39] B. Elsner, *Hermann Weyls Abiturarbeit*, "Mitteilungen der Mathematischen Gesellschaft in Hamburg" 2002, vol. XXI, no. 2, p. 7 ff.

[40] Ibidem, p. 8.

[41] Ibidem, p. 14.

[42] Ibidem, p. 6.

the 19th c., such as those in Darmstadt, Dresden, Hannover, Karlsruhe, Munich, etc. (in alphabetical order). Contrary to this approach, Prussia had special types of schools for different areas of application for a long time. For example, in Berlin the *Gewerbeschule* / *Gewerbeakademie* (the commercial academy) and the *Bauakademie* (the construction academy) were merged to the *Technische Hochschule Charlottenburg* only in 1879, with the *Bergakademie* (the mining academy) joining this institution even as late as in 1916. The first polytechnic institution founded by Prussia was the *Rheinisch-Westfälische Polytechnische Schule* in Aachen in 1870, even if the Polytechnic at Hannover was Prussian since the occupation of the Kingdom of Hannover in 1866.

A large part of the students at the (poly)technical schools had not graduated from the neohumanistic gymnasium, which has been discussed up to now, but came from other types of secondary schools instead. At first, they came mainly from the *Realschulen*, where *Real* referred to the *Realien*: modern languages (English, French), natural sciences, and mathematics. But during the course of the 19th c., both the social status and the academic level of the studies offered at the (poly)technical schools increased. This made it necessary to upgrade the schools that could grant their graduates the right to study at a (poly)technical school or, later on, even higher institutions. So, in 1859 the school type *Realschule erster Ordnung* was created, which was later renamed *Realgymnasium* (by law in 1882) and, since the 1870s, the *Oberrealschule* also emerged.

Whereas the neohumanistic gymnasium could award its graduates the right to matriculate for all kinds of university studies, these other secondary schools could grant only restricted rights: graduates from *Realschulen erster Ordnung* were allowed to study at polytechnic schools since 1859 and even, beginning in the 1870s, at universities but had then to restrict to scientific disciplines like mathematics and the natural sciences. The same opportunities were opened for the graduates of the *Oberrealschulen* some years later: studies at polytechnic schools from 1882 onwards and at universities from 1892 onwards, again with restrictions concerning the specific studies that could be taken. (One has to admit that the neohumanistic gymnasium dominated the system still around 1900 if all scientific disciplines were taken together. Then about seven times as many students had graduated from gymnasia than from *Realgymnasia*, whereas the number of those who had graduated from *Oberrealschulen* was almost negligible.)

Characteristic features of the three different types of secondary schools were that at the neohumanistic gymnasia both Latin and Greek were taught, at *Realschulen erster Ordnung* / *Realgymnasien* only Latin, and at *Oberrealschulen* no classical languages at all; the total number of mathematics hours during the whole time at school was 34 for the gymnasium, 44 for the *Realschule erster Ordnung* / *Realgymnasium* and 49 for the *Oberrealschule*.

These facts indicate that the curricula and the timetables both for the *Realschulen erster Ordnung* / *Realgymnasien* and for the *Oberrealschulen* were considerably more oriented towards mathematics and away from the classical languages than those for the gymnasia. Taking into account that a conflict between these two school subjects had led to the ban of infinitesimal calculus, one could hope that in these two types of schools the situation would have been better.

However, in 1856, just three years before the constitution of the *Realschule erster Ordnung*, a new *Lehrplan*, authored by Ludwig Wiese (1809–1900), was issued, in which he explicitly wrote:

> In m a t h e m a t i c s, the requirements must be kept exactly within the limits set by curriculum for the gymnasia.[43]

Of course, this specific stress on restriction in mathematics implies in turn that there were quite a few mathematics teachers who did not accept the limitations for their school subject.

In fact, when the regulations for the graduation from the *Realschulen erster Ordnung* were issued on 6 October 1859, topics like plane trigonometry, stereometry, elements of descriptive geometry and conic sections were listed, but still infinitesimal calculus was absent, which amounted to a ban on it even in this type of school[44].

Again, however, there were teachers and even entire schools that would not accept this situation with respect to infinitesimal calculus (and also to analytic geometry of space). In 1882 the roles of the three types of secondary schools discussed above were formally settled. This distinction necessitated a further revision of the curricula and the regulations for the graduation, which did not change much with respect to the curricula and the number of mathematics hours per week. But the elements of integral calculus were completely eliminated for all three types of schools, whereas differential calculus (and analytic geometry of space) was left as permissible for the *Oberrealschulen*, but only for them. A further revision in 1892, however, removed even this exception.

Concluding remarks

It is not surprising that not all the 'big ideas' of a great reform of an educational system will become reality when the reform is carried out. But in the case of the Humboldtian Reform of the Prussian gymnasium the influence of the teachers of Latin is rather remarkable: They were able not only to regain their leading

[43] L. Wiese, *Das höhere Schulwesen in Preussen. Historisch-statistische Darstellung*, vol. 1, Wiegandt und Grieben, Berlin 1864, p. 498.

[44] See W. Lietzmann, *Methodik des mathematischen Unterrichts. 1. Teil: Organisation, Allgemeine Methode und Technik des Unterrichts*, Quelle & Meyer, Leipzig 1926 (Handbuch des naturwissenschaftlichen und mathematischen Unterrichts, vol. 7, part 1), p. 254–255.

position over Greek in the classical languages but also to claim dominance over mathematics.

But the Humboldtian Reform also intended to treat mathematics teachers as scholars. Therefore, the events to the advantage of only one school subject, Latin, seemingly provoked mathematics teachers to oppose the rules set up by the administration by trying to circumvent them. That even a prominent person like Schellbach did so, in printed form, indicates once more that there is no direct dependence of the situation in the classroom on the decrees issued by the authorities of the educational system.

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Peter Ullrich – held university positions for mathematics, its didactics and its history in Münster, Gießen, Augsburg, Siegen, and Koblenz. He is mainly interested in the history of theoretical mathematics, in particular algebra and analysis, in the 19th and early 20th c., including some side glances at other mathematical topics and into the 16th and 17th c. Among other things, he is the editor of the book series *Schriften zur Geschichte der Mathematik und ihrer Didaktik*, WTM-Verlag.
e-mail: ullrich@uni-koblenz.de