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MODELING ANDANALYSIS OF FREE VIBRATION OF STEEL-CONCRETECOMPOSITE BEAMS BY FINITE ELEMENT METHOD

Tomasz Wróblewski, Agnieszka Pełka-Sawenko, Małgorzata Abramowicz, Stefan Berczyński

S u m m a r y

Most technological machines generate vibrations which are transferred to either support systems or foundations. To ensure an object's safe operation, it is necessary to have adequate knowledge about dynamic properties of both machines and a supporting construction. A steel-concrete composite floor is an example of a supporting construction. It consists of steel beams connected with a reinforced concrete slab in a way that enables mating of both elements. This paper presents a discreet model of a steel-concrete composite beam that takes into account flexibility of the connection. An analysis of the beam's natural vibrations was conducted and the results were compared with those of experimental studies. Tests were performed on two sets of beams. In the first group of beams B1 a connection that consisted of steel stud connectors was used whereas perforated steel slats was used in the second groups of beams B2. The present paper is a report from the analysis that was conducted on the beams from group B2. The beam model was developed on Abaqus platform using deformable finite element method. Matlab system was used for the analysis and its environment was applied to control the model development and to identify the model's selected parameters. The beam model was made in two versions that differ in the approach to modelling connection. The developed model, after parameter identification, yields highly consistent results with those of experimental tests.

Keywords: Composite structures, identification, Abaqus, Matlab, SPRING2

Modelowanie i analiza drgań własnych stalowo-betonowych belek zespolonych metodą elementów skończonych

S t r e s z c z e n i e

Większość urządzeń technicznych wytwarza drgania, które przekazywane są na konstrukcje wsporcze lub fundamenty. W celu zapewnienia bezpiecznej eksploatacji obiektu niezbędne jest uwzględnienie właściwości dynamicznych zarówno maszyn, jak i konstrukcji wsporczej. Konstrukcją wsporczająest na przykład stalowo-betonowy strop zespolony.Składa się z belek stalowych połączonych z płytążelbetową w sposób umożliwiający współpracę obydwu tych elementów. W artykule przedstawiono model obliczeniowy stalowo-betonowej belki zespolonej uwzględniający podatność zespolenia.Prowadzono analizę jej drgań własnych ijej wyniki porównano z wynikami wykonanych badań doświadczalnych dla dwóch grup belek. W pierwszej grupie B1 stosowano zespolenie stalowymi sworzniami zespalającymi.W drugiej natomiast B2 stosowano zespolenie stalowymi listwami

Address: Prof. Stefan BERCZYŃSKI, Tomasz WRÓBLEWSKI, Ph.D. Eng., Agnieszka PEŁKA-SAWENKO, M.Sc.Eng., Małgorzata ABRAMOWICZ, M.Sc.Eng., West Pomeranian University of Technology of Szczecin,Faculty of Civil Engineering and Architecture, Piastów 50, 70-311 Szczecin, e-mails: Stefan.Berczynski@zut.edu.pl, aps@zut.edu.pl, wroblewski@zut.edu.pl, mabramo-wicz@zut.edu.pl.

perforowanymi. W pracy analizie poddano belki z grupy B2. Model belki opracowano w systemieAbaqus wykorzystującym metodę odkształcalnych elementów skończonych. Do analizystosowano oprogramowanie Matlabsterowania procesem przygotowania modelu oraz identyfikacją wybranych jego parametrów. Model belki opracowano w dwóch wersjach różniących sięsposobem modelowania zespolenia. Użyto go także do identyfikacji parametrów. Uzyskano wyniki symulacji częstotliwości drgańo dużej zgodności z wynikami badań doświadczalnych.

Słowa kluczowe: konstrukcje zespolone, identyfikacja, Abaqus, Matlab, SPRING2, MAC

1. Introduction

Steel-concrete composite beams are very often used in public space and industrial building engineering as elements of floors or in bridge engineering as main carrying girders. Special attention should be paid in each case to dynamic properties of a designed construction. In public buildings, floor vibration control is required due to meet Serviceability Limit States that ensure comfort of users of a building. In industrial buildings, machines are often placed on floors. Machines generate vibrations of various frequency which are transferred to supporting constructions. Precision machines require a stable floor with defined and known dynamic characteristics. In bridge engineering, dynamic load is common and particular attention should be paid to small and medium span beam bridges that are found along high-speed rail tracks where trains can travel with speeds exceeding 300 km/h.

Over the past decades many studies on steel-concrete composite constructions have been published. Elements are modelled usually using Euler or Timoshenko beam theories [1-5]. Biscontin et al. [1] analysed a composite beam with Euler beam theory considering connection flexibility along the direction parallel to the beam's axis. Analysis results were compared with those from experimental tests. Flexibility of the connection in two directions was investigated by Dilena and Morassi [2]. They developed a model to analyse dynamic properties of composite beams without and with local damage of connections. In [3] Dilena and Morassi analysed the frequency of flexural and axial vibrations of a free beam with two connection types as well as their modal damping ratios and vibration modes. The analysis was conducted for both undamaged and partly damaged beams. Berczynski and Wróblewski et al. [5] presented results of experimental tests conducted on three composite beams with different connections between a steel beam and a reinforced concrete slab. On the basis of the results, parameters of two computational models were identified. The first, continuous model was based on Timoshenko beam theory and is discussed in-depth elsewhere [4].Motion behavior of the model is described by a system of partial differential equations. It is difficult and time-consuming to find a solution of such a system of equations even for one-dimensional systems. The second model was created in convention of Rigid Finite Element Method – RFE. The method is often used to developed and identified parameters of the machine tool supporting systems models [6,7], however, it can also successfully be used to analyze the steel-concrete composite elements of the civil engineering structures. Both models took into account two-directional stiffness of a connection. The results obtained from the two models were highly consistent with those of experimental tests.

The discrete model, defined by many scientists, provides an alternative for the continuous model. The most popular method is the Finite Element Method (FEM). FEM provides an efficient technique for finding numerical solutions. Extended, commercial systems, such as Abaqus, Nastran and Ansys, based on FEM can be used to conduct many comprehensive analyses of three-dimensional models. Many researchers [8-18] analysed composite beams using FEM. Some used two-dimensional numerical models, e.g. Gattesco [8] and Pi et al. [9], others applied layer models in which an element is divided into several layers (Szajna [10], Madaj [11]). However, most scholars prefer three-dimensional models [10-16]. A numerical model of a concrete slab and a steel beam can be defined in many different ways. Thevendran et al. [12] proposed a model in which a slab and a beam were modelled with shell elements. Prakash et al. [13] and Vasdravellis et al. [14] developed a model with solid elements to model both a slab and a beam. A composite beam model which combined solid and shell elements was used by Mirza and Uy [15], Queiroz et al. [16] and Baskar et al. [17] – solid elements defined the concrete slab and shell elements defined the steel beam. Luo et al. [18] applied another approach. They modelled a concrete slab using solid elements and a steel beam was defined with beam elements.

Various approaches to modelling connection were applied. In [12] and [17] a connection was modelled as beam elements connecting nodes of elements modelling a concrete slab with independent nodes of steel beam model. A comprehensive model, in which connection studs were defined using solid elements, was applied in [15] and [12]. Likewise, in [18] an (adhesive) connection was defined with solid elements.To account for shearing, non-linear spring elements were additionally introduced between the nodes of a concrete slab and the connection. Spring elements were also used by other authors [14] and [16]. The complexity degree of a model and model selection depend on the character of planned analysis. Many aspects of composite elements were discussed earlier, including beam deflection analysis with partial and full connections [16], curved plain beams [12], behaviour of beams and connections in high temperatures [15], stress and slide analysis in concrete [12] and many others. The issues of dynamic analysis of composite beams have not, however, been frequently discussed.

The present study contains results of frequency of natural vibration analysis conducted on steel-concrete composite beams. Beam models were defined in FEM environment. The beam was modelled with both shell and solid finite elements to model the steel part and reinforced concrete section. The connection was modelled using one-dimensional finite elements. Stiffness of the connection in the perpendicular and parallel direction to the beam's axis was considered and discussed. The model was developed on Abaqus platform. Selected parameters of the model were identified on the basis of experimental tests that had been conducted earlier. Perforated steel slats was used as the beam's connection. The parameter identification process was controlled in Matlab environment. The developed model can be used in the future for damage detection purposes in composite beams.

2. Dynamic equation of motion

The dynamic equation of motion can be written as

$$\mathrm{M}\ddot{\mathrm{q}} + \mathrm{C}\dot{\mathrm{q}} + \mathrm{K}\mathrm{q} = \mathrm{P} \tag{2.1}$$

where M is inertia matrix, C is damping matrix, K is the matrix of system stiffness, Pis the vector of outside forces acting on the system, and q is the vector of generalised coordinates. For free vibration, where damping and the influence of external load is neglected, Equation (2.1) takes the following form

$$\mathrm{M}\ddot{\mathrm{q}} + \mathrm{K}\mathrm{q} = 0 \tag{2.2}$$

The frequency of natural vibration for the modelled composite beam is the solution of the eigenvalue problem of the following matrix:

$$\det\left(\mathrm{K} - \omega^2\mathrm{M}\right) = 0 \tag{2.3}$$

where ω is the frequency of natural vibration. The vector of natural vibration mode can be found using Equation (2.4).

$$\left(\mathrm{K} - \omega^2\mathrm{M}\right)\Phi = 0 \tag{2.4}$$

3. Experimental tests of composite beams

Experimental tests were conducted on six composite beams: three from series B1 with connection made using headed steel studs and further three beams from series B2 in which perforated steelslats welded to the top flange of a structural steel section was used as the connection. Series B2 beams only are analysed in the present study. The beam and its cross-section are presented in Fig. 1. The beam, 3200 mm in length, consists of a reinforced concrete slab 600 mm long and 60 mm thick and a structural steel section IPE 160. The slab was made from C30/37 concrete, the structural steel section and the connection slats from S235 steel.

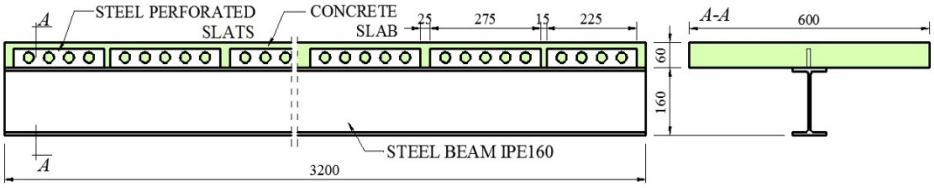

Fig. 1. Composite beam from B2 group: longitudinal view and cross-section

Initially, the beams were tested under static load at the level of 40% of elastic load capacity. The aim of the test was to check whether or not the elements were properly manufactured and to prepare the elements for real tests conducted to find dynamic characteristics. The characteristics were defined for a beam with both free ends – the beam was suspended on two steel frames using steel cable. The suspension points were assumed to overlap the nodes of basic mode of flexural vibrations of the beam. Impulse excitation was applied and acceleration was measured. The tests were conducted according to the procedure described elsewhere [4]. A grid of measurement points is presented in Fig. 2. Acceleration measurements were conducted within the grid using triaxial piezoelectric sensors. Points were excitation was applied are marked in Fig. 2 with **X** symbols.

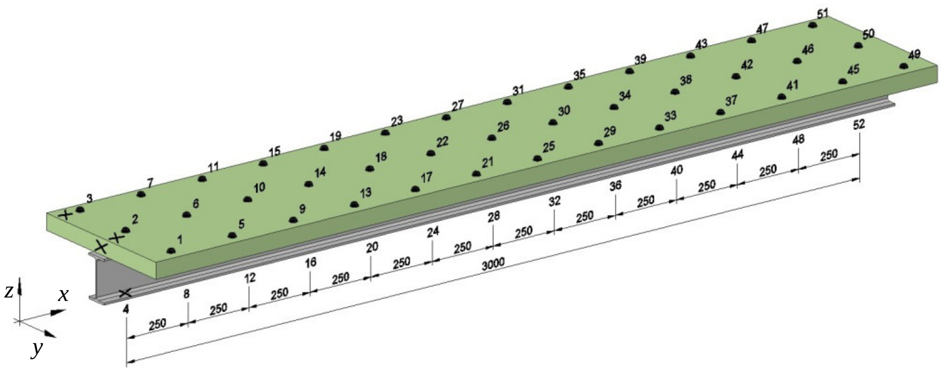

Fig. 2. Grid of measurement points

4. FEM model of composite beams

While defining the computational model, it was assumed that the beam has a static scheme with free ends and that its cross-section is constant at all its length. The model was defined on Abaqus platform as a spatial system with independently modelled reinforced concrete slab, steel beam and connection.

A discussion of possible approaches to how a numerical model of the beam could be developed was presented in the introduction. Given the planned analysis it was decided that the reinforced concrete slab should be modelled with solidelements of the solid type. C3D8I solid first-order elements were used. These are eight-node elements whose standard shape function was enhanced with so-called "bubble functions". C3D8I elements are not affected by *"hourglassing"* which results in unnatural deformations of the finite element mesh. Two elements were used on the thickness of the slab. On the width of the slab, a division into 14 elements was applied with a thickening in the central area where the slab interacts with the structural steel section. The mesh along the beam's axis consisted of elements 50 mm in length.

The steel beam made from structural steel section IPE 160 was modelled with shell elements (S8R). Three shells were defined in the model. The first shell defines web of beamwhile the other two the top and bottom flanges. The FEM model of the composite beam as well as its cross-section are presented in Fig. 3.

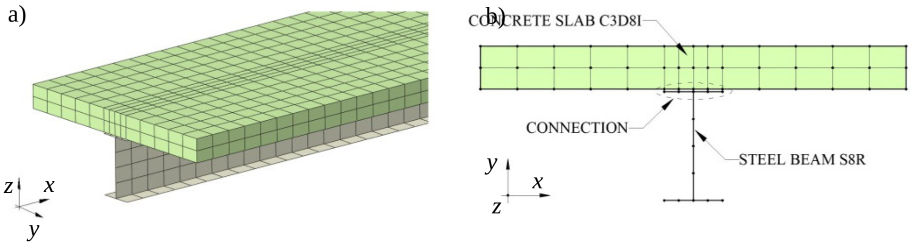

Fig. 3. FEM model of the analysed composite beam: a) a view of the beam, b) cross-section

The connection between the reinforced concrete slab and the steel beam was modelled in two different ways (Fig. 4) so effectively two independent models were constructed. In both cases one-dimensional elements were used. In a model denoted as MB, beam elements defined in Abaqus library as B31 were used whereas in a model denoted as MS, SPRING2 elements were applied. In both cases elements that modelled the connection connected shell elements S8R that modelled the upper flange of the structural steel section with the lower nodes of C3D81 elements which modelled the reinforced concrete slab. The elements that modelled the connection were evenly distributed along the whole area of the upper flange of the steel beam, i.e. 5 elements along the width, every 50 mm, along the beam's axis at every node of the mesh. This distribution of the elements ensured a continuous connection between the upper flange and the lower edge of the slab which is consistent with the character of the connection used in B2 beams. The connection stiffness was determined independently for two directions. The stiffness along the horizontal direction (axis z) was denoted asK_h – in this case the connection is sheared. The stiffness in the vertical direction (axis y) was denoted asK_v. In MB model the stiffness of the connection was changed by changing parameters that defined B31 beam elements, i.e. a change of the cross-sectional area A_{conn} reflected a change in stiffness$K_v$whereas a change of moment of inertia $J_{y,conn}$ reflected a change in stiffnessK_h. In MS model two independent groups of spring elements SPRING2 were used. The first group was responsible for stiffnessK_h(along axis z direction) while the second group was responsible for the interaction between steel and concrete along the vertical direction (axis y) – stiffnessK_v. The stiffness of the connection in axis x direction (horizontal, perpendicular to the beam's axis) was neglected owing to the scope of analysed modes of the beam's vibrations.

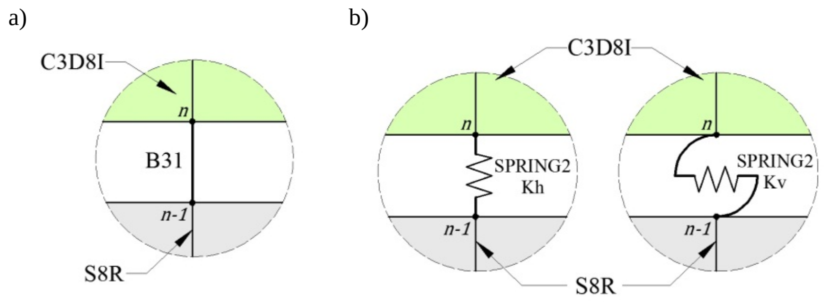

Fig. 4. FEM model of connection for composite beam: a) connection in MB model – beam elements; b) connection in MS model – spring elements

5. Parameter identification of the models

Most parameters for the developed models were obtained from the literature or from an inventory of three B2 beams. The inventory results were averaged so that to produce a cumulative model of B2 beams (in fact two models that differed from each other in the approach to modelling of the connection). As no precise enough data were available for three parameters, they were identified. These included substitute Young's modulus of the reinforced concrete slab$E_c$which accounts for the influence of the slab's longitudinal reinforcement, the stiffness of connectionK_h and K_v. In MB model the stiffness of connection was indirectly identified by determination of cross-sectional area A_{conn} and moment of inertia $J_{y,conn}$ for B31 elements.

The best fit of computational and experimental dynamic characteristics was assumed to be the consistency criterion. Consequently, index S can be minimised and can be given by

$$S = \sum_{i=1}^{5} w_{i\_flex} \left(\frac{f_{i\_flex}^{exp} - f_{i\_flex}^{com}}{f_{i\_flex}^{exp}} \right)^2 + w_{1\_long} \left(\frac{f_{1\_long}^{exp} - f_{1\_long}^{com}}{f_{1\_long}^{exp}} \right)^2 \tag{5.1}$$

where S is the sum of squares of relative deviations of computational and experimental frequencies of the first five modes of flexural vibrations (f_{i_flex}) and frequency of fundamental mode of axial vibration (f_{1_long}). Additionally, weight index$w_{i_flex}$was introduced which had the following values: 0.5 for the first frequency of flexural vibration, 0.2 for the second frequency and 0.1 for every successive frequency. The adopted value of index w_{1_long} was 0.1.

Parameter identification was conducted in Matlab environment. To enable parameter optimisation, the communication between Matlab environment and Abaqus platform had to be ensured. A script prepared in Python environment was used to this end. The script was used to create the beam's model, send it over to Abaqus for calculations to be conducted, and sendreceived results to Matlab, where optimisation procedures were used to decide what possible changes had to be made to the values of sought parameters. A new set of sought parameters was sent to the script to update the beam's model and make its new calculations. This fully automated process was repeated until the minimum of index S was reached. Table 1 presents a comparison of results of experimental tests with those obtained using MB and MS models with identified parameters.

The first four modes of flexural vibrations for MB model are presented in Fig. 5.

After identification, a comparison of the first four modes of flexural vibrations was made using MAC (*Modal Assurance Criterion*) which can be given by

$$MAC = \frac{(\boldsymbol{\psi}_{exp}{}^{H}\boldsymbol{\psi}_{com})(\boldsymbol{\psi}_{com}{}^{H}\boldsymbol{\psi}_{exp})}{\boldsymbol{\psi}_{exp}{}^{H}\boldsymbol{\psi}_{exp}\boldsymbol{\psi}_{com}{}^{H}\boldsymbol{\psi}_{com}} \tag{5.2}$$

where $\boldsymbol{\psi}_{exp}$ is the modal vector for mode of natural vibration obtained in the experimental tests and $\boldsymbol{\psi}_{com}$ is the modal vector for mode obtained from the computational model. The computational modal vector $\boldsymbol{\psi}_{com}$ was determined for selected nodes of the model marked in Fig. 5 (red dots). The nodes were selected so that they overlapped with points on the measurement grid used in the experimental tests. Modal vector components in the vertical direction (axis y) were analysed. A comparison of normalized experimental and computational (MB model) modal vectors corresponding to second flexural mode of vibrations is presented in Fig. 6.

Table 1. Identification results – frequencies, relative errors, identified parameters

Model →	**MB**			**MS**		
Mode of vibration ↓	**Experimental frequency $f_{i,exp}$, Hz**	**Computational frequency $f_{i,com}$, Hz**	**Relative error Δ, %**	**Experimental frequency $f_{i,exp}$, Hz**	**Computational frequency $f_{i,com}$, Hz**	**Relative error Δ, %**
1_f – first flexural	75.70	76.01	-0.4	75.70	75.36	0.5
2_f – second flexural	180.00	178.45	0.9	180.00	179.49	0.3
3_f – third flexural	290.00	288.42	0.5	290.00	290.35	–0.1
4_f – fourth flexural	393.30	394.28	–0.3	393.30	394.21	–0.2
5_f – fifth flexural	495.60	497.92	–0.5	495.60	494.07	0.3
1_a – first axial	588.80	594.48	–1.0	588.80	588.80	0.0
S	$2.864 \cdot 10^{-5}$			$1.345 \cdot 10^{-5}$		
E_c, MPa	$2.947 \cdot 10^{4}$			$2.927 \cdot 10^{4}$		
A_{conn}, m^2	$5.629 \cdot 10^{-6}$			–		
$J_{y,conn}$, m^4	$1.133 \cdot 10^{-11}$			–		
K_h, MPa	–			337,3		
K_v, MPa	–			104,4		

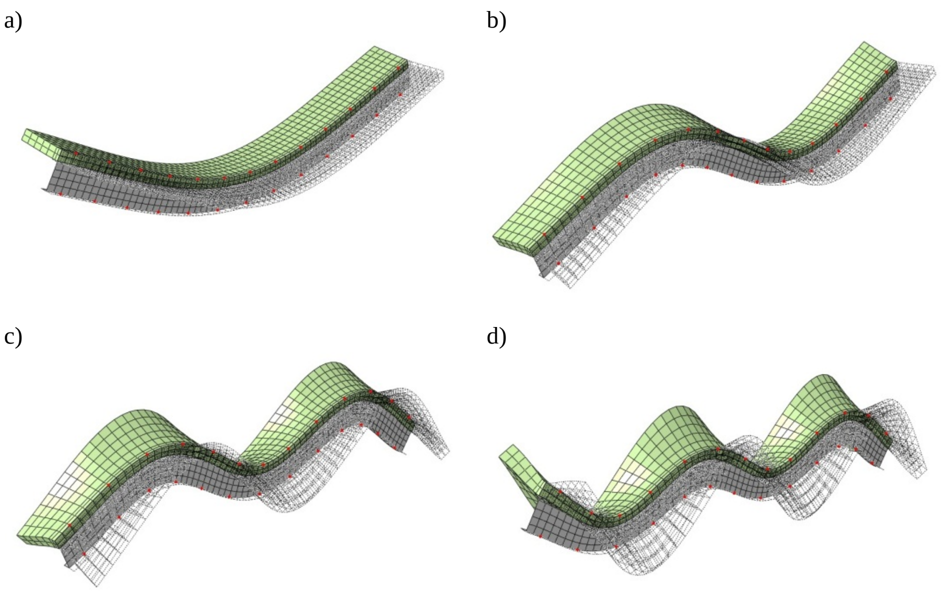

Fig. 5. Modes of flexural vibrations of beam in MB model: a) first, b) second, c) third, d) fourth

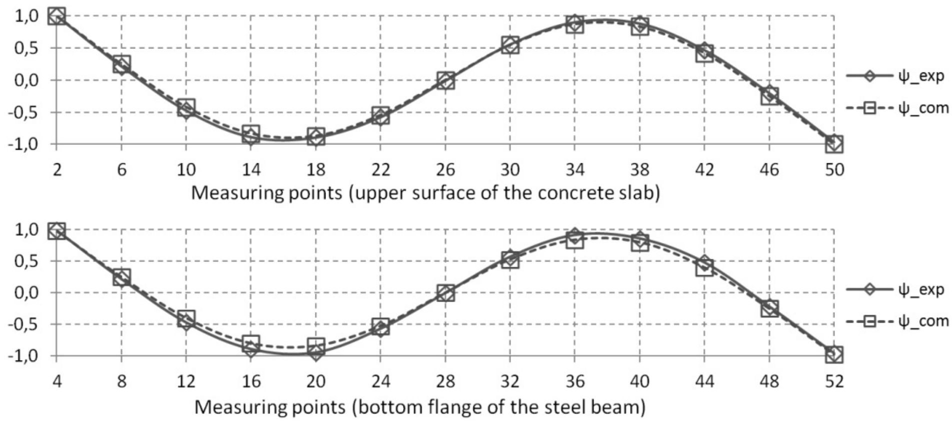

Fig. 6. Comparison of normalizedmodal vectors, second mode of flexural vibrations – MB model, vertical direction– axis*y*

MAC values determined for MB and MS models are presented in Fig. 7.

a)b)

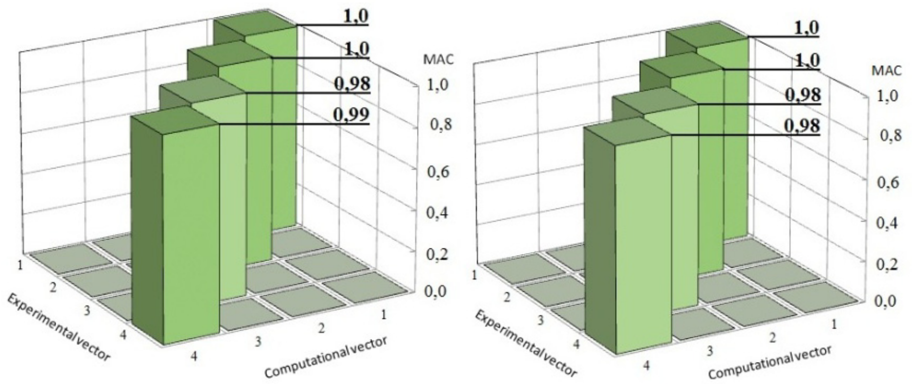

Fig. 7. Histogram of MAC values – direction *y*: a) MB model, b) MS model

6. Results

The computational models of steel-concrete composite beams with connection made ofperforated steelslats and modelled with FEM are highly consistent with the real object. Moreover, highly accurate fit of frequency in experiment and simulation (the maximum relative error Δ = 0.96%) was obtained. The MAC values which compare experimental and computational modal vectors of flexural vibrations also confirm the models' high consistency with experimental results.

While comparing the computational models, it can be noticed that MS model (spring elements) provides more accurate fit than MB model (beam elements). The degree of fit for vectors of vibration modes is almost at the same level. It was easier and faster to model beams in Python and Abaqus environments using MS model. The MS model was also much faster during one computation loop for the MS model required significantly less time than the MB model which significantly reduced time of identification process.

The developed computational models of composite beams can be used in future research for the purposes of analysis and of damage detection in composite beam components and connection.

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