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Two theorems about Lorentz method for asymmetrical anisotropic regions

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Abstract. The paper has dealt with two theoretical problems of calculation of electromagnetic force or torque. The first problem considers the magnetically anisotropic and conductive region. The theorem about equivalence of both Maxwell and Lorentz methods has been presented. The second problem deals with the independence from the integration surface of force or torque calculated by the Maxwell method. The second theorem which presents the sufficient condition for an independence problem in the anisotropic and nonconductive region has been formulated.

Key words: asymmetrical magnetic anisotropy, electromagnetic force and torque, models of electromechanical converters for benchmarks.

1. Electromagnetic field forces

The electromagnetic field forces [1, 2] theoretical analysis is still a vital problem [3–5]. For evaluation of the electromagnetic force density the following form is applied

$$\vec{f} = \vec{f}_{\mathrm{L}} + \vec{M}, \tag{1}$$

where $\vec{M}$ is so-called anisotropy component [4, 5]. The proof of (1) bases on the Lorentz force density $\vec{f}_{\mathrm{L}}$ formula, Maxwell's equations, and assumptions that displacement current (that leads to electromagnetic field impulse force [5, 6]) and hysteresis phenomenon can be neglected (nonhomogeneous component $\vec{N} = 0$). The force density component $\vec{M}$ in (1) means the anisotropy force component. For magnetic field it takes the form of

$$\vec{M} = \frac{1}{2}(\nu_{vu} - \nu_{uv})B_v \mathrm{grad}(B_u). \tag{2}$$

Total electromagnetic force can be calculated by the following equation

$$\vec{f} = -\vec{i}_u \mathrm{div}_u(\vec{\sigma}_u) - \vec{\Delta}, \tag{3}$$

where $\vec{i}_u$ denotes versor for u^{th} coordinate, vectors $\vec{\sigma}_u$ $(u = 1, 2, 3)$ are built of Maxwell's stress tensors and $\vec{\Delta}$ is the residual vector [4, 5]. The total force or torque can also be calculated by means of the coenergy method.

2. The first theorem – surface-integral force or torque representation in conductive and magnetically anisotropic regions

The first theorem considers the equivalence between both volume (Lorentz force density integral) and surface (Maxwell stress tensor) integrals representations for Lorentz force or torque – Fig. 1. This problem is one of these which are called *surface-integral representation problems*. This problem is analogous to the surface-integral representation of total electric charge which is placed in a closed region due to the Gaussian law. It is well-known that the surface-integral representation for electromagnetic field forces can be introduced for electromagnetic field regions if the Maxwell stress tensor is symmetrical [1, 2, 4, 5]. However, the symmetry of the Maxwell stress tensor is guaranteed only for isotropic media. The first theorem extends this statement for anisotropic media if symmetric anisotropy of reluctivity occurs. There is a question: whether the surface-integral of the Lorentz force or the torque representation for anisotropic media is possible?

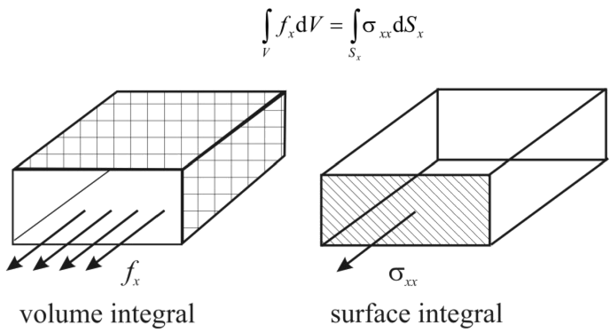

Fig. 1. Equivalence of volume and surface integrals

The answer is positive under a certain condition. Namely, the surface-integral representation of force or torque is possible for isotropic and also for anisotropic media exhibiting normal anisotropy feature i.e. reluctivity matrix is symmetrical. For isotropic medium the reluctivity (permeability) matrix is diagonal one and all pivot values are equal to each other. For media that exhibit normal anisotropy the reluctivity matrix can be presented as diagonal having different pivot values. There are also media and structures for these the reluctivity matrix is asymmetrical. This case considers the first theorem.

For magnetic field region where

– nonhomogeneous force vanishes (no reluctance force),
– hysteresis phenomenon does not appear,

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the Lorentz's force (volume integral) and Maxwell (surface integral) methods lead to the equal results for magnetically anisotropic region if reluctivity matrix is symmetrical i.e. if for $u \neq v$ is satisfied

$$\nu_{vu} = \nu_{uv}. \tag{4}$$

The condition (4) is satisfied for both isotropic and normal anisotropic media. It should be pointed out that if the condition (4) is not satisfied i.e.

$$\nu_{vu} \neq \nu_{uv} \tag{5}$$

the surface-integral representation of Lorentz force/torque is not possible.

The media for the condition (5) are satisfied, therefore they can be called active structures. As an example, the active feature results form magnetization of the region by magnets or rotation in a magnetic field [7, 8]. In generally, the asymmetry of reluctivity appears if in a certain way the energy is supplied to the medium.

The mathematical proof of this theorem is derived from Eq. (3). For u^{th} forces components of Eq. (3) the following can be obtained:

$$f_u = f_{\mathrm{L}u} + M_u = -\mathrm{div}_{|u|}(\vec{\sigma}_u) - \Delta_u. \tag{6}$$

Because, if (4) is satisfied the anisotropy component vanishes $M_u = 0$ due to (2). For the properly chosen coordinate system often $\Delta_u = 0$ (e.g. for the Cartesian coordinate system $\Delta_x = \Delta_y = \Delta_z = 0$, for the cylindrical system $\Delta_\alpha = 0$, for the spherical system $\Delta_\varphi = 0$). Hence, according to (3) it can be written

$$L_{|u|} f_u = L_{|u|} f_{\mathrm{L}u} = -\mathrm{div}(\vec{\sigma}_u), \tag{7}$$

where L_u are Lame coefficients (no summation over $|u|$ is provided). Applying the Gaussian theorem for (7) the following equation is obtained

$$\int\limits_V L_{|u|} f_u \mathrm{d}V = -\int\limits_S \vec{\sigma}_u \mathrm{d}\vec{S}, \tag{8}$$

which proves the first theorem. Namely, this means that for regions with symmetric reluctivity matrix (either isotropic or normal anisotropic) the Lorentz force or torque can be presented by surface-integral of the Maxwell stress tensor. In other words, the surface-integral representation is possible for media with symmetric reluctivity matrix.

The force density leads to the electromagnetic force or torque acting in electromechanical converters such electrical machines [9–11]. The examples for applying the first theorem for electromechanical converter models linear (force calculation) and cylindrical (torque calculation) have been widely presented in [5].

3. The second theorem – surface-integral force or torque representation in nonconductive regions

Let us consider electromagnetic force or torque that is exerted in the finite conductive region Ω of an electromechanical converter. It is the moving part e.g. rotor, carriage. Force or torque can be evaluated by surface integral over the surface S placed in the nonconductive region Ω_{out} which surrounds the conductive region Ω. The outer region Ω_{out} is the gap of an electromechanical converter. It is the nonconductive region Ω_{out} that usually does not exert electromagnetic force or torque. The integration surface S in the gap Ω_{out} can be placed in different way – Fig. 2 (e.g. its radius can be different).

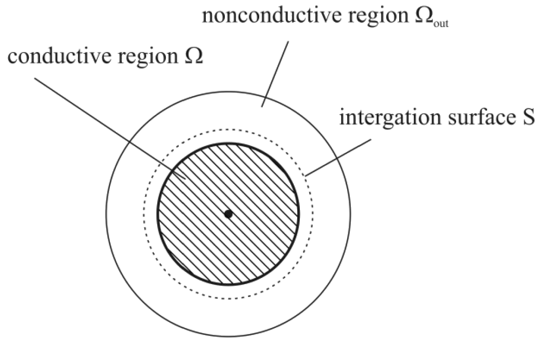

Fig. 2. The electromagnetic torque evaluation by surface integral over S

Mostly, for electromechanical converters, such as rotating electric machines [9–11], the electromagnetic torque value given by surface integral does not depend on the radius of surface S placed in the gap. The independence of surface integral results from a magnetic feature of the gap which is the air-gap (the isotropic gap), usually.

The problem is well-known in electrostatics while calculating total electric charge with the help of the Gauss law. The outer surface (Gaussian surface) must be spread so as to surround the whole charge independently from its shape, but can surround the greater region.

The gap which surrounds the active region Ω (e.g. rotor, carriage) nonconductive but which could be magnetically active medium e.g. could be filled with magnetic fluids (paragraphs 5 and 6). As a consequence, the gap region Ω_{out} could be magnetically either isotropic or anisotropic.

The second theorem considers the sufficient condition for a surface-integral representation for different magnetic features of the nonconductive region Ω_{out} (the gap). The region Ω exerting electromagnetic force or torque is placed in volume V of both conductive and nonconductive regions

$$V = \Omega \oplus \Omega_{\mathrm{out}}, \tag{9}$$

where Ω_{out} is the region which surrounds the conductive region Ω. The integrals which lead to total electromagnetic force and torque values exerted in the conductive Ω are

$$\vec{F}_V = \int\limits_V \vec{f} \mathrm{d}V = \int\limits_\Omega \vec{f} \mathrm{d}V + \int\limits_{\Omega_{\mathrm{out}}} \vec{f} \mathrm{d}V \tag{10a}$$

and

$$\vec{T}_V = \int\limits_V (\vec{r} \times \vec{f}) \mathrm{d}V = \int\limits_\Omega (\vec{r} \times \vec{f}) \mathrm{d}V + \int\limits_{\Omega_{\mathrm{out}}} (\vec{r} \times \vec{f}) \mathrm{d}V \tag{10b}$$

respectively. The total electromagnetic force and torque can be denoted as follows

$$\vec{F}_V = \vec{F}_{\rm e} + \int\limits_{\Omega_{\rm out}} \vec{f} {\rm d}V = \vec{F}_{\rm e} + \Delta\vec{F}_{\rm out} \tag{11a}$$

and

$$\vec{T}_V = \vec{T}_{\rm e} + \int\limits_{\Omega_{\rm out}} (\vec{r}\times\vec{f}) {\rm d}V = \vec{T}_{\rm e} + \Delta\vec{T}_{\rm out} \tag{11b}$$

where $\vec{F}_V$, $\vec{T}_V$ are force and torque calculated over the volume V of both regions, $\vec{F}_{\rm e}$, $\vec{T}_{\rm e}$ are total electromagnetic force and torque, $\Delta\vec{F}_{\rm out}$, $\Delta\vec{T}_{\rm out}$ denote the force and torque exerted in nonconductive region $\Omega_{\rm out}$.

The second theorem answers to the question: whether the electromagnetic force or torque arising in region Ω is equal to the value for surface-integral over surface S? In other words: whether the electromagnetic force or torque can be evaluated by surface-integral over different surfaces S which surrounds the conductive region Ω. If the answer is positive the total electromagnetic force/torque can be calculated for any surface S i.e. for different surfaces S_1 and $S_2 \neq S_1$ as shown in Fig. 3.

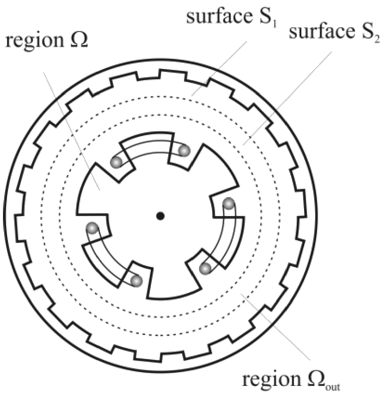

Fig. 3. The electromagnetic torque does not depend on integral-surface position in the isotropic gap

The solution of the problem results from (1), (2). If the nonconductive region $\Omega_{\rm out}$ is also homogeneous, and there is no hysteresis phenomenon, hence only an anisotropic component can give a contribution to residual integrals in (11a,b). The total electromagnetic force and torque satisfy the following equivalences

$$\vec{F}_V = \vec{F}_{\rm e} \quad \Leftrightarrow \quad \Delta\vec{F}_{\rm out} = 0 \tag{11c}$$

and

$$\vec{T}_V = \vec{T}_{\rm e} \quad \Leftrightarrow \quad \Delta\vec{T}_{\rm out} = 0. \tag{11d}$$

In other words, only magnetic anisotropy of the nonconductive region (the gap region is indexed by δ) does not influence on electromagnetic force or torque, when the anisotropy component (2) is equal to zero i.e. if for $u \neq v$ is satisfied.

$$\nu_{vu\delta} = \nu_{uv\delta}. \tag{12}$$

Hence, the second theorem could be formulated.

For magnetic field region where

– nonhomogeneous force vanishes (no reluctance force),
– hysteresis phenomenon does not appear

the Lorentz force or torque does not depend on a surface position in the nonconductive and magnetically anisotropic region (the gap δ) if the equality (12) is satisfied.

These two theorems about surface-integral representation of the Lorentz force/torque are important from both theoretical and computational points of view. In order to present them subsequently three models of electromechanical converters with conductive and nonconductive magnetically anisotropic movable parts are considered.

4. Electromagnetic force and torque calculations

In order to present the theoretical results some models of electromechanical converters are considered. There are: linear, cylindrical and spherical converter models. Applying these models enable us to omit numerical errors. The necessary simplifications of the converter geometry are assumed. The analyses of electromagnetic field can be provided with the help of the variable separation method [1, 2, 5, 8]. This way of analysis is chosen for giving also a precise insight into electromagnetic force or torque calculations.

The electromagnetic force and torque given by the Maxwell's method are as follows

$$F_{\rm e} = \int\limits_{\partial V} H_2 B_1 {\rm d}S, \tag{13a}$$

$$T_{\rm e} = \int\limits_{\partial V} L_2 H_2 B_1 {\rm d}S, \tag{13b}$$

where indices for linear, cylindrical and spherical coordinate systems denote: $1 \leftrightarrow x, r, r$; $2 \leftrightarrow y, \alpha, \phi$; $3 \leftrightarrow z, z, \theta$, respectively [5].

The total electromagnetic force and torque can be calculated also by means of coenergy W_c as follows

$$F_{\rm e} = \left.\frac{\partial W_C}{\partial y}\right|_{j=\rm const} = \int\limits_V \left(\vec{j}\frac{\partial\vec{A}}{\partial y} + \vec{B}\frac{\partial\vec{H}}{\partial y}\right) {\rm d}V, \tag{14a}$$

$$T_{\rm e} = \left.\frac{\partial W_C}{\partial x_2}\right|_{j=\rm const} = \int\limits_V \left(\vec{j}\frac{\partial\vec{A}}{\partial x_2} + \vec{B}\frac{\partial\vec{H}}{\partial x_2}\right) {\rm d}V, \tag{14b}$$

where $x_2 = \alpha$ for cylindrical converter or $x_2 = \phi$ for spherical converter. The both Maxwell and coenergy methods give **in any case** (**if electromagnetic field impulse force can be neglected**) the same results for isotropic as well as anisotropic region V. The force and torque evaluated by the Lorentz method are given by the formulas

$$F_{\rm L} = \int\limits_{\Omega} (j_z B_x - j_x B_z)\, {\rm d}V, \tag{15a}$$

$$T_{\rm L} = \int\limits_{\Omega} L_2\, (j_3 B_1 - j_1 B_3)\, {\rm d}V. \tag{15b}$$

The anisotropy force and torque component [5] are equal to

$$F_{\mathrm{M}} = \frac{1}{2}\int\limits_{V} (\nu_{xy} - \nu_{yx}) B_x \frac{\partial B_y}{\partial y} \mathrm{d}V, \tag{16a}$$

$$T_{\mathrm{M}} = \tfrac{1}{2}\int\limits_{V} (\nu_{12} - \nu_{21}) B_1 \tfrac{\partial B_2}{\partial x_2} \mathrm{d}V \tag{16b}$$

and do not vanish for regions where asymmetrical magnetic anisotropy appears, i.e. $\nu_{12} \neq \nu_{21}$.

The results of force and torque calculations can also be treated as benchmarks for models of linear, cylindrical and spherical converters [12–14]. For the linear converter an acting force is calculated. For cylindrical and spherical converters electromagnetic torques are calculated.

The first theorem was widely discussed in [5, 15]. For the second theorem examples of linear, cylindrical and spherical electromechanical converters are developed.

5. Linear motor – force calculation

Firstly, an exemplary linear motor is considered – Fig. 4. The magnetic medium can appear in the gap in order to enhance the force value [16]. The ratings are $\gamma = 30 \cdot 10^6$ S/m (carriage conductivity), $a = 0.02$ m (conductive layer width), $l = 1.0$ m (rotor length), $g = 0.01$ m (the gap width), $\Theta_1 = 4870$ A (magnetomotive force first harmonic), $Y = 1$ m (pair-pole length), $\nu_{xx} = 0.4\nu_0$ (cross-layer axis reluctivity), $\nu_{yy} = 0.4\nu_0$ (move direction axis reluctivity), $\nu_{xy\delta} = 0.1\nu_0$, $\nu_{yx\delta} = 0.0$ and different gap reluctivities $\nu_{xy\delta}$, $\nu_{yx\delta}$ (Table 1).

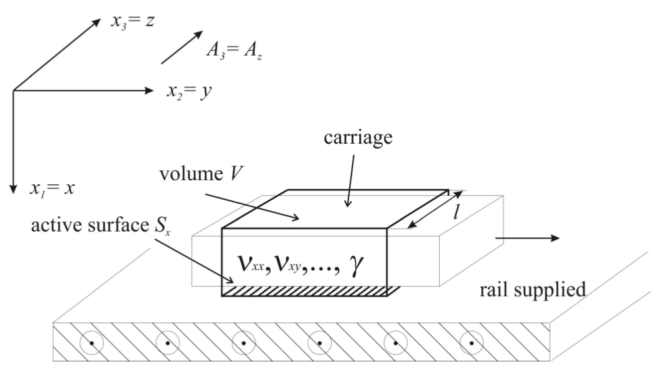

Fig. 4. Linear electromechanical converter

Table 1
Examples for torques evaluation for the second theorem presentation – linear motor

	Reluctivities	The second theorem
a)	$\nu_{xy\delta} = 0 \quad \nu_{yx\delta} = 0$	(12) is satisfied – Fig. 5a
b)	$\nu_{xy\delta} = 0.1\nu_0 \quad \nu_{yx\delta} = 0$	(12) is **not** satisfied – Fig. 5b

Table 1 and Figs. 5a,b confirm that if the condition (12) is satisfied the second theorem thesis for force is fulfilled. Otherwise, the force depends on the position of the integration surface in the case b). It shows that if condition (12) is not fulfilled the second theorem for forces cannot be applied – Fig. 5b.

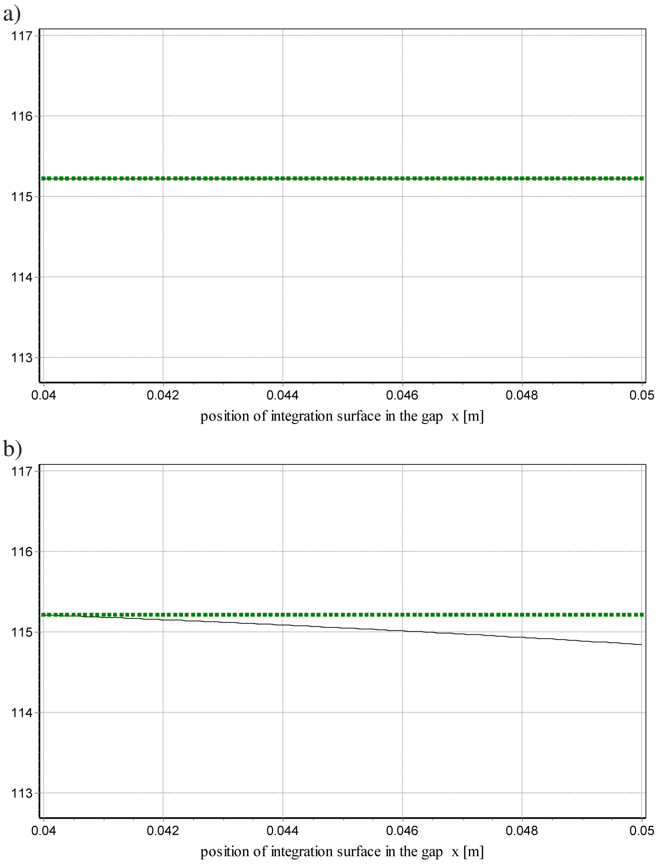

Fig. 5. Electromagnetic force vs. position of integral surface in the gap of linear motor (Maxwell and coenergy method – line Lorentz method - points)

6. Cylindrical motor – torque calculation

For presenting these theorem for electromagnetic torque calculations the induction motor with solid rotor (Fig. 6) and ferrofluid in the gap [17] is taken into account with parameters: $\gamma = 35 \cdot 10^6$ S/m (rotor conductivity), $a = 0.02$ m (conductive rotor layer width), $R = 0.05$ m (rotor outer radius), $l = 0.25$ m (rotor length), $g = 0.0005$ m (the gap width), $\Theta_1 = 1504$ A (magnetomotive force first harmonic), $p = 2$ (pair pole number), $\nu_{rr} = \nu_0/3$ (radial reluctivity), $\nu_{\alpha\alpha} = \nu_0/2$ (tangential reluctivity), $\nu_{r\alpha} = 0.2\nu_0$, $\nu_{\alpha r} = 0.2\nu_0$ and gap reluctivities $\nu_{r\alpha\delta}$, $\nu_{\alpha r\delta}$ as shown in Table 2.

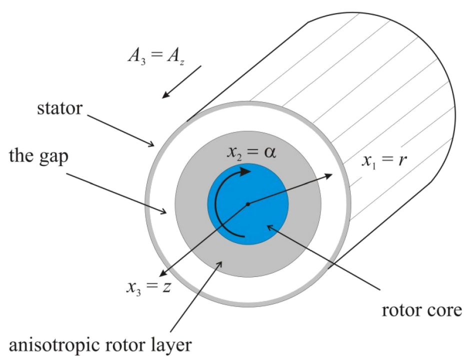

Fig. 6. Cylindrical electromechanical converter

Table 2
Examples of torques evaluation for the second theorem presentation – cylindrical motor

	Reluctivities	The second theorem
a)	$\nu_{r\alpha\delta} = 0 \quad \nu_{\alpha r\delta} = 0$	(12) is satisfied – Fig. 7a
b)	$\nu_{r\alpha\delta} = 0.3\nu_0 \quad \nu_{\alpha r\delta} = 0$	(12) is **not** satisfied – Fig. 7b

Table 2 and the Fig. 7a confirm that if condition (12) is satisfied the second theorem for electromagnetic torques is fulfilled. Otherwise, in the case b), it is shown that if condition (12) is not satisfied thus the second theorem thesis is not satisfied. Figure 7b presents that the electromagnetic torque calculated by the Maxwell's and coenergy method for different radius of the integration surface (cylindrical surface) can be different for radius change from $r = R$ (conductive rotor outer surface) up to $r = R + g$ (inner stator surface).

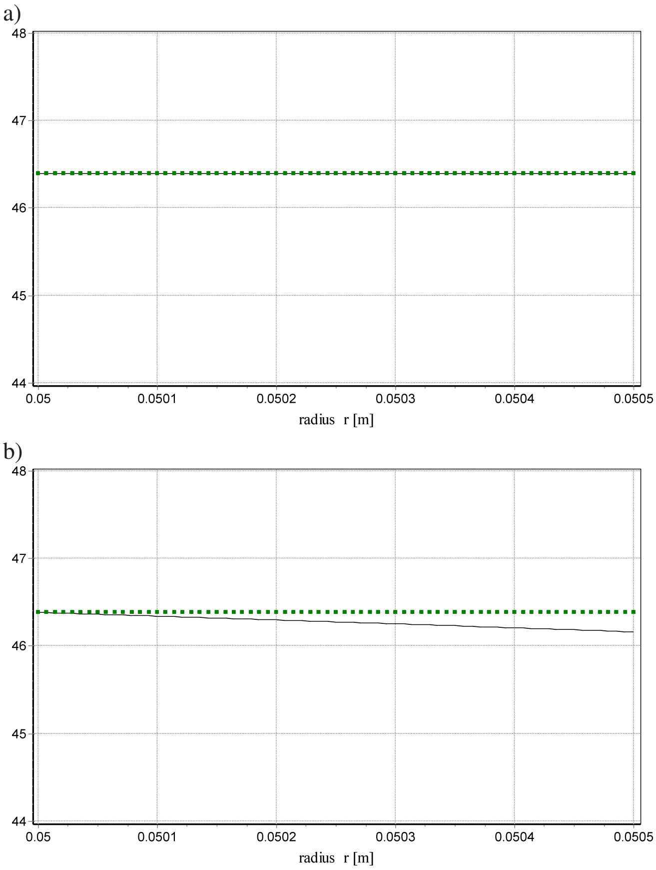

Fig. 7. Electromagnetic torque vs. radius for the gap region (Maxwell and coenergy method – line, Lorentz method – points)

7. Spherical motor – torque calculation

For an induction motor with a spherical rotor the second theorem is also satisfied for electromagnetic torque. Exemplary, a spherical motor is considered (Fig. 8) with solid rotor $\gamma = 56 \cdot 10^6$ S/m (rotor conductivity), $a = 0.02$ m (conductive rotor layer width), $R = 0.03$ m (rotor outer radius), $g = 0.0005$ m (the gap width), $\Theta_1 = 300$ A (magnetomotive force first harmonic), $p = 2$ (pair pole number), $\nu_{rr} = 0.7\nu_0$ (radial reluctivity), $\nu_{\phi\phi} = 0.7\nu_0$ (tangential reluctivity), $\nu_{r\phi} = 0.1\nu_0$, $\nu_{\phi r} = 0.1\nu_0$ and gap reluctivities $\nu_{r\phi\delta}$, $\nu_{\phi r\delta}$ (Table 3).

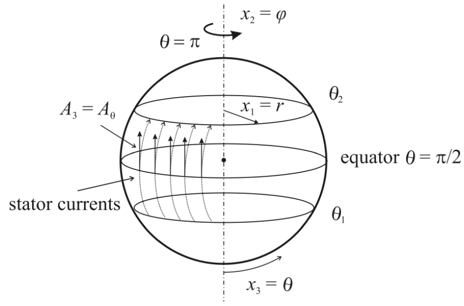

Fig. 8. Spherical electromechanical converter

Table 3
Examples for torques evaluation for the second theorem presentation – spherical motor

	Reluctivities	The second theorem
a)	$\nu_{r\phi\delta} = 0 \quad \nu_{\phi r\delta} = 0$	(12) is satisfied – Fig. 9a
b)	$\nu_{r\phi\delta} = 0.15\nu_0 \quad \nu_{\phi r\delta} = 0$	(12) is **not** satisfied – Fig. 9b

Figures 9 a,b present the electromagnetic torque value given by the Maxwell's method for different radii of spherical integration surface. The radius changes from $r = R$ (conductive rotor outer surface) up to $r = R + g$ (inner stator surface). Table 3 and Fig. 9a confirm that if condition (12) is satisfied the second theorem for electromagnetic torque of spherical converter. In opposite, in the case b) it is shown that if condition (12) is not satisfied the second theorem cannot be applied – Fig. 9b.

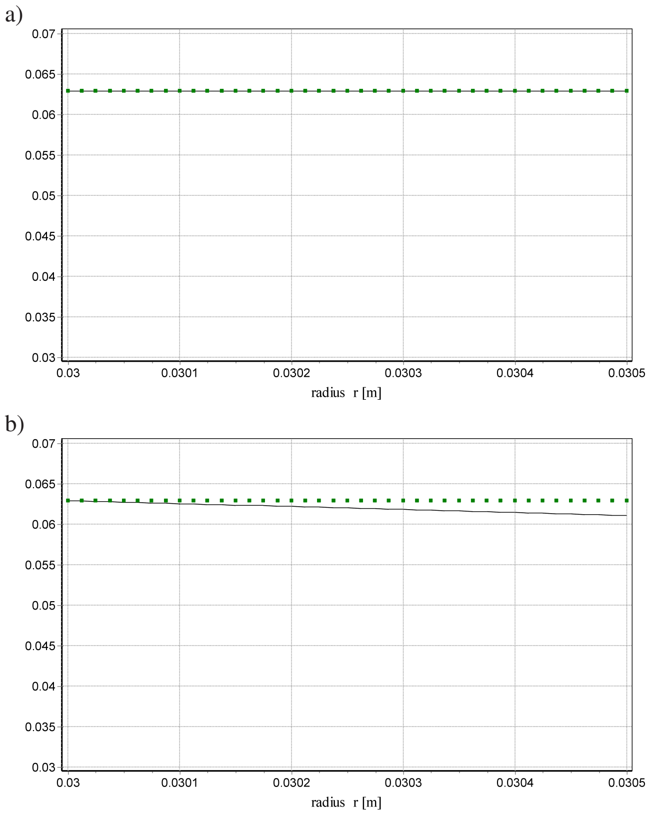

Fig. 9. Electromagnetic torque vs. radius for the gap region (Maxwell and coenergy method – line, Lorentz method – points)

8. Conclusions

There two theorems about electromagnetic force or torque for anisotropic media have been presented. The first theorem for conductive regions states:

For magnetic field region where

– nonhomogeneous force vanishes (no reluctance force),
– hysteresis phenomenon does not appear,

the Lorentz's force (volume integral) and Maxwell (surface integral) methods lead to the equal results for magnetically anisotropic region if reluctivity matrix is symmetrical i.e. if for $u \neq v$ is satisfied $\nu_{vu} = \nu_{uv}$.

The second theorem for nonconductive regions states:

For magnetic field region where

– nonhomogeneous force vanishes (no reluctance force),
– hysteresis phenomenon does not appear

the Lorentz force/torque value does not depend on surface position in the nonconductive and magnetically anisotropic region (the gap δ) if for $u \neq v$ is satisfied $\nu_{vu\delta} = \nu_{uv\delta}$.

In order to present these two theorems three models of electromechanical converters with conductive (carriage, rotor) and nonconductive (the gap) magnetically anisotropic regions have been considered. Particularly, the linear, cylindrical and spherical induction motors have been taken into account. The theorems have been applied and presented. The results of forces and torques calculations can also be treated as benchmarks.

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