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Axi-symmetric deformation in the micropolar porous generalized thermoelastic medium

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Abstract. In the present article we studied the thermodynamical theory of micropolar porous material and derived the equations of the linear theory of microploar porous generalized thermoelastic solid. Then the general solution to the field equations for plane axi-symmetric problem are obtained. The Laplace and Hankel transforms have been employed to study the problem, which are inverted numerically by using numerical inversion technique. An application of normal force and thermal source has been taken to show the utility of the approach. The technique developed in the present paper is simple, straightforward and convenient for numerical computation. Effect of micropolarity and porosity on the components of stress, temperature distribution and volume fraction field together with the effect of generalized theory of thermoelasticity have been depicted graphically for a specific model. Some particular cases are also deduced from the present problem.

Key words: micropolar thermoelastic solid, concentrated source, microrotation vector, microinertia, volume fraction field.

1. Introduction

The classical mechanics deals with the basic assumption that the effect of the microstructure of a material is not essential for describing mechanical behavior. Such an approximation has been shown in many well-known cases. Often, however, discrepancies between the classical theory and experiments are observed, indicating that the microstructure might be important. For example, discrepancies have been found in the stress concentrations in the areas of holes, notches and cracks; elastic vibrations characterized by a high frequency and small wavelengths, particularly in granular composites consisiting of stiff inclusions embedded in a weaker matrix, fibers or grains; and the mechanical behavior of complex fluids such as liquid crystals, polymeric suspensions, and animal blood. In general, granular composites, for example porous materials, are widely used in the area of passive noise control as sound absorbers and the effect of acoustical waves characterized by high frequencies and small wavelengths becomes significant.

A simple theory, in which some consideration is given to the microstructure, is the theory of micropolar continuum mechanics. A homogeneous isotropic micropolar material is a material, characterized by a continuum in which rigid grains of infinitesimal size are uniformly distributed in an elastic matrix. Homogenity and isotropy are macro-properties of the medium. The effect of granular structure becomes important in transmitting waves of small wavelength and high frequency. When the wavelength is comparable with the average grain size, the motion of the grains must be taken into account. Eringen [1, 2] gave a complete description of the elements of the linear theory of micropolar elasticity. Such a micropolar medium undergoing a homogeneous deformation can support couple stress and spin inertia. This work suggests a method to identify the micropolar elastic moduli on the basis of the vibrating data. A special micropolar material was fabricated in which uniformly distributed rigid aluminium shot was cast in an elastic epoxy matrix. Gauthier [3] found this aluminum-epoxy composite to be micropolar material and investigated the values of the relevant parameters based on specimen of aluminium-epoxy composite. Hsia et. al. [4, 5] discussed the longitudinal plane wave propagation in an elastic micropolar media and transverse wave propagation and its its reflection and transmission from a plane interface between two different elastic-micropolar porous interfaces in perfect contact, and solved completely by syntactic foam, respectively for numerical computation.

The linear theory of micropolar thermoelasticity was developed by Eringen [6] and Nowacki [7] to include thermal effects and is known as micropolar coupled thermoelasticity.

Goodman and Cowin [8] established a continuum theory for granular materials, whose matrix material (or skeletal) is elastic and interstices are voids. They introduced the concept of distributed body, which represents a continuum model for granular materials (sand, grain, powder, etc) as well as porous materials (rock, soil, sponge, pressed powder, cork, etc.). Nunziato and Cowin [9] in 1979, developed the non-linear theory of elastic materials with void, underlying the basic concept that the bulk density of the material is written as the product of two fields, the density field of the matrix material and the volume fraction field (the ratio of volume occupied by grains to the bulk volume at a point of the material). This representation of the bulk density of the material introduces an additional kinematic variable in the theory. They developed the constitutive equations for solid like material which are non-conductor of heat and discussed restrictions imposed on these constitutive equations by thermodynamics. They showed that the volume that changes in the volume fraction causes an internal dissipation in the material and this internal dis-

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sipation leads to relaxation property in the material which is similar to that associated with viscoelastic materials. The linearized theory of elastic materials with voids has been presented by Cowin and Nunziato [10], which is generalization of classical theory of elasticity and reduces to it when the dependence on change in volume fraction and its gradient are suppressed. In this theory, the volume fraction corresponding to void volume is taken as an independent kinematical variable. Some authors [11, 12] derived the constitutive relations in porous thermoelasticity, by assuming that the material possesses mechanical dissipation and included the time rate of volume fraction field $\dot{\varphi}$ in order to satisfy the dissipation inequality. When we study the systems where the dissipation mechanism is sufficiently strong the localization of solution in the time variable can hold. This means that the decay of the solution is sufficiently fast to guarantee that they vanish after a finite time.

Since a microelastic medium may also contain a distribution of vacuous pores and such types of materials are abundantly available in the crustal layers of the Earth, discussion of their joint effect is extremely necessary in various problems of deformation. In view of this, in the present problem, we obtained the expressions for stresses, change in volume fraction field and temperature distribution due to applied mechanical and thermal sources.

2. Derivation of basic equations

Let $ß_o$ and $\partial ß_o$ be the region and its boundary of the physical 3-dimensional ($\equiv \Re^3$) occupied by a micropolar porous elastic budy in a refernce configuration, and ß and ∂ß be the region and its boundary in the current configuration.

The motion of the body is referred to the reference configuration and a fixed orthonormal frame in $\Re^3$. We shall denote the tensor components of order $p \geq 1$ ranging over $\{1, 2, 3\}$. Superposed dot or subscript preceded by a comma will mean partial derivative with respect to the time or the corresponding coordinate. In this connection, we disregard regularity questions, simply understanding a degree of smoothness sufficient to make sense everywhere.

We consider a model of micropolar porous elastic material that is governed by the following local balance laws: **Balance of energy.**

$$\rho(\dot{E} - S) = t_{ij}\dot{e_{ij}} + m_{ij}\dot{\chi_{ij}} - g\dot{\varphi} + h_i\dot{\varphi}_{,i} + q_{i,i},$$

Entropy inequality.

$$\rho\left(\dot{\eta} - \frac{S}{\Phi}\right) - \left(\frac{q_i}{\Phi}\right)_{,i} \geq 0, \tag{1}$$

where e_{ij}, χ_{ij} – deformation tensor and wryness tensor, t_{ij}, m_{ij} – stress tensor and couple stress tensor, h_i, g – equilibrated stress tensor and intrinsic equilibrated body force, q_i – heat flux vector, E, S – internal energy and strength of internal heat source, ρ, φ – bulk mass density and change in volume fraction (from the reference configuration), η – entropy density and Φ is a strictly positive function.

If we introduce the scalar function (Helmholtz's free energy)

$$\psi = E - \eta\Phi, \tag{2}$$

then we get by system (1),

$$\rho\dot{\psi} = t_{ij}\dot{e_{ij}} + m_{ij}\dot{\chi_{ij}} - g\dot{\varphi} + h_i\dot{\varphi}_{,i} + q_{i,i} + \rho S - \rho\eta\dot{\Phi} - \rho\dot{\eta}\Phi,$$

$$\rho\dot{\psi} \leq t_{ij}\dot{e_{ij}} + m_{ij}\dot{\chi_{ij}} - g\dot{\varphi} + h_i\dot{\varphi}_{,i} + \frac{q_i\Phi_{,i}}{\Phi} - \rho\eta\dot{\Phi}, \tag{3}$$

Constitutive equations. We consider the set of independent variables e_{ij}, χ_{ij}, φ, $\varphi_{,i}$, T, $T_{,i}$ and we consider the following constitutive relations:

$$\psi = \psi(\Upsilon), \qquad t_{ij} = t_{ij}(\Upsilon),$$
$$m_{ij} = m_{ij}(\Upsilon), \qquad g = g(\Upsilon),$$
$$\pi_i = \pi_i(\Upsilon), \qquad \eta = \eta(\Upsilon),$$
$$q_i = q_i(\Upsilon), \qquad \Phi = \Phi(\Omega),$$

with

$$\Upsilon = \Upsilon(e_{ij}, \chi_{ij}, \varphi, \varphi_{,i}, T, T_{,i})$$

and

$$\Omega = \Omega(\varphi, T, \dot{T})$$

so that,

$$\dot{\psi} = \frac{\partial\psi}{\partial e_{ij}}\dot{e}_{ij} + \frac{\partial\psi}{\partial\chi_{ij}}\dot{\chi}_{ij} + \frac{\partial\psi}{\partial\varphi}\dot{\varphi} + \frac{\partial\psi}{\partial\varphi_{,i}}\dot{\varphi}_{,i} + \frac{\partial\psi}{\partial T}\dot{T} + \frac{\partial\psi}{\partial T_{,i}}\dot{T}_{,i}, \tag{4}$$

$$\dot{\Phi} = \frac{\partial\Phi}{\partial\varphi}\dot{\varphi} + \frac{\partial\Phi}{\partial T}\dot{T} + \frac{\partial\psi}{\partial\dot{T}}\ddot{T}.$$

Using Eq. (4) in the Eq. (3) and neglecting the higher order partial derivatives, we obtain

$$\left(t_{ij} - \rho\frac{\partial\psi}{\partial e_{ij}}\right)\dot{e_{ij}} + \left(m_{ij} - \rho\frac{\partial\psi}{\partial\chi_{ij}}\right)\dot{\chi_{ij}} + \left(\rho\frac{\partial\psi}{\partial\varphi} + g + \rho\eta\frac{\partial\Phi}{\partial\varphi}\right)\dot{\varphi} + \left(h_i - \rho\frac{\partial\psi}{\partial\varphi_{,i}}\right)\dot{\varphi}_{,i} + \rho\left(\frac{\partial\psi}{\partial T} + \eta\frac{\partial\Phi}{\partial T}\right)\dot{T} - \rho\frac{\partial\psi}{\partial T_{,i}}\dot{T}_{,i} + q_{i,i} + \rho S - \rho\dot{\eta}\Phi = 0,$$

$$\left(t_{ij} - \rho\frac{\partial\psi}{\partial e_{ij}}\right)\dot{e_{ij}} + \left(m_{ij} - \rho\frac{\partial\psi}{\partial\chi_{ij}}\right)\dot{\chi_{ij}} + \left(\rho\frac{\partial\psi}{\partial\varphi} + g + \rho\eta\frac{\partial\Phi}{\partial\varphi}\right)\dot{\varphi} + (h_i - \rho\frac{\partial\psi}{\partial\varphi_{,i}})\dot{\varphi}_{,i} + \rho\left(\frac{\partial\psi}{\partial T} + \eta\frac{\partial\Phi}{\partial T}\right)\dot{T} + \left(\rho\frac{\partial\psi}{\partial T_{,i}} + \frac{\partial\Phi}{\partial\dot{T}}\right)\dot{T}_{,i} + - \frac{q_i}{\Phi}\left(\frac{\partial\Phi}{\partial\varphi}\varphi_{,i} + \frac{\partial\Phi}{\partial T}T_{,i}\right) \geq 0, \tag{5}$$

The relations (5) and (6) can be satisfied for arbitrary $\dot{e}_{ij}$, $\dot{\chi}_{ij}$,$\dot{\varphi}$, $\dot{\varphi}_{,i}$, $\dot{T}$ and $\dot{T}_{,i}$ if and only if the corresponding coefficients do vanish. Consequently we have the following:
(i) ψ determines t_{ij}, m_{ij}, g, h_i and η through the following constitutive equations of the heat- flux dependent micropolar porous thermoelasticity:

$$t_{ij} = \rho\frac{\partial\psi}{\partial e_{ij}}, \qquad m_{ij} = \rho\frac{\partial\psi}{\partial\chi_{ij}},$$

$$h_i = \rho\frac{\partial\psi}{\partial\varphi_{,i}}, \qquad \rho\eta = -\frac{\partial\psi}{\partial\dot{T}}\Big/\frac{\partial\Phi}{\partial\dot{T}}, \tag{6}$$

$$\frac{q_i}{\Phi} = -\rho\frac{\partial\psi}{\partial T_i}\Big/\frac{\partial\Phi}{\partial\dot{T}},$$

and

$$g = -\rho\frac{\partial\psi}{\partial\varphi} - \rho\eta\frac{\partial\Phi}{\partial\varphi} + F, \tag{7}$$

where F is called the dissipation function. Note that for the case having no dependence on rate of change in volume fraction, the equation for g given by (7) does not contain the term F.

In the context of linear theory, the dissipation function F must be a linear function in the independent constitutive variables. Following [10], we have

$$F = \omega_*^*\dot{\varphi},$$

where ω_*^* defines the porous dissipation.

(ii) the constitutive fields should satisfy the following inequality

$$\frac{q_i}{\Phi}\left(\frac{\partial\Phi}{\partial\varphi}\varphi_{,i} + \frac{\partial\Phi}{\partial T}T_{,i}\right) - \left(\rho\frac{\partial\psi}{\partial T_{,i}} + \frac{\partial\Phi}{\partial\dot{T}}\right)\dot{T}_{,i} \geq 0, \tag{8}$$

By using Eq. (7) in Eq. (5), we can also obtain the equation of energy balance of the form

$$\rho\frac{\partial\psi}{\partial T_{,i}}\dot{T}_{,i} + \rho\Phi\dot{\eta} = \rho S + q_{i,i}. \tag{9}$$

Note that entropy-inequality is satisfied in the following sense: Using Eq. (2) and together with the help of Eqs. (4) and (7) in first equation of (1), we obtain

$$\rho\left(\dot{\eta} - \frac{S}{\Phi}\right) - \left(\frac{q_i}{\Phi}\right)_{,i} = \frac{q_i\Phi_{,i}}{\Phi^2} \geq 0.$$

Note that if the terms of higher order of Φ are ignored, the entropy inequality (second equation of (1)) is satisfied in approximate form.

Now, expanding ψ and Φ in Maclaurin's series, we obtain

$$\begin{aligned}
\rho\psi &= \rho\psi_o - a_oT - b_o\dot{T} + C_{ij}e_{ij} + \Gamma_{ij}\chi_{ij} + \\
&+ p\varphi + p_i\varphi_{,i} - \frac{1}{2}d_oT^2 - c\dot{T}T - \frac{1}{2}f\dot{T}^2 = \\
&= -a_iTT_{,i} - \alpha_ob_i\dot{T}T_{,i} + \\
&-\frac{1}{2}\alpha_ok_{ij}T_{,i}T_{,j} - a_{ij}e_{ij}T - a_{ijk}e_{ij}T_{,k} - b_{ij}e_{ij}\dot{T} + \\
&- b_{ijk}\chi_{ij}T_{,k} - c_{ij}\chi_{ij}T - d_{ij}\chi_{ij}\dot{T} - bT\varphi + \\
&- \tau\dot{T}\varphi - c_iT_{,i}\varphi + f_{ij}e_{ij}\varphi + g_{ij}\chi_{ij}\varphi + \\
&+ \frac{1}{2}a\varphi^2 + d_i\varphi\varphi_{,i} - e_iT\varphi_{,i} - f_i\dot{T}\varphi_{,i} - h_{ij}T_{,i}\varphi_{,j} + \\
&+ F_{ijk}e_{ij}\varphi_{,k} + G_{ijk}\chi_{ij}\varphi_{,k} + \frac{1}{2}p_{ij} = \varphi_{,i}\varphi_{,j} + \\
&+ \frac{1}{2}A_{ijrs}e_{ij}e_{rs} + B_{ijrs}e_{ij}\chi_{rs} + \frac{1}{2}C_{ijrs}\chi_{ij}\chi_{rs},
\end{aligned} \tag{10}$$

$$\Phi = T_o - a'T - \tau_1\dot{T} - \beta_oT\dot{T} - \frac{1}{2}\gamma_o\dot{T}^2 - \delta\dot{T}\varphi, \tag{11}$$

Without loss of generality we shall take $\psi_o = 0$.

Since we are considering homogeneous material, the coefficients of (10) and (11) are all constants. Also, the following symmetric properties are evident

$$A_{ijrs} = A_{rsij}, \qquad C_{ijrs} = C_{rsij},$$

$$k_{ij} = k_{ji}, \qquad p_{ij} = p_{ji}.$$

If we assume that in the reference (natural) configuration the material is free from stress and couple stress, and has zero entropy and heat flux rate, then as a consequence of (7) and inequality (8), we have the following restrictions on the coefficients of (10)–(11),

$$\begin{aligned}
&a_i = 0, \qquad a_{ijk} = 0, \\
&b_{ijk} = 0, \qquad c_i = 0, \\
&h_{ij} = 0, \qquad b = \tau_1 a, \\
&\tau_1 a_{ij} = b_{ij}, \qquad \tau_1 b_o = \tau + a_o\delta, \\
&d_{ij} = \tau_1 c_{ij}, \qquad f_i = \tau_1 e_i, \\
&\tau_1 d_o = c - a_o\beta_o.
\end{aligned}$$

Thus, we obtain the following linear constitutive equations:

$$\begin{aligned}
t_{ij} &= A_{ijrs}e_{rs} + B_{ijrs}\chi_{rs} + f_{ij}\varphi + \\
&+ F_{ijr}\varphi_=, r - a_{ij}(T + \tau_1\dot{T}), \\
m_{ij} &= B_{ijrs}e_{rs} + C_{ijrs}\chi_{rs} + g_{ij}\varphi + \\
&+ G_{ijr}\varphi_=, r - c_{ij}(T + \tau_1\dot{T}),
\end{aligned} \tag{12}$$

$$\begin{aligned}
\rho\eta &= -(a + dT + h\dot{T} - a_{ij}e_{ij} - c_{ij}\chi_{ij} + \\
&- b\varphi - e_i\varphi_{,i}), \\
q_i &= T_o(b_i\dot{T} + k_{ij}T_{,j}), \\
g &= -f_{ij}e_{ij} - g_{ij}\chi_{ij} - a\varphi - d_i\varphi_{,i} + \\
&- b(T + \tau_1\dot{T}) + \omega_*^*\dot{\varphi},
\end{aligned} \tag{13}$$

$$h_i = d_i\varphi + p_{ij}\varphi_{,j} - e_i(T + \tau_1\dot{T}),$$

The material coefficients τ_1 and k_{ij} are interpreted as thermal relaxation time and thermal conductivity tensor, respectively.

The linearized forms of the local balance laws are

`Balance of momentum.`

$$t_{ij,j} + \rho f_i = \rho\ddot{u}_i,$$

`Balance of angular momentum.`

$$m_{ij,j} - \epsilon_{irs}t_{rs} + \rho l_i = \rho j\ddot{\phi}_j, \tag{14}$$

`Balance of equilibrated stress.`

$$h_{i,i} + g + \rho l = \rho X\ddot{\varphi}.$$

In above equations, f_i and l_i are the body force and body couple, j is micro-inertia, l and X are extrinsic body force and equilibrated inertia, respectively.

In the context of the linear theory and in the absence of internal heat source, the energy Eq. (9) reduces to

$$\rho T_o \dot{\eta} = q_{i,i}. \tag{15}$$

For an isotropic material, the constitutive relations become

$$\begin{gathered} A_{ijrs} = \lambda\delta_{ij}\delta_{rs} + (\mu + K)\delta_{ir}\delta_{js} + \mu\delta_{is}\delta_{jr}, \\ f_{ij} = \xi\delta_{ij}, \qquad a_{ij} = \nu\delta_{ij}, \qquad d_o = \rho C^*, \\ C_{ijrs} = \alpha\delta_{ij}\delta_{rs} + \gamma\delta_{ir}\delta_{js}\beta\delta_{is}\delta_{jr}, \\ g_{ij} = \varsigma\delta_{ij}, \qquad p_{ij} = d\delta_{ij}, \\ k_{ij} = K^*\delta_{ij}, \qquad h = d_o\tau_o, \\ B_{ijrs} = F_{ijr} = G_{ijr} = \\ = c_{ij} = e_i = d_i = b_i = a_o = 0, \end{gathered} \tag{16}$$

where, δ_{ij} is the Kronecker delta and λ and μ are the usual Lame constants, K^* is the thermal conductivity, τ_o is the thermal relaxation time, ν is the coefficient of linear thermal expansion, and K, α, γ, β, ν, b, a, d, k, J are the constitutive modulii for the theory.

Accordingly, the constitutive equations are reduced to

$$\begin{gathered} t_{ij} = \lambda = e_{rr}\delta_{ij} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijk}\phi_k) + \\ - \nu\delta_{ij}(T + \tau_1\dot{T}) + \xi\varphi\delta_{ij}, \\ m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i} + \varsigma\varphi\delta_{ij}, \\ g = -\xi\delta_{ij}e_{ij} - \varsigma\delta_{ij}\chi_{ij} - a\varphi - b(T + \tau_1\dot{T}) + \omega_*^*\dot{\varphi}, \\ h_i = d\varphi_{,i}, \\ \rho\eta = \nu e_{ii} - b\varphi + \rho C^*\left(1 + \tau_o\frac{\partial}{\partial t}\right)T, \end{gathered} \tag{17}$$

$$q_{i,i} = K^* T_{,ii}. \tag{18}$$

In the case Lord and Shulman (L-S) [13] theory, we take $h = 0$ in Eq. (16) and instead of Eq. (18) we use

$$\left(1 + \tau_o\frac{\partial}{\partial t}\right)q_{i,i} = K^* T_{,ii}. \tag{19}$$

Now, the field Eqs. (14) and (15) with the help of Eqs. (17)–(19) for the micropolar porous generalized thermoelastic solid take the form,

$$\begin{gathered} (\lambda + 2\mu + K)\nabla(\nabla\cdot\vec{u}) - (\mu + K)\nabla\times\nabla\times\vec{u} + \\ + K(\nabla\times\vec{\phi}) + \xi\nabla\varphi - \nu\left(1 + \tau_1\frac{\partial}{\partial t}\right)\nabla T = \rho\frac{\partial^2\vec{u}}{\partial t^2}, \end{gathered} \tag{20}$$

$$\begin{gathered} (\alpha + \beta + \gamma)\nabla(\nabla\cdot\vec{\phi}) - \gamma\nabla\times(\nabla\times\vec{\phi}) + \\ + K(\nabla\times\vec{u}) - 2K\vec{\phi} + \varsigma\nabla\varphi = \rho\jmath\frac{\partial^2\vec{\phi}}{\partial t^2}, \end{gathered} \tag{21}$$

$$\begin{gathered} d\nabla^2\varphi - \xi\nabla\cdot\vec{u} - \varsigma\nabla\cdot\vec{\phi} - \omega_*^*\frac{\partial\varphi}{\partial t} + \\ - a\varphi - b\left(1 + \tau_1\frac{\partial}{\partial t}\right)T = \rho X\frac{\partial^2\varphi}{\partial t^2}, \end{gathered} \tag{22}$$

$$\begin{gathered} K^*\nabla^2 T = \rho C^* T_0\left(\frac{\partial T}{\partial t} + \tau_0\frac{\partial^2 T}{\partial t^2}\right) + \\ + T_0\left(1 + n_0\tau_0\frac{\partial}{\partial t}\right)\left(\nu\,\mathrm{div}\,\vec{u} - b\frac{\partial\varphi}{\partial t}\right). \end{gathered} \tag{23}$$

For Lord and Shulman (L-S) [13] theory $\tau_1 = 0$, $n_0 = 1$ and for Green and Lindsay (G-L) [14] theory $\tau_1 \geq \tau_0 > 0$, $n_0 = 0$.

3. Formulation of the problem

We considered a micropolar porous generalized thermoelastic medium in cylindrical coordinate system $(r,\ \theta,\ z)$ with z-axis pointing upwards along axis of the cylinder. For two dimensional problem, we assume the components of displacement and microrotation vector of the form,

$$\vec{u} = (u_r, 0, u_z), \qquad \vec{\phi} = (0, \phi_\theta, 0), \tag{24}$$

where we had taken symmetry about z-axis, so all partial derivatives with respect to the variable θ would be zero. Also, we introduce the non-dimensional quantities defined by the expressions

$$\begin{gathered} (r', z') = \frac{\omega^*}{c_1}(r, z), \\ (u'_r, u'_z) = \frac{\rho\omega^* c_1}{\nu T_0}(u_r, u_z), \\ t'_{zz} = \frac{t_{zz}}{\nu T_0}, \qquad t'_{rr} = \frac{t_{rr}}{\nu T_0}, \\ \varphi' = \left(\frac{\rho c_1^2}{\nu T_0}\right)\varphi, \qquad \phi'_\theta = \left(\frac{\rho c_1^2}{\nu T_0}\right)\phi_\theta, \\ m'_{z\theta} = \left(\frac{\omega^*}{c_1\nu T_0}\right)m_{z\theta}, \qquad T' = \frac{T}{T_0}, \\ t' = \omega^* t, \qquad \tau'_0 = \omega^*\tau_0, \\ \tau'_1 = \omega^*\tau_1, \end{gathered}$$

where

$$c_1^2 = \frac{\lambda + 2\mu + K}{\rho}, \qquad \omega^* = \frac{\rho C^* c_1^2}{K^*}. \tag{25}$$

Introducing the potential functions ψ_1, ψ_2 and Γ of the form

$$\begin{gathered} u_r = \frac{\partial\psi_1}{\partial r} + \frac{\partial^2\psi_2}{\partial r\partial z}, \\ u_z = \frac{\partial\psi_1}{\partial z} - \left(\nabla^2 - \frac{\partial^2}{\partial z^2}\right)\psi_2, \\ \phi_2 = -\frac{\partial\Gamma}{\partial r}, \end{gathered} \tag{26}$$

and using Laplace and Hankel transform defined by

$$\overline{f}(p) = \int_0^\infty e^{-pt} f(t)dt, \tag{27}$$

$$\widehat{f}(q,z,p)=\int_0^\infty \overline{f}(r,z,p)rJ_n(rq)dr, \tag{28}$$

where $J_n()$ is the Bessel function of the first kind of index n, together with the help of Eqs. (24) and (25), in Eqs. (20)–(23), we obtain

$$\left(\frac{d^4}{dz^4}+A\frac{d^2}{dz^2}+B\right)(\widehat{\psi_2},\widehat{\Gamma})=0, \tag{29}$$

$$\left(\frac{d^6}{dz^6}+C\frac{d^4}{dz^4}+D\frac{d^2}{dz^2}+E\right)(\widehat{\psi_1},\widehat{\varphi},\widehat{T})=0, \tag{30}$$

where

$$A=-\frac{K}{(\mu+K)}\left[\frac{2q^2(\mu+K)}{K}+\frac{p^2\rho c_1^2}{K}-+\right.$$
$$\left.-\frac{\rho\jmath p^2c_1^2(\mu+K)}{\gamma K}-\frac{2Kc_1^2(\mu+K)}{\gamma\omega^{*2}K}+\frac{Kc_1^2}{\gamma\omega^{*2}}\right],$$

$$B=\frac{K}{(\mu+K)}\left[\left(\frac{q^2(\mu+K)+p^2\rho c_1^2}{K}\right)\right.$$
$$\left.\left(-q^2+\frac{\rho\jmath c_1^2p^2}{\gamma}+\frac{2Kc_1^2}{\gamma\omega^{*2}}\right)+\frac{Kc_1^2q^2}{\gamma\omega^{*2}}\right],$$

$$a_1=q^2-\frac{\omega_*^*c_1^2p}{d\omega^*}+\frac{ac_1^2}{d\omega^{*2}}+\frac{\rho Xp^2}{d\omega^{*2}},$$

$$a_2=\frac{b\rho c_1^2(1-\tau_1p)}{d\omega^{*2}\nu},$$

$$\epsilon_1=\frac{\nu^2T_o}{K^*\rho\omega^*},$$

$$\epsilon_2=\frac{\nu bT_o}{K^*\rho\omega^*},$$

$$a_3=p-n_0\tau_0p^2,$$

$$a_4=q^2-a_3,$$

$$C=-a_1-a_4-p^2-q^2+\frac{\xi^2}{\rho\omega^{*2}},$$

$$D=a_1a_4+p\epsilon_2a_2a_3+(p^2+q^2)(a_1+a_4)-\frac{\xi^2a_4}{\rho\omega^{*2}}+$$
$$+\frac{\xi c_1^2p\epsilon_2a_3(1-\tau_1p)}{\omega^{*2}}+\epsilon_1a_3(1-\tau_1p),$$

$$E=(p^2+q^2)(a_1*a_4+p\epsilon_2a_2a_3)+$$
$$-\frac{\xi\epsilon_1a_2a_3}{\rho c_1^2}-\epsilon_1a_1a_3(1-\tau_1p).$$

Let us assume that the roots of Eq. (29) and (30) are q_1, q_2, q_3 and q_4, q_5 respectively. The solution of Eqs. (29) and (30) satisfying radiation condition are

$$(\widehat{\psi}_1,\widehat{\varphi},\widehat{T})=\sum_{i=1}^{3}A_i(1,r_i,s_i)e^{-q_iz},$$
$$(\widehat{\psi}_2,\widehat{\Gamma})=\sum_{j=4}^{5}B_i(1,t_i)e^{-q_iz}, \tag{31}$$

where

$$r_i=\frac{\dfrac{\xi c_1^2q_i^2}{\omega^{*2}}+\dfrac{b\rho c_1^2}{d\nu\omega^{*2}}(q_i^2-p^2-q^2)}{q_i^2-q^2+b1},$$

$$s_i=\frac{\dfrac{\xi r_i}{\rho c_1^2}+q_i^2-p^2-q^2}{(1-\tau_1p)},\quad i=1,\,2,\,3,$$

$$t_j=-\frac{(\mu+K)}{K}\left[q_j^2-q^2-\frac{p^2\rho c_1^2}{(\mu+K)}\right],\ j=4,\,5.$$

4. Boundary conditions

The boundary conditions for a load applied at the surface $z=0$ are:

$$t_{zz}=-P_1f_1(r,t),$$
$$t_{zr}=0,$$
$$m_{z\theta}=0, \tag{32}$$
$$\frac{\partial T}{\partial z}=P_2f_2(r,t),$$
$$\frac{\partial\varphi}{\partial z}=0,$$

where P_1 is the magnitude of force and P_2 is the constant temperature applied on the boundary and $f_1(r,t)$, $f_2(r,t)$ are the known functions, defined below in the manuscript.

5. Application

In this section, we consider the different type of sources,

$$f(r,t)=\begin{cases}f_1(r,t) & \text{for mechanical force,}\\ f_2(r,t) & \text{for thermal source.}\end{cases} \tag{33}$$

Applying Laplace and Hankel transform defined by (27) and (28) on (33), we obtain

$$\widehat{f}(\xi,p)=\begin{cases}\widehat{f_1}(\xi,p) & \text{for mechanical force,}\\ \widehat{f_2}(\xi,p) & \text{for thermal source.}\end{cases} \tag{34}$$

Case 1. Concentarted normal source.

In order to determine the displacement and stress components due to concentrated force/ thermal point source (described by Dirac's delta)

$$f(r,t)=\frac{\delta(r)\delta(t)}{2\pi r},$$

must be used with

$$\widehat{f}(\xi,p)=\frac{1}{2\pi}. \tag{35}$$

Case 2. Normal source over the circular region.

In order to determine the displacement and stress components due to normal source over the circular region of non-dimensional radius 'a' is obtained by setting

$$f(r,t) = \frac{1}{\pi a^2} H(a-r)\delta(t),$$

with

$$\widehat{f}(\xi,p) = \frac{J_1(a\xi)}{\pi a\xi}. \tag{36}$$

Making use of Eqs. (17), (24)–(26) in the boundary conditions (32) and applying Laplace and Hankel transforms defined by Eqs. (27)–(28) together with the help of (31) in the resulting expressions, we obtain the expressions for stresses, volume fraction field and temperature distribution as

$$(\widehat{t}_{zz}, \widehat{t}_{zr}) = \frac{1}{\Delta}\sum_{k=1}^{5} \Delta_k(a_k^*, b_k^*) e^{-q_k z},$$

$$(\widehat{\varphi}, \widehat{T}) = \frac{1}{\Delta}\sum_{l=1}^{3} \Delta_l(r_l, s_l) e^{-q_l z}, \tag{37}$$

$$\widehat{m}_{z\theta} = \frac{1}{\Delta}(\Delta_4 c_4^* e^{-q_4 z} + \Delta a_5 b_5^* e^{-q_5 z}),$$

where

$$\triangle = \begin{vmatrix} a_1^* & a_2^* & a_3^* & a_4^* & a_5^* \\ b_1^* & b_2^* & b_3^* & b_4^* & b_5^* \\ 0 & 0 & 0 & c_4^* & c_5^* \\ s_1q_1 & s_2q_2 & s_3q_3 & 0 & 0 \\ r_1q_1 & r_2q_2 & r_3q_3 & 0 & 0 \end{vmatrix},$$

and Δ_i, $i = 1, ..., 5$, is obtained from Δ by replacing i^{th} column of Δ with $|-\widehat{P}_1\widehat{f}_1(\xi,p) \;\; 0 \;\; 0 \;\; \widehat{P}_2\widehat{f}_2(\xi,p) \;\; 0|^T$. Taking $P_1 = 1$ and $P_2 = 0$, we obtain the solutions for mechanical force and when we take $P_1 = 0$ and $P_2 = 1$, the corresponding solutions for the case of thermal source can be obtained.

$$a_i^* = q_i^2 - \lambda q^2 + \frac{\xi r_i}{\rho c_1^2} - (1+\tau_1 p)s_i,$$

$$a_{4,5}^* = \frac{(2\mu + K)q^2}{\rho c_1^2},$$

$$b_i^* = \frac{(2\mu + K)qq_i}{\rho c_1^2}, \tag{38}$$

$$b_{4,5}^* = \frac{-(qq_{4,5}^2(\mu+K) + q^3\mu + Kqt_{1,2})}{\rho c_1^2},$$

$$c_{1,2}^* = \frac{-\gamma\omega^* q\, t_{1,2}\, q_{4,5}}{\rho c_1^4}, \quad i = 1,2,3.$$

To obtain the expressions obtained for concentrated source and source applied over circular region is obtained by replacing the value of $\widehat{f}(\xi,p)$ from Eqs. (35) and (36) in Eq. (38).

6. Particular case

Neglecting the porosity effect (i.e. substituting ξ, ς, d, a, b, $\omega_*^* = 0$), we obtain the corresponding expressions in the case of micropolar generalized thermoelastic solid and our results tally with those obtained by Kumar et al. [15].

7. Inversion of the transform

To obtain the solution to the problem in the physical domain, we must invert the transforms in Eq. (37). Transformed stresses and temperature distribution are functions of r, the parameters of Laplace and Hankel transforms p and ξ, respectively, and hence are of the form $\tilde{f}(\xi, r, p)$. To get the function in the physical domain, we invert the Laplace and Hankel transform by using the method described in [15].

8. Numerical results and discussion

We take the case of magnesium crystal [16] like material (micropolar elastic solid) subjected to mechanical disturbances for numerical calculations. The physical constants used are:

$$\rho = 1.74 \text{ Kg/m}^3,$$

$$\lambda = 9.4 \times 10^{10} \text{ N/m}^2,$$

$$\mu = 4.0 \times 10^{10} \text{ N/m}^2,$$

$$K = 1.0 \times 10^{10} \text{ N/m}^2,$$

$$j = 0.2 \times 10^{-15} \text{ m}^2,$$

$$\gamma = 0.779 \times 10^{-9} \text{ N}.$$

The thermal parameters are given as

$$T_0 = 293 \text{ K},$$

$$K^* = 1.7 \times 10^2 \text{ J/sec m}^\circ\text{C},$$

$$C^* = 1.04 \times 10^3 \text{ J/kg}^\circ\text{C},$$

$$\tau_o = 0.04\text{s},$$

$$\tau_1 = 0.07 \text{ s}.$$

The void parameters are taken as,

$$d = 3.668 \times 10^{-9} \text{ N},$$

$$b = 1.13849 \times 10^{10} \text{ N/m}^2,$$

$$a = 1.475 \times 10^{10} \text{ N/m}^2,$$

$$\omega_*^* = 0.787 \times 10^{-4} \text{ N sec/m}^2,$$

$$X = 1.753 \times 10^{-19} \text{ m}^2.$$

The variations of normal stress t_{zz}, tangential stress t_{zr}, tangential couple stress $m_{z\theta}$. Volume fraction field φ and temperature distribution T with radial distance $'r'$ at the plane $z = 1.0$ for L-S and G-L theories have been depicted graphically with:

a) solid and broken line without center symbol for micropolar porous thermoelastic solid (MPTS),
b) solid and broken line with center symbol ($- \circ - \circ -$) for micropolar thermoelastic solid (MTS),
c) solid and broken line with center symbol ($-\triangle - \triangle -$) for porous thermoelastic solid (PTS),
d) solid and broken line with center symbol ($- \times - \times -$) for thermoelastic solid (TS),
e) dotted line for the case of micropolar porous solid (MPS).

Figures 1–9 show the variations of stresses, volume fraction field and temperature distribution when mechanical forces are applied, while Figs. 10–18 show the corresponding variations when thermal sources are applied. Figures 1–4, 10–14 show the variations of normal stress, tangential stress, tangential couple stress and temperature distribution due to applied concentrated source, while Figs. 5–8, 15–17 show the corresponding variations when the source is applied over a circular region. In these figures the comparison is shown for classical thermoelasticity with those of micropolar one together with comparison of L-S and G-L theories. Figures 9 and 18 show the variations of volume fraction field when mechanical and thermal sources are applied. In these figures comparison is shown for L-S and G-L theory together with comparison of concentrated source (C) and source applied over a circular region (CR).

Figure 1 depicts that the value of normal stress t_{zz} initially decreases and then oscillate with increase in radial distance r for MPTS, whereas for MTS its value start with sharp initial increase over the interval (0.3), then decreases, after that oscillate with small amplitude about the zero value. However, in the case of MPS, its value sharply increases and then oscillates with decreasing amplitude near the origin. Also, in the case of PTS its value oscillate about origin with very small amplitude, and for TS its value initially increases and then oscillate about origin. The variation is similar for both L-S and G-L theories. It is evident from Fig. 2 that the value of tangential stress t_{zr}, for MPTS, start with sharp initial decrease and then oscillate with large amplitude about the origin, whereas for MTS its value increases sharply and then oscillates with decreasing amplitude. Also in the case of PTS, its value oscillate with constant amplitude, while in the case of TS the variation is similar to that of MTS. It is observed from Fig. 3 that the values of tangential couple stress $m_{z\theta}$ oscillate with increasing amplitude for MPTS, while for MTS its value sharply decreases, then sharply increases and then oscillates with increase in radial distance r. It is evident from the Figs. 2 and 3 that, the variation pattern of t_{zr} and $m_{z\theta}$ for MPS is similar to the variation of MTS, with difference in their amplitude. Figure 4 shows that the value of temperature distribution T in the case of MPTS, initially decreases and then oscillate with increase in radial distance r whereas in the case of MTS its value starts with sharp initial increase, then oscillate with decreasing amplitude about the origin. The variation of T, in the case of PTS is similar to that of MPTS with difference in its amplitude, while for TS, its behavior is opposite to that of MTS. The variation of normal stress t_{zz}, tangential stress $m_{z\theta}$, tangential couple stress $m_{z\theta}$ and temperature distribution T on application of point force applied over the circular region is shown in Figs. 5–8. From Fig. 5 it is observed that the trend of variation of normal stress t_{zz} for MPTS, the amplitude of oscillation of normal stress, near the point of application of the force is higher for G-L theory as compared to that of L-S. However for the remaining cases its value remains almost similar in nature. It is evident from Fig. 6 that the variation pattern of tangential stress is similar for MTS, PTS and TS and opposite for MPTS. However, the variation pattern of temperature distribution T is similar for MPTS, PTS and TS as shown in Fig. 8. The value of tangential couple stress $m_{z\theta}$ for MPTS and for both the theories oscillate about origin with increasing amplitude, while for MTS, its value decreases within the range $0 \leq x \leq 3$ and then increases with increase in radial distance r. It is also depicted from Figs. 5–7 that the neglecting the effect of generalized theories of thermoelastcity, the values of t_{zz}, t_{zr} and $m_{z\theta}$ vary with small amplitude.

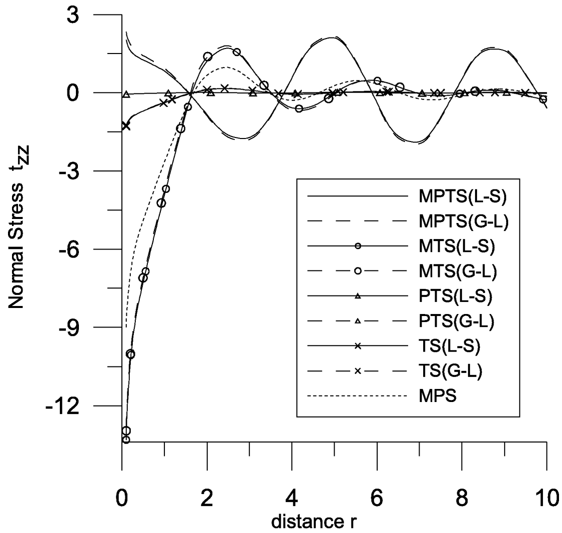

Fig. 1. Variations of t_{zz} with distance r due to concentrated force

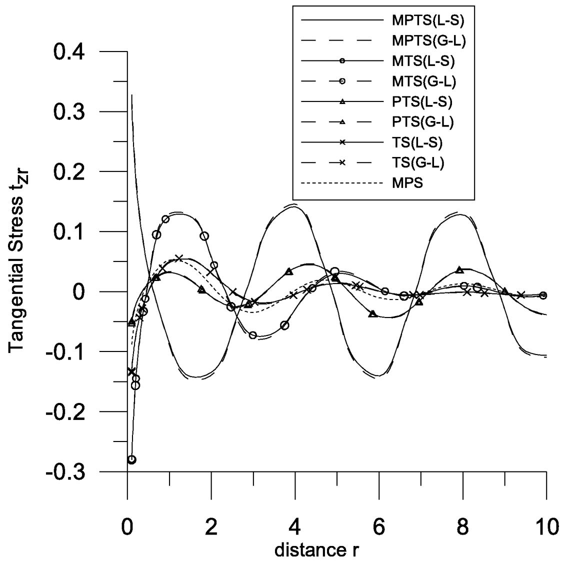

Fig. 2. Variations of t_{zr} with distance r due to concentrated force

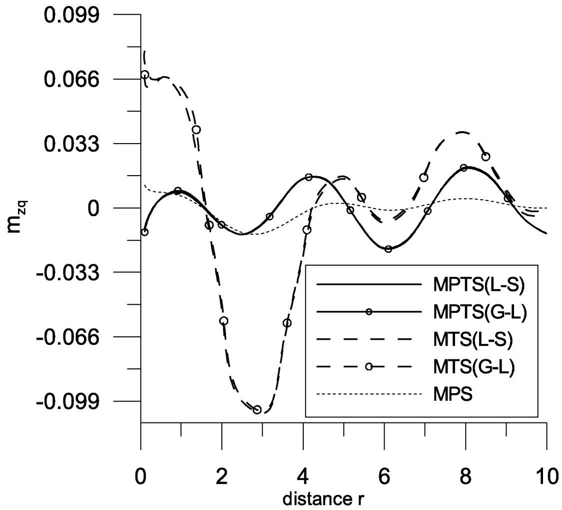

Fig. 3. Variations of $m_{z\theta}$ with distance r due to concentrated force

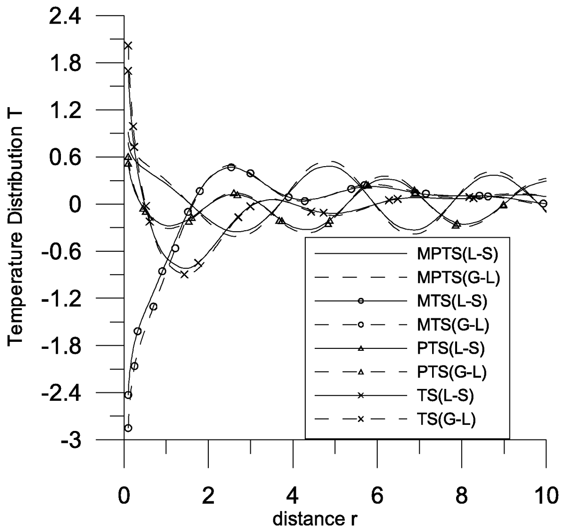

Fig. 4. Variations of T with distance r due to concentrated force

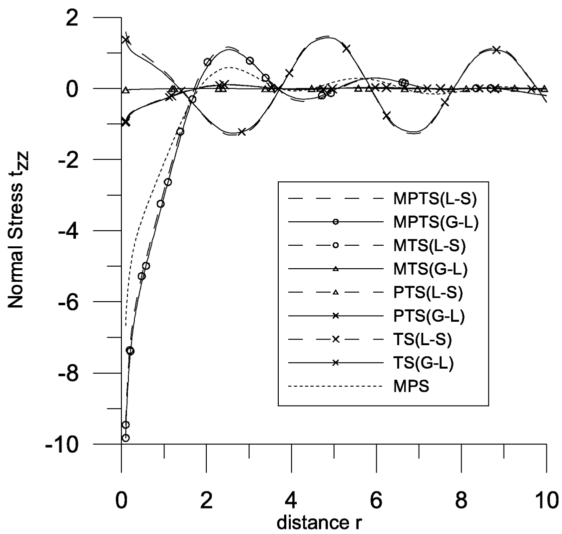

Fig. 5. Variations of t_{zz} with distance r due to source over the circular region

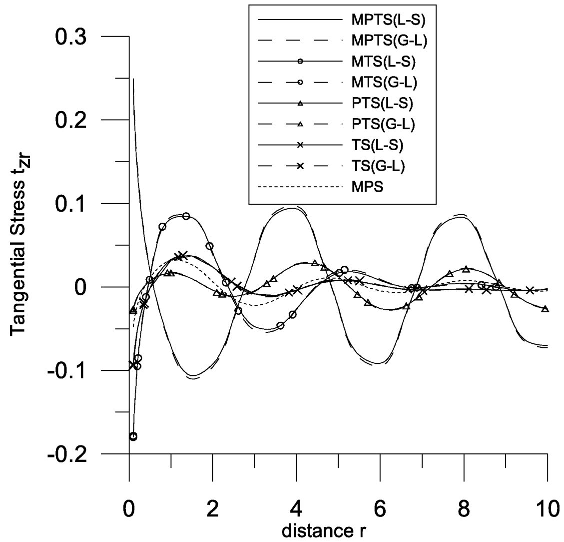

Fig. 6. Variations of t_{zr} with distance r due to source over the circular region

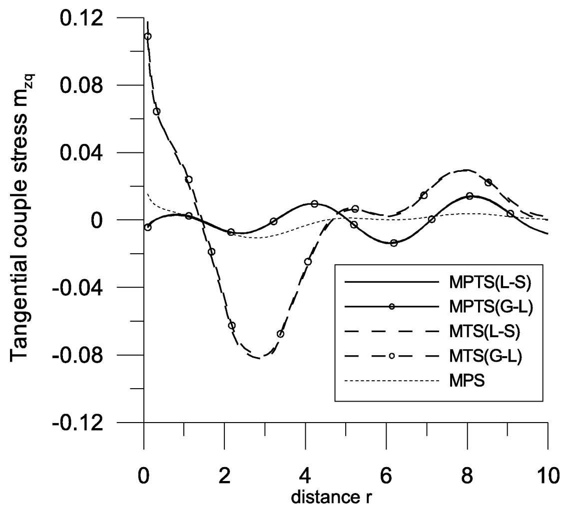

Fig. 7. Variations of $m_{z\theta}$ with distance r due to source over a circular region

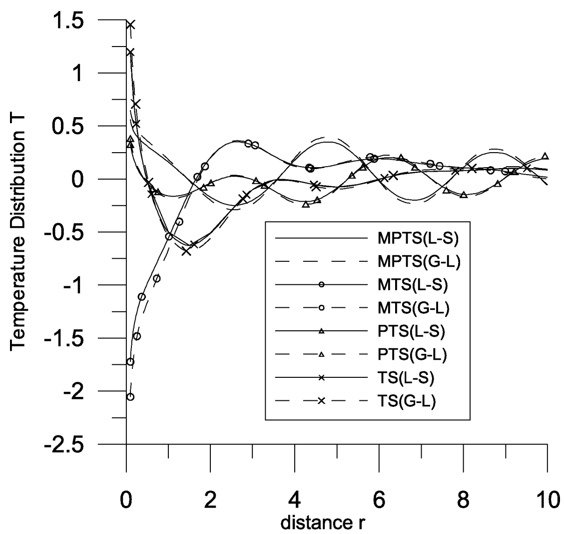

Fig. 8. Variations of T with distance r due to source over the circular region

Figure 9 shows the variation of volume fraction field φ, with radial distance r. In this graph, the comparison of coupled thermoelasticity with those of micropolar theory, together with the comparison of forces applied is shown. It is evident from figure that on the application of both the forces and for MPTS, the value of volume fraction field sharply increases and then oscillate about origin with large amplitude, however, for PTS its value oscillates with increasing amplitude near the point of application of the force. Also for MPS, its values starts with sharp initial initial decrease over the interval (0,3.2) and then oscillate with very small amplitude.

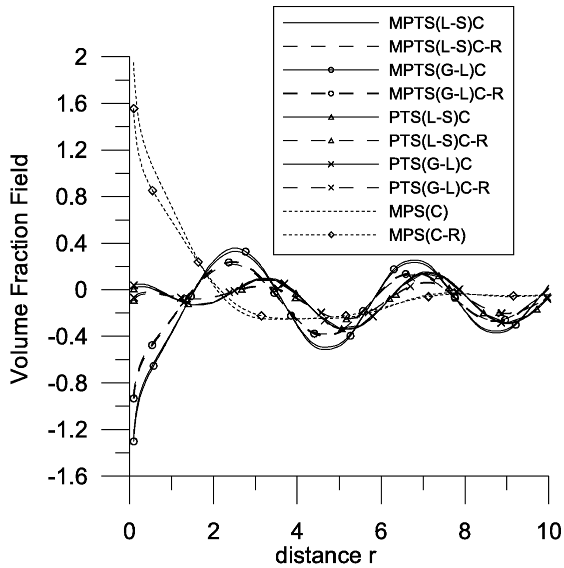

Fig. 9. Variation of volume fraction field with respect to distance r

It is evident from Fig. 10 that the value of normal stress t_{zz}, in the case of MPTS, initially increases, then decreases and then oscillates with very large amplitude about the origin, while for PTS its value oscillate with very small amplitude about the origin. In the case of MTS, its value sharply increases for the range $0 \leq x \leq 3$, and then oscillates with decreasing amplitude, while reverse behavior is observed in the case of TS. The value of tangential stress t_{zr} for all the cases is oscillatory in nature. For micropolar solid these variations are of higher amplitude, while the variations are of small amplitude for the solid without microplar. These variations are shown in Fig. 11. It is evident from Fig. 12 that the value of $m_{z\theta}$ starts with sharp initial decrease, increases and then oscillates with further increase in radial distance r for MPTS. However, for MTS its value initially decrease, then increases and then oscillates with further increase in r. It is observed from Fig. 13 that the variation of temperature distribution is oscillatory in nature. It is observed form the figure that for MTS and TS, its value starts with sharp initial increase and oscillates with decreasing amplitude, while for the remaining cases, near the point of application of the load, its value oscillates with constant amplitude.

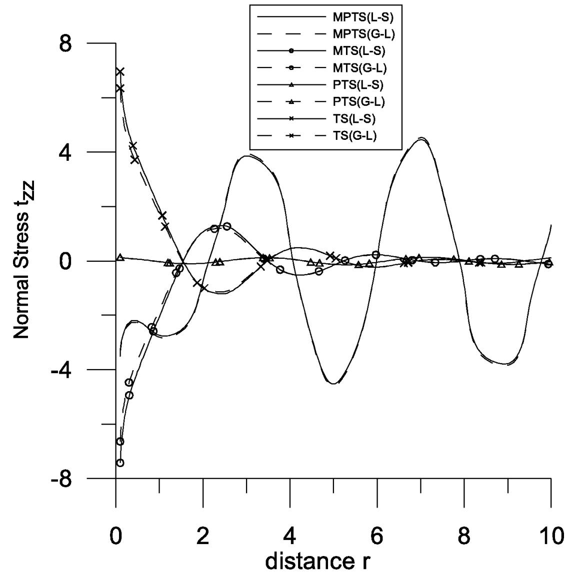

Fig. 10. Variations of normal stress with distance r due to concentrated source

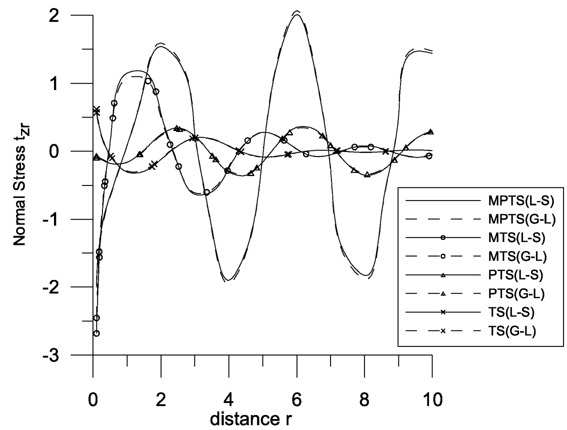

Fig. 11. Variations of t_{zr} with distance r due to concentrated source

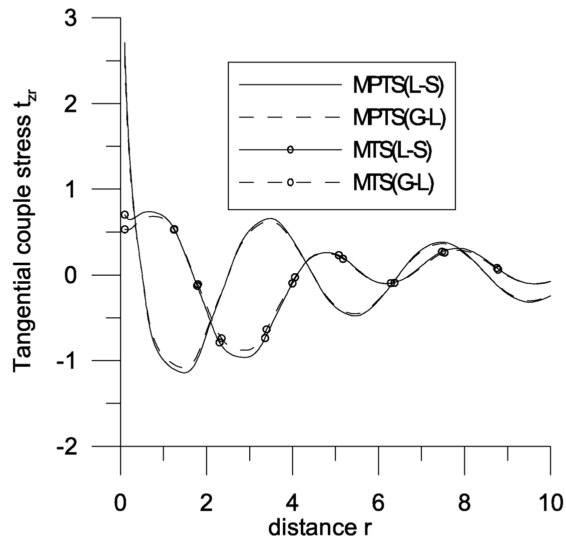

Fig. 12. Variations of tangential couple stress with respect to distance r

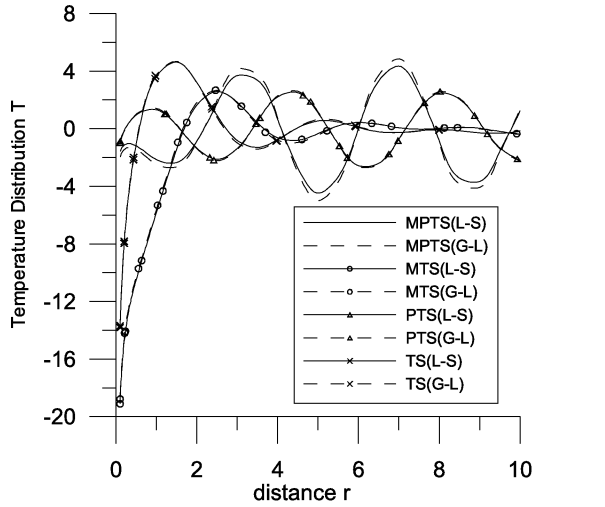

Fig. 13. Variations of T with distance r due to concentrated thermal source

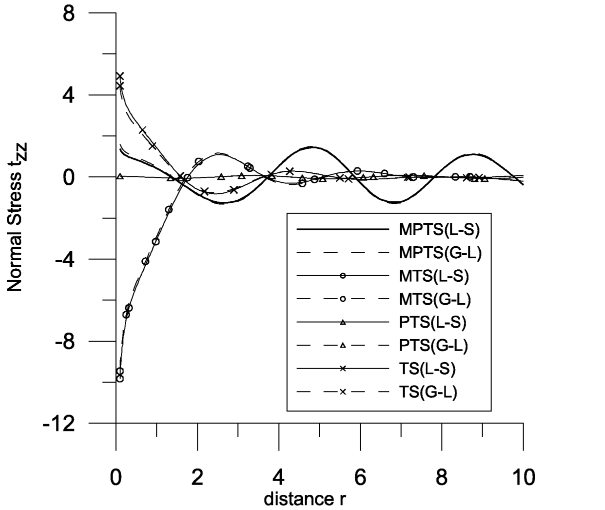

Fig. 14. Variations of t_{zz} with distance r due to thermal source over the circular region

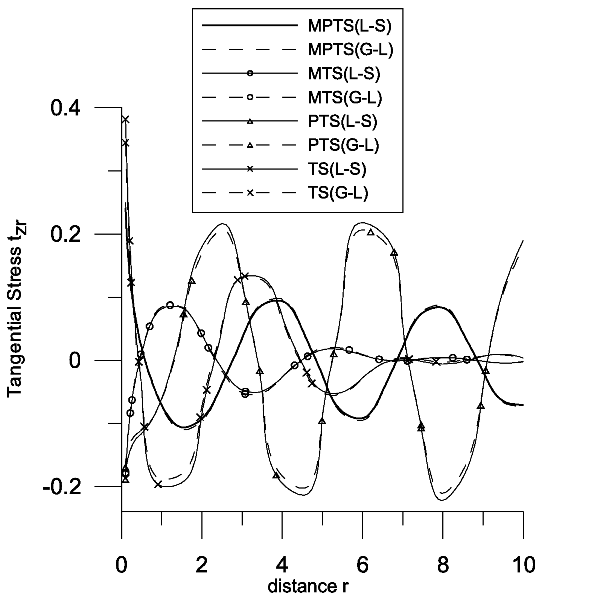

Fig. 15. Variations of t_{zr} with distance r due to thermal source over the circular region

Figures 14–17 show the variations of stresses and temperature distribution when source applied over a circular region. The value of normal stress t_{zz} is more for all the cases except that of MTS initially decreases and then oscillates with decreasing amplitude, where its value starts with sharp initial increase and then oscillates, which is observed from Fig. 14. Figure 15 shows that the variations of tangential stress is oscillatory in nature. The amplitude of oscillation are higher for the micropolar solid as compared to that of thermoelastic solid. It is observed from Fig. 16 that the value of tangential couple stress $m_{z\theta}$ is higher for MPTS as compared to that of MTS for both L-S and G-L theory. Figure 17 shows that for MPTS, its value oscillates with very small amplitude near the point of application of the load, while for MTS its value initially increases and then oscillates with very small amplitude about the origin. However, for PTS, its value oscillates with increasing amplitude and for TS, its value starts with sharp increase, and then oscillates with decreasing amplitude. The variations of volume fraction field φ is shown in Fig. 18.

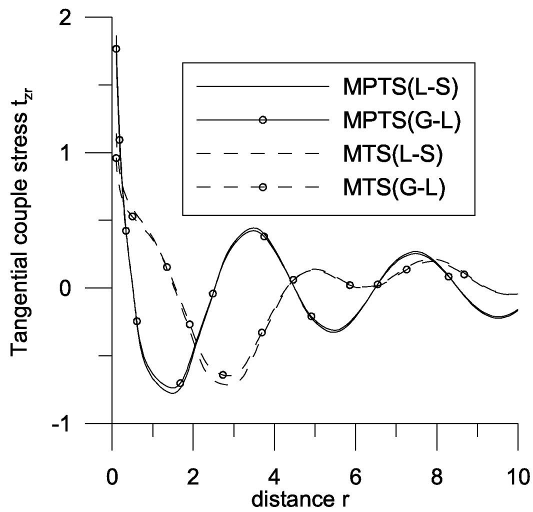

Fig. 16. Variations of $m_{z\theta}$ with distance r due to source over circular region

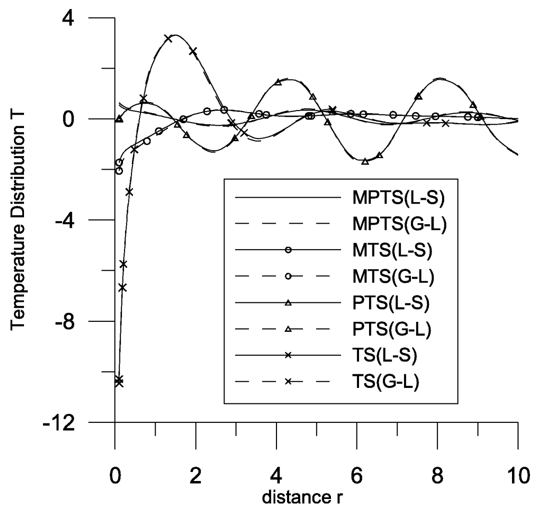

Fig. 17. Variations of T with distance r due to thermal source over the circular region

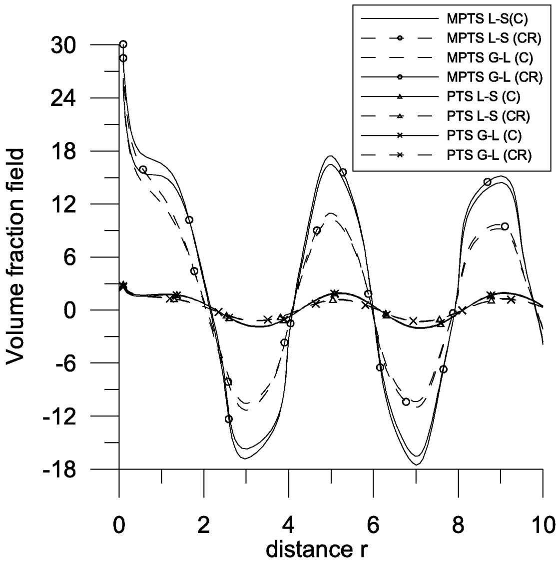

Fig. 18. Variations of volume fraction field with respect to distance r

For all the cases of micropolar solid, its value initially decreases and then oscillates with very large amplitude about the origin, while in the case of thermoelastic solid, its values oscillates with very small amplitude.

9. Conclusions

Deformation in micropolar porous generalized thermoelastic medium is investigated due to the application of normal load. An appreciable effect of micropolarity and porosity is observed on the stresses, volume fraction field and temperature distribution. Also, appreciable effect of thermoelasticty is observed on the stresses, volume fraction field. It is concluded that on the application of all the sources except for concentrated thermal source, the trend of variations of normal and tangential stresses for porous thermoelastic material is opposite to that of thermoelastic material. However, the variation pattern remains same for the both micropolar porous generalized thermoelastic material and micropolar generalized thermoelastic material. The amplitudes of oscillation of tangential couple stress on the application of mechanical forces get increased due to presence of porosity in the material. The variation of temperature distribution get decreased due to the presence of porosity, on the application of thermal source over the circular region. Also, the value of volume fraction field, is initially less for concentrated source as compared to those when the source is applied over the circular region, but afterwards its value get increased. However, its value in the case of thermoelastic porous solid is less as compared to its value in the case of micropolar porous generalized thermoelastic solid. It is also concluded that neglecting the thermoelastic effect from the micropolar porous thermoelastic solid, the value of stresses initially get, reversed and then the amplitude of oscillations gets decreased. So, we can say that introduction of generalized thermoelasticity meliorated the results, obtained for micropolar theory.

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