Coherence investigations of DC voltage group standard

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Abstract. A method of checking coherence of comparison results of primary DC voltage group standards is described in the paper. Usefulness of the proposed method was proved by coherence investigation of DC voltage group standard kept in the Central Military Calibration Laboratory (CWOM, Warsaw, Poland). Results obtained for the group of five Fluke DC voltage standards prove very good coherence of voltage differences results and confirmed accuracy of DC voltage unit in the CWOM at the level of 10^{-6}.

Key words: coherence, comparison, group standard, voltage standard.

1. Introduction

Group standards are commonly used to maintain units of different physical quantities (for example: DC voltage, mass, resistance, inductance) at many National Metrology Institutes (NMI's) and Designated Institutes [1, 2]. The idea of the group standard is based on fundamental assumption that in stable conditions any change of individual value of a standard is stochastic and independent of changes in the values of other members, however an average value of several changes tends to zero. This basic assumption implies that group value is constant although each individual member of the group can change its value. Taking into consideration stochastic changes of real values of n members forming the group, the variance of the group value is n-times smaller than for a single standard.

The difference measurements between the group members taken in pairs are necessary to determine the group value. Direct difference measurements are very advantageous because the influence of residual parameters is minimized in this case.

The coherence test is a suitable method of checking the systems used to measure differences of physical quantities. This method allows to conclude about the correctness of measurements and accuracy of measuring equipment. Coherence tests are frequently carried out for newly designed equipment and systems. Examples are the KWL impedance bridges [3, 4], designed by a team of scientists from the Institute of Measurement Science, Electronics and Control of Silesian University of Technology in the years 1997–2009, or automatic bridge designed by a team from Ukraine [5, 6].

To clearly explain the need of the coherence test let us examine the following example:

1. There are three numbers representing values of three similar standards: $S_1 = 1.001$; $S_2 = 1.003$; $S_3 = 1.006$.
2. Calculated differences of the numbers:

$S_{12} = S_1 - S_2 = -0.002; \quad S_{21} = S_2 - S_1 = 0.002;$

$S_{23} = S_2 - S_3 = -0.003; \quad S_{32} = S_3 - S_2 = 0.003;$

$S_{31} = S_3 - S_1 = 0.005; \quad S_{13} = S_1 - S_3 = -0.005.$

3. Let us calculate mean difference for every pair of numbers, according to the definition:

$$\overline{S}_{12} = 0.5(S_{12} - S_{21}) = -0.002;$$

$$\overline{S}_{23} = 0.5(S_{23} - S_{32}) = -0.003;$$

$$\overline{S}_{31} = 0.5(S_{31} - S_{13}) = 0.005.$$

4. The mean differences fulfill the obvious condition: $\overline{S}_{12} + \overline{S}_{23} + \overline{S}_{31} = 0$.
5. However, if differences are measured directly, the results are affected by errors and the above sum is non-zero: $\overline{S}_{12} + \overline{S}_{23} + \overline{S}_{31} \neq 0$.

If the above sum is non-zero, the instrumentation, connections and measurement procedure should be checked carefully against error sources.

In the paper the above mentioned idea is applied to the DC voltage sources, forming the voltage group standard composed of five units. In a typical procedure of the group standard maintenance, the voltage differences are measured to calculate group value. In the paper we would like to show how the difference results can be used to confirm lack of major errors, both systematic and random.

This paper is organized as follows. The theoretical basics of the coherence test, in general terms, are presented and discussed in Sec. 2. Section 3 is focused on the application of the method to the voltage group standard. Compressed measurement results are presented here. These results are important for experienced measurement laboratory staff and measurement science specialists for continuous improvement of measurement methods and procedures.

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2. Theoretical basis of coherence investigation

To check the coherence of comparison results the group consisting of at least three standards is suitable, therefore, coherence tests are sometimes called "triangle method" (Fig. 1a). For groups containing more standards, comparison schemes are shown graphically in Fig. 1b and Fig. 1c. In order to eliminate systematic errors occurring in the measurement system (e.g. caused by thermoelectric power and voltage drops on the connection cables) each comparison is carried out twice, directly and after changing places of S_i and S_j standards. Considering the comparison scheme shown in Fig. 1a, six raw difference measurements results are obtained. These differences are marked: S_{12}, S_{21}, S_{13}, S_{31}, S_{23}, S_{32}. The three final results $\overline{S}_{ij}$, free of systematic errors, are calculated as a half of the difference between two corresponding raw results according to the following equations:

$$\overline{S}_{12} = 0.5(S_{12} - S_{21}), \tag{1}$$

$$\overline{S}_{23} = 0.5(S_{23} - S_{32}), \tag{2}$$

$$\overline{S}_{31} = 0.5(S_{31} - S_{13}). \tag{3}$$
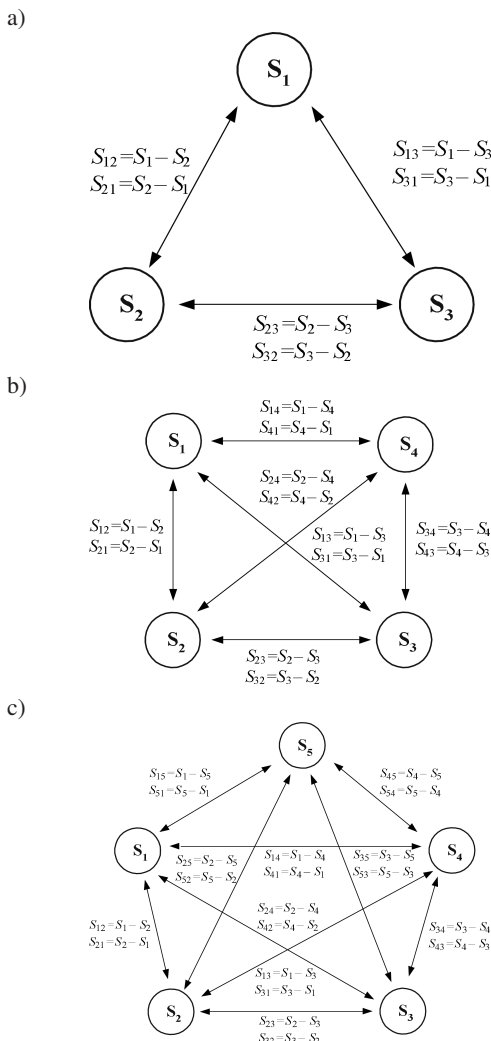
a)

b)

c)

Fig. 1. Comparison scheme: a) for a group of 3 standards, b) for a group of 4 standards, c) for a group of 5 standards

The coherence test of the comparison results consists in checking the sum of the three differences calculated from the Eqs. (1)–(3). In a perfect measuring circuit the triangle of differences is closed, thus

$$\left|\overline{S}_{12} + \overline{S}_{23} + \overline{S}_{31}\right| = 0. \tag{4}$$

In practice, due to any imperfections of the measuring system the sum above is non-zero. Therefore there is allowed some inequality at which it is assumed that the relation (4) is satisfied enough. The maximum relative difference between right and left side of the Eq. (4) is further marked by λ, and is called a level of coherence of the comparison results. λ should be arbitrarily assumed by the laboratory staff, according to the measurement capabilities of the laboratory. Thus, the difference results for considered group of standards should meet the following relation:

$$\left|\overline{S}_{12} + \overline{S}_{23} + \overline{S}_{31}\right|/S_n \le \lambda, \tag{5}$$

where λ is arbitrarily assumed level of coherence, e.g. $\lambda = 1\ \mu V/V$, and S_n is nominal value of standards compared. The assumed level of coherence should always correspond to the uncertainty of the measurement system being checked.

If the test results do not satisfy the relation (5) it means there are non-identified errors in the circuit which should be detected and removed. Then the measurement must be repeated. Only a coherent result series can be the basis to determine the value of standards.

The analysis presented above can be generalized to any number of standards. If a group consists of four members, as shown in Fig. 1b, not six, as before, but twelve raw measurements of differences and six final results $\overline{S}_{ij}$ are obtained. In this case coherence should be checked for the entire group (quadrangle of differences), and also for each of the four subgroups that can be created from the whole four-element group. Therefore, the investigation of coherence in this case consists in checking if the following inequalities are satisfied:

$$\left|\overline{S}_{12} + \overline{S}_{23} + \overline{S}_{31}\right|/S_n \le \lambda, \tag{6}$$

$$\left|\overline{S}_{12} + \overline{S}_{24} + \overline{S}_{41}\right|/S_n \le \lambda, \tag{7}$$

$$\left|\overline{S}_{13} + \overline{S}_{34} + \overline{S}_{41}\right|/S_n \le \lambda, \tag{8}$$

$$\left|\overline{S}_{23} + \overline{S}_{34} + \overline{S}_{42}\right|/S_n \le \lambda, \tag{9}$$

$$\left|\overline{S}_{12} + \overline{S}_{23} + \overline{S}_{34} + \overline{S}_{41}\right|/S_n \le \lambda, \tag{10}$$

In the group consisting of five standards (Fig. 1c) there can be distinguished five subgroups containing four members and up to ten subgroups containing three members. Together with the group of five standards sixteen equations for coherence checking are obtained in this case.

3. Coherence of DC voltage standard

3.1. Tested object. Coherence tests were performed in the Central Military Calibration Laboratory (CWOM, Warsaw, Poland), which is metrology centre of Polish Ministry of National Defence. The main task of the Centre is to ensure traceability and uniformity of measures in all organizational units

of the Ministry of National Defence, which is extremely important for efficient and reliable operation of weapon systems and military equipment in Poland. The CWOM key role in this field includes the maintenance of military primary standards for electrical and non-electrical quantities, relating them to international and national standards, and transfer of units of measures to secondary standards used by the district Military Calibration Laboratories (WOM).

The voltage standard is one of the four key standards of electrical units (besides resistance, capacitance and inductance standard) maintained in laboratories of Polish Ministry of National Defense. The primary standard of DC voltage in the CWOM is a group standard consisting of five semiconductor voltage reference sources Fluke 732B type. These sources based on Zener diodes are characterized by very high time stability of output voltage, up to $\pm$ 0.2 ppm/year [7–11] and very small temperature coefficient [12, 13]. The individual members of the group are calibrated once a year in the Central Office of Measures[1] in Warsaw. Semiconductor DC voltage group standard is maintained in the laboratory from 2008. Earlier the same role was played by electrochemical group standard 9152/12 by Guildline company, consisting of 12 Weston cells placed in a common thermostat. The currently used semiconductor DC voltage group standard is maintained at air-conditioned room with restricted access. The air conditioning system, working continuously, regularly maintained, enables to keep the required temperature $\theta = (23 \pm 1)°C$, relative humidity RH = $(45 \pm 15)\%$ and overpressure in a laboratory room (important to ensure low level of dust in the air). The temperature, relative humidity and barometric pressure are monitored and recorded using a suitable computer system. The history of changes in the temperature and humidity is analyzed during the measurements, and for several hours before they start, as well. The failure to comply with the requirements of environmental conditions results in the decision to suspend or cancel the already made measurements.

3.2. Differences measurements. To check the level of coherence of the DC voltage group standard kept in the CWOM, the measurements of the voltage differences between each pair of the standard were performed (Fig. 2).
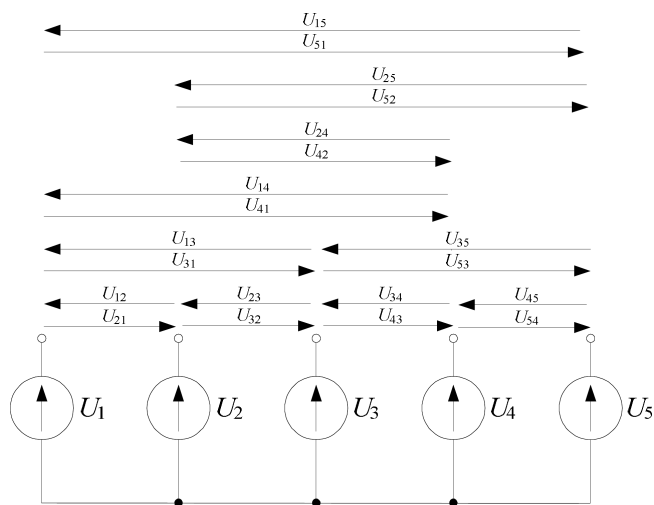

Fig. 2. Signs of the voltage differences measured between each pair of sources

For this purpose the nanovoltmeter Keithley 182, having 1 nV resolution in 3 mV range, was used. Each difference was measured twice for the two polarizations of the standards in relation to the input terminal of the nanovoltmeter. For both polarizations the voltmeter automatically measured each difference 100 times. Average value of the series was calculated and displayed by the instrument. The standard deviation, whose value is given in Tables 1 and 2, was also calculated and displayed by the nanovoltmeter.

Table 1
Voltage differences measured between each pair of the group standard 1.018 V

Numbers of standards used $i-j$	Average value $\bar{U}_{ij}$ (from 100 results) [μV]	Standard deviation σ [μV]	Numbers of standards used $i-j$	Average value $\bar{U}_{ij}$ (from 100 results) [μV]	Standard deviation σ [μV]
1–2	−71.513	0.020	2–1	71.368	0.036
1–3	−33.224	0.052	3–1	33.255	0.017
1–4	−52.444	0.010	4–1	52.506	0.059
1–5	−97.503	0.019	5–1	97.563	0.037
2–3	38.282	0.023	3–2	−38.289	0.005
2–4	19.049	0.004	4–2	−19.113	0.015
2–5	−26.002	0.033	5–2	26.078	0.007
3–4	−19.279	0.005	4–3	19.274	0.085
3–5	−64.342	0.007	5–3	64.308	0.006
4–5	−45.045	0.005	5–4	45.121	0.007

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[1]Central Office of Measures (Polish abbreviation: GUM) is Polish National Metrology Institute.

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Table 2
Voltage differences measured between each pair of the group standard 10 V

Numbers of standards used $i-j$	Average value $\overline{U}_{ij}$ (from 100 results) [μV]	Standard deviation σ [μV]	Numbers of standards used $i-j$	Average value $\overline{U}_{ij}$ (from 100 results) [μV]	Standard deviation σ [μV]
1–2	30.508	0.133	2–1	−30.755	0.212
1–3	10.567	0.125	3–1	−10.690	0.201
1–4	7.143	0.159	4–1	−6.958	0.185
1–5	−166.765	0.261	5–1	166.723	0.212
2–3	−20.331	0.187	3–2	20.310	0.165
2–4	−23.876	0.200	4–2	24.012	0.250
2–5	−197.955	0.215	5–2	198.006	0.211
3–4	−3.379	0.163	4–3	3.554	0.169
3–5	−177.586	0.227	5–3	177.551	0.170
4–5	−173.854	0.204	5–4	174.164	0.221

It should be noted that all changes in the measuring circuit in the course of measurements were made using Scanner 160A Data Proof company (Fig. 3). This kind of solution is characterized by low and constant level of thermoelectric power generated across connections, which enables to determine their impact on the measurement results.
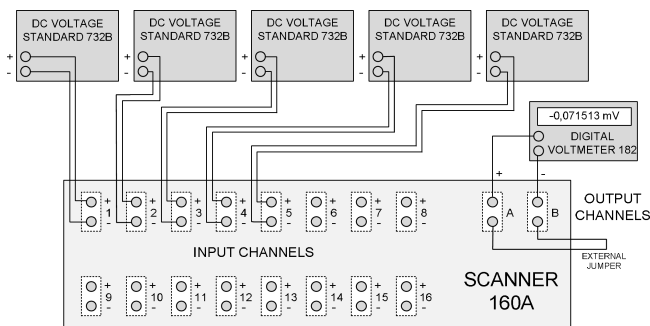

Fig. 3. Measuring circuit used for comparison of the group standard

A view of the measuring station used for coherence measurements is shown in Fig. 4. The results of checking are shown in Tables 1 and 2. Measurements were made for two groups, each group consisted of five DC voltage standards.
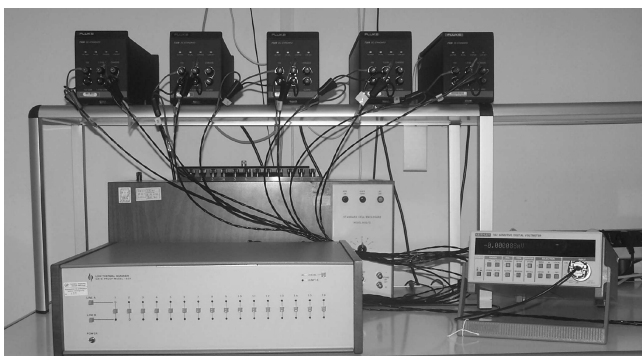

Fig. 4. View of the measuring station for comparison of voltage standards Fluke 732B

The first group contained standards of nominal voltage 1.018 V and the other – of nominal voltage 10 V. The ex-

panded uncertainty of the difference measurements, whose main components are: type A uncertainty associated with the scatter of results in the series and the type B uncertainty associated with the instrumental errors, does not exceed 0.25 μV (with coverage factor $k = 2$).

3.3. Coherence results. On the basis of difference measurements presented in Subsec. 3.2 the coherence level was assessed. In Table 3 the final results of coherence test for the groups of 1.018 V and 10 V are presented. The formulas used for calculations are placed in the first column of Table 3. It can be seen that coherence was examined by analyzing all possible variants of the closing of the figures which can be created for comparison scheme containing five standards (Fig. 1c).

The values obtained confirm a very good coherence of the difference results. The level of the coherence for the group of 1.018 V is better than 0.2 ppm, and for the group of 10 V, better than 0.1 ppm. The results confirmed competences of the CWOM in the area of DC voltage comparisons.

3.4. Reliability test. The difference measurements performed to evaluate coherence of the comparison method were also used for a reliability test. Such a test is usually performed in the CWOM once a month. The test is a method introduced by the CWOM to control its own voltage standards in reference to the values from the calibration certificates given by the Polish NMI (Central Office o Measures) [14].

Together with the coherence investigation the reliability tests provide information about accuracy of the comparison method.

In the CWOM Laboratory a group voltage standard is considered reliable if each measured voltage difference satisfies the relation (11):

$$\left| \frac{\overline{U}_{ij} - U_{ij,\text{cor}}}{U_n} \right| \le 2 \ \mu\text{V/V}, \qquad (11)$$

where nominal voltage $U_n = 1.018$ V or $U_n = 10$ V, according to the compared group of standards.

Table 3
Results of coherence test for two group standards

Formula	Coherence level for group of 1.018 V	Coherence level for group of 10 V
	[μV/V]	[μV/V]
$\left\|\overline{U}_{12} + \overline{U}_{23} + \overline{U}_{34} + \overline{U}_{45} + \overline{U}_{51}\right\|/U_n$	0.018	0.042
$\left\|\overline{U}_{12} + \overline{U}_{23} + \overline{U}_{34} + \overline{U}_{41}\right\|/U_n$	0.043	0.021
$\left\|\overline{U}_{23} + \overline{U}_{34} + \overline{U}_{45} + \overline{U}_{52}\right\|/U_n$	0.033	0.018
$\left\|\overline{U}_{12} + \overline{U}_{23} + \overline{U}_{35} + \overline{U}_{51}\right\|/U_n$	0.052	0.051
$\left\|\overline{U}_{12} + \overline{U}_{24} + \overline{U}_{45} + \overline{U}_{51}\right\|/U_n$	0.089	0.058
$\left\|\overline{U}_{13} + \overline{U}_{34} + \overline{U}_{45} + \overline{U}_{51}\right\|/U_n$	0.065	0.010
$\left\|\overline{U}_{12} + \overline{U}_{23} + \overline{U}_{31}\right\|/U_n$	0.083	0.032
$\left\|\overline{U}_{23} + \overline{U}_{34} + \overline{U}_{42}\right\|/U_n$	0.071	0.016
$\left\|\overline{U}_{34} + \overline{U}_{45} + \overline{U}_{53}\right\|/U_n$	0.034	0.009
$\left\|\overline{U}_{12} + \overline{U}_{24} + \overline{U}_{41}\right\|/U_n$	0.113	0.036
$\left\|\overline{U}_{12} + \overline{U}_{25} + \overline{U}_{51}\right\|/U_n$	0.052	0.061
$\left\|\overline{U}_{13} + \overline{U}_{35} + \overline{U}_{51}\right\|/U_n$	0.031	0.020
$\left\|\overline{U}_{14} + \overline{U}_{45} + \overline{U}_{51}\right\|/U_n$	0.025	0.021
$\left\|\overline{U}_{23} + \overline{U}_{35} + \overline{U}_{52}\right\|/U_n$	0.001	0.009
$\left\|\overline{U}_{24} + \overline{U}_{45} + \overline{U}_{52}\right\|/U_n$	0.037	0.002
$\left\|\overline{U}_{13} + \overline{U}_{34} + \overline{U}_{41}\right\|/U_n$	0.040	0.011

In the Eq. (11) minuend and subtrahend are calculated from the following equations:

$$\overline{U}_{ij} = \frac{U_{ij} - U_{ji}}{2}, \qquad (12)$$

$$U_{ij,\text{cor}} = U_i + c_i - (U_j + c_j), \qquad (13)$$

where U_{ij} – voltage difference occurring between i-th and j-th member of the group standard ($U_{ij} = U_i - U_j$), U_{ji} – voltage difference occurring between j-th and i-th member of the group standard ($U_{ji} = U_j - U_i$), $\overline{U}_{ij}$ – averaged final result of voltage difference occurring between i-th and j-th member of the group standard, $U_{ij,\text{cor}}$ – corrected value of voltage difference occurring between i-th and j-th member of the group standard; calculated from the results of the recent calibration of the group members done at the Central Office of Measures (U_i, U_j), taking into consideration corrections of time drift from the recent calibration (c_i, c_j).

In Tables 4 and 5 in the following columns there are provided: final results determined from the relation (12) based on the data from Tables 2 and 3, the corrected voltage differences determined on the basis of the actual calibration in the Central Office of Measures, and the periodic evaluation of the group standard.

For all possible pairs of the standard connections divergences between measured and corrected value less than 2 ppm were obtained, so the test has passed the periodic control.

During the periodic check, in addition to Eq. (11), measurement results of the voltage difference must also meet an additional condition:

$$\left\| U_{ij} \right| - \left| U_{ji} \right\| \le 1 \ \mu\text{V}. \qquad (14)$$

The fulfillment of this condition shows that in the measurement system there are no significant differences in the results due to thermoelectric forces after changing standards' positions. The data presented in Tables 4 and 5 show that the relation (14) is satisfied for all variants of standards' connections. The maximum difference in absolute values of voltage obtained for two different polarizations is 0.247 μV and occurred at the 1–2 configuration of the group standard of 10 V (Table 5).

Table 4
Results of periodic control of group standard 1.018 V

Numbers of standards used i–j	Averaged final result $\overline{U}_{ij}$	Corrected value $U_{ij,\text{cor}}$	Result of control $\left\|\frac{\overline{U}_{ij} - U_{ij,\text{cor}}}{U_n}\right\|$
	[μV]	[μV]	[μV/V]
1–2	−71.4405	−72.6950	1.2
1–3	−33.2395	−34.7610	1.5
1–4	−52.475	−53.0830	0.6
1–5	−97.533	−97.7600	0.2
2–3	38.2855	37.9340	0.3
2–4	19.081	19.6120	0.5
2–5	−26.04	−25.0650	1.0
3–4	−19.2765	−18.3220	0.9
3–5	−64.325	−62.9990	1.3
4–5	−45.083	−44.6770	0.4

Table 4
Results of periodic control of group standard 10 V

Numbers of standards used i–j	Averaged final result $\overline{U}_{ij}$	Corrected value $U_{ij,\text{cor}}$	Result of control $\left\|\frac{\overline{U}_{ij} - U_{ij,\text{cor}}}{U_n}\right\|$
	[μV]	[μV]	[μV/V]
1–2	30.6315	26.6109	0.4
1–3	10.6285	7.3000	0.3
1–4	7.0505	4.9029	0.2
1–5	−166.744	−162.0260	0.5
2–3	−20.3205	−19.3109	0.1
2–4	−23.944	−21.7080	0.2
2–5	−197.981	−188.6369	0.9
3–4	−3.4665	−2.3970	0.1
3–5	−177.569	−169.3260	0.8
4–5	−174.009	−166.9289	0.7

4. Conclusions

Coherence measurements are powerful method for assess the accuracy of differential calibration systems. Usefulness of the method was proved by investigation of DC voltage group standards kept in the CWOM. Very high level of coherence achieved for measurements of DC voltage differences in the Central Military Calibration Laboratory confirms the removal of the systematic errors and larger random errors in the measurement system. It increases the reliability of measurements performed for DC voltage group standard and confirm accuracy of DC voltage unit in the CWOM on the level of 10^{-6}.

The method for checking coherence of comparison results presented here in relation to the DC voltage standard is very

versatile and can be applied to any other type of standard (not only electrical unit), if differential method is used.

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