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INSTRUMENTS OF PROBABILISTIC OPTIMISATION OF LOAD BEARING CAPACITY AND RELIABILITY OF STATICALLY INDETERMINATE COMPLEX STRUCTURES

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The paper presents the method of probabilistic optimisation of load bearing capacity and reliability of statically indeterminate bar structures, and of coupling of members in kinematically admissible failure mechanisms (**KAFM**), which contain minimal critical sets of elements (**MCSE**). The latter are characterised by the fact that if only a single element is operational, the whole set is operational too. A method of increasing load bearing capacity and reliability of **KAFM** built from bars dimensioned in accordance with the code is presented. The paper also shows estimation of load bearing capacity and reliability of **KAFM** of the optimised structures containing elastic-plastic bars with quasi-brittle connections with nodes. The necessity of increasing connection of load bearing capacity and reliability in relation to bar reliability in order to prevent bars from being excluded from **MCSE** due to connection fracture is estimated.

Keywords: load bearing capacity, reliability, probabilistic optimisation, complex structures

1. Introduction

In recent years, many failures caused by fracture of member connections occurred in prestigious steel structures. Fracture of high strength bolts in semi-rigid end-plate connections is seen to be of the utmost importance. In truss structures, the dominant failure mechanism is the fracture of welds in gussets in welded and welded/bolted connections. When substantial welding and assembly stresses are found, fracture of connections in truss structures occurs already during the assembly.

According to the General Office of Building Control [1], 3770 collapses, including 999 brought about by non-random causes, and 2771 due to random causes, happened in Poland in the years 1995–2008. The highest number of collapses occurred in the years 2007–2008, which constitutes 43.3% of such events in the 1995–2008 period. The situation results from a lack of methods for estimating the load bearing capacity and

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reliability of structures assembled in accordance with reliability indexes recommended by the code [12].

Statically determinate structures are characterised by as many kinematically admissible failure mechanisms as there are bars in the structure. The higher the number of members, the lower load bearing capacity and reliability of such structures. In the study [7], a method for restoring load bearing capacity and reliability of such structures to those recommended in the code [12] for individual reliability classes. In practice, however, checking and restoring recommended structural reliability is not performed, which leads to an excessive number of failures and collapses.

An economical method of purposefully increasing load bearing capacity and reliability of building structures involves applying the instruments of probabilistic optimisation of load bearing capacity and reliability. Such instruments can be used both for designing new structures and structurally upgrading of those already existing, generally without increasing cross-sections of the main structural members dimensioned in accordance with the code. Those instruments involve coupling a larger number of principal decisive elements of the structure in kinematically admissible failure mechanisms **(KAFM)** and estimating increased load bearing capacity and reliability resulting from an increased number of coupled elements in the minimal critical sets of elements **(MCSE)**.

2. Instruments of probabilistic optimisation of load bearing capacity and reliability

In the concept of estimating and restoring recommended load bearing capacity and reliability of complex statically indeterminate bar structures, the following information was employed: 1) statistical distributions of load bearing capacity of the structure decisive bars are described by normal distribution, 2) the structural decisive elements, computed and dimensioned in accordance with standards validated in practice, satisfy the recommendations [12] on their reliability, 3) in complex statically indeterminate structures, it is possible to determine the structural kinematically admissible failure mechanisms (**KAFM**) and the scope of virtual collapse, 4) the structural elements, dimensioned in accordance with the code, describe the object function and its load bearing capacity and reliability as **the structural eigenvalue, measured with quantiles of load bearing capacity N_k** and **reliability indexes $t = \beta$** determining the probabilities of meeting **$p = Pr\{N(w) > N_k\}$ the structural random load bearing capacity $N(w)$**. The structure described in such a manner can be affected by different types of random loads **$P_i(w)$**. Then, structural reliability is estimated from formula **$Pr\{N(w) > P_i(w)\}$**.

The structural elements dimensioned on the basis of static load calculations and standards gain **eigenvalue** in the form of load bearing capacity quantile (equivalent of computational load bearing capacity) with probability of meeting the reliability index recommended by the code [12] **(Appendix B, p. B3.11:…. Presently, reliability**

requirements are referred to the elements of the object structure). It provides a basis for the analysis of topology impact on reliability of complex structures. The information given above was employed to develop instruments of probabilistic optimisation. Those are intended to adjust load bearing capacity and reliability of complex structures to reliability indexes recommended for reliability classes **RC1 (t = 3.3)**, **RC2 (t = 3.8)**, **RC3 (t = 4.3)** [12].

In complex structures composed of elastic-plastic decisive elements many kinematically admissible failure mechanisms **(KAFM)** containing **minimal critical sets of elements (MCSE)** occur**,** the failure of which results in a failure of the structure, or of its part. The following can be found: a) independent sets, b) dependent sets with common elements, c) sets containing elements connected in series.

In statically determinate structures, only single-element **KAFM** occurs. It should be taken into account that bars and their connections with nodes, which are found in **KAFM**, constitute integrated decisive elements, most frequently composed of bars and their two connections with nodes.

3. Load bearing capacity and reliability of KAFM built from the members of known load bearing capacity and reliability

Let us consider minimal critical sets of elements (**MCSE**) coupled in **KAFM**, which can be used to increase the structural load bearing capacity and reliability above load bearing capacity and reliability of elements dimensioned in accordance with relevant standards.

Figure 1 shows an example of tilt **KAFM** of columns with heads coupled with longitudinal roof bracings. The columns are coupled in one **MCSE**. Virtual collapse is possible when all columns coupled with roof bracings fail on one side of the hall (e.g. Figure 1a).

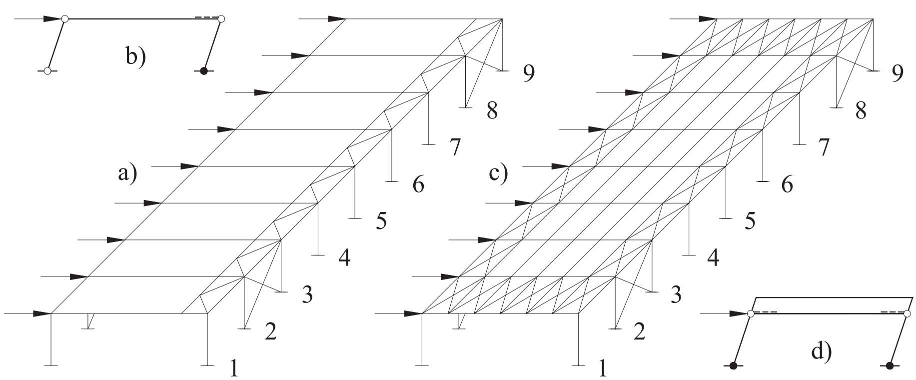

Fig. 1. Examples of tilt KAFM of columns

The quantile of load bearing capacity and reliability of **KAFM** of the structure containing **n** decisive elements in one **MCSE** can be estimated on the basis of known parameters [6], [7] of the structure coupled elements in accordance with a generalised algorithm below:

Expected load bearing capacity **E(N)** of set **MCSE** amounts to (3.1):

$$E(N) = \sum a_i E(N_i), \tag{3.1}$$

where: a_i – dimensionless weight of the i-th element in **KAFM**, $E(N_i)$ – expected (mean) load bearing capacity of the i-th element.

Variance $D^2(N)$ of the load bearing capacity of **MCSE** coupled in **KAFM** of elements is:

$$D^2(N) = \sum a_i^2 D^2(N_i), \tag{3.2}$$

where: $D^2(N_i)$ -- variance of the load bearing capacity of the i-th element.

Standard deviation **D(N)** of the load bearing capacity of **MCSE** is:

$$D(N) = [\sum a_i^2 D^2(N_i)]^{0.5}. \tag{3.3}$$

Variation coefficient **v** of load bearing capacity of **MCSE** is:

$$v = D(N) / E(N) = [\sum a_i^2 D^2(N_i)]^{0.5} / \sum a_i E(N_i). \tag{3.4}$$

Quantile N_k of load bearing capacity of **n** elements coupled in **MCSE** is [6], [8]:

$$N_k = E(N) [1 - t v], \tag{3.5}$$

where: reliability index **t** in accordance with normal distribution [15].

If **n** identical elements are coupled in **KAFM,** formulas (3.1) to (3.5) take on the forms (3.1a) to (3.5a):

(3.1a) $E(N) = n E(N_1)$ – expected load bearing capacity of **n** elements coupled in parallel in **MCSE**, each with expected load bearing capacity $E(N_1)$.

(3.2a) $D^2(N) = nD^2(N_1)$ variance of load bearing capacity of **n** coupled elements.

(3.3a) $D(N) = [n D^2(N_1)]^{0.5}$ – standard deviation of **n** coupled elements.

(3.4a) $v = v_1 / n^{0.5}$ variation coefficient **v** of load bearing capacity of **n** coupled elements.

(3.5a) $N_k = n E(N_1)[1 - t v_1 / n^{0.5}]$ – the quantile of load bearing capacity of **MCSE** of **n** identical coupled elements.

Increase **s** in load bearing capacity of **MCSE** of **n** identical coupled elements when compared with the load bearing capacity of **n** separate elements is determined from Eq. (3.6a):

(3.6a) $$\mathbf{n\ E(N_1)\ [1 - t\ v_1 / n^{0.5}] = s\ n\ E(N_1)\ [1 - t\ v_1]}.$$

From equation (3.6a), formula (3.7a) for increase **s** in load bearing capacity of elements coupled in parallel is obtained:

(3.7a) $$\mathbf{s = [1 - t\ v_1 / n^{0.5}] / [1 - t\ v_1]}.$$

Example 1. Set $\mathbf{n = 9}$ of identical columns of **RC2 class** [12] is given, which has: **RC2** reliability index $\mathbf{t = 3.8}$, reliability $\mathbf{p_1 = 0.999\ 927\ 652}$ [15], failure ratio: $\mathbf{q_1 = 1 - p = 0.000\ 072\ 348}$, variation coefficient of load bearing capacity $\mathbf{v_1 = 0.09}$. The weight of elements in **MCSE:** $\mathbf{a_i = 1}$. The set of columns is coupled with the roof bracing in one tilt **KAFM** shown in Figure **1a**. It is necessary to compute the quantile of the load bearing capacity of the minimal critical set of **9** columns coupled in one **KAFM**.

The quantile of the load bearing capacity N_{1k} of a simple column is computed from formula (3.5) and equals:

$$\mathbf{N_{1k} = E(N_1)[1 - tv_1] = E(N_1)[1 - 3.8\ x\ 0.09] = 0.658\ E(N_1)},$$

where: $\mathbf{v_1 = 0.09}$ – variation coefficient of the load bearing capacity of a single column, $\mathbf{E(N_1)}$ – expected (mean) load bearing capacity of one column.

Variation coefficient **v** of the load bearing capacity of the minimal critical set $\mathbf{n = 9}$ of identical columns, for $\mathbf{a = 1}$, amounts to (3.4a):

$$\mathbf{v = D(N) / E(N) = v_1 / n^{0.5} = 0.09 / 3 = 0.03}$$

Quantile $\mathbf{N_k}$ of load bearing capacity of $\mathbf{n = 9}$ identical decisive elements coupled in one **KAFM** estimated from formula (3.5a) is:

$$\mathbf{N_k = n\ E(N_1)\ [1 - tv_1 / n^{0.5}] = 9\ E(N_1)\ (1 - 3.8\ x\ 0.03) = 9\ x\ 0.886\ E(N_1)}.$$

Increase in the load bearing capacity of $\mathbf{n = 9}$ columns coupled in KAFM, estimated from formula (3.7a), equals: **s** = 9 x 0.886 $\mathbf{E(N_1)}$ / 9 x 0.658 $\mathbf{E(N_1)}$ = **1.3465**.
Conclusion: load bearing capacity and reliability of **9** columns coupled in **KAFM** is substantially increased above recommendations in the code [12]. If faulting columns are left without head roof bracings, their failure ratio increases, which is shown in example **2**.

Example 2. Failure ratio of **9** columns connected in series amounts to: **q = nq$_1$ = 9q$_1$ = = 9 0.000 072 348 = 0.000 651 132**. The reliability of columns decreases and amounts to: **p** = **1 – q = 1 – 0.000 651 132 = 0.999 348 868**. Reliability index decreases and amounts to: **t = 3.2155 < 3.8.**

In example **3**, it is shown that already for **4** columns coupled in parallel, an increase trend in load bearing capacity is strongly manifested.

Example 3. A set of **n = 4** identical columns of **RC2** class, which have the same parameters as in Example **1** is given. The variation coefficient of load bearing capacity of the minimal critical set of **n = 4** identical columns is: **v = D(N) / E(N) = v$_1$ / n$^{0.5}$ = 0.09 / 2 = 0.045**

Quantile **N$_k$** of load bearing capacity of the set of **n = 4** identical columns coupled in one **KAFM**, estimated from formula (3.5a) [6], is:

$$N_k = = nE(N_1)[1 - tv_1 / n^{0.5}] = 4\ E(N_1)\ (1 - 3.8 \times 0.045) = 4 \times 0.829\ E(N_1).$$

An increase in load bearing capacity of **MCSE of 4** coupled columns is: **s =** 0.829 / 0.658 **= 1.26**.

If **4** faulting columns are left, failure ratio increases **q = 4q$_1$** = 4 x 0.000 072 348 **= 0.000 289 392**. The reliability of columns falls and it amounts to: **p** = **1 – q** = 1 – 0.000 289 868 **= 0.999 710 600.** Reliability index **[15]** decreases: **t = 3.4414 < 3.8**.
Conclusion. An increase in number **n** of elements coupled in parallel in one **KAFM** significantly increases load bearing capacity and reliability of the examined **KAFM** above the recommendations of the code [12].
General conclusion: It is necessary to increase the number of elements in **KAFM** and reduce the number of **KAFM** in the structure.

Table **1** shows, as computed from formula (3.7a), exemplary increase in load bearing capacity of **MCSE** containing up to **16** elements coupled in parallel in **KAFM**, with element variation **v$_1$ = 0.06 to 0.1.**

Table 1

Increasing load bearing capacity of MCSE sized up to n = 16 for v_1 = from 0.06 to 0.1

Item	v_1	s_2	s_3	s_4	s_5	s_6	s_7	s_8	s_9	s_{12}	s_{16}
1	**0.06**	**1.087**	**1.125**	**1.148**	**1.163**	**1.175**	**1.184**	**1.191**	**1.197**	**1.210**	**1.222**
2	**0.07**	**1.106**	**1.153**	**1.181**	**1.200**	**1.214**	**1.225**	**1.234**	**1.242**	**1.258**	**1.272**
3	**0.08**	**1.128**	**1.185**	**1.218**	**1.241**	**1.259**	**1.272**	**1.282**	**1.291**	**1.311**	**1.328**
4	**0.09**	**1.152**	**1.220**	**1.260**	**1.287**	**1.308**	**1.323**	**1.336**	**1.347**	**1.370**	**1.390**
5	**0.10**	**1.180**	**1.259**	**1.307**	**1.339**	**1.363**	**1.383**	**1.396**	**1.409**	**1.436**	**1.460**

In **KAFM** of statically indeterminate complex structures, bars integrated with connections in series can occur. In such cases, it is necessary to perform the procedure, described in section **4**, which prevents connection failure prior to the loss of load bearing capacity of bars. The procedure is aimed to prevent the exclusion of decisive elements from **MCSE** when the structural load increases in service.

4. LOAD BEARING CAPACITY AND RELIABILITY OF KAFM BUILT FROM BARS CONNECTED IN SERIES WITH NODES

Decisive elements, connected in series, which are found in MCSE are characterised by reduced load bearing capacity and reliability. Such elements most frequently consist of bars and their connections with nodes (Figure 2). Connections often generate failures and collapses of complex bar structures when their load bearing capacity is lower than that of connected bars.

Figure 2 shows an example of bar 3 connected with nodes in series and the static equilibrium path for connections 1 and 2, and bar 3. Such complex decisive elements most often occur in **KAFM**.

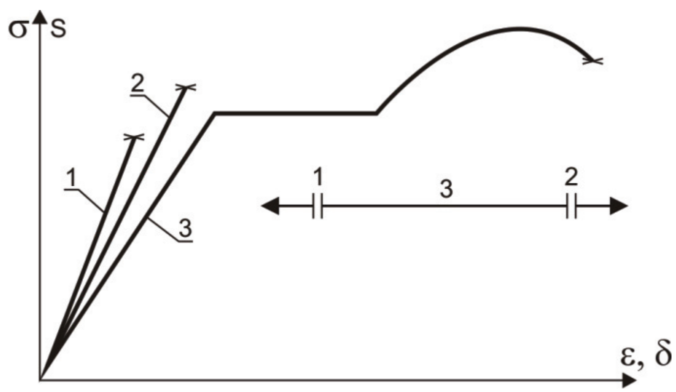

Fig. 2. Static equilibrium paths of connections 1 and 2 and bar 3

A general requirement that has to be satisfied when elements are coupled in **KAFM** is as follows: **minimal critical set of elements MCSE is such a set of elements coupled in KAFM, in which if only one element is operational, the whole set is operational, too**.

If connections having lower load bearing capacity than that of bars occur, a bar can be excluded from **MCSE** when the structure is in service, thus lowering the load bearing capacity of **KAFM**. That would be equivalent to the failure of the whole system.

Connections and bars shown in Figure **2** have qualitatively different static equilibrium paths **(SEP)**. Bars maintain their strain capability along the whole of the static equilibrium path until their load bearing capacity is exhausted. Their connections, however, have significantly lower strain capability. In order to prevent bar exclusion from **KAFM** prior to the exhaustion of elastic-plastic load bearing capacity of bars, **postulate 2** must be satisfied: **Load bearing capacity and reliability of connections should be greater than load bearing capacity and reliability of bars**.

An assumption can be made that the load bearing capacity and reliability of bar **3** and connections **1** and **2** was dimensioned in accordance with the present state-of-the-art and standards, and it also satisfies the recommendations on reliability indexes [12] $\mathbf{t = \beta}$ for one of three structural reliability classes: $\mathbf{t = 3.3}$ for **RC1** class, $\mathbf{t = 3.8}$ for **RC2** class, and $\mathbf{t = 4.3}$ for **RC3** class.

The reliability **p** of each decisive element (**1,2,3**), integrated in series in accordance with **Figure 2**, which occurs in KAFM, amounts to:

$$\mathbf{p = p_1\, p_2\, p_3}, \tag{4.1}$$

where: $\mathbf{p_1}$ and $\mathbf{p_2}$ – reliability of connections, $\mathbf{p_3}$ – reliability of the bar.
Failure ratio of an integrated bar is:

$$\mathbf{q = 1 - p = q_1 + q_2 + q_3} \tag{4.2}$$

From postulate **2**, it follows that inequalities (4.3) should hold:

$$\mathbf{q_1 + q_2 < q_3 \qquad q_1 < q_3 \qquad q_2 < q_3} \tag{4.3}$$

Assume that failure ratio of connections is the same**:** $\mathbf{q_1 = q_2}$. Then, the following is obtained (4.4):

$$\mathbf{2q_1 < q_3}. \tag{4.4}$$

To adjust load bearing capacity and reliability of a complex decisive element to code recommendation, it is required to satisfy conditions (4.5) and (4.6):

$$\mathbf{p = p_1\, p_2\, p_3 > p(t)}, \tag{4.5}$$

$$\mathbf{p_1\, p_2 > p_3 \text{ at the assumption that } p_1 = p_2} \tag{4.6}$$

Adjusting load bearing capacity and reliability of bars with connections, in accordance with formulas (4.4), (4.5), (4.6), to the reliability index recommended by the code [12] should be applied to all bar structural systems, both statically determinate and statically indeterminate, in which bars, integrated with connections in series, occur.

5. ESTIMATION OF LOAD BEARING CAPACITY AND RELIABILITY OF BARS AND CONNECTIONS WITH NODES IN RC2 CLASS STRUCTURES

When estimating load bearing capacity and reliability of bars and connections integrated with bars in order to adjust them to code recommendations [12], relations **4.1 to 4.6** were employed**.** The adjustment algorithm was shown on the example of **RC2** class structure, for which reliability index $\mathbf{t = \beta = 3.8}$ is recommended. From relation (4.5), the recommended inequality (5.1) of reliability of complex elements of **RC2** class structure is given:

(5.1) $$\mathbf{p = p_1\ p_2\ p_3 => p(t = 3.8) => 0.999\ 927\ 652}$$

Failure ratio of an integrated bar should not exceed (5.2):

(5.2) $$\mathbf{q = 1 - p = q_1 + q_2 + q_3 =< 0.000\ 072\ 348}$$

Taking into account the fact that the total failure ratio of connections $\mathbf{q_1} = \mathbf{q_2}$ should be lower than failure ratio of a bar, we have: $\mathbf{2q_1 < q_3}$. By substituting $\mathbf{q_3 = 2q_1}$, from Eq. (5.3), failure ratio if an integrated bar is received.

(5.3) $$\mathbf{q = q_1 + q_1 + 2\ q_1 = 4q_1 =< 0.000\ 072\ 348}.$$

Failure ratio of connections should not exceed: $\mathbf{q_1 = q / 4 < 0.000\ 018\ 087}$.

Reliability of connections should be greater than or equal to:

(5.4) $$\mathbf{p_1 = 1 - q_1 => 0.999\ 981\ 913}.$$

Reliability index of connections should be increased to $\mathbf{t = \beta = 4.13066}$ [15].

The quantile of load bearing capacity of bar connections in RC2 class structures has to be computed from formula:

(5.5) $$\mathbf{N_{1k} = E(N_1)(1 - 4.13066\ v_1)}$$

That leads to increasing load bearing capacity of connections in relation to the load bearing capacity of a separate bar.

Failure ratio of a bar amounts to: $\mathbf{q_3 = 2q_1 = 0.000\ 036\ 174}$. Reliability is**:** $\mathbf{p_3 = 0.999\ 963\ 826}$. Reliability index is: t = **3.869**. The quantile of load bearing capacity should be computed from formula (5.6):

(5.6) $$\mathbf{N_{3k} = E(N_3)(1 - 3.869v_3)}.$$

To adjust quantile $\mathbf{N_{3k}}$ of load bearing capacity of a bar, only a minor correction is needed. A substantial increase in load bearing capacity $\mathbf{N_{1k}}$ of connections is required, though. Computational load bearing capacity $\mathbf{N_{1k}}$ of connections, estimated in accordance with the code, needs to be increased in relation to computational load bearing capacity $\mathbf{N_{3k}}$ of a bar, using correction coefficient $\mathbf{k} = \mathbf{E(N_1)\ /\ E(N_3)}$, computed from Eq. (5.7):

$$E(N_1)\,(1 - 4.13066\,v_1) = k\,E(N_3)\,(1 - 3.869v_3). \tag{5.7}$$

From Eq. (5.7), coefficient **k** (5.8) is derived, which corrects load bearing capacity of connections of **RC2** class structures compared with the load bearing capacity of bars.

$$k = E(N_1) / E(N_3) = (1–3.869v_3) / (1–4.13066\,v_1). \tag{5.8}$$

Example 4. A bar, with variation coefficient of load bearing capacity $\mathbf{v_3 = 0.08}$, with connections having variation coefficient $\mathbf{v_1 = 0.1}$ is given. It is necessary to compute coefficient **k** of increasing load bearing capacity of the connection. From formula (5.8), the following is obtained: $\mathbf{k} = (1 – 3.869 \times 0.08) / (1 – 4.13066 \times 0.1) = \mathbf{1.1764}$.

Table **2** presents the matrix of coefficients **k**, which depends on variation coefficients of load bearing capacity $\mathbf{v_1}$ and $\mathbf{v_3}$ at the assumption that variation in the load bearing capacity of connections is greater than variation in the load bearing capacity of a bar $\mathbf{v_1 > v_3}$ computed from formula (5.8). The least correction coefficients **k** proposed in the study are shown in bold type.

Table 2

Coefficients **k** of increasing the load bearing capacity of RC2 class bar connections, **t = 3.8**

$v_1 =$	**0.06**	**0.07**	**0.08**	**0.09**	**0.100**
$\mathbf{v_3 = 0.06}$	1.021	**1.080**	1.147	1.222	1.308
$\mathbf{v_3 = 0.07}$		1.026	**1.089**	1.161	1.242
$\mathbf{v_3 = 0.08}$			1.031	**1.099**	1.176
$\mathbf{v_3 = 0.09}$				1.038	**1.111**
$\mathbf{v_3 = 0.10}$					1.045

Conclusion. The reliability of bars integrated with connections, estimated on the basis of the procedure presented above, makes it possible to treat them as decisive bars with class **RC2** reliability index $\mathbf{t = \beta = 3.8}$. The procedure of increasing the load bearing capacity of nodes should be applied to both statically determinate and statically indeterminate structures.

6. ESTIMATION OF LOAD BEARING CAPACITY AND RELIABILITY OF BARS AND CONNECTIONS WITH NODES FOR RC3 CLASS STRUCTURE

Estimating an increase in load bearing capacity and reliability of connections in relation to load bearing capacity and reliability of connected bars of **RC3** class structure is intended to lower the probability of occurrence of failures and collapses. Those are caused by excluding bars from **KAFM** due to the failure of bar connections prior to exhaustion of the load bearing capacity of bars.

The recommended inequality of reliability of **RC3** class bars, integrated with connections, with index $\mathbf{t = 4.3}$ is *(6.1):*

(6.1) $$\mathbf{p = p_1\, p_2\, p_3 = > 0.999\,991\,460\,095}$$

The maximum failure ratio **q** of an integrated bar of **RC3** class amounts to (6.2):

(6.2) $$\mathbf{q = 1 - p = 4q_1 < = 0.000\,008\,539\,905}.$$

The maximum failure ratio $\mathbf{q_1}$ of connections should amount to:

(6.3) $$\mathbf{q_1 = q / 4 = 0.000\,002\,134\,976}.$$

The minimum reliability $\mathbf{p_1}$ of connections should be:

(6.4) $$\mathbf{p_1 = 1 - q_1 = > 0.999\,997\,865\,024}.$$

Reliability index [15] of connections is: $\mathbf{t = \beta = 4.578 > 4.3}$

The quantile of the load bearing capacity (computational load bearing capacity) of bar connections with nodes is (6.5):

(6.5) $$\mathbf{N_{1k} = E(N_1)(1 - 4.578\, v_1)}$$

To satisfy the requirement that load bearing capacity of connections should be greater than that of bars, it is necessary to increase the expected value $\mathbf{E(N_1)}$ of load bearing capacity of a connection above the expected load bearing capacity $\mathbf{E(N_3)}$ of a bar.

Failure ratio of a separate bar is: $\mathbf{q_3 = 2q_1 = 0.000\,004\,269\,953}$.

Recommended reliability **p** of a bar without connections is: $\mathbf{p = 1 - q_3 = 0.999\,995\,730\,047}$
The reliability index **t** of a separate bar is: $\mathbf{t = 4.4512}$.

Quantile of load bearing capacity of a separate bar, adjusted to **RC3** class reliability, should be calculated from formula (6.6):

(6.6) $$\mathbf{N_{3k} = E(N_3)(1 - 4.4512 v_3)}.$$

Computational load bearing capacity $\mathbf{N_{1k}}$ of a connection estimated in accordance with the code should be increased in relation to the computational load bearing capacity of the bar $\mathbf{N_{3k}}$ using a correction coefficient k = **E(N$_1$) / E(N$_3$)** computed from Eq. (6.7):

$$\mathbf{E(N_1)(1 - 4.578\ v_1) = k\ E(N_3)\ (1 - 4.4512 v_3),} \tag{6.7}$$

From Eq. (6.7), formula (6.8) is obtained:

$$\mathbf{k = E(N_1) / E(N_3) = (1 - 4.4512 v_3) / (1 - 4.578\ v_1)} \tag{6.8}$$

Example 4. A bar with load bearing capacity variation coefficient $\mathbf{v_3 = 0.08}$ is given**,** with connections having variation coefficient $\mathbf{v_1 = 0.1.}$ It is necessary to compute correction coefficient **k** of increasing the load bearing capacity of connections:

$$\mathbf{k = (1 - 4.4512x0.08) / (1 - 4.578\ x\ 0.1) = 1.188}$$

Table 3 presents the matrix of coefficients **k** from formula (6.7) at the assumption that the coefficient of load bearing capacity of connections $\mathbf{v_1}$ is greater than the coefficient $\mathbf{v_3}$ of load bearing capacity of the bar**:** $\mathbf{v_1 > v_3}$.

Table 3

Coefficients **k** of increase in load bearing capacity of connections of **RC3** class structures**, t = 4.3**

$v_1 =$	**0.06**	**0.07**	**0.08**	**0.09**	**0.100**
$\mathbf{v_3 = 0.06}$	1.010	**1.079**	1.156	1.247	1.352
$\mathbf{v_3 = 0.07}$		1.013	**1.086**	1.171	1.270
$_3$ = 0 v.08			1.016	**1.095**	1.188
$\mathbf{v_3 = 0.09}$				1.019	**1.105**
$\mathbf{v_3 = 0.10}$					1.023

Conclusion: The principle that load bearing capacity and reliability of connections have to be higher than those of bars should be applied to all structural classes.

7. SUMMARY, REMARKS AND CONCLUSIONS

Instruments of probabilistic optimisation are of primary importance for preventing failures and collapses of complex structures, and for increasing, without additional outlay, their load bearing capacity and reliability. These instruments change, from reliability standpoint, element connections in series into connections in parallel. They provide a substantially stronger mechanism to increase the structure load bearing capacity and reliability compared with the weakening that occurs in statically determinate structures with an increase in the number of decisive elements. Additionally, in those cases, in which the deterministic collaboration of elements is advantageous, the effect of increasing load bearing capacity due to probabilistic optimisation can be observed.

Decisive elements that occur in **MCSE** are most frequently composed of decisive elements, such as bars, beams or tensioned ties, and of connections of those with nodes. Static equilibrium paths of basic elements and their connections show qualitative differences (Figure 2), therefore they should satisfy the **postulate of increased load bearing capacity and reliability of connections with nodes compared with load bearing capacity of bars**. That is indented to eliminate quasi-brittle fracture of connections, which is a direct cause of failures of prestigious steel structures. Connection fracture results in the exclusion of bars from **KAFM** of statically indeterminate structures. That refers, in particular, to fracture of bolts in semi-rigid end-plate connections and fracture of welds in welded and welded/bolted connections.

General conclusions: 1. The basic instrument of probabilistic optimisation of load bearing capacity and reliability of complex bar structures is, **from the reliability standpoint, parallel coupling of the structural decisive elements** in kinematically admissible failure mechanisms (**KAFM**), associated with minimal critical sets of elements (**MCSE**).

2. **Connections** of decisive bars that occur in kinematically admissible failure mechanisms of trusses **should have higher load bearing capacity and reliability than bars.** Failure ratio of connections should be at least twice smaller than failure ratio of separate bars.

REFERENCES

1. BARYŁKA J., Collapses in Poland caused by climatic phenomena (in Polish). GUNB, Warszawa 30.09.2009. www.koalicjaklimatyczna.org/lang/pl/page/ conference materials /id/8/.
2. GWÓŹDŹ M., MACHOWSKI A., Selected Problems and Building Structure Calculations with Probabilistic Methods (in Polish). Wyd. Politechniki Krakowskiej, Kraków 2011.
3. DZIUBDZIELA W., KOPOCIŃSKI B., KOWAL Z., Ultimate bearing capacity of structural systems with minimal critical sets having connection elements in pairs. Archives of Mechanics 25, 5 / 1973, pp. 719-731.
4. KOPOCIŃSKI B., Outline of Theory of Rehabilitation and Reliability (in Polish), PWN, Warszawa 1973.
5. KOWAL Z.,Zuverlassigkeit von Konstruktionssystemen. Wissenschaftliche Zeitschrift der Technischen Universitat Dresden 25 / 1976 H. 1 / 2, pp. 265-270.
6. KOWAL Z., Probabilistic Optimisation of Bearing Capacity of Columns in Conventional Halls. ZNPR nr 283, Z. 59 (2/2012/II), pp. 185-192.
7. KOWAL Z., On Adjusting the Load Bearing Capacity of Decisive Members to Reliability Classes of Statically Determinate Complex Structures. Archives of Civil Engineering, LIX, 1, 2013, pp. 132-142.
8. KOWAL Z., The formation of space bar structures supported by the system reliability theory, Arch. of Civil and Mechanical Engineering, Vol. XI, No. 1, 2011. Polish Ac. of Sciences – Wrocław Branch., Wrocław Univ. of Technology.
9. KOWAL Z., Hazards associated with the load-bearing capacity of bar space structures during assembly and performance. Promysłowe budiwnyctwo ta inżenerni sporudy, Kiev, 2011, Nr 3, pp. 34-40.

10. Kowal Z., Dimensioning Bar Connections in Trusses from the Structure Reliability Standpoint (in Polish). Conference paper, Krynica 2010, T. Problemy badawcze budownictwa, pp. 675-682.
11. Murzewski J., Reliability of Engineering Structures (in Polish), Arkady, Warszawa 1989.
12. Eurocode PN-EN – 1990:2002 Fundamentals of Structural Design with Appendix B.
13. PN-EN 1991-1-5:2005, Eurocode 1: Actions on Structures.
14. Eurocode PN-EN – 1993 Design of Steel Structures.
15. Zieliński R., Statistical Tables (in Polish), PWN, Warszawa 1972.

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