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Identification of emitter sources in the aspect of their fractal features

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Abstract. This article presents the procedure of identification radar emitter sources with the trace distinctive features of original signal with the use of fractal features. It is a specific kind of identification called *Specific Emitter Identification*, where as a result of using transformations, which change measure points, a transformation attractor was received. The use of linear regression and the Lagrange polynomial interpolation resulted in the estimation of the measurement function. The method analysing properties of the measurement function which has been suggested by the authors caused the extraction of two additional distinctive features. These features extended the vector of basic radar signals' parameters. The extended vector of radar signals' features made it possible to identify the copy of radar emitter source.

Key words: fractal feature, pattern of radar, signal processing, Specific Emitter Identification (SEI).

1. Introduction

The word "fractal" was introduced and popularized by Benoit Mandelbrot in the book The Fractal Geometry of Nature [1]. This term concerns a wide range of example geometric objects such as Cantor set [2], Helge'a von Koch curve [3], Sierpinski carpet [4], Julia Mandelbrot's sets and many more. In her early works Mandelbrot describes fractals using three basic properties i.e. defined by the recursive relation and not by a formula, having the self-similarity feature (a part is similar to the entire object) and their dimension is not an integer. However, it has to be said that the properties of fractals cannot be a base for an accurate mathematical definition as the same objects may be defined in different ways. The term "recursive" is also one of many others as some "typical fractal" may be defined in an algebraic way. A good example of it is the Cantor set. The self-similarity feature is also difficult to define especially while taking into account the fact that the simplest geometric objects have this feature. One should focus its attention on the dimension of geometric object especially when it is difficult to define it by an integer. Therefore, one of the most common number characteristics of fractals is the dimension. At the same time examples of sets such as Cantor set, Peano curves [5] and Hilbert curves [6] resulted in a new definition of dimension [7]. Also Hausdorff dimension [8] and the definition of Menger topological dimension [9] have a direct influence on the Mandelbrot definition. The researchers of the field of science often use the definition of dimension by Minkowski [10], which is also called a *box dimension.*

2. Specific Identification of emitter sources, extraction of Fractal features

Identification of signals with the use of classic methods (classic identification methods) based on the statistic analysis of basic measurement parameters such as radio frequency (RF), amplitude (A), pulse width (PW) or pulse repetition interval (PRI) is not enough for SEI problems. Therefore, what is often adopted in the process of definition are methods using for instance, out-of-band radiation or extraction of distinctive features which increase the explicitness of the results received in identification of emitter sources [11, 12].

One of the ways to increase the number of details of definition is a specific identification of electromagnetic emitter sources SEI which extracts distinctive features in the process of signal transformation. The distinctive features may be a result of the received transformations of measurement data sets. New data sets will have fractal features which will make it possible to define clearly the source of emission. The fractal features and the theory of fractals is adopted by researchers especially in the field of SAR (*Synthetic Aperture Radar)* image transformation [13, 14], acoustic signal transformation and the analysis of radar signals. New possibilities of Digital Signal Processing (DSP) in Frequency Modulated Continuous Wave (FMCW) radar and fractal image compression is a promising brand new compression method [15, 16]. As the authors of this article claim the identification of emitter sources based on classical methods of the analysis of basic parameters is currently inefficient. The methods of SEI [17, 18] should be used in order to identify a radar copy of the same type more precisely.

2.1. Measurement points transformation. The easiest way to make fractals is using a set of affine transformations which are contractions or narrowing transformations. In this case the set of affine transformations is Iterated Function System (IFS). The authors of this article made a recording of radar signal where further frequency values, for which the recording was made, correspond to particular measurement points. By

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transforming the sets of measurement points within the scope of their symmetry or left-side/right-side asymmetry what was received was the attractor of transformation which can be a fractal in a special case. As a result the attractor of the generalized measurement function appeared which was the result of the procedure of SEI emitter identification described here. While doing the analytical procedure of defining the attractor of the measurement function the authors assigned right-side measurement vectors $\mathbf{p}^{\mathrm{r}}$ and left-side ones $\mathbf{p}^{\mathrm{l}}$, with the beginning in the particular point of reference f_0, so that: $\mathbf{p}^{\mathrm{r}} = [p_1^{\mathrm{r}}, p_2^{\mathrm{r}}, ..., p_N^{\mathrm{r}}]^T$ and $\mathbf{p}^{\mathrm{l}} = [p_1^{\mathrm{l}}, p_2^{\mathrm{l}}, ..., p_M^{\mathrm{l}}]^T$. In order to define the desirable selective features the $T\colon \mathbf{p}^{\mathrm{r}} \to \mathbf{t}$ transformation was done. In this transformation $\mathbf{t}$ is the image of the $\mathbf{p}^{\mathrm{r}}$ vector in the form of a vector with coordinates corresponding to the $\mathbf{p}^{\mathrm{l}}$ vector. For the transparent record of the transformation above with the use of vectors: $\mathbf{p}^{\mathrm{r}}$ and $\mathbf{p}^{\mathrm{l}}$, the mapping was written in the Euclidean plane, that is $T : E^1 \to E^2$. In the issue, which is considered here, these transformations are linear mappings, so they can be written in the matrix form as $\mathbf{t} = T(\mathbf{p}^{\mathrm{p}}, \mathbf{A})$, in which $\mathbf{A}$ is the matrix of a given transformation. Depending on the received symmetry or asymmetry (right/left-hand) of measurement points they create three different dispersion graphs. These dispersion graphs were presented in [19].

2.2. The attractor of transformation, extraction of distinctive features. In further analytical procedure of making an attractor of the measurement function the authors used the method of linear regression in order to define regression equations and coordinates of characteristic points. The next step estimated the characteristic points (which are solutions of regression equations) used to estimate the measurement function running across these points in the form of Lagrange interpolation polynomial. Measurement points presented in Fig. 1, transformed and depicted together, form the so-called measurement function $K(f_n)$. Figure 1 shows the coordinate plane, where an abscissa (the value of x) is marked as a f_n^x and an ordinate (the value of y) is marked as a f_n^y. On the basis of distinctive streaks which were formed, such hypothesis can be proposed: functions $g_A(f_n)$, $g_B(f_n)$, $g_C(f_n)$ and $g_D(f_n)$ belong to the class of linear functions, in which $g_A(f_n)$, $g_B(f_n)$, $g_C(f_n)$ and $g_D(f_n)$ are the regression lines for the streaks formed through the measurement points [19, 20].

A linear equation of regression for the presented case is defined with the following equation $\mathbf{g}(f_n) = \boldsymbol{\alpha} \cdot f_n + \boldsymbol{\beta}$, in which $\boldsymbol{\alpha}$ can be expressed as a vector $[\alpha_A, \alpha_B, \alpha_C, \alpha_D]^T$ and $\boldsymbol{\beta}$ can be expressed as a vector $[\beta_A, \beta_B, \beta_C, \beta_D]^T$ and $\mathbf{g}(f_n)$ can be expressed as a vector $[g_A(f_n), g_B(f_n), g_C(f_n), g_D(f_n)]^T$. To define the value of $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ Eq. (1) should be minimalized.

$$E\left[f_n^Y - \boldsymbol{\alpha} \cdot f_n - \boldsymbol{\beta}\right]^2 = \min \tag{1}$$

$$\begin{cases} \frac{\partial}{\partial \alpha} E\left[f_n^Y - \boldsymbol{\alpha} \cdot f_n - \boldsymbol{\beta}\right]^2 = -2E\left[\left(f_n^Y - \alpha \cdot f_n - \boldsymbol{\beta}\right) f_n\right] \\ \frac{\partial}{\partial \beta} E\left[f_n^Y - \boldsymbol{\alpha} \cdot f_n - \boldsymbol{\beta}\right]^2 = -2E\left[\left(f_n^Y - \alpha \cdot f_n - \boldsymbol{\beta}\right)\right] \end{cases} \tag{2}$$

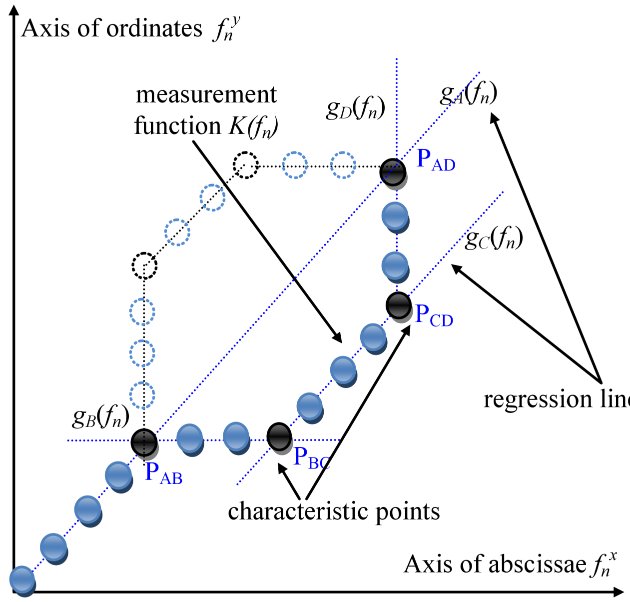

Fig. 1. A graph of measurement points dispersion after transformation in two-dimensional Euclidean plane E^2 in combined depicting – attractor of transformation

After comparing the calculated derivatives (2) to zero, the system of normal equations appears in which after replacing the expected values with particular moments of equation systems the following relations (3) can be written,

$$\begin{cases} \boldsymbol{\alpha} \cdot m_{20} + \boldsymbol{\beta} \cdot m_{10} = m_{11} \\ \boldsymbol{\alpha} \cdot m_{10} + \boldsymbol{\beta} = m_{01} \end{cases} \tag{3}$$

in which m_{10} and m_{01} are sample 1-*th* moments, m_{20} is sample 2-*th* moment and m_{11} is mixed sample 1-*th* moment. After further transformations the regression equation is as follows,

$$\begin{aligned} g(f_n) &= \frac{\mu_{11}}{\mu_{20}} \cdot f_n + \left(m_{01} - \frac{\mu_{11}}{\mu_{20}} m_{10}\right) \\ &= \boldsymbol{\alpha}_{21} f_n + \boldsymbol{\beta}, \end{aligned} \tag{4}$$

where

$$\begin{aligned} \boldsymbol{\alpha}_{21} &= \left[\frac{\mu_{11}^A}{\mu_{20}^A}, \quad \frac{\mu_{11}^B}{\mu_{20}^B}, \quad \frac{\mu_{11}^C}{\mu_{20}^C}, \quad \frac{\mu_{11}^D}{\mu_{20}^D}\right]^T \\ &= \left[\alpha_A, \quad \alpha_B, \quad \alpha_C, \quad \alpha_D\right]^T, \end{aligned} \tag{5}$$

$$\begin{aligned} \boldsymbol{\beta} = \Bigg[& m_{01}^A - \frac{\mu_{11}^A}{\mu_{20}^A} m_{10}^A, \quad m_{01}^B - \frac{\mu_{11}^B}{\mu_{20}^B} m_{10}^B, \\ & m_{01}^C - \frac{\mu_{11}^C}{\mu_{20}^C} m_{10}^C, \quad m_{01}^D - \frac{\mu_{11}^D}{\mu_{20}^D} m_{10}^D \Bigg]^T \\ = & \left[\beta_A, \quad \beta_B, \quad \beta_C, \quad \beta_D\right]^T \end{aligned} \tag{6}$$

and μ_{11} means mixed 2-*th* central moment and μ_{20} means 2-*th* central moment.

As a result of further transformations four equations of linear regression have been received. Then it is possible to draw a measurement function $K(f_n)$ in the form of a product degree k, given $k+1$ characteristic points, defined by the interpolation Lagrange's formula in accordance with the following relation,

$$K(f_n) = a_k f_n^k + a_{k-1} f_n^{k-1} \\ +a_{k-2} f_n^{k-2} + \ldots + a_0, \tag{7}$$

where a_k, a_{k-1}, ..., a_0 are characteristic parameters of a generalized measurement function $K(f_n)$. The formalized notation of the measurement function $K(f_n)$ allows to extract distinctive features through defining the space area under the measurement function and the arc length of the function, which appeared for the SEI process. The feature $\widehat{S}$, is the value of the space area of a closed surface expanding from the generalized measurement function $K(f_n)$ in the bracket $\langle f_n^{\min}, f_n^{\max}\rangle$, respecting the relation (8)

$$\widehat{S} = \int_{f_n^{\min}}^{f_n^{\max}} K(f_n) df_n = \\ \int_{f_n^{\min}}^{f_n^{\max}} \left(a_k f_n^k + a_{k-1} f_n^{k-1} + a_{k-2} f_n^{k-2} + \ldots + a_0\right) df_n. \tag{8}$$

Simultaneously, the arc length of the generalized measurement function $K(f_n)$ as the second distinction feature of the radar emission source is represented through the arc length $\widehat{L}$ of the function $K(f_n)$in the brackets $\langle f_n^{\min}, f_n^{\max}\rangle$ respecting the following Eq. (9).

$$\widehat{L} = \int_{f_n^{\min}}^{f_n^{\max}} \left[1 + \left(\frac{\partial K(f_n)}{\partial f_n}\right)^2\right]^{\frac{1}{2}} df_n = \\ \int_{f_n^{\min}}^{f_n^{\max}} \left[1 + \left(k a_k f_n^{k-1} + (k-1) a_{k-1} f^{k-2} + \ldots + a_1\right)^2\right]^{1/2} df_n. \tag{9}$$

The image of measurement points' transformation which was received is a „peculiar" attractor of transformation and the further analysis makes it possible to define two additional distinctive features in the form of area under the measurement function and the length of its arc. The additional distinctive features received in that way modify the vector of basic measurable parameters of a radar signal i.e. RF, PW, A and PRI and further process of identification based on the extended vector of features makes it possible to identify every copy of radar emission source.

3. Results of estimations

In the researching and measuring procedure which was carried out by the authors about 400 radar emissions from the same type of several radar copies were analysed. On the basis of the registered measurement vectors with the use of holdout method [21, 22] (it divides the set of measurement data into two separate sets i.e. the set used to teach the classificator and the set used to test the classificator- usually the division is: 2/3 available data is the teaching set and 1/3 is the testing set), what was received were the standards of radar classes and testing vectors.

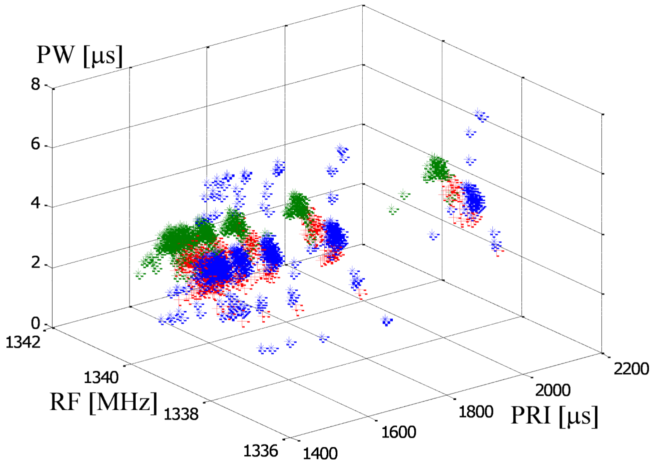

Fig. 2. 3-D graphic depicting of RF, PRI and PW for three selected copies of the same type of radars marked by blue, red and green colour

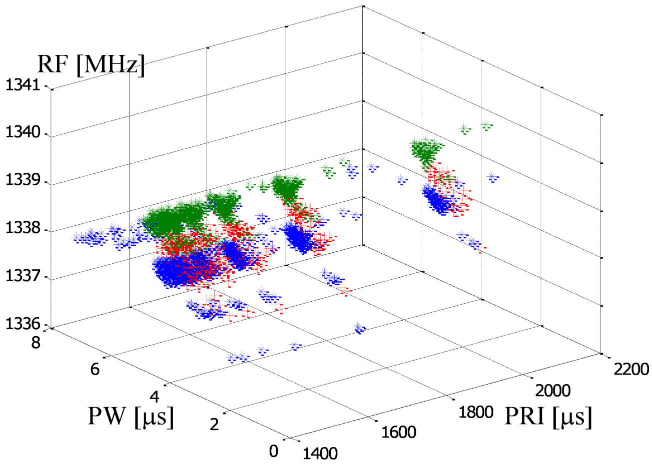

Fig. 3. 3-D graphic depicting of PW, PRI and RF for three selected copies of the same type of radars marked by blue, red and green colour

On the basis of the recordings and initial analysis to further process of identification only these copies were admitted whose basic measurable parameters i.e. RF, PW and PRI were much the same – see Figs. 2 and 3. Figures 4–7 present the biggest similarity of the radar signal parameters which those sources generated.

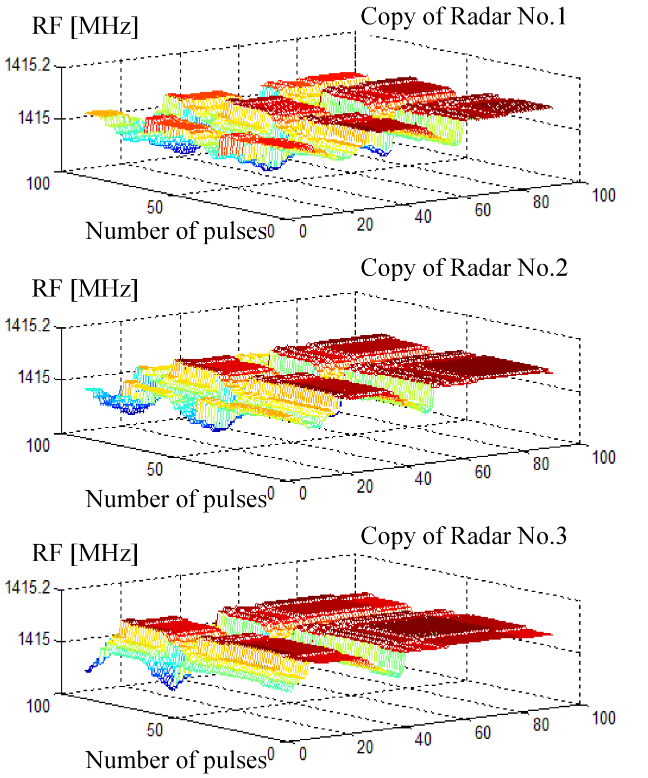

Fig. 4. 3-D graphic depicting of RF for three selected copies of the same type of radars

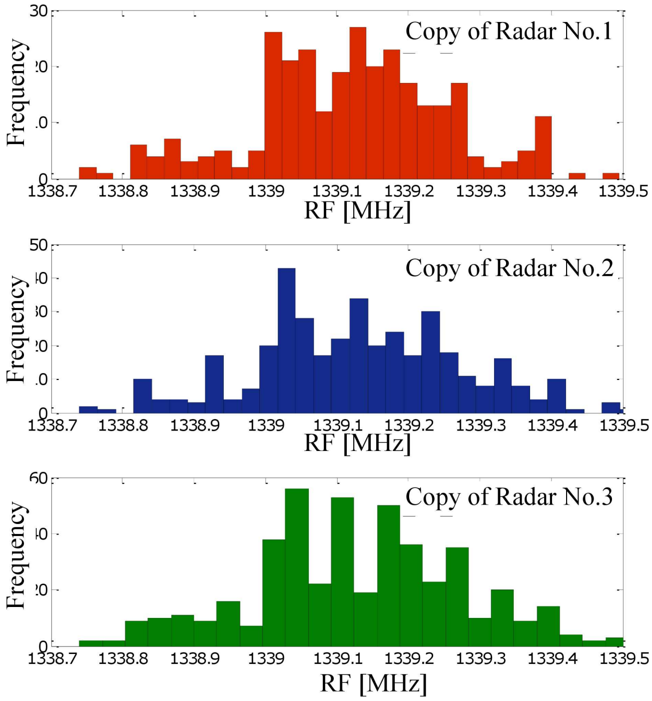

Fig. 5. RF histogram for three selected copies of the same type of radars

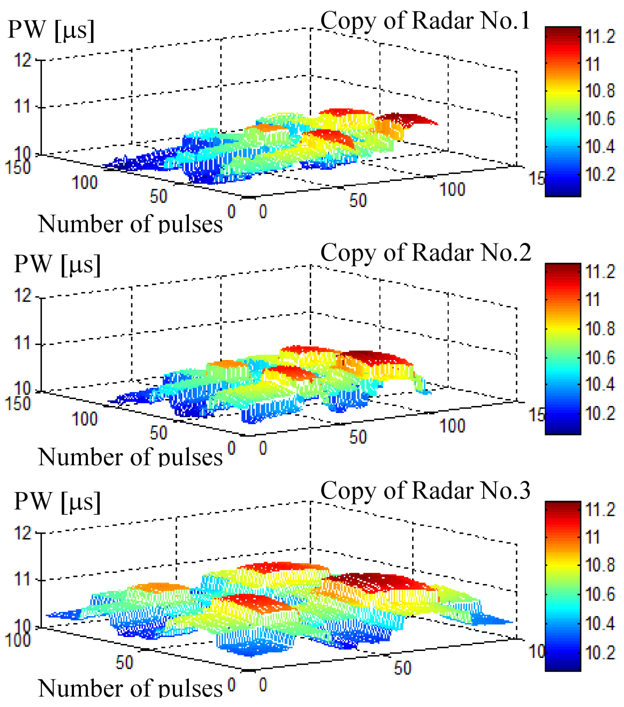

Fig. 6. 3-D graphic depicting of PW for three selected copies of the same type of radars

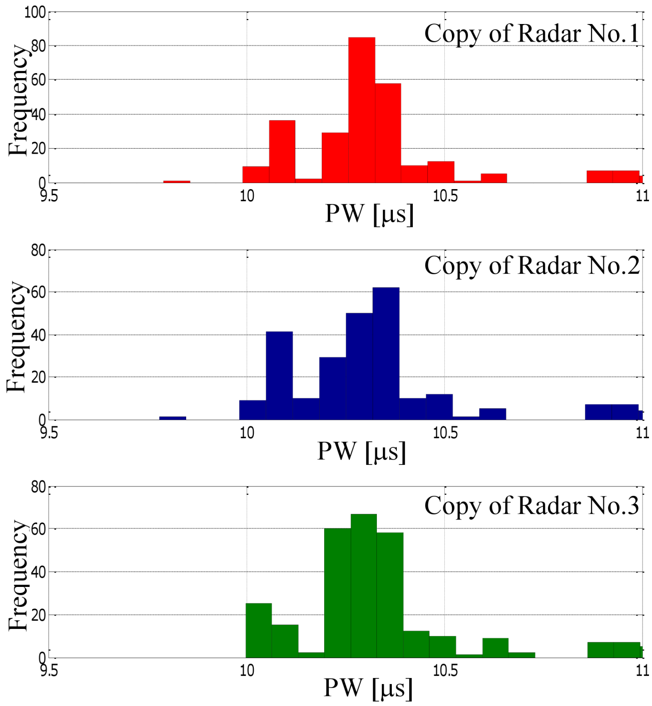

Fig. 7. PW histogram for three selected copies of the same type of radars

Figure 5 presents the histogram (graphic composition) of the radio frequency (RF) for three copies radars of the same type. Figure 7 presents the histogram of the pulse width (PW) for three copies of the same type of radar. Packets of radar signals consisting of 500 impulses were analysed. Figures 4 and 6 present 3-D depicting of radio frequency (RF) and pulse width (PW) of radar signals. These graphs were received with the use of 'mesh' function in the MatLab software. This program makes it possible to receive a solid net which depicts RF and PW values of the analysed radar signals in three dimensions. These graphs were made for a hundred of consecutive impulses. As it can be easily seen in Figures 4÷7 basic measurable radar parameters filter through each other. Classical methods to define these emissions fail in identification the particular radar copy. And these three radar copies underwent the process of identification with the use of fractal features. The process of identification was made on the basis of length measurement and the decision about the criterion of minimal distance classification. The functional of conformity assessment of tests with particular class was Mahalanobis, Euclidean and Hamming distances (metrics) [21, 22]. The criterion of classification was the criterion of "the nearest neighbour" which was used as one of basic threshold criteria [23]. In order to assess the quality of the classification/identification process the Correct Identification Coefficient (CIC) was defined. This CIC is the quotient of the number of correct classification to the number of all identification tests. As the process of making measurement vectors, estimating distances between classes, defining the coefficient and the criteria used are not the main problem of this article their precise description is in [19].

The first step was to define the vectors of the basic measurable parameters i.e. RF, PW and PRI. With the use of Mahalanobis, Euclidean and Hamming distances the correct identification coefficient CIC was estimated. The value of CIC was precisely: CIC = 0.169 for Mahalanobis distance, CIC = 0.118 for Euclid distance and CIC = 0.202 with the use of Hamming distance. The results of received were not enough to identify exactly the copies of radars. Then the transformation of measurable points was done and with the use of the linear regression method the coordinates of characteristic points and measurement functions $K(f_n)$ were defined and as a result the attractor of these transformations was received. The results were presented in Figs. 8–10. The presented method of features extraction makes it possible to estimate numerical surface areas under the measurement functions (feature $\widehat{S}$) and the distance of the arc of these functions (feature $\widehat{L}$). Then the vector of basic measurable parameters of a radar signal was extended with two additional features $\widehat{S}$ and $\widehat{L}$. Then the received vectors of features underwent the aforementioned process of transformation. As a result of the analysis: CIC = 0.916 for Mahalanobis distance, CIC = 0.967 for Hamming and Euclidean distances.

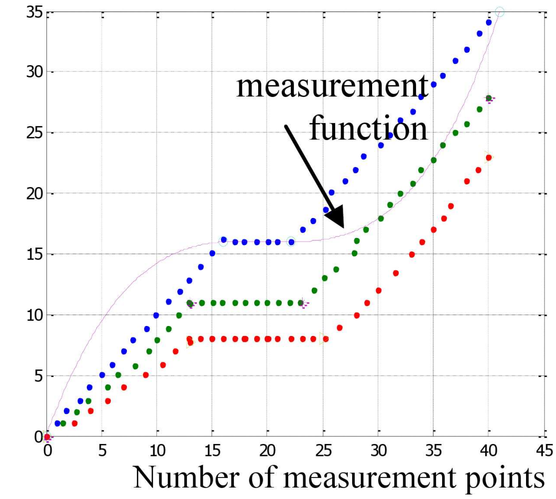

Fig. 8. Attractor of transformation for Copy of Radar No.1

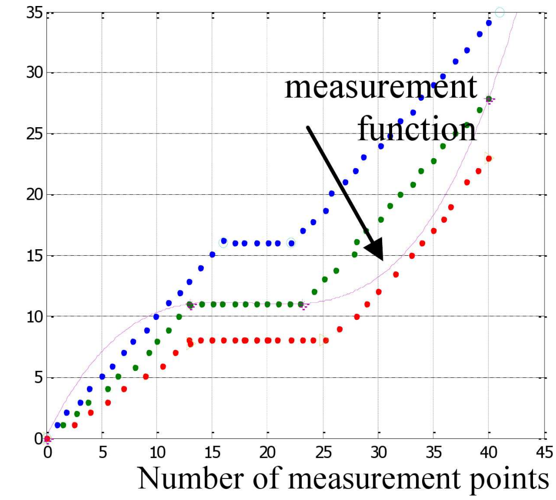

Fig. 9. Attractor of transformation for Copy of Radar No.2

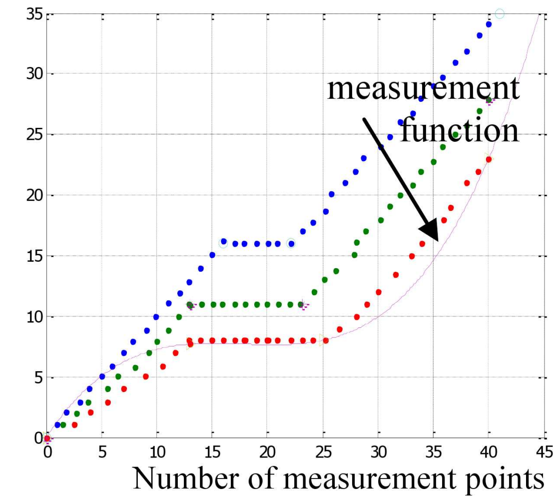

Fig. 10. Attractor of transformation for Copy of Radar No.3

4. Conclusions

Taking into account the results received it can be concluded that the process of identification of radar copies of the same type based on the basic radar signal parameters i.e. PW, RF and PRI is rather impossible. The possibility of classification, that is identification of types which are in most identification systems, is not enough.

With the use of the linear regression method it is possible to formalize the written form of the measurement function $K(f_n)$ and the extraction of two distinctive features $\widehat{S}$ and $\widehat{L}$. As a result there is a possibility to enter the additional features, which modify the vector of basic measurable parameters of a radar signal, to the description of the radar copy. The features are a piece of distinctive information which is a good measure of separation in order to distinguish well the exact copy of the emission source. Simultaneously, as a result of the use of transformation sets with measurement points what was received was the transformation attractor of the generalized measurement function. The "transformation fractal" which was received will be used in further research in order to optimalize the procedure of specific identification of radar copies of the same type.

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