

A NEW METHOD FOR DETERMINING AND EVALUATING THE ACCURACY OF THE CUTTING TOOL WEAR CURVE INDEPENDENT OF THE CUTTING PARAMETERS IN PROFILE TURNING PROCESSES WITH POINT-TIP TOOLS

Józef Zawora

Summary

The major goal of this work is to present a new method for determining the relationship of the cutting tool wear $VB_B = f(t/T)$ independent of the cutting parameters and to perform local statistical evaluation of its accuracy. It is connected with contour turning processes employing form tools, in varying machining conditions. The method is valid for a particular machine tool-clamping device-part-tool system, and accepted range of machining parameters: cutting speed, feed rate, depth of cut taken as perpendicular to the axis, angle of the contour. Processes of contour turning with point-tip tool may be implemented on CNC lathes, auto-lathes and tracer lathes with hydraulic control. The wear function which would be independent of the cutting parameters is founded on the sets of experimental results using the technique of experimental design. Box Wilson 5-level experimental design of second order, orthogonal, rotatable, compositional, for four-dimensional factor space was employed. Analytical description of the function was adopted, as a certain degree polynomial satisfying a condition of passing through the point at the co-ordinate system origin. Tool wear characteristics was determined by two methods: averaging of polynomials representing individual experiments and the approximation of all the tests together. The following requirements were taken into account: positive value of the function derivative and the lowest possible curve undulations. The averaging the coefficients of polynomials was selected as the best method. The local assessment of tool point wear curve accuracy has been performed by analyzing the course of the confidence level local values in the measurement points determined using dimensionless coordinate t/T . The possible applications of tool wear curve were mentioned. VB_B – tool edge wear level, the wear land on the flank in the B zone.

Keywords: dimensionless wear curve, accuracy assessment

Nowa metoda wyznaczania i oceny dokładności krzywej zużycia ostrza niezależnej od parametrów skrawania w procesach toczenia kształtowego narzędziami punktowymi

Streszczenie

W pracy zaprezentowano nową metodę wyznaczania krzywej zużycia ostrza $VB_c = f(t/T)$ niezależnej od parametrów skrawania i statystyczną lokalną ocenę jej dokładności. Dotyczy procesów toczenia kształtowego narzędziami punktowymi w zmiennych warunkach obróbki. Metoda jest prawidłową dla określonego układu OUPN i przyjętego zakresu wartości parametrów obróbki: prędkości skrawania, posuwu wypadkowego, głębokości skrawania oraz kąta zarysu. Procesy toczenia kształtowego narzędziami punktowymi są

Address: Józef ZAWORA, Ph.D. Eng., Institute of Manufacturing Technology, Warsaw University of Technology, 00-663 Warsaw, Al. Niepodległości 222, e-mail: jzawora@meil.pw.edu.pl

realizowane na tokarkach i automatach tokarskich CNC oraz tokarkach kopiarkach ze sterowaniem hydraulicznym. Podstawą do wyznaczenia krzywej zużycia niezależnej od parametrów skrawania są zbiory wyników badań doświadczalnych z zastosowaniem techniki planowania eksperymentów. Wprowadzono pięciopoziomowy plan doświadczeń Boxa Wilsona drugiego rzędu, ortogonalny, rotatabilny, kompozycyjny dla czterowymiarowej przestrzeni czynnikowej. Przyjęto analityczny opis krzywej w postaci wielomianu określonego stopnia z zadaniem warunkiem przejścia krzywej przez punkt początku układu współrzędnych. Charakterystykę zużycia ostrza wyznaczono dwoma metodami: uśredniania wielomianów poszczególnych doświadczeń oraz aproksymacji wszystkich doświadczeń łącznie. Uwzględniono warunki: dodatniej wartości pochodnej i możliwie najmniejszego pofalowania. Jako najlepszą wybrano metodę uśredniania współczynników wielomianów. Dokonano lokalnej oceny dokładności krzywej zużycia ostrza za pomocą przebiegu lokalnych wartości poziomu ufności w punktach pomiarowych bezwymiarowej współrzędnej t/T . Wskazano na zastosowanie krzywej zużycia, VB_B – współczynnik zużycia ostrza – wysokość starcia na powierzchni przyłożenia w strefie B .

Słowa kluczowe: bezwymiarowa krzywa zużycia, ocena dokładności

1. Introduction

In accordance with the adopted plan of the experiment, tool life T was measured, by measuring wear parameter VB_B of the cutting edge in time t for $N = 31$ experiments [1], for various parameters of form turning: v_c, f, a_w, α_z until a critical tool wear $VB_{B,kr} = 0.3$ mm occurred. Depending on machining parameters, the measurements were carried out at intervals, Δt ranging from several seconds to several minutes. In order to take into account the impact of wear of the tool edge, the other process variables such as selected surface roughness parameters, components of cutting forces in motion in the stable motion and dynamic components in the instant motion, the power of cutting, etc., the time variable should be additionally introduced into the factor space as an independent variable. After entering the time variable, the number of experiments would almost double when compared with the number of experiments in previous space of four factors, which would result in much greater consumption of test material, and hence a significant increase in costs is involved. In order to avoid this, a special kind of characteristics was introduced describing a ratio of current quantity values to their average value and they became related to dimensionless ratio t/T where t is tool point operation time and T is tool point life value.

$$\frac{y_i}{y_{i, \bar{s}r}} = f_i \left(\frac{t}{T} \right), \quad i = Rz, Ra, F_x, F_y, F_z, F_{xd}, F_{yd}, F_{zd}, P_s. \quad (1)$$

It has been shown in the available references [2-6] that the tool edge wear depends on the dimensionless ratio t/T , or the ratio of the tool point operation to the tool point life T and describes a fraction or percentage of tool point life used. These considerations predominantly concern the longitudinal turning. An analysis of the applicability of this relationship in determining the tool point wear increments in varying process conditions has been referenced [2]. This can

be attributed, inter alia, to the similarity of geometric tool edge wear functions $VB_j = f_j(t)$ (where $j = 1, 2, \dots, N$) obtained at different process parameters for a particular machine tool-clamping device-part-tool system. It is obviously valid for the second range of tool life vs. cutting speed function when the tool life decreases with the cutting speed increase. A significant increase in the geometric similarity of wear curves determined from experiments performed according to the experimental plan for different process parameters can be obtained by presenting them by a dimensionless coordinate t/T (abscissa axis). Such functions can be determined by dividing the amount of time after which individual wear values have been measured for the given tool edges by the relevant tool life values. The equations describing individual wear curves for all the experiments within experimental design can be achieved using computer programs based on polynomial approximation using the minimum sum of squares condition. Individual wear curves can also be obtained by approximations of measurement points in the natural coordinates (VB_i, t_i) , and then applying the scaling of polynomials coefficients. The scaling is aimed at determining the polynomials approximating the wear curve shape using dimensionless independent variable $t/T \in [0, 1]$. Given the very high degree of geometric similarity of these curves it is advisable to replace them with a single curve which would be independent of parameters of cut in the process of profile turning by the point – tip tools. Purpose of this part of the work is to propose methodology for such a curve construction and the evaluation of wear estimation accuracy. Basic requirements to be adopted for the construction of such a curve are as follows: the curve should be a monotone function, increasing, smooth as possible without waving, passing through the origin $[0, 0]$ and the most accurately reflect the possible physical side of the course of wear. For this purpose, a special algorithm was developed for construction of such a curve, and principle establishment for estimating its accuracy.

The experiments have been performed on tracer lathe TGC 8, with machined samples made of carbon steel C45 (203HB) and the tool made of the tungsten carbide S20S. The following cutting parameters ranges were applied: the cutting speed $v_c = 84 \div 220$ m/min, feed rate $f = 0.125 \div 0.6$ mm/rev, the depth of cut taken as perpendicular to the spindle axis $a_n = 1.0 \div 4.0$ mm and the sample profile angle $\alpha_z = 160 \div 255$ deg.

2. Methodology and algorithm for wear curve computing

Data needed to develop a wear curve which is independent of the parameters of cut for a particular machine tool-clamping device-part-tool system consist of $N = 31$ sets containing pairs of values $(VB_{B,i}, t_i)_j$, where $VB_{B,i}$ is the tool edge wear and t_i are the corresponding time values for each experimental element j , where: $i = 1, 2, \dots, N_p$ is an indicator of subsequent measurements in a

specific experimental element j , and N_p is their number. N_p , the number of measurements in subsequent experiments is not constant because it depends on values of cutting parameters and is contained within the following limits: $5 \leq N_p \leq 17$. The minimum allowable number of measurements in the individual experimental elements of the experiment is $N_p = 4$. The measurement points in this case are approximated by a polynomial of degree $r = N_p - 1 = 3$ and in this case a method of interpolation is used. Each pair $(VB_{B,i} t_i)$ can be called a node. The assumed degrees r of the approximating polynomials for individual wear curves are given the values: $3 \leq r \leq 9$. The initial values for $VB_{B,0} \dots j$ and $t_0 \dots j$ are zeros. When choosing the degree of the polynomial, the following convention has been assumed: The minimum degree of the polynomial $r = 3$, and the maximum r_m is odd and such that first derivative $d(VB_B)/dt$ for all nodes $t \dots$ is zero or positive. The primary objective of the first stage is to define degrees of approximating polynomials for each experimental element in the experiment. In the first stage for each experiment, the following computations are performed:

- a set of $VB_B \dots$ is approximated by a polynomial of an odd degree r (starting with the smallest permissible value) having in mind possible small undulation of the curve defined by the following relationship;

$$r \leq 2 \cdot \sqrt{N_p - 1} \quad (2)$$

- the polynomial coefficients $A \dots r$, the polynomial $VB_{B,0} \dots r$, the first derivative $d(VB_B)/dt$ at the nodes $t \dots$ and the differences between wear obtained from the measurements and those computed basing on the determined polynomials are calculated; $EVB_B \dots r = VB_B \dots r - VB_{B,0}$; a statistics of the following structure:

$N_p = n$ – number of points in a given experimental element,

$r_m = N_p - 1$ – the maximum possible degree of the polynomial (interpolation),

r – the applied odd polynomial degree,

DV – the length of the interval $t_{max} - t_0$ [min],

SUM – sum of the approximation errors (zero for the case of interpolation),

$ESUM$ – mean error (zero for the interpolation),

NE – standard error estimation or the sum of error squares (zero for the case of interpolation),

VE – variance of the approximation error $NE/(N_p - 1)$ (zero for the case of interpolation),

SD – standard deviation (zero for the case of interpolation).

$$SD = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (y_i - \hat{y}_i)^2} \quad (3)$$

When the set of the first derivatives satisfies the assumed convention for the adopted degree of the polynomial r , calculations are carried out for the degree $r + 2$, the quantities specified above are recalculated and again, the result is checked for meeting the convention. If it is not met, the level of r is assumed to be finally selected.

If N_p is an even number and for $r = N_p - 1$ "canon" is met, then we just have the case of the interpolation instead of approximation and the statistical errors are not calculated.

The main objective of the second stage is the approximation of measuring point in each experimental element by a polynomial of r degree determined in the first stage, and so the following is to be done:

- the approximation $VB_B \dots r$ sets under the imposed interpolation condition for the initial node, supplemented by the coordinates $[0, 0]$;
- calculation of coefficients $A \dots r$ of the polynomial, the polynomial values $VB_{B,1} \dots r$, the first derivatives $d(VB_B)/dt$ at the nodes $t \dots$, the differences

$$EVB_{B,1} \dots r = VB_B \dots r - VB_{B,1} \dots r.$$

- one can calculate a statistics with a similar structure as one in the first stage. An example of the results of calculations is given in Fig. 1.

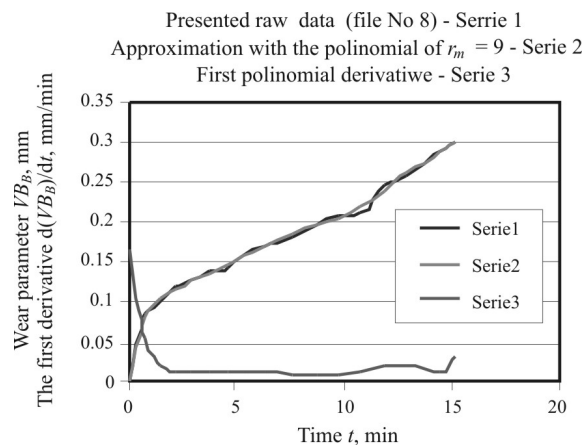


Fig. 1. The results of calculations for the experimental element No. 8

Since it is not possible to complete the cutting edge wear measurement in all the experimental elements exactly up to the value of $VB_B = VB_{B,kr} = 0.3$ mm, which corresponds to the time T equal to tool life, therefore, this fact shall be taken into account in the third stage as follows:

- Using the coefficients of polynomials from the previous step $A \dots r$, the tool life values $T \dots r$ are calculated (with an assumed accuracy of 10^{-3}) for which the value of wear level becomes critical $VB_B = VB_{B,kr}$. Tool life is calculated using the coefficients of polynomials by the determined degree r for the given experimental elements, especially when $VB_{B,Np} \neq VB_{B,kr}$.

- Dividing the values of the time at which value of wear was measured by the tool life values of cutting edges in each experimental element one can obtain dimensionless tool life $t/T = 1$ which are not dependent on the cutting parameters. Thus new sets of data $[VB_i, (t/T)_i]_j$ are obtained, for which approximations can be made with the same polynomials r to obtain the $N = 31$ curves with the dimensionless cutting edge wear coordinate (the abscissa) t/T (Fig. 2). These curves can also be obtained by scaling previously determined polynomials using size scaling which is the quotient of $T \dots r / (t_{VBkr} / T) \dots r$.

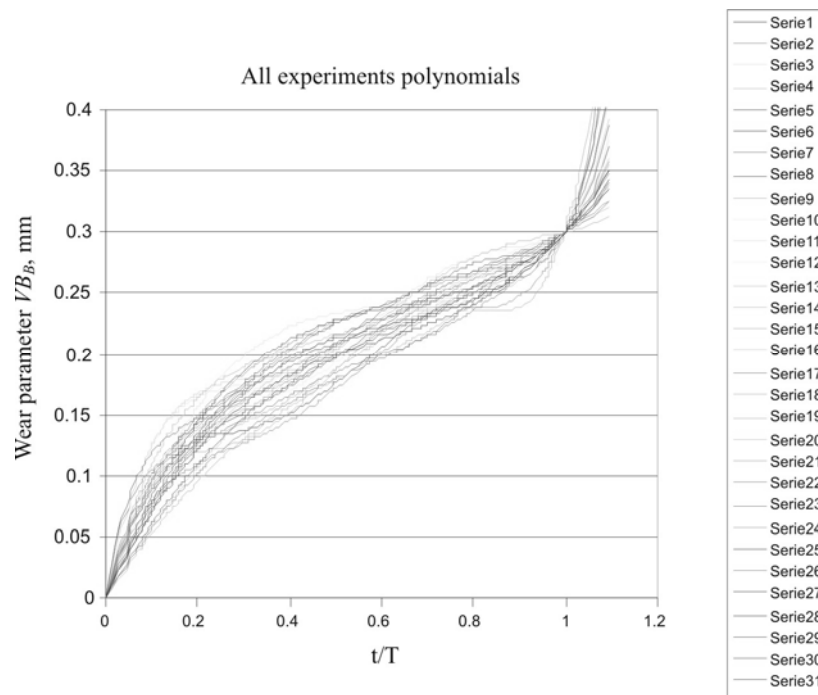


Fig. 2. Curves of wear presented in the dimensionless coordinate t/T for all experimental elements $N = 31$

Replacement of all wear curves in dimensionless coordinate $t/T \dots N_p$ obtained for all the experimental elements of the experiment with one curve can be done by two methods:

1. Averaging the coefficients of polynomials of all curves.
2. Polynomial approximation of all points throughout the experiment after having rearranged the dimensionless coordinate values of t/T , ($t/T \in [0, 1]$).

The first method is founded on the arithmetic mean-based averaged polynomials, namely the calculation of the arithmetic averages of all the coefficients of polynomials, placed by consecutive powers and then computing the values necessary to evaluate an averaged polynomial. The implementation of these tasks is the fourth stage of the procedure and shall include:

- Forming the sets of coefficients for consecutive powers of polynomials in terms of wear curves representing each experimental element $0N$, $1N$, $2N$, ..., $r_m N$, where $r_m = 9$ is the largest degree of the polynomial occurring in the experiment. If in a given experiment the degree of the polynomial is not present, the value of this coefficient is 0. The set $0N$ is a collection of zero values for each polynomial.
- Calculating the values of arithmetic mean for coefficients of polynomials, placed by consecutive powers. These average values become coefficients of the averaged polynomial representing the wear curve which is not dependent on the parameters of cut and this curve is common to all experimental elements.

$$VB_{B,u} = \sum_{i=0}^{r_m} a_{u,i} \cdot \left(\frac{t}{T}\right)^i \quad (4)$$

Combining all data sets with different numbers of wear measurement points from 31 experimental elements of experimental design in a single set with dimensionless coordinate t/T and calculating the number of nodes (total number of wear measurement points) $n = 184$:

- Organizing a set of nodes (VB_B , t/T) in view of the coordinate t/T , for all values of $0 < (t/T) < 1$ using the coefficients of the averaged polynomial.
- Combining all data sets with different numbers of wear measurement points.
 - Calculating the first derivative $d(VB_B)/d(t/T)$ for all nodes.
 - For values: $0 \leq (t/T) \leq 1$ in each node, the cutting edge wear values were calculated $VB_{Bn} \dots N$ using polynomial coefficients determined for each of $N = 31$ experiments in the third stage of the computing procedure.
 - Assuming the calculated values of the tool edge wear, contained in the set $VB_{B,u} \dots n$, as average values u_i , for each node t/T referred to above, the differences were calculated between the wear values contained in the sets ($VB_{Bn} \dots N$), and the corresponding mean values; $i = n - 2 =$ number of the nodes.
 - For each node, the value of the variance and standard deviation σ_i were calculated, values of the sums: the mean $\pm 3\sigma$ and the minimum and maximum wear values $x_{min,i} = VB_{Bmin,i}$ and $x_{max,i} = VB_{Bmax,i}$ were selected which is necessary

for the calculation of cumulative distribution function and for the envelope determining (Fig. 3).

• The mean value and standard deviation are the parameters of the Gaussian normal distribution density, i.e. the probability density of wear VB_B ranging from $VB_{B,min,i}$ to $VB_{B,max,i}$. This probability can be expressed as follows:

$$P(VB_{B,min} \leq VB_B \leq VB_{B,max}) = F(VB_{B,max}) - F(VB_{B,min}) \quad (5)$$

where F – value of the cumulative distribution function at certain points.

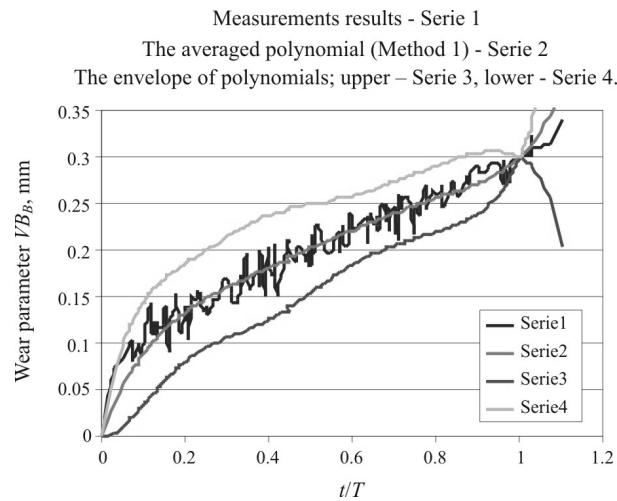


Fig. 3. Envelopes of all the curves of wear

Probability distribution (determined on the basis of the experiment) implies assumption that all the curves presented in dimensionless coordinate t/T are the same and can be replaced by a single curve which has been determined by constructing normal distribution of wear in vertical cross-section of the curve beam passing through each measurement point (node) defined by the value of coordinate t/T (Fig. 4). After determining the minimum and maximum values of wear in all $n - 2$ cross sections, which lie on the lower and upper envelope values, distribution functions corresponding to the maximum and minimum values of consumption were calculated (Fig. 5). Two points were assumed as constant, namely, first passing through the co-ordinate system origin and the second, which defines critical wear for the value of dimensionless $VB_{B,kr}$ coordinate $t/T = 1$.

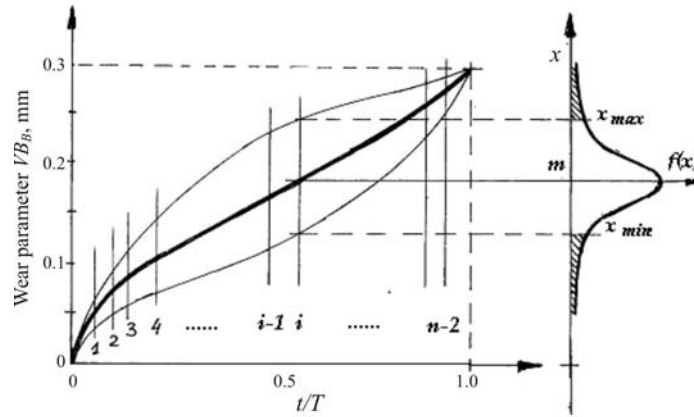


Fig. 4. Determining the wear distributions in cross-sections of the curve beam obtained in variable machining conditions

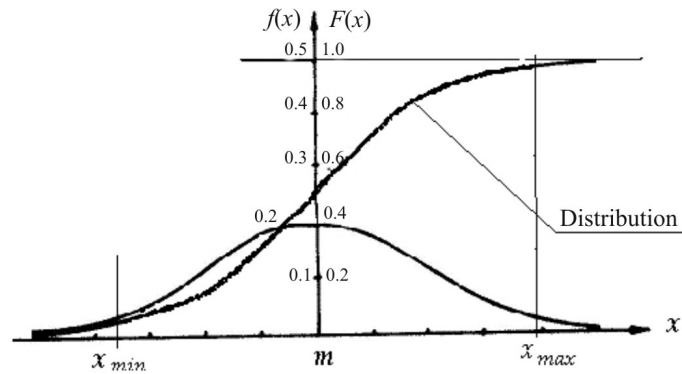


Fig. 5. Explanation of the cumulative distribution determining in cross-sections of the curve beam

Can be used to calculate the probability of the corresponding level of confidence.

Probability density distribution of the normal (Gaussian) distribution:

$$p(x) = \frac{1}{\sigma(2\pi)^{0.5}} e^{-\frac{(x-m)^2}{2\sigma^2}} \quad (6)$$

Cumulative distribution function for the normal distribution for variable x is:

$$P(X \leq x) = F(x) = \int_{-\infty}^x p(y)dy = \frac{1}{\sigma(2\pi)^{0.5}} \int_{-\infty}^x e^{-\frac{(y-m)^2}{2\sigma^2}} dy \quad (7)$$

The probability integral for the variable x is:

$$P(-x + m \leq X \leq x - m) = F(x - m) - F(-x + m) = 2F(x - m) - 1 \quad (8)$$

Level of confidence:

$$\begin{aligned} P(x_{MIN} \leq X \leq x_{MAX}) &= F(x_{MAX}) - F(x_{MIN}) = \\ &= \frac{1}{\sigma(2\pi)^{0.5}} \int_{-\infty}^{x_{MAX}} e^{-\frac{(y-m)^2}{2\sigma^2}} dy - \frac{1}{\sigma(2\pi)^{0.5}} \int_{-\infty}^{x_{MIN}} e^{-\frac{(y-m)^2}{2\sigma^2}} dy \end{aligned} \quad (9)$$

Level of significance:

$$P(\overline{x_{MIN} \leq X \leq x_{MAX}}) = 1 - P(x_{MIN} \leq X \leq x_{MAX}) = 1 - [F(x_{MAX}) - F(x_{MIN})] \quad (10)$$

The significance level is the probability of an event contrary to one described by level of confidence. The probability integral has to be in this case, symmetric with respect to the variable m . The confidence level distribution along the t/T coordinate is shown in Fig. 6. Probability of the occurrence that the VB_B variable stays within $VB_{Bmin} \leq VB_B \leq VB_{Bmax}$ (the level of confidence).

The value of the probability which is equal to unity at the starting point can be considered correct, while at the point with coordinate $t/T = 1$ it results only from the given assumptions. In fact, it should be assumed that the level of confidence at this point is of the same order as in other parts of the wear curve.

The second method of substitution of wear curves in dimensionless coordinate $t/T \dots N_p$ by a single curve consists in the polynomial approximation of all points throughout the experiment, ordered with respect of dimensionless coordinate values of t/T , ($t/T \in [0, 1]$). Accomplishment of this task is the fifth stage of the proceedings, which includes the following points:

- Approximation of all measurement points of the experiment by the polynomial of the highest degree $r_m = 9$ established in the first stage, adding and a condition for the first interpolation point $[0, 0]$.

- Computing the approximating polynomial coefficients $A_0 \dots r_m$ and its values $VB_{B,A} \dots n$, then the first derivatives $d(VB_{B,A})/d(t/T) \dots n$ in the measurement points $0 < (t/T) \leq 1$.
- Similar as in the fourth stage, are calculated the differences in wear between the values calculated using the polynomial coefficients determined for the individual experimental elements $N = 31$ in the third stage of the experiment, but this time taking the value of the approximating polynomial as mean tool edge wear values at $VB_{B,A} \dots n$ nodes.
- Calculation of all other values for quantities specified in the fourth stage in order to compare them with the values obtained using the method of averaging the coefficients of polynomials.

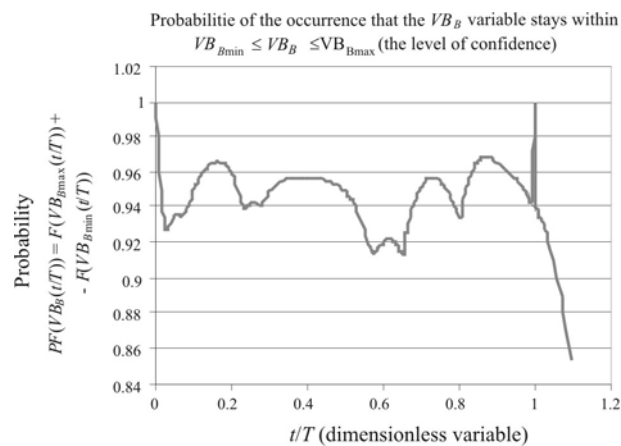


Fig. 6. Change in the confidence level of the tool edge wear calculated using the averaged polynomial

In the sixth stage, the two methods were compared using the shapes of the two tool edge wear curves in dimensionless coordinate t/T (Fig. 7) the shapes of the first derivative functions for both methods, taking the experimental measurement points as the background (Fig. 8). Basing on the course for changes of the first derivative of the polynomial which was determined by the method of averaging individual polynomials resulting from the successive experiments and improved wear curve smoothness as well as lower curve fit errors, a method of polynomial averaging was chosen as a better one and it was used to present the results of calculation. The value 0.95 of the confidence level obtained through the prevailing range of dimensionless variable and locally 0.92, should be regarded as a good result. The resulting curve can be used for the optimum selection of cutting parameters in varying machining conditions. The coefficients of polynomials representing the tool edge wear curve as a function

of coordinate t/T , obtained by averaging the polynomials and then applying approximation are the following (Table 1).

Table 1. The values of the coefficients of polynomials $VB_B = f(t/T)$.

The method of averaging	The method of approximation
$A_0 = 0.0$	$A_0 = 0.0$
$A_1 = 1.291090111658747$	$A_1 = 2.716159237126472$
$A_2 = -5.30165755825663$	$A_2 = -28.82581899623517$
$A_3 = 16.01155471803400$	$A_3 = 174.1841673619219$
$A_4 = -35.40864343758312$	$A_4 = -613.9150534494922$
$A_5 = 61.31013255360121$	$A_5 = 1331.396922382793$
$A_6 = -79.14534997053460$	$A_6 = -1801.6182964549280$
$A_7 = 68.19952210450566$	$A_7 = 1481.7000829162890$
$A_8 = -34.16310440077529$	$A_8 = -677.0937554445126$
$A_9 = 7.506455848277964$	$A_9 = 131.7559437575929$

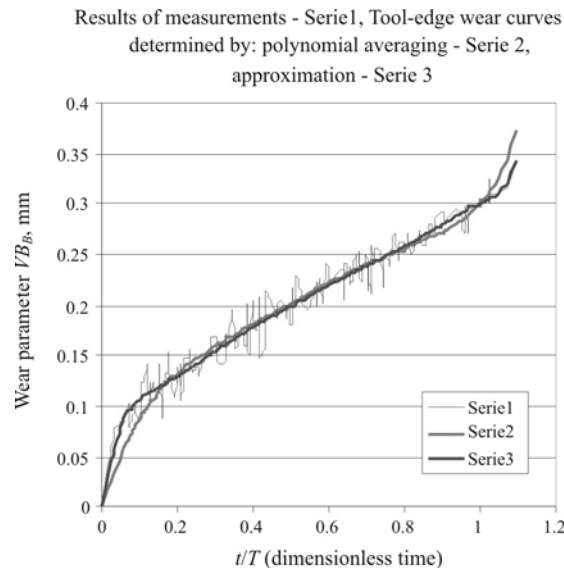


Fig. 7. Wear curves determined by a method 1 and 2 and the dispersion of actual tool edge wear values VB_B resulting from measurements

An analysis of the following sets of differences, and their statistics was performed:

- Differences between the values of tool edge wear measurements and values calculated by averaging the polynomial coefficients.

• Differences between the values of tool edge wear $VB_{B,ij}$ ($i = 1, 2, \dots, N_p$, $a j = 1, 2, \dots, N$) calculated using 31 polynomials determined for all experimental elements N and the values obtained by averaging the polynomial coefficients. The presentation of these results is shown in Fig. 9.

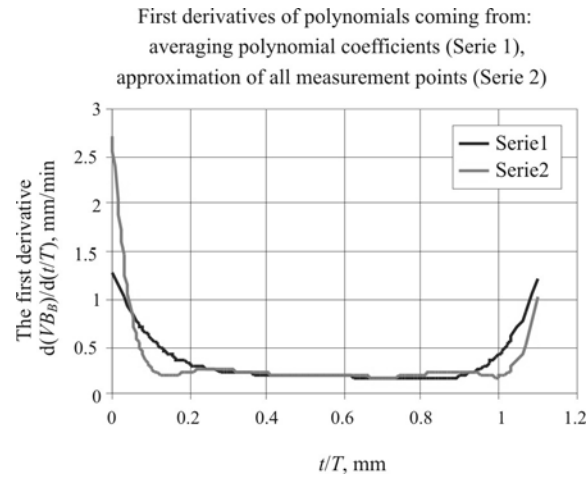


Fig. 8. Graphs of the first derivatives for both methods

Averaging errors - Serie 1. Dispersion OW values ($N = 31$) for individual polynomials: upper - Serie 2, lower - Serie 3.

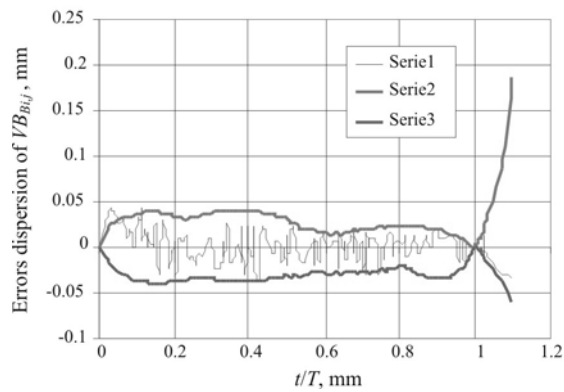


Fig. 9. Comparison of averaging errors and errors of individual dispersions of averaged polynomials relative to the averaged polynomial

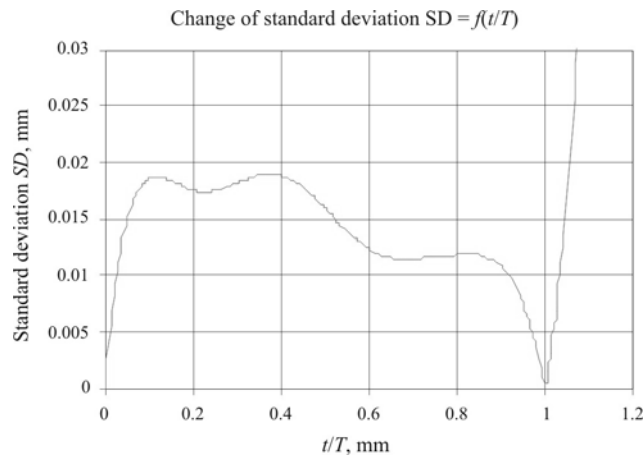


Fig. 10. The standard deviation as a measure for dispersion of measurement results relative to the curve calculated by polynomial averaging

Changes to the standard deviation of tool edge wear values as a function of dimensionless coordinate t/T is shown in Fig. 10.

3. Conclusions

The investigation results show that the method of polynomial averaging is preferable for determining wear function which is independent of cutting parameters in varying conditions of profile turning. The highest odd polynomial coefficient r_m is equal to 9. The average value of the confidence level for the determined curve equals 0.95 while the local boundaries fall within 0.92-0.97. The highest level of the standard deviation for the tool edge wear VB_B according to the computed curve is 0.018 mm while its average value is around 0.015 mm. The curve passes through the coordinate system origin for a brand new tool. The major conclusion is that the elaborated method is suitable for determining the wear curve which can be used for computing the tool life in varying conditions of turning complex profiles with the point-tip tools at confidence level $\alpha = 0.05$ for the given assumptions and ranges of machining parameters.

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