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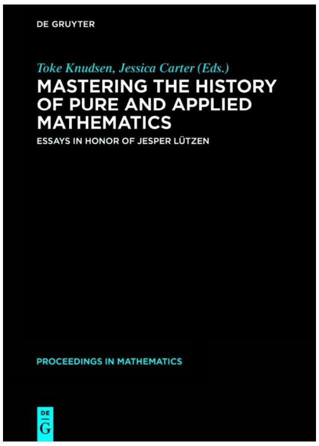

BOOK REVIEW

MASTERING THE HISTORY OF PURE AND APPLIED MATHEMATICS: ESSAYS IN HONOR OF JESPER LÜTZEN, ED. BY TOKE KNUDSEN, JESSICA CARTER, DE GRUYTER, BERLIN–BOSTON 2024.

Mastering the History of Pure and Applied Mathematics is a Festschrift dedicated to Jesper Lützen, professor emeritus at the University of Copenhagen and a distinguished historian of mathematics. The title, which echoes Lützen's seminal monograph *Joseph Liouville 1809–1882: Master of Pure and Applied Mathematics*[1], reflects his lifelong engagement with the history of mathematics, its intersection with physics, and its pedagogical implications.

The contributions gathered in this volume range from Hellenistic astronomy to the history and sociology of mathematics in the 21st c., testifying both to Lützen's scholarly impact and to his role in the international community of historians and philosophers of mathematics.

The volume opens with an introduction by the editors, Toke Knudsen (State University of New York at Oneonta) and Jessica Carter (Aarhus University). Both are Lützen's colleagues and members of the lively Danish and international community of historians and philosophers of mathematics, to whose growth Jesper Lützen has greatly contributed. The introduction outlines Lützen's academic trajectory and includes personal reflections on the editors' collaboration with him.

[1] J. Lützen, *Joseph Liouville 1809–1882: Master of Pure and Applied Mathematics*, Springer, New York, Berlin [etc.] 1990.

The eleven chapters composing this homage not only honor Jesper Lützen, his research, and his legacy over the past forty years, but they also provide insightful perspectives on the evolution of the history of mathematics as a field, particularly its changing relationship to both mathematics and history.

The explicit thread connecting the chapters is biographical: the contributions engage with aspects of Lützen's research and reflect the authors' collaboration with him. Simultaneously, the collection offers a snapshot of the directions in both content and methodology taken by the contemporary history of mathematics.

This twofold character of the volume, personal and programmatic, can be best captured by starting with the last chapter, Henrik Kragh Sørensen's 'What is history of mathematics to twenty-first century mathematicians?'. Sørensen provides an overview of the major historiographical tensions of the 19th and 20th c. As he remarks, citing the late mathematics historian Ivor Grattan-Guinness, the history of mathematics is a relatively young discipline, yet it risks being relegated to a 'residual category' (p. 235). In the 19th and early 20th c., it was primarily regarded as a subfield of mathematics. Historians of mathematics typically entered the field from a background in mathematical research, with modern mathematics as their main focus. This trajectory was regarded by the eminent mathematician and historian of mathematics André Weil as not merely common but an essential feature of the discipline. This necessity stemmed from its perceived nature and aims at the time: the history of mathematics was conceived mostly as a search for precursors to modern mathematical ideas, a task that could be accomplished only by experts familiar with contemporary mathematics.

A significant shift occurred with a later generation of scholars, ushered in by Sabetai Unguru, whose work on Greek mathematics argued for the need to rewrite its history to study earlier mathematical cultures according to their own standards. Remarkably, this shift became manifest in the 1970s, when the history of mathematics began to assert itself as an autonomous academic discipline at the intersection of mathematics and history. The tension over disciplinary identity, with the history of mathematics occupying a grey area between the sciences and humanities, has led to significant debates that remain unresolved today.

The chapters in this volume may be read as different responses to this historiographical conflict. While many, if not all, of the contributions offer a view of the field as centered on mathematical concepts, they all take great care not to detach these from the broader networks of historical, social, and political contexts in which they are embedded. In other words, a shared conviction emerges that mathematical concepts are not separable from the mathematicians who thought about and used them. Understanding these concepts requires recovering their past uses and meanings.

A compelling illustration of this shift in the historiographical approach is provided by Bruno Belhoste and Karine Chemla's chapter on 'Ideal chords in Poncelet's work, from the Saratov notebooks to the *Traité des propriétés projectives des figures*'. This chapter explores the distinction Poncelet made between 'ideal' and 'imaginary' elements in his mathematical practice, a nuance often overlooked by later commentators. It thus serves as a case study cautioning against the tendency to anachronistically impose later frameworks, such as Hilbert's theory of ideal elements. Drawing on Poncelet's 1822 treatise and preparatory manuscripts, the authors reconstruct the subtleties of his geometrical thinking and its sources of inspiration. Rather than seeking precursors to modern ideas, Belhoste and Chemla aim to recover displaced or forgotten trajectories, the study of which can reveal unexpected legacies.

Connected to these themes, I would like to highlight another major strand in Lützen's work: the history of mathematical impossibility, the subject to which he has devoted several papers and a book published in 2023[2].

Philip Beeley's chapter, 'Revisiting Fermat: John Wallis and the English reception of number theory in the second half of the seventeenth century', addresses Fermat's well-known theorem concerning the impossibility of expressing a cube as the sum of two cubes. Fermat issued this 'negative theorem' (this expression was used by his contemporaries, such as Wallis) as a challenge to his English correspondents: echoing Lützen's thesis that impossibility results often stimulate rather than hinder mathematical progress, Beeley shows that Fermat's challenge spurred British mathematicians to develop number theory in new directions. Evidence of this change can be found in Wallis's manuscripts, which contain a flawed attempt at an impossibility proof. Their content and style are discussed in Beeley's chapter in this volume.

In his long-term study of impossibility theorems, Lützen argued that in the 19th c., such results, which were previously formulated as metamathematical or normative principles guiding mathematicians' activities, became proper mathematical theorems. Leo Corry's chapter, 'Von Neumann and impossibility, from Goedel to EDVAC', explores this theme in John von Neumann's work. Corry persuasively argues that four key impossibility results (namely Gödel's incompleteness theorem, von Neumann's own no-hidden-variable result in quantum mechanics, Turing's solution to the *Entscheidungsproblem*, and limitations in early computing) were central to von Neumann's intellectual development. These challenges did not impede his work; rather, they acted as catalysts for innovation.

Before the rise of 19th c. mathematics, algebra and number theory had already offered tools for proving impossibility results, unlike geometry. Jens Høyrup's chapter, 'Which groups, which forces transformed abbacus algebra, thus creating intellectual infrastructure for the scientific revolution? Reflections on the possibility to transfer the Zilsel thesis', revisits the birth of 'modern' algebra through

[2] J. Lützen, *A history of mathematical Impossibilities*, Oxford University Press, Oxford 2023.

the lens of Edgar Zilsel's thesis that science arose when early modern intellectuals adopted the empirical methods of artisans. It should be noted that the adjective 'modern' is used here in contrast to the algebra of the abbacus masters of the previous centuries. Høyrup identified three groups involved in shaping early modern algebra – abbacus masters, humanists, and university scholars – and traced how their interactions led to Viète and Descartes's symbolic algebra. This new algebra functioned as a kind of 'metatheory' offering a framework to assess the solvability (and sometimes the insolvability) of classical geometric problems, such as cube duplication, angle trisection, and the circle-squaring problem. Incidentally, regarding impossibility results in geometry (e.g., angle trisection and cube duplication), Lützen made the interesting discovery that Descartes sought a purely geometric proof, apparently overlooking the potential of algebra for dealing with 'impossible proofs'[3].

The theme of impossibility in classical geometry reflects both the sophistication of ancient mathematics and its conceptual transformation over time. This connects two contributions focused on 19th c. geometry: Jeremy Gray's chapter on Liouville's "Journal de Mathématiques Pures et Appliquées" ('Liouville's *Journal and the Journal de l'École Polytechnique*: a brief comparison') and David Rowe's chapter on the legacy of classical Greek geometry in the 19th c. ('Geometrical themes from Greek antiquity: how they resonated in the nineteenth century'). Both build on Lützen's historical research on 19th c. mathematics. Gray's essay, which draws explicitly on Lützen's monograph on Liouville, examines the "Journal de mathématiques pures et appliquées" and the "Journal de l'École Polytechnique", showing how these competing journals helped shape French mathematical culture. Liouville's journal was particularly important for circulating new conceptions of mathematics, such as Poinsot's view of it as a science of number, order, and measure, a framework that paved the way for modern structuralist conceptions of mathematics. Significantly, the journal also published Wantzel's proof of the impossibility of doubling the cube and trisecting an angle by ruler and compass – two classic problems of antiquity. It should be noted that Liouville himself contributed to the history of the third 'famous problem', namely the squaring of the circle, with an article on the existence of transcendental numbers published in 1844 in "Comptes rendus de l'Académie des Sciences".

Rowe highlights how the study of conic sections had a 'renaissance' in the 19th and early 20th c. owing to developments in projective geometry and geometrical optics. At the same time, British mathematicians maintained a strong interest – which Rowe calls a 'fascination' – in Euclid's *Elements*, above all for pedagogical reasons. Figures like Charles Dodgson (better known as Lewis Car-

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[3] See J. Lützen, *The algebra of geometric impossibility: Descartes and Montucla on the impossibility of the duplication of the cube and the trisection of the angle*, "Centaurus" 2010, vol. 52, no. 1, p. 4–37.

roll) defended Euclid's *Elements* as the best model for teaching formal reasoning, while others called for reform. A key insight of Rowe's contribution is that these 19th c. debates shaped the emerging historiography of mathematics, as seen in the works of Zeuthen, Neugebauer, and van der Waerden, thus underscoring the mutual influence of historical and contemporary mathematical inquiry.

Mikkel Willum Johansen's chapter, 'Cognitive artifacts in the history of mathematics', introduces a novel dimension to the recent history of mathematics resulting from a more intense dialogue with the philosophy of mathematics, especially the philosophy of mathematical practice: the study of cognitive artifacts and concrete tools used to represent abstract ideas. These include not only physical instruments such as calculators and geometric devices, but also symbols and diagrams. Johansen's two central theses are compelling. First, he argues that cognitive artifacts make otherwise intractable ideas manageable, and so they change mathematical practice. For example, it is sufficient to consider the algebra of complex numbers and how this enables the solution of impossible problems in geometry. The second thesis of this chapter is that cognitive artifacts in mathematics can themselves become objects of mathematical investigation. According to Johansen, the use of geometric diagrams in graph theory is an example of how cognitive tools can be employed to study other cognitive artifacts, namely polynomial equations. Another example, not mentioned in Johansen's paper, is the mathematical theory of geometric machines introduced by Alfred Kempe in the 19th c., in which algebra was used to study the behaviour of geometric artifacts (compasses).

Finally, three chapters stand out for their focus on mathematical interactions with other sciences: physics, biology, and astronomy. A common thread among them is that they all discuss models as 'epistemic resources', namely as 'tools used by scientists in producing knowledge' (see the chapter by T.H. Kjeldsen and A. Loettgers, p. 182).

In 'How Heinrich Hertz mapped the radiation field' (Chapter 3), Jed Buchwald analyzes the interplay between experimental practice and mathematical abstraction in Hertz's detection and mapping of the electromagnetic field. Hertz represents the behaviour of radiation fields using geometric diagrams. In this formalization of the electromagnetic field, the material oscillator – that is, the physical device used by Hertz to detect the field – is not represented. This deliberate omission suggests a change in the relationship between the mathematical models and their physical background. It can be concluded that, for Hertz, the nature of the electromagnetic field consists of equations describing the mathematical field and its geometrical representation, without the need for supplementary mechanical sources.

In Chapter 8, Tinne Hoff Kjeldsen and Andrea Loettgers examine: 'Nicolas Rashevsky and Alfred Lotka: different modeling strategies in the beginning of mathematical biology in the early twentieth century'. Nicolas Rashevsky (1899–1972) used a strategy of 'explorative modelling' (an expression introduced by A. Gelfert). To explain a biological phenomenon, such as cell division, he introduced an analogy with a physical situation, such as the behavior of droplets. The goal of the droplet model was to transfer the mathematical equations that describe the physics of droplet division to the case of the cell. The goal of the model was to explain the latter behavior using the same mathematical apparatus elaborated for a different context. However, the model did not offer any insight into the biological mechanisms that govern cellular division. The reductionist character of Rashevsky's model has led to criticism from biologists. By contrast, Alfred Lotka (1880–1949), best known for the Lotka-Volterra equations that describe population dynamics, adopted a top-down approach to modelling biological phenomena. Lotka began with the abstract formal structures common to physical, chemical, and biological systems rather than empirical situations, and described them using mathematical formalism. This approach is in direct opposition to Rashevsky's bottom-up model.

In Erhard Scholz's chapter, 'From heliocentrism to epicycles: a commentary on pre-Ptolemaic astronomy', the models concern planetary astronomy before Ptolemy. Scholz challenges the thesis that ancient heliocentrism was merely speculative and conjectures that the necessity to explain the apparent motion of planets and the Sun motivated 3rd c. BCE astronomers, between Aristarchos of Samos and Apollonius, to construct epicyclical planetary models. These models could also have provided quantitative predictions – for example, about the distances of the planets to the Sun under a heliocentric hypothesis. While heliocentrism was later abandoned in favor of geocentrism, the geometric machinery of epicycles was retained, as well as conceptions that could be legacies of a pre-Ptolemaic heliocentric theory, such as the order of the planets.

To conclude, I will return to the starting point of this review, namely Sørensen's question, which also gives the title to his chapter: What is the history of mathematics to 21st c. mathematicians? Sørensen's empirical analysis of a database formed by online communities reveals an ambiguous attitude shown by mathematicians toward the history of mathematics, or maybe toward historians generally. While mathematicians show a genuine interest in the history of their discipline, they are also reluctant to engage with scholars in the history of mathematics and with the scholarly literature, which is seldom quoted in online discussions. There may be several reasons behind this diffidence: a certain enduring belief among mathematicians in the universality and cultural neutrality of mathematical thought, but also the over-specialization and professionalization of today's historians, which often leads them to write primarily for their peers on subjects that do not interest mathematicians. Against this backdrop, this volume stands out as a bridge between the two communities, much like Jesper Lützen's teaching and research career, which combines the study of mathematics with sensitivity to the historical context.

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