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Comparison of four state observer design algorithms for MIMO system

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A state observer is a system that models a real system in order to provide an estimate of the internal state of the system. The design techniques and comparison of four different types of state observers are presented in this paper. The considered observers include Luenberger observer, Kalman observer, unknown input observer and sliding mode observer. The application of these observers to a Multiple Input Multiple Output (MIMO) DC servo motor model and the performance of observers is assessed. In order to evaluate the effectiveness of these schemes, the simulated results on the position of DC servo motor in terms of residuals including white noise disturbance and additive faults are compared.

Key words: Luenberger observer, Kalman observer, unknown input observer, sliding mode observer

A. Introduction

A state observer is typically a computer implemented mathematical model and it provides the estimation of the internal states of the system. In most practical cases, the physical state of the system cannot be determined by direct observation. Instead, indirect effects of the internal state are observed by way of the system outputs. Luenberger Observer possesses a relatively simple design that makes it an attractive general design technique [1]. Later, the Luenberger observer was extended to form a Kalman filter [2]. Although the Kalman filter is in use for more than 30 years and has been described in many papers and books, its design is still an area of concern for many researches and studies. It could be argued that the Kalman filter is one of the good observers against a wide range of disturbances.

The problem of estimating a state of a dynamical system driven by unknown inputs has been the subject of a large number of studies in the past three decades. An observer that is capable of estimating the state of a linear system with unknown inputs can also be of tremendous use when dealing with the problem of instrument fault detection, since

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in such systems most actuator faults can be generally modeled as unknown inputs to the system [3, 4]. A new methodology for fault detection and identification subject to plant parameter uncertainties is presented in [5]. A full-order unknown input and output structure is used in order to generate residuals, which can be used to detect fault and isolate on a vertically taking-off and landing aircraft dynamic model in [7]. Designing the unknown input and output observer was reported by considering the unknown constant disturbance of parameters in chaotic systems in [8]. However, when the number of sensors and unknown inputs are equal, the observer may not exist. Hence, the unknown input observer method is not always feasible for fault detection.

A key feature in the Utkin observer [9] is the introduction of a switching function in the observer to achieve a sliding mode and also stable error dynamics. This sliding mode characteristic which is a consequence of the switching function is claimed to result in system performance which includes insensitivity to parameter variations, and complete rejection of disturbances. In this paper, a DC servo motor is considered as multiple inputs and multiple outputs (MIMO) model. The model is controllable and observable [11]. Moreover, the continuous linear system has been discretized [12] with the sampling time of 0.1 second.

The organization of the paper is as follows. In Section 2 the modeling of DC Servo Motor is given in detail. The various models of state observers are briefed in section 3 The simulation results to a DC Servo Motor system are reported in section 4. Finally the comparison among the four observers is given in the conclusion.

B. Modeling of DC servo motor

A DC motor is a second order system with multiple input and multiple outputs. The model [6] is designed according to the parameters, armature resistance, armature inductance, magnetic flux, voltage drop factor, inertia constant and viscous friction. It is studied as a linear system. The inputs are the armature voltage $U_A(t)$ and the load torque $M_L(t)$. In simulation the armature voltage is given as a step function while the load torque is given a fixed value of 0.1. The measured output signals are the armature current $I_A(t)$ and the speed of the motor $\omega(t)$. Fig. 1 shows the model of the DC motor.

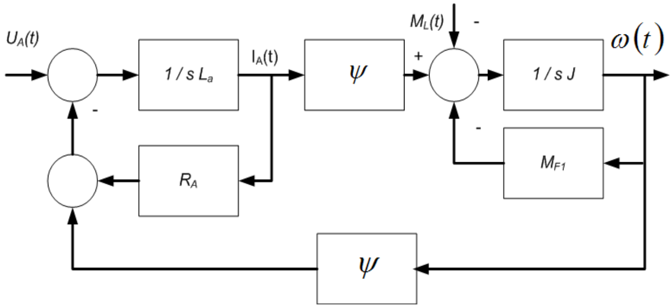

Figure 1. Signal flow diagram of the considered DC motor.

The values of parameters are as follows. Armature resistance R_a = 1.52 Ω. Armature inductance $L_a = 6.82 \cdot 10^{-3}$ Ω s. Magnetic flux $\psi = 0.33$ V s. Inertia constant $J = 0.0192$ kg m^2. Viscous friction $M_{Fl} = 0.36 \cdot 10^{-3}$ Nms.

The armature current $I_A(t)$ and armature speed $\omega(t)$ are represented as in the following

$$\begin{aligned} L_a \dot{I}_A(t) &= -R_a I_A(t) - \psi\omega(t) - U_A(t) \\ J\dot{\omega}(t) &= \psi I_A(t) - M_{Fl}\omega(t) - M_L(t) \end{aligned} \tag{1}$$

The general continuous state space form with faults or disturbance is represented as

$$\dot{x}(t) = Fx(t) + Gu(t) + L_l f_l(t) \tag{2}$$

$$y(t) = Cx(t) + Du(t) + M_m f_m(t) \tag{3}$$

where $x(t) \in R^n$ is a state vector, $u(t) \in R^m$ represents control input vector, $y(t) \in R^p$ is a measurement output vector, F, G, C and D are known constant matrices. The continuous time system (2) can be discretised using 0.1 second sampling time to obtain the discrete time model represented as follows.

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Lf_l(k) \\ y(k) &= Cx(k) + Du(k) + Mf_m(k) \end{aligned} \tag{4}$$

where $A = e^{FT}$, T is sampling time; $B = TAG$, $L = TFL_l$ and $M = M_m$.

The continuous time model of a DC motor as a state space form is thus of the form

$$\begin{bmatrix} \dot{I}_A(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} -R_a/L_a & -\psi/L_a \\ \psi/J & -M_{F1}/J \end{bmatrix} \begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 1/L_a & 0 \\ 0 & -1/J \end{bmatrix} \begin{bmatrix} U_A(t) \\ M_L(t) \end{bmatrix} \tag{5}$$

$$\begin{bmatrix} y_1(t) \\ y_2(t) \end{bmatrix} = \begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} I_A(t) \\ \omega(t) \end{bmatrix} + [0\ \ 0] \begin{bmatrix} U_A(t) \\ M_L(t) \end{bmatrix} \tag{6}$$

C. Models of observers

C.1. Luenberger observer

For continuous time linear system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) \\ y(t) &= Cx(t) \end{aligned} \tag{7}$$

where $x \in R^n$, $u \in R^m$, $y \in R^r$, a linear system observer equation is given by

$$\begin{aligned} \dot{\hat{x}}(t) &= A\hat{x}(t) + Bu(t) + L[y(t) - \hat{y}(t)] \\ \hat{y}(t) &= C\hat{x}(t). \end{aligned} \tag{8}$$

Observer error is defined as

$$e = x - \hat{x}. \tag{9}$$

Differentiating equation (9)

$$\dot{e} = \dot{x} - \dot{\hat{x}} \tag{10}$$

and substituting equations (7), (8) into (10) we get

$$\dot{e} = Ax + Bu - [A\hat{x} + Bu + L(y - \hat{y})] \tag{11}$$

or

$$\dot{e} = A(x - \hat{x}) - L(y - \hat{y})] \tag{12}$$

and with (7)

$$\dot{e} = A(x - \hat{x}) - LC(x - \hat{x})] \tag{13}$$

$$\dot{e} = (A - LC)e. \tag{14}$$

Solution of eqn. (14) is given by

$$e(t) = e^{A-LC}e(0) \tag{15}$$

The eigenvalues of the matrix $A - LC$ can be made arbitrary by appropriate choice of the observer gain L, when the pair $[A, C]$ is observable (i.e. observability condition holds). So the observer error $e \rightarrow 0$ when $t \rightarrow \infty$.

The given system is observable if and only if the $n \times n$ matrix, $[C^T \; A^T C^T \; \dots \; (A^T)^{(n-1)} C^T]$ is of rank n. This matrix is called the observability matrix.

Design of plant gain and observer gain

For plant gain, characteristic equation becomes

$$|sI - A + BK| = (s - \mu_1)(s - \mu_2)\dots(s - \mu_n) \tag{16}$$

where μ_1, μ_2, ..., μ_n, are desired closed loop poles of the plant. K is state feedback gain matrix.

For observer gain, characteristic equation becomes

$$|sI - A + BK| = (s - \mu_{1ob})(s - \mu_{2ob})\dots(s - \mu_{nob}) \tag{17}$$

where μ_{1ob}, μ_{2ob}, ..., μ_{nob}, are desired closed loop poles of the observer.

The observer gain is chosen in such a way that observer responds 5 to 10 times faster than plant response. One should consider that system which responds faster requires more energy to control and heavier actuator to control. Desired dominant pole location should be far away from the $j\omega$ axis. Fig. 2 shows the block diagram of Luenberger observer.

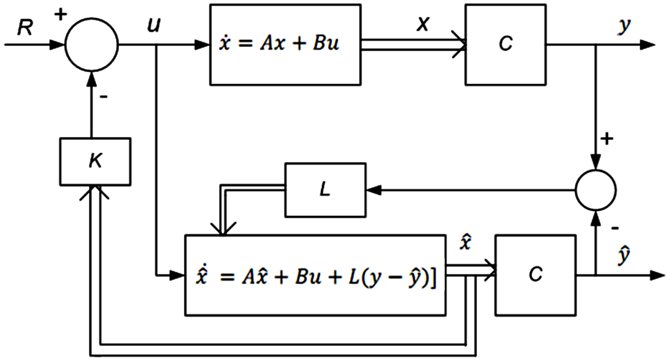

Figure 2. Block diagram of Luenberger observer.

C.2. Kalman observer

Kalman observer is a recursive predictive filter that is based on the use of state space techniques and recursive algorithms, i.e. only the estimated state from the previous time step and the current measurement are needed to compute the estimate of the current state. The Kalman filter operates by propagating the mean and covariance of the state through time. The notation $\hat{X}_{n|m}$ represents the estimate of the state vector X at time n given observations up to and including time m. The state of the filter is represented by two variables:

- $\hat{X}_{k|k}$, the *a posteriori* state estimate at time k given observations up to and including at time k,
- $P_{k|k}$, the *a posteriori* error covariance matrix (a measure of the estimated accuracy of the state estimate).

The Kalman filter has two distinct phases, prediction and correction. The prediction phase uses the state estimate from the previous time step to produce an estimate of the state at the current time step. This predicted state estimate is also known as the a priori state estimate because, although it is an estimate of the state at the current time step, it does not include observation information from the current time step. In the correction phase, the current apriori prediction is combined with current observation information to refine the state estimate. This improved estimate is termed the a posteriori state estimate. The block diagram of Kalman observer is shown in Fig. 3.

Typically, the two phases alternate, with the prediction advancing the state until the next scheduled observation, and the correction incorporating the observation. However, this is not necessary, if an observation is unavailable for some reason, the update may be skipped and multiple prediction steps are performed. Consider a linear time invariant discrete system given by the following equation

$$X_{k+1} = FX_k + Bu_k \tag{18}$$

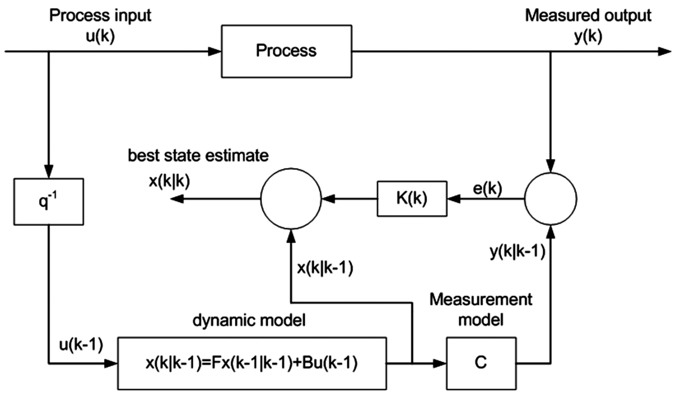

Figure 3. Block diagram of Kalman observer.

$$Z_{k+1} = HX_{k+1} + V_{k+1} \tag{19}$$

where F is the state transition matrix, B is the control input matrix, W_k is the process noise with zero mean multivariate normal distribution having covariance Q_k, H is the observation matrix, V_{k+1} is the observation noise which is zero mean Gaussian white noise having covariance? R_k, u_k is the control input.

Prediction (time update) equations

Predicted state estimate is as follows

$$\hat{X}_{k|k-1} = F\hat{X}_{k-1|k-1} + Bu_k \tag{20}$$

and predicted estimate covariance

$$P_{k|k-1} = FP_{k-1|k-1}F_k^T + Q_k. \tag{21}$$

Correction (measurement update) equations

Innovation or measurement residual is as follows

$$\widetilde{y}_k = Z_k - H\hat{X}_{k|k-1} \tag{22}$$

and innovation (or residual) covariance

$$S_k = HP_{k|k-1}H^T + R_k. \tag{23}$$

Optimal Kalman gain is then

$$K_k = P_{k|k-1}H^T S_k^{-1}. \tag{24}$$

Updated (*a posteriori*) state estimate takes a formula

$$\hat{X}_{k|k} = \hat{X}_{k|k-1} + K_k \widetilde{y}_k \tag{25}$$

and updated (*a posteriori*) estimate covariance

$$P_{k|k} = (I - K_k H) P_{k|k-1} \tag{26}$$

Unknown input observer

Consider a continuous linear time invariant steady space model of the system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t) + Ed(t) \\ y(t) &= Cx(t) \end{aligned} \tag{27}$$

where $x \in R^n$ is the state vector, u is input vector, y is sensor output, A is system coefficient matrix, B is input coefficient matrix, C is output coefficient matrix, $d \in R^q$ is the unknown input vector, and $E \in R^{n \times q}$ is the unknown input distribution matrix. The structure of the unknown input observer is described as [5]

$$\begin{aligned} \dot{z}(t) &= Fx(t) + TBu(t) + Ky(t) \\ \hat{x}(t) &= z(t) + Hy(t) \end{aligned} \tag{28}$$

where $\hat{x} \in R^n$ is the estimated state vector, $T \in R^{n \times n}$, $K \in R^{n \times n}$ and $H \in R^{n \times n}$ are matrices satisfying certain requirements.

The block diagram of unknown input observer is shown in Fig. 4.

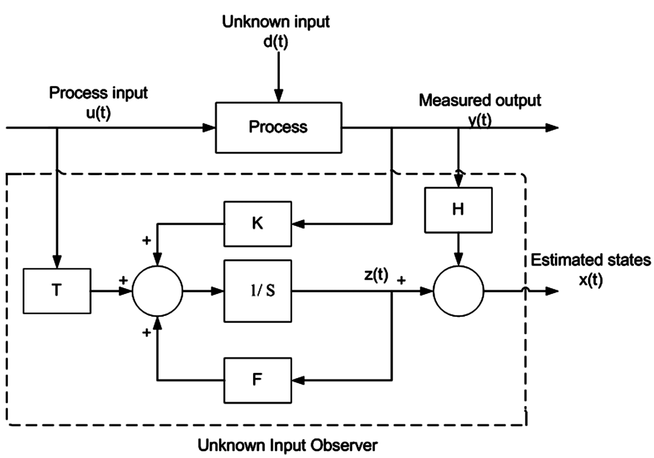

Figure 4. Block diagram of unknown input observer.

The error vector is given by

$$e(t) = x(t) - \hat{x}(t). \tag{29}$$

The state estimation error is governed by the following equation

$$\begin{aligned} e(t) &= x(t) - \hat{x}(t) = x(t) - z(t) - Hy(t) = x(t) - z(t) - HCx(t) \\ &= (I - HC)x(t) - z(t). \end{aligned} \tag{30}$$

Using eqn. (30), derivative of the vector is obtained

$$\begin{aligned} \dot{e}(t) &= (A - HCA - K_1C)e(t) + (A - HCA - K_1C)z(t) + (A - HCA - K_1C)Hy(t) \\ &+ (I - HC)Bu(t) + (I - HC)Ed(t) - Fz(t) - TBu(t) - K_2y(t) = \\ &= (A - HCA - K_1C)e(t) + [F - (A - HCA - K_1C)]z(t) \\ &- [K_2 - (A - HCA - K_1C)]Hy(t) - [K_2 - (A - HCA - K_1C)]Hy(t) \end{aligned} \tag{31}$$

The following relations hold true:

$$(HC - I)E = 0 \tag{32}$$

$$T = (I - HC) \tag{33}$$

$$F = A - HCA - K_1CK_2 \tag{34}$$

$$K_2 = FH \tag{35}$$

$$K = K_1 + K_2. \tag{36}$$

Derivative of the error vector (31) will be $\dot{e}(t) = Fe(t)$ and then the solution of the error vector is $e(t) = e^{Ft}e(0)$. If F is chosen as a Hurwitz matrix, the solution of the error equation goes to zero asymptotically. So, $\hat{x}$ converges to x.

Necessary and sufficient conditions for the observer (28) to be a unknown input observer for defined system in (27) are [7]:

(i) $\mathrm{rank}(CE) = \mathrm{rank}(E)$,

(ii) (C, A_1) is a detectable pair,

where $A_1 = A - E[(CE)^TCE]^{-1}(CE)^TCA$. A flow chart of design procedure is shown in Fig. 5.

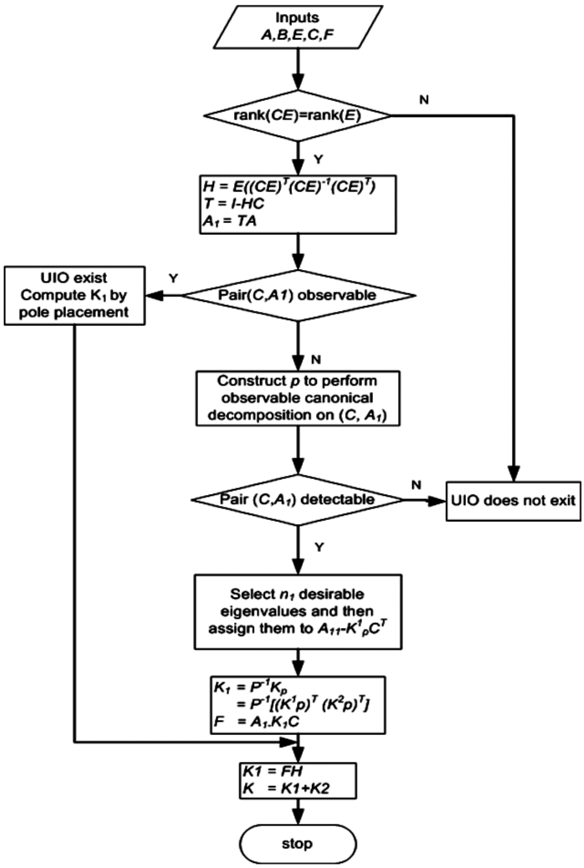

Figure 5. Flow chart of design procedure.

C.3. Sliding mode observer

A brief review of Utkin observer [9, 10] is presented here. Consider a continuous time linear system described by

$$\dot{x}(t) = Ax(t) + Bu(t) \tag{37}$$

$$y(t) = Cx(t) \tag{38}$$

where $A \in R^{n\times n}$, $B \in R^{n\times m}$ and $p \leqslant m$. Assume that the matrices B and C are of full rank and pair (A,C) is observable. As the outputs are to be considered, it is logical to effect a change of coordinates so that the outputs appear as components of the states. One possibility is to consider the transformation $x \to T_c x$, where

$$T_c = \begin{bmatrix} N_c \\ 0 \end{bmatrix} \tag{39}$$

and the columns of $N_c \in R^{n\times(n-p)}$ span the null space of C. This transformation is non-singular, and with respect to this new coordinate system, the new output distribution matrix is

$$CT_c^{-1} = [0 \quad I_p] \tag{40}$$

where p is the number of system output and n is the order of the system.

If the other system matrices are written as

$$T_cAT_c^{-1} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{41}$$

$$T_cB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \tag{42}$$

then the nominal system can be written as

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}y(t) + B_1u(t) \tag{43}$$

$$y(t) = A_{21}x_1(t) + A_{22}y(t) + B_2u(t) \tag{44}$$

where

$$T_cx = \begin{bmatrix} x_1 \\ y \end{bmatrix}. \tag{45}$$

The observer proposed by Utkin has the form

$$\dot{x}_1(t) = A_{11}x_1(t) + A_{12}y(t) + B_1u(t) + Lv \tag{46}$$

$$y(t) = A_{21}x_1(t) + A_{22}y(t) + B_2u(t) - v \tag{47}$$

where $(\hat{x}_1, \hat{y}_1)$ represent the state estimates for $(x_1, y_1), L \in R^{(n\times p)\times p}$ is a constant feedback gain matrix and the discontinuous vector v is defined component wise by

$$v_i = M\mathrm{sgn}(\hat{y}_i - y_i) \tag{48}$$

where $M \in R_+$. The errors between the estimates and the true states are $e_1 = \hat{x}_1 - x_1$ and $e_y = \hat{y} - y$.

D. Simulation results

Figures 6-9 present results of simulations of observers with additive fault. Figures 10-13 present results of simulations of observers with disturbance (white noise). Figures 14-17 present plots of simulated state variable (position).

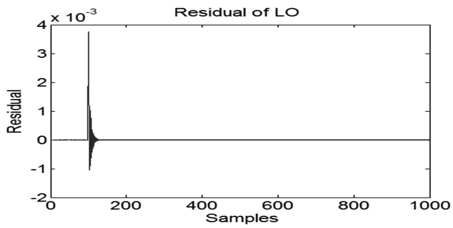

Figure 6. Residual of Luenberger observer.

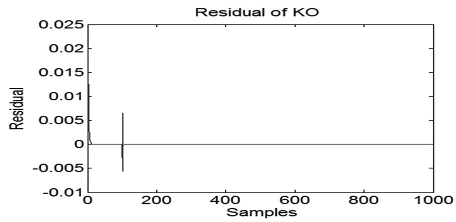

Figure 7. Residual of Kalman observer.

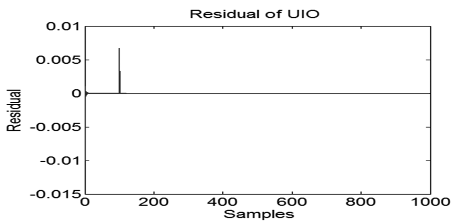

Figure 8. Residual of unknown input observer.

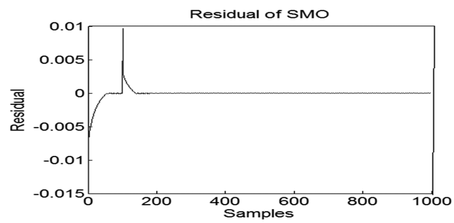

Figure 9. Residual of sliding mode observer.

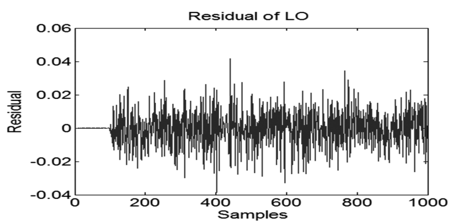

Figure 10. Residual of Luenberger observer.

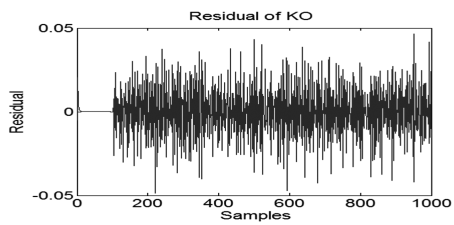

Figure 11. Residual of Kalman observer.

Comparisons of the results

Table 1 shows the comparison factors depending on the results of the simulations of the designed observers. The effectiveness of each observer is compared according to the additive faults and the disturbance. Comparing according to the additive faults, range of the residual of Luenberger observer is the least one while the residual of Kalman observer is the highest and comparing according to the disturbance, range of the residual of sliding mode observer is the least one while the residual of Luenberger observer is the highest.

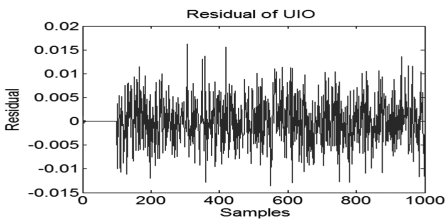

Figure 12. Residual of unknown input observer.

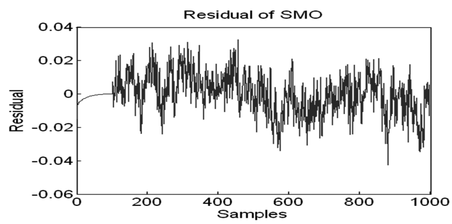

Figure 13. Residual of sliding mode observer.

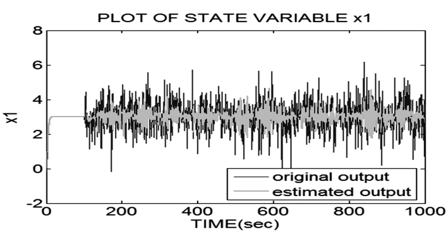

Figure 14. State variable x_1 of Luenberger observer. i

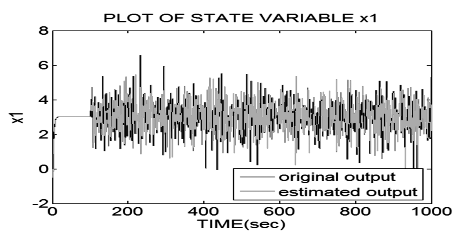

Figure 15. State variable x_1 of Kalman observer.

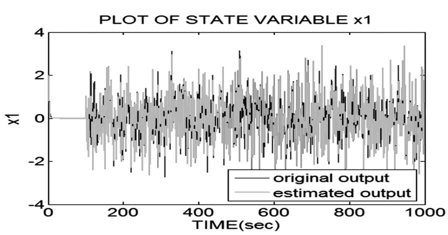

Figure 16. State variable x_1 of unknown input observer.

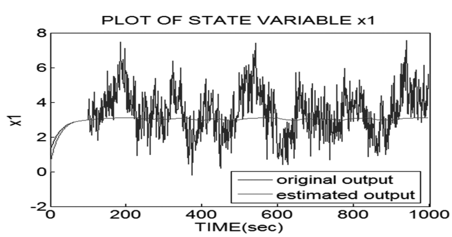

Figure 17. State variable x_1 of sliding mode observer.

Table 9. Comparison factors of observers

Observer	Design of gain matrix	Residual (with additive faults)	Residual (with disturbance)
Luenberger	Pole placement	-0.002 to 0.004	-0.04 to 0.06
Kalman	Using correction matrix	-0.01 to 0.025	-0.05 to 0.05
unknown input	Pole placement	-0.015 to 0.01	-0.015 to 0.02
sliding mode	Pole placement	-0.015 to 0.05	-0.01 to 0.01

E. Conclusion

In this paper, a comparative study on four kinds of state observers (Luenberger observer, Kalman observer, unknown input observer and sliding mode observer) for a MIMO based DC servo motor system is presented. The computational criterion chosen is the amplitude of residual for both white noise disturbance and additive faults. The performance is assessed by considering the same, assumed values of disturbance and additive faults. The simulated results show that the performance of sliding mode observer is superior in terms of the disturbance rejection compared with other observers.

References

[1] R. ISERMANN: Fault-Diagnosis Systems: An Introduction from Fault Detection to Fault Tolerance. Springer, 2006.

[2] G.B. WELCH: An Introduction To The Kalman Filter. Copyright 2001 by ACM, Inc., 2001.

[3] M.DAROUACH: On the novel approach to the design of unknown input observer. *IEEE Trans. on Automatic Control*, **39**(3) (1994), 698-699.

[4] F. YANG and R.W. WILDE: Observers for linear systems with unknown input. *IEEE Trans. on Automatic Control*, **33**(7), (1988), 677-668.

[5] E. KIYAK and O.C. AYSE KAHVECIOGLU: Aircraft sensor fault detection based on unknown input observers. *An Int. J. Aircraft Engineering and Aerospace Technology*, **80**(5), (2008), 545-548.

[6] A.H. AI-BAYATI: A comparative study of linear observers applied to a DC servo motor. *Int. Conf. on Modelling Identification and Control*, 2010.

[7] J. ZAREI and J. POSHTAN: Sensor fault detection and diagnosis of a process using unknown input observer. *Mathematical and Computational Applications*, **16**(1), (2011), 31-42.

[8] J. CHEN and R.J. PATTON: Robust model-based fault diagnosis for dynamic systems. Massachusetts: Kluwer Academic Publishers, 1999.

[9] S.DRAKUNOV and V. UTKIN: Sliding mode observers. Tutorial. *34th IEEE Conf. on Decision and Control*, **4** (1998), 3376-3378.

[10] C. EDWARDS and S.K. SPURGEON: Sliding mode control: theory and applications. **7** Systems and Control Book Series. Taylor & Francis, London, pp.128-147, 1998.

[11] B.C. KUO and M.F. GOLNARAGHI: Automatic Control Systems. Hoboken Wiley, 8th ed. 2003.

[12] B.C. KUO: Digital Control Systems. Oxford University Press, 1992.