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Dedicated to Professor Jan Dorosz
on the occasion of his 65th birthday

Producibility of the ion-exchange method in manufacturing gradient refractive index in glass

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Abstract. This paper presents the technological aspect of application of the ion exchange method in producing gradient refractive index in glass. The possibility of predictable and repeatable producing of the changes in glass refraction with the use of this method has been presented, as well as the method of *in situ* control of the process of diffusion doping of glass based on the measurement of the temperature. This method is based on simultaneous (to the carried process) solving the nonlinear diffusion equation modeling the spatio-temporal changes in normalized concentration of the admixture ions in glass. For this purpose the knowledge of temperature characteristics of diffusion coefficients of exchanged ions is used. The result of such control of diffusion processes is information on the current (temporary) refractive index profile of the resulting waveguide. The presented method of control has been confirmed by experimental results, which concern modeling and measurements of planar waveguide structures of slab type. The proposed methodology can also be used to control the diffusion processes of producing another type of two- and three-dimensional gradient structures. According to the author's knowledge the method mentioned above has not been described in literature before.

Key words: ion exchange in glass, diffusion equation, planar gradient waveguides, effective indicies.

1. Introduction

The technological method of ion exchange in glass has been known for a long time. Its implementation for producing gradient changes in refraction of glass in order to produce a waveguide structure was first published in the work of Izawa and Nakagone in 1972 [1]. This method has found its widest application in relation to the oxide glasses. The physical basis of this method is the phenomenon of ionic conductivity of the glass. In the multicomponent glasses the modifiers are incorporated into its composition at the stage of preparation of the glass. After the vitrification process the ions of these modifiers are associated with glass skeleton by weak Coulomb bonds [2]. At sufficiently high temperatures, they gain energy allowing their migration in the glass skeleton [3]. The directions of migration of mobile modifier depend on the local environment of polar units. After dissociation each of them is capable of capturing mobile cations. Because the structural units in the glass forming the glass skeleton do not produce the ordering of long range, a random network of most probable directions of hoppings is formed. In the presence of an external electric field a structured drift of modifiers' ions forming current of ionic conductivity occurs.

The ion exchange processes are initiated in the glass as a result of the appearance of so-called admixture ions in the surface area of the glass. This situation occurs when at a sufficiently high temperature the glass is in contact with the phase of the source of these ions (most often it is a liquid phase). As liquid source of dopant ions, the molten salts of nitrates, sulfates, chlorides or the corresponding metals can be used. Among these groups, nitrate salts have relatively low melting points, and their strong oxidizing properties do not place the glass surface cleanliness requirements. For this reason, they are the most commonly used sources of admixture ions. Suitably high temperature of the molten salt provides the dissociation of its molecules on the metal cation and metalloid oxide anion. Then alkaline cations pass the liquid phase into glass, and the mobile modifiers' cations leave the glass passing to the liquid. This provides the condition of electrical neutrality of the glass. As the effect of the appearance of the new admixture ions at the glass surface, with simultaneous reduction in concentration of its modifier in this area, two oppositely directed streams of exchanged ions caused by the concentration gradients are produced in the glass. Due to the diffusive nature of these phenomena they are called diffusion processes. The liquid phase of the source of admixture has a number of great advantages. Such phase can be regarded as a virtually unlimited source of admixture which allows for the adoption of assumption about the temporary stability of the concentration of ions entering from the source at the glass surface. By applying a sufficiently large capacity of the bath and the use of its continuous mixing such a source can be used for long-term (several hundred hours) processes. Another advantage of this type of source is ensuring the stability of the glass temperature of the entire glass sample. During the total immersion of the glass substrate in a bath it is isolated from the influence of the environment.

As the admixture ions the following are used: silver ions Ag^+ [4–13], potassium ions K^+ [14–23], rarely: cesium ions Cs^+ [24, 25], copper ions Cu^{2+} [26–28], thallium ions Tl^+ [1, 29, 30] and lithium ions Li^+ [31].

Among the above-mentioned admixture ions the most commonly used are: silver ions Ag^+ (silver nitrate $AgNO_3$

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source) and potassium ions K^+ (potassium nitrate KNO_3 source).

Oxide glasses, in which it is possible to carry out the ion exchange processes, must demonstrate a good ionic conductivity. They contain a large amount of modifiers weakly bound to the skeleton of the glass. Such modifier is generally a sodium ion Na^+ as a result of adding the sodium oxide Na_2O to the glass composition [32]. As the effect of different properties, such as electric polarizability and the ionic radius, the dopant ions introduced into the glass alter locally its optical properties (refractive index) [33, 34]. Due to the diffusive nature of the ion exchange phenomenon and the fact that the admixture is introduced into the glass by its surface, the resulting changes in glass refraction are located at its surface and the change in the refractive index in the glass has the gradient character.

2. Applications of producing gradient refractive index in glass

The nature of changes in the glass refraction and the extent of these changes achieved as a result of ion exchange determine the specificity of this technological method's purpose. The kinetics of ion exchange processes at the set temperature of the process depends on the type of glass and the type of admixture ions. The duration of the ion exchange processes determines the range (depth) of obtained changes in glass refraction. These depths may vary in range from a fraction of μm to tens and even hundreds of μm. Ion exchange method has been used primarily in planar optics. With the use of this method the waveguide structures with the gradient changes of the refractive index are manufactured. They can be either one-dimensional (planar slab type) as well as two-dimensional structure (planar waveguides of channel type). Taking as a reference the system in Fig. 1 the waveguide structures can be classified on the basis of the vector of gradient of refraction changes ∇n. In the first case, the vector has only one coordinate g_x (Fig. 1a). In the second case it is a vector with coordinates g_x and g_y (Fig. 1b). For both single-and two-dimensional structures they may be single or multimode waveguides [35]. In such waveguide structures, the wave vector $\vec{k}$ of a propagating wave has a direction parallel to the plane of the glass surface (Fig. 1a, Fig. 1b). On the basis of such structures various elements of waveguide planar optics may be designed, such as splitters of Y type [36], 1xN splitters [37], multiplexers [38–42], directional couplers [43], ring resonators [44, 45], planar polarizers [46], planar interferometers [47, 48], gradient structures of multimode interference couplers MMI [49–51]. These elements cooperate with fiber waveguides. Hence the need to optimize the distribution of refraction of channel waveguides in terms of minimizing the loss of optical power at their connections with fiber waveguides [52, 53]. In addition to the passive elements of planar optics, the laser structures and optical amplifiers can also be produced by the ion exchange method. The glasses used for this purpose are doped with the rare-earth ions [54–60]. On the basis of gradient planar waveguide structures the optical sensors are also produced [61–65]. In the gradient waveguides produced by ion exchange in glasses doped with semiconductors the nonlinear effects are also observed [66–68].

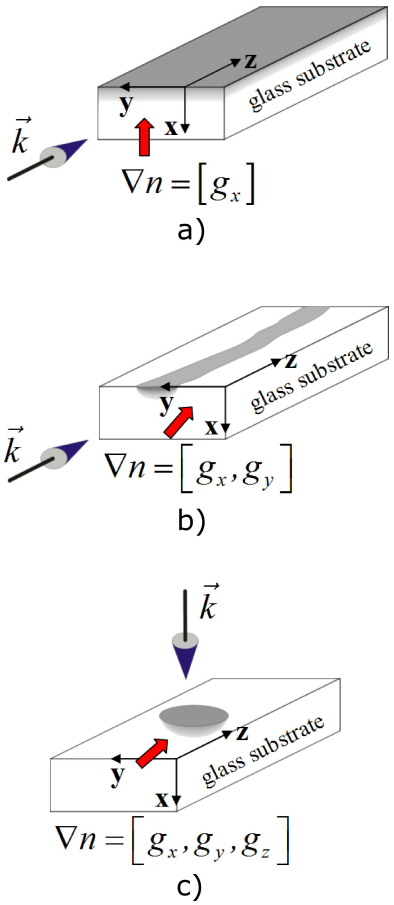

Fig. 1. The classification of gradient structures in glass: a) planar of slab type, b) planar of channel type, c) refractive

Beside the waveguide structures in the surface area of glass, the refraction structures are also produced by the ion exchange method [69–71]. In this case the wave vector $\vec{k}$ of a wave passing through such structure (Fig. 1c) may be perpendicular to the plane of the glass surface in which it was made. The vector of gradient of changes in refraction in this case has three coordinates g_x, g_y and g_z (Fig. 1c). Refraction structures can be in this case, for example, microlenses or microlens arrays [72].

The ion exchange in glass phenomenon is also accompanied by geometrical changes (volume changes) of the admixture area. This is a result of the difference of radii of dopant ions and the glass modifier ions involved in the exchange process. In this case the changes in refractive index of glass are characterized by the birefringence being the effect of the stress generated in the glass [73]. The magnitude of these stresses can reach values of the order of hundreds N/mm^2. Such situation occurs for example in *BK7* glass doped with the potassium ions (K^+) [74, 75]. The extent of these stresses is comparable to the range of changes of refraction of glass. Thus the gradients of these stresses reach very large values in

the superficial area of the glass. Because the produced stresses are of compressive nature, the ion exchange method can be used for surface hardening of glasses [76–78].

3. Ion exchange in glass as a technological method

An important advantage of the ion exchange method is its low cost both in terms of materials used as well as the equipment required for its implementation. The classification of this method to the category of technological methods require obtaining predictable and repeatable results by its use. Predictability of technological method requires the formulation of a mathematical model describing its effects. Such model with regard to the obtained technological effects allows to set certain physical quantities characterizing the technological process. This is done by fitting a model of the process to set (on the basis of measurements) quantities characterizing the obtained technological effect. In turn, obtaining reproducibility of the technological method requires the determination of the impact of all the parameters of the process on its final effect. Then it is possible to determine the tolerance of technological effect resulting from the tolerance of implementation of the parameters of the process.

A mathematical model in the case of the ion exchange phenomenon is the description of the process of diffusion of the admixture ions introduced into the glass. The technological result here are the changes of the refractive index of the glass dependent on the glass and admixture types as well as the method of realization of the technological process. These effects are described by means of certain functions under the physical basis of this phenomenon. These functions represent the dependence of the refractive index change of the glass from the spatial coordinates. Depending on the type of produced gradient structure there may be one, two or three coordinates (Fig. 1). Changes of glass refraction which are the result of ion exchange are the easiest to be examined in the case of planar waveguide structures, slab type (Fig. 1a). The rationale here is the easiness of taking measurements while ensuring their high precision. The method applied here is the waveguide method (m-line method). It uses a prism coupler to selectively stimulate the waveguide modes. The measuring effect are the synchronous angles φ_m (where m – mode row), on the basis of which the normalized propagation constants of the waveguide modes (effective refractive indices N_m) are calculated. The measurement uncertainties ΔN_m of the effective refractive indices designated by this method are of the order of 10^{-4} [80]. On the basis of the determined set of effective refractive indices of the modes $\{N_m : m = 0, 1..., M-1\}$ (where M – the number of stimulated modes of the waveguide) the approximate refractive index profile of the waveguide is reconstructed. This profile is a set of points $\{(x_m, N_m) : m = 0, 1..., M-1\}$, where x_m marks a turning point of the mode with a N_m effective refractive index. The procedure for determining the x_m turning points on the basis of a set of effective refractive indices of modes has been described for the first time in paper [79]. The modes equation binding monotonic refractive index profile of the gradient waveguide to the effective refractive indices of modes is as follows:

$$k_0 \int_0^{x_m} \sqrt{n^2(x) - N_m^2}\,dx = \pi\left(m + \frac{1}{4}\right) + \operatorname{arctg}\left(r\sqrt{\frac{N_m^2 - n_c^2}{n_s^2 - N_m^2}}\right), \tag{1}$$

where k_0 – wave number of the light in free space, x_m – the position of the turning point of the m-th mode, $n(x)$ – refractive index profile of the waveguide, N_m – effective refractive index of the m-th mode, n_c – refractive index of the environment (coverage) of the waveguide, n_s – refractive index of the waveguide at the glass surface, $r = 1$ (for the TE polarization), $r = (n_s/n_c)^2$ (for the TM polarization). The second component on the right side of the equation takes into account the phase change upon wavelength reflection (mode) from the glass surface, depending on its state of polarization.

Fidelity to the reconstruction of the refractive profile using this procedure increases with the number of modes conducted through the waveguide [80].

Theoretical description of changes in refractive index of glass is based on a mathematical model of a two-component ion exchange [80, 81]. It assumes the existence of only two types of ions involved in the exchange process. These are the ions introduced to the glass (ions of A type) and the glass modifier ions (ions of B type). In this model, the diffusion coefficients of ions exchanged in the glass are functions of their normalized concentration and temperature, in the form of:

$$\begin{aligned} D_A &= D_A(u,T) = D_{0A}(T)\cdot\exp(Au), \\ D_B &= D_B(w,T) = D_{0B}(T)\cdot\exp(Bw), \end{aligned} \tag{2}$$

where D_A, D_B – diffusion coefficients, u and w – normalized concentrations of ions of A and B type, T – temperature of the process (K), D_{0A}, D_{0B}, A and B – coefficients determined experimentally.

The dependence of the D_{0A} and D_{0B} coefficients on the temperature is described by the Arrhenius equations:

$$D_{0i}(T) = D_{0i}^* \cdot \exp\left(-\frac{\Delta Q_i}{RT}\right) \qquad (i = A, B), \tag{3}$$

wheret ΔQ_i – activation energy of the i-th ion type, R – universal gas constant.

In Eq. (3) the constants: D_{0i}^* and ΔQ_i are experimentally determined for a given glass and admixture type.

The equation describing the spatio-temporal changes in normalized concentration of admixture ions introduced into the glass in the one-dimensional thermal diffusion process has the form [80]:

$$\frac{\partial u}{\partial t} = \frac{D_{0A}\exp(Au)}{1-\alpha u}\cdot\frac{\partial^2 u}{\partial x^2} + \frac{D_{0A}\exp(Au)[\alpha+(1-u)A]-u(1-\alpha)^2 D_{0B}B\exp[B(1-u)]}{(1-\alpha u)^2}\left(\frac{\partial u}{\partial x}\right)^2, \tag{4}$$

where $\alpha = 1 - \dfrac{D_{0A}}{D_{0B}}\cdot\exp\left[u(A+B) - B\right]$.

The function $u(x,t)$ which is a solution to this equation is transformed into one-dimensional refractive index profile $n(x)$ according to the relation:

$$n(x) = n_b + \Delta n_s \cdot u(x). \tag{5}$$

In the equation above n_b is the refractive index of undoped glass, Δn_s is the maximum change in the refractive index of the doping area of the glass (for the preliminary diffusion – at the glass surface).

For a theoretical description of the formation of the changes of glass refraction as a result of the ion exchange process (diffusion of the admixture ions) it is necessary to know the diffusion coefficients of exchanged ions (2). These coefficients, which are the functions of normalized concentration of exchanged ions, require knowledge of four parameters: D_{0A}, D_{0B}, A and B. Proceedings to their designation consists of three stages [80]. The first implements a process of diffusion of dopant ions to the given type of glass at a fixed temperature T_{diff} (the average temperature of the process) in a known time t_{diff}. In the second phase the measurement of the effective refractive indices of the modes of the produced waveguide is made, as well as the reconstruction of the refractive index profile (1). In the third step the solution of the diffusion Eq. (4) transformed into the refractive index profile (5) is fitted to the points of the profile reconstructed in the second stage. The parameters D_{0A}, D_{0B}, A and B giving the best fit are taken as parameters determining the diffusion coefficients for a given temperature [81]. Carrying out this procedure for different temperatures of diffusion processes the D^*_{0i} and ΔQ_i values can be determined based on the Arrhenius Eq. (3). Thus the temperature dependences $D_{0A}(T)$ and $D_{0B}(T)$ of diffusion coefficients of exchanged ions are obtained.

Knowledge of these relationships allows the *in situ* control of the diffusion processes of glass doping. This method [81] is to solve the diffusion Eq. (4) in real time of the process. The directly measured value here is the temperature. This procedure is presented in Fig. 2.

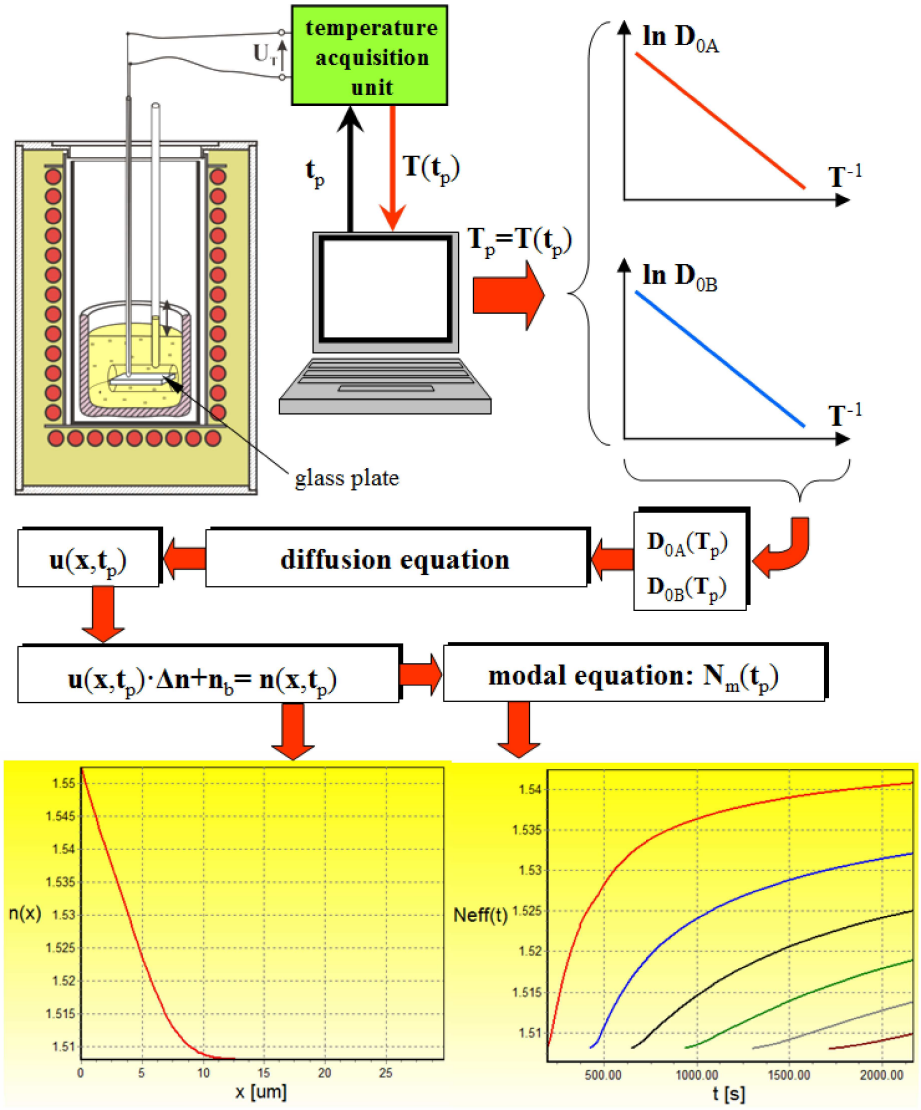

Fig. 2. The principle of diffusion process control in real time

The crucible, in which there is the molten salt (the source of admixture), is placed inside the furnace. The glass plate (substrate) in which the diffusion process is carried out, is placed in a special holder that allows continuous mixing of the surrounding salt. This ensures that the thermal homogeneity of the source of admixture. Temperature measurement is performed using thermocouple located in the immediate vicinity of the glass plate (Fig. 3). The diffusion Eq. (4) is integrated over the time domain with the time step Δt. Solving this equation is carried out in parallel with the implementation of the diffusion process. Measurement of the temperature in the crucible occurs at certain moments of time t_p, which are integer multiples of the time step Δt. The diffusion coefficients appearing in this equation are calculated at points of time t_p based on knowledge of their temperature dependences. In this way the course of the process temperature $T_p = T(t_p)$ through the values of diffusion coefficients $D_{0A}(T_p)$ and $D_{0B}(T_p)$ affects the form of the solution $u(x, t_p)$ of the diffusion Eq. (4). Using Eq. (5) a correlation of the function describing the distribution of refraction in the glass $n(x, t_p)$ is obtained. In turn, for the modes Eq. (1), for the assumed wavelength and the polarization state, the values of the effective refractive indices of modes $N_m(t_p)$ corresponding with the refractive profile $n(x, t_p)$ are calculated. Visualization of the current refractive profile $n(x, t_p)$ and the resulting set of effective refractive indices of the modes $N_m(t_p)$ allow a direct control of the diffusion process. After completion of the process (with a t_{diff} duration) a registered final profile of the glass refraction $n(x, t_{diff})$ is obtained as well as a set of effective refractive indices $\{N_m(t_{diff}) : m = 0, 1..., M - 1\}$ (M – number of modes).

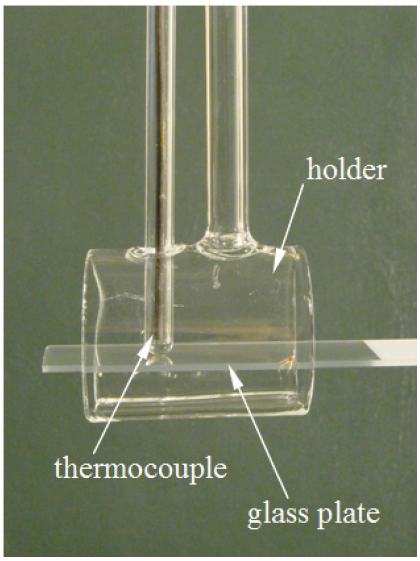

Fig. 3. A method for measuring the temperature of the glass substrate

Faithfulness of this modeling process based on the course of its temperature is determined by both the temperature measurement (thermocouple closest to the glass plate) and by the accuracy of determination of the temperature dependences of the diffusion coefficients. Such a method of controlling the diffusion process also allows taking into account the cooling stage of the glass substrate after removing it from the crucible (ensuring its continued contact with the thermocouple). This is important in cases of carrying short diffusion processes. Of course, the cooling speeds of the substrate and the thermocouple differ. This is because the thermocouple is placed in the shield isolating it from contact with the salt in the crucible.

The results of this method of process control can be verified by measuring. This includes comparison of a set of calculated effective refractive indices of modes $\{N_m(t_{diff}) : m = 0, 1..., M_{calc}-1\}_{\text{calc}}$ with a set of measured effective refractive indices of modes $\{N_m(t_{diff}) : m = 0, 1..., M_{meas}-1\}_{\text{meas}}$. The effects of such process allow us to conclude the predictability and repeatability of the results obtained by this method and thus its recognition as a technological method. According to the Author's knowledge such method has not been described in literature before. The next chapter presents the results of such comparisons, which were made for selected glass-admixture systems.

4. The results of calculations and measurements

The method of controlling the diffusion processes, which was described in the previous section, has been verified for the three types of glass substrate and two types of admixture ions. The research was carried out for soda-lime glass (Menzel-Glasser company), *BK7* (Schott) and Pyrex (Borosilicate 33 of Corning company). Among these the Pyrex glass has the lowest refractive index, close to the one of silica. The admixture ions used here was the silver ions Ag^+ (source: silver nitrate $AgNO_3$) and potassium ions K^+ (source: potassium nitrate KNO_3). Figure 4 presents the characteristics of the temperature dependences of diffusion coefficients of exchanged ions for the glass-admixture systems mentioned above. These characteristics were determined on the basis of Eqs. (3). Linearized form of these s allows the determination of the coefficients $\ln(D_{0i}{}^*)$ and $\Delta Q_i/R$ determining the temperature diffusion coefficients. Table 1 summarizes the types of glass and admixture ions, for which the temperature characteristics of diffusion coefficients have been designated. The measurements of effective refractive indices of the waveguide modes were performed for a wavelength of $\lambda = 677$ nm (TE polarization).

Tables 2–4 present the comparison of the values of the effective refractive indices obtained in the process of numerical simulations (N_{calc}) with the values determined on the basis of the measurement (N_{meas}). The measure of such comparisons are the absolute values of the differences of these values $|N_{calc} - N_{meas}|_m$ calculated for each row of modes m. These comparisons were performed for the glass-admixture systems listed above. Table 2 contains the results on $Ag^+ \leftrightarrow Na^+$ ion exchange processes realized in the soda-lime glass with the use of $AgNO_3$ as a source of admixture. In this type of glass these processes occur very quickly. This is evidenced by the number of supported modes in waveguides manufactured in diffusion processes of several minutes duration. For the $Ag^+ \leftrightarrow Na^+$ ion exchange in this glass the largest absolute

values of differences $|N_{calc} - N_{meas}|_m$ are of the order of 10^{-3}. In the case of the $K^+ \leftrightarrow Na^+$ ion exchange realized in *BK7* glass (with KNO_3 as the source of admixture) the kinetics of this process is the smallest (comparing to the other ones). The production of the multimode waveguides in the temperature of the diffusion process of about 400° requires duration of order of tens (to over a hundred) hours. However, with this type of the ion exchange the control of diffusion processes gives very good results. The absolute differences $|N_{calc} - N_{meas}|_m$ for the effective refractive indices of the modes in the produced waveguides are in this case are at the level of 10^{-4} (Table 3). The $Ag^+ \leftrightarrow Na^+$ ion exchange realized in Pyrex glass from the $AgNO_3$ source of admixture has also small kinetics (but slightly greater than the $K^+ \leftrightarrow Na^+$ ion exchange in *BK7* glass). In this case also the absolute differences $|N_{calc} - N_{meas}|_m$ are of the order of 10^{-4}. This even applies to the processes that produce single- and two-mode waveguides (Table 4).

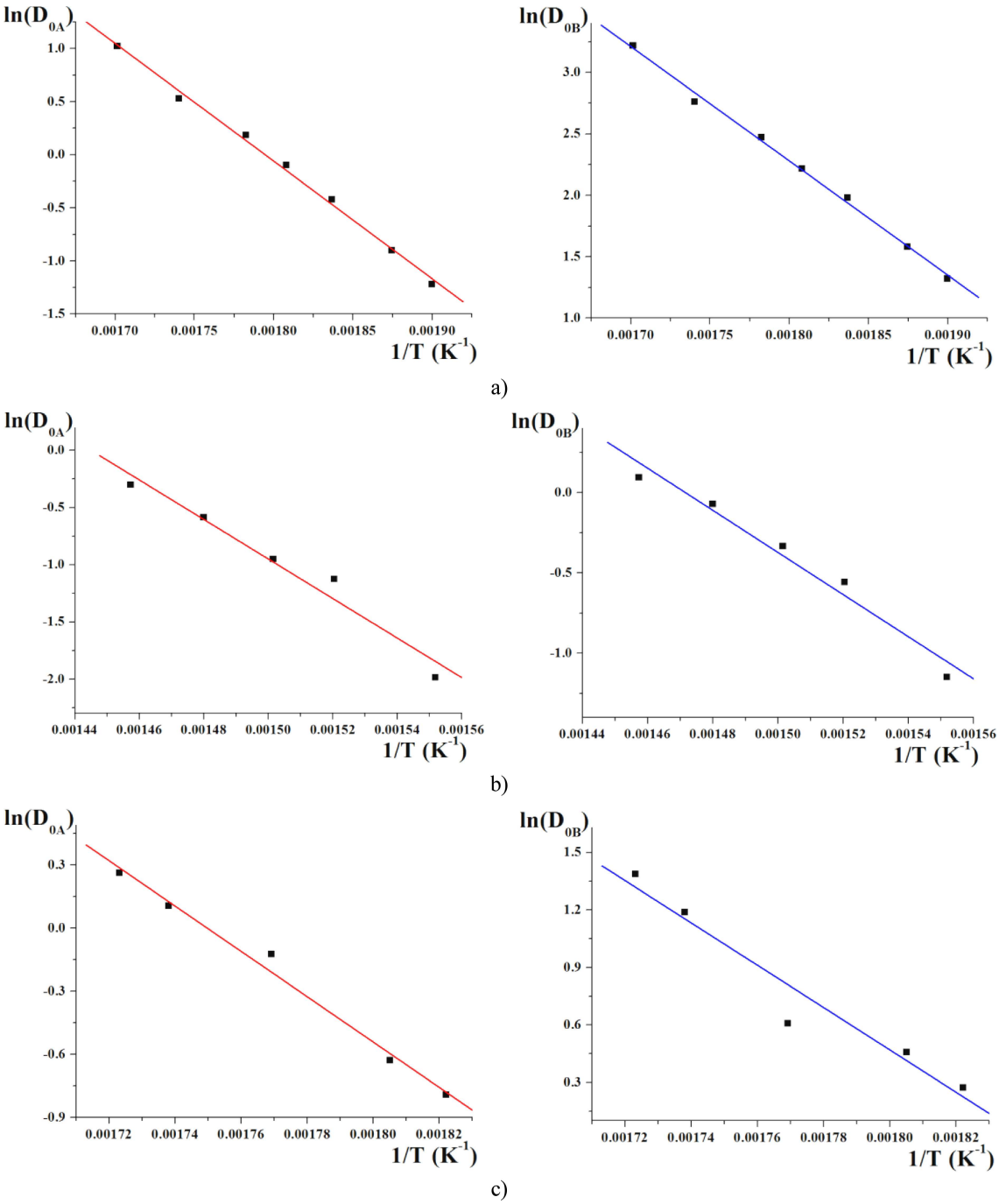

Fig. 4. Determined by Eq. (3) temperature dependences of diffusion coefficients $D_{0A}(T)$ and $D_{0B}(T)$ for the glass-admixture systems of Table 1. a) Soda-lime glass: ion exchange $Ag^+ \leftrightarrow Na^+$ (source of admixture ions: $AgNO_3$), b) BK7 glass: ion exchange $K^+ \leftrightarrow Na^+$ (source of admixture ions: KNO_3), c) Pyrex glass: ion exchange $Ag^+ \leftrightarrow Na^+$ (source of admixture ions: $AgNO_3$)

Table 1
The types of glass and admixture ions

Type of glass	Ion exchange type	Source of admixture	Diffusion coefficients				$n_{b,677}$ nm	Δn_{677} nm
			$\Delta Q_A/R$ (K)	$\ln(D_{0A}{}^*)$	$\Delta Q_B/R$ (K)	$\ln(D_{0B}{}^*)$		
Soda-lime	$Ag^+ \leftrightarrow Na^+$	$AgNO_3$	$-1.1086 \cdot 10^4$	19.894	$-9.2988 \cdot 10^3$	19.017	1.5105	0.0952
BK7	$K^+ \leftrightarrow Na^+$	KNO_3	$-1.5786 \cdot 10^4$	22.740	$-1.3376 \cdot 10^4$	19.688	1.5137	0.0079
Pyrex	$Ag^+ \leftrightarrow Na^+$	$AgNO_3$	$-1.0771 \cdot 10^4$	18.847	$-1.1044 \cdot 10^4$	20.349	1.4678	0.0190

Table 2
Diffusion processes in soda-lime glass. Ion exchange $Ag^+ \leftrightarrow Na^+$. Source of admixture ions: $AgNO_3$

	$t_{diff} = 3$ h, $T_{ave} = 296$ C			$t_{diff} = 3$ h, $T_{ave} = 288$ C			$t_{diff} = 6$ h, $T_{ave} = 253$ C		
m	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$
0	1.5935	1.5948	0.0013	1.5925	1.5939	0.0014	1.5907	1.5899	0.0008
1	1.5844	1.5848	0.0004	1.5826	1.5833	0.0007	1.5793	1.5778	0.0015
2	1.5771	1.5773	0.0002	1.5748	1.5752	0.0004	1.5702	1.5683	0.0019
3	1.5709	1.5707	0.0002	1.5679	1.5682	0.0003	1.5620	1.5599	0.0021
4	1.5651	1.5647	0.0004	1.5616	1.5619	0.0003	1.5543	1.5518	0.0025
5	1.5598	1.5592	0.0006	1.5557	1.5558	0.0001	1.5467	1.5442	0.0025
6	1.5546	1.5536	0.0010	1.5499	1.5499	0.0000	1.5390	1.5361	0.0029
7	1.5496	1.5485	0.0011	1.5441	1.5440	0.0001	1.5312	1.5280	0.0032
8	1.5445	1.5433	0.0012	1.5383	1.5382	0.0001	1.5234	1.5198	0.0036
9	1.5395	1.5380	0.0015	1.5325	1.5323	0.0002	1.5159	1.5124	0.0035
10	1.5345	1.5328	0.0017	1.5267	1.5265	0.0002			
11	1.5295	1.5277	0.0018	1.5210	1.5206	0.0004			
12	1.5244	1.5224	0.0020	1.5155	1.5151	0.0004			
13	1.5195	1.5172	0.0023						
14	1.5148	1.5126	0.0022						
15	1.5111	1.5088	0.0023						
Short time diffusion processes									
	$t_{diff} = 15'$, $T_{ave} = 300$ C			$t_{diff} = 8'$, $T_{ave} = 300$ C			$t_{diff} = 4'$, $T_{ave} = 302$ C		
m	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$
0	1.5805	1.5811	0.0006	1.5751	1.5760	0.0009	1.5712	1.5690	0.0022
1	1.5615	1.5621	0.0006	1.5513	1.5520	0.0007	1.5430	1.5389	0.0041
2	1.5454	1.5458	0.0004	1.5295	1.5310	0.0015	1.5154	1.5109	0.0045
3	1.5294	1.5297	0.0003						
4	1.5140	1.5143	0.0003						

Table 3
Diffusion processes in *BK7* glass. Ion exchange $K^+ \leftrightarrow Na^+$. Source of admixture ions: KNO_3

	$t_{diff} = 91$ h, $T_{ave} = 413$ C			$t_{diff} = 118$ h, $T_{ave} = 385$ C			$t_{diff} = 117$ h, $T_{ave} = 393$ C		
m	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas} \rvert_m$
0	1.5197	1.5202	0.0005	1.5193	1.5195	0.0002	1.5195	1.5195	0.0000
1	1.5183	1.5187	0.0004	1.5177	1.5178	0.0001	1.5180	1.5178	0.0002
2	1.5173	1.5176	0.0003	1.5164	1.5165	0.0001	1.5168	1.5166	0.0002
3	1.5164	1.5167	0.0003	1.5154	1.5155	0.0001	1.5158	1.5157	0.0001
4	1.5156	1.5160	0.0004	1.5145	1.5147	0.0002	1.5150	1.5149	0.0001
5	1.5150	1.5153	0.0003	1.5139	1.5140	0.0001	1.5143	1.5142	0.0001
6	1.5144	1.5147	0.0003						
7	1.5140	1.5143	0.0003						
8	1.5137	1.5138	0.0001						

Table 4
Diffusion processes in *Pyrex* glass. Ion exchange $Ag^+ \leftrightarrow Na^+$. Source of admixture ions: $AgNO_3$

	t_{diff} = 23 h, T_{ave} = 302 C			t_{diff} = 18 h, T_{ave} = 307 C			t_{diff} = 18 h, T_{ave} = 295 C		
m	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas}\rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas}\rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas}\rvert_m$
0	1.4831	1.4832	0.0001	1.4830	1.4832	0.0002	1.4824	1.4833	0.0009
1	1.4802	1.4802	0.0000	1.4800	1.4801	0.0001	1.4790	1.4798	0.0008
2	1.4778	1.4778	0.0000	1.4776	1.4776	0.0000	1.4762	1.4771	0.0009
3	1.4757	1.4756	0.0001	1.4755	1.4754	0.0001	1.4738	1.4746	0.0008
4	1.4739	1.4737	0.0002	1.4735	1.4735	0.0000	1.4717	1.4725	0.0008
5	1.4721	1.4719	0.0002	1.4718	1.4717	0.0001	1.4698	1.4707	0.0009
6	1.4706	1.4704	0.0002	1.4702	1.4701	0.0001	1.4684	1.4693	0.0009
7	1.4693	1.4691	0.0002	1.4689	1.4688	0.0001			
8	1.4683	1.4681	0.0002	1.4680	1.4680	0.0000			
Short time diffusion processes									
	t_{diff} = 3 h, T_{ave} = 286 C			t_{diff} = 2.9 h, T_{ave} = 287 C			t_{diff} = 1 h, T_{ave} = 289 C		
m	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas}\rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas}\rvert_m$	N_{calc}	N_{meas}	$\lvert N_{calc} - N_{meas}\rvert_m$
0	1.4780	1.4777	0.0003	1.4781	1.4791	0.0010	1.4750	1.4742	0.0008
1	1.4713	1.4714	0.0001	1.4713	1.4718	0.0005			

The present control method of the process of diffusion doping of glass on the basis of the temperature measurement is also used in the aspect of reproducibility of obtained refraction changes. The values of the effective refractive indices of the modes recorded on an ongoing basis can be compared with the reference values N_{norm}. The set of reference values of the effective refractive indices can be calculated by modeling the planar structure or may be the result of measurement of the waveguide produced by another diffusion process. Along with the duration of the process the calculated values N_{calc} increase. The rate of this increase depends on the temperature. The temperature increase reduces the time of the diffusion process, in which the controlled N_{calc} values reach the reference values N_{norm}, and vice versa. The diffusion process (its duration time and temperature) can therefore be directed so that it gets close up to the reference values.

The above statements have been verified experimentally. The object of the study was soda-lime glass, in which the diffusion processes were carried out by the $Ag^+ \leftrightarrow Na^+$ ion exchange from the $AgNO_3$ admixture source. Six diffusion processes at different temperatures have been carried out (Fig. 5a). The result of the first process was a planar waveguide leading 5 modes of the *TE* polarization for a wave of $\lambda = 677$ nm. The aim of each of the next processes was to produce the same waveguide with the same effective refractive indices. The thermo-temporal characteristics presented in Fig. 5a show the correctness of shortening the diffusion time at higher temperatures (or vice versa) while the end result of the process remains the same. Figure 5b presents the ranges of the measured values of the effective indices of the modes for each row of modes for all six produced waveguides. The smallest dispersion of the measured values occurs here for the fundamental modes $\Delta N_0 = 0.0008$. This is a result of adopting effective indices of fundamental modes in the produced and reference waveguides as a comparative criterion.

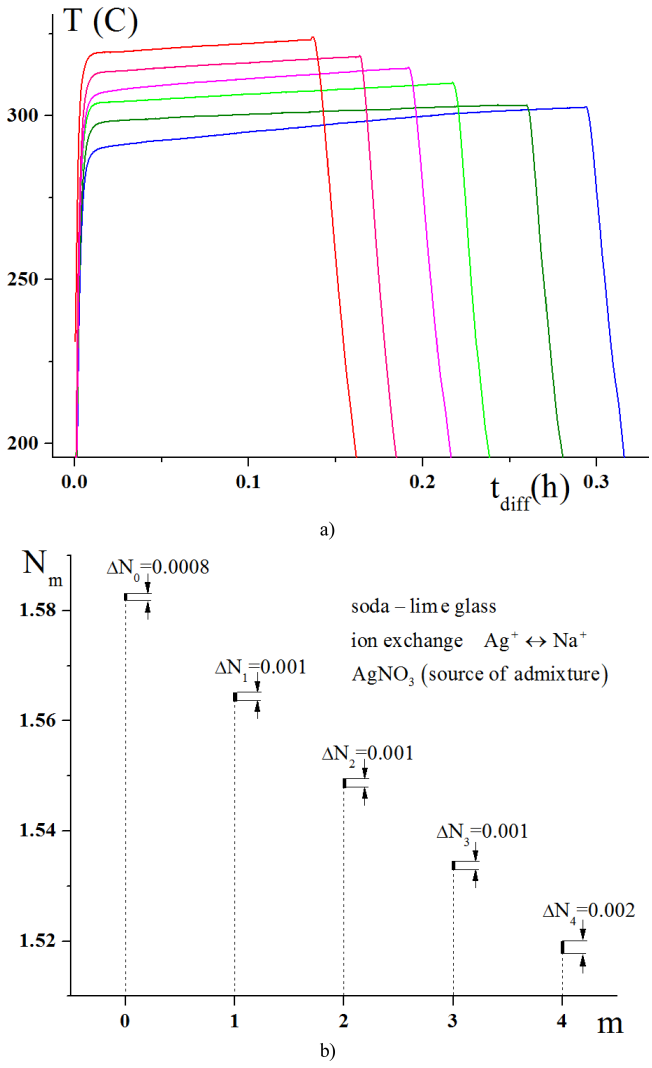

Fig. 5. The thermo-temporal characteristics of the diffusion processes a), the measured values (and their scattering) of the effective indices of modes for waveguides produced by the diffusion processes b)

5. Conclusions

The presented results indicate the ability to control the ion exchange processes in real-time of diffusion processes. In the case of preparation of planar waveguides of slab type, a repeatability (tolerance) of the effective indices of the modes at the level of $\Delta N \sim 10^{-3}$ can be guaranteed. In the glasses, in which the kinetics of ion exchange processes is small (e.g. *BK7* glass, ion exchange: $K^+ \leftrightarrow Na^+$), this repeatability may be of the 10^{-4} order.

For the planar waveguides of the slab type the calculation time (associated with the simulation of one-dimensional refractive profiles and designation on their basis of a set of effective indices) is short. This allows to obtain information about the process with the time step of the $\Delta t = 1$ s order. It is a quite sufficient period of time for modeling processes whose duration is of the order of minutes. In the case of simulation of two-dimensional waveguide structures (e.g. planar waveguides of the channel type) the calculation time of the refractive profile will definitely be longer. Additional calculation on the basis of such profile of modes propagation constants (with the use of the simplest effective index method) significantly extends the time step of modeling.

In this case, however, the modeling of the equivalent planar waveguide of slab type can be applied, which could be formed during the identical (in terms of time and temperature) diffusion process. The same procedure can be adopted for an even more complex simulations of the three-dimensional structures.

The proposed method of control of diffusion processes can also be transferred to the annealing processes. In this case, the gradient distribution of changes of refractive index obtained in the glass is subject to the thermal diffusion process in the absence of flow of external admixture. Control principle remains the same here. In the numerical simulations only the initial and boundary conditions for the distribution of admixture in the glass are changed.

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