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Optimization of thermo-electric coolers using hybrid genetic algorithm and simulated annealing

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Thermo-electric Coolers (TECs) nowadays are applied in a wide range of thermal energy systems. This is due to their superior features where no refrigerant and dynamic parts are needed. TECs generate no electrical or acoustical noise and are environmentally friendly. Over the past decades, many researches were employed to improve the efficiency of TECs by enhancing the material parameters and design parameters. The material parameters are restricted by currently available materials and module fabricating technologies. Therefore, the main objective of TECs design is to determine a set of design parameters such as leg area, leg length and the number of legs. Two elements that play an important role when considering the suitability of TECs in applications are rated of refrigeration (*ROR*) and coefficient of performance (*COP*). In this paper, the review of some previous researches will be conducted to see the diversity of optimization in the design of TECs in enhancing the performance and efficiency. After that, single-objective optimization problems (SOP) will be tested first by using Genetic Algorithm (GA) and Simulated Annealing (SA) to optimize geometry properties so that TECs will operate at near optimal conditions. Equality constraint and inequality constraint were taken into consideration.

Key words: thermo-electric coolers, genetic algorithm, simulated annealing, coefficient of performance, rate of refrigeration, fitness function

1. Introduction

1.1. Mud-pulse high-temperature measurement while drilling (MWD)

Mud-Pulse High-Temperature Measurement While Drilling (MWD) is a system developed to perform drilling related measurements down-hole and transmit information to the surface while drilling a well. MWD systems can take several measurements like a natural gamma ray, directional survey, tool face, borehole pressure, temperature, vibra-

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tion, shock and torque. Maintaining optimal payload temperatures in a typical down-hole environment of 230^{o}C requires that the MWD cooling system is capable of pumping a significant load and requires a low thermal resistance path on the heat rejection (hot) side of the TEC. The application in the extreme environment of high temperature, high pressure, mechanical shock and vibration require the use of high temperature TEC materials and assemblies. A typical High Temperature MWD tool is shown in Fig. 1.

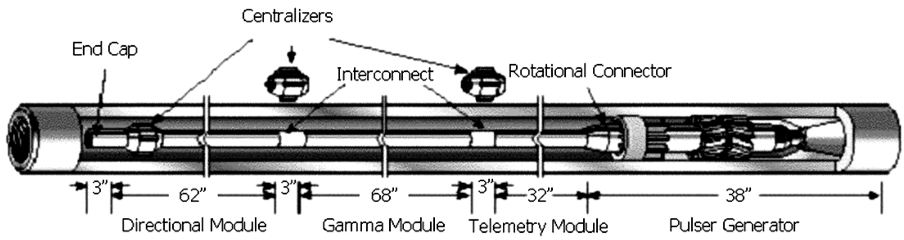


Figure 1. High-temperature MWD tool.

Cooling of electronic components inside MWD housing is crucial for maintaining optimal operating conditions in the MWD. It has been identified that this can be accomplished using thin-film thermo-electric cooling devices.

1.2. Thermoelectric coolers (TECs)

TECs are solid state cooling devices that use the Peltier effect through p-type and n-type semiconductor elements (unlike vapor-cycle based refrigerators). These types of coolers are used to convert electrical energy into a temperature gradient. Thermo-electric coolers use no refrigerant and have no dynamic parts which make these devices highly reliable and require low maintenance. These coolers generate no electrical or acoustical noise and are ecologically clean. These coolers are compact in terms of size, light weight and have high precision in temperature control. However, for this application the most attractive features of TECs that they have the capacity for cooling instruments such as MWDs under extreme physical conditions. A diagram of a standard TECs unit is shown in Fig. 2.

TEC can be a single-stage or multi-stages type. The commercially available single-stage TECs (STECs) can produce a maximum temperature difference of about 60-70K when the hot side remains at room temperature. Nevertheless, when a large temperature difference is required for some special applications, the STECs will not be qualified. To enlarge the maximum temperature difference of TEC we use two-stage TECs (TTECs) or multi-stage TECs which have STECs stacked on the top of the others. As mentioned previously, the application of TECs has been partitioned by their relatively low energy conversion efficiency and ability to dissipate only a limited amount of heat flux. Two parameters play a crucial role in characterization of TECs are rate of refrigeration (*ROR*) and coefficient of performance (*COP*). Thermo-electric coolers operate at about 5-10%

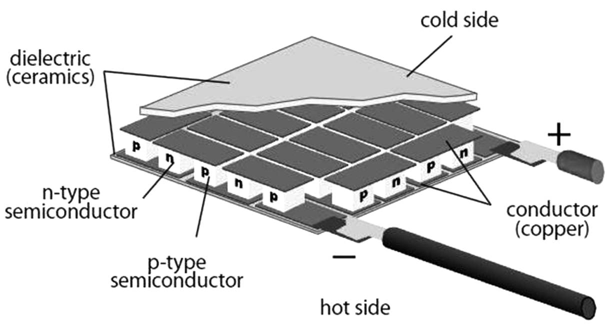


Figure 2. Single-stage TEC unit.

of Carnot cycle *COP* where as compressor based refrigerators normally operate at more than 30%.

1.3. Parametric enhancement of performance and efficiency

Several intelligent techniques that can be used for engineering design optimizations are discussed in Deb [18]. However, one of the most effective and non-traditional method used as an optimization technique for TECs is the Non-dominated Sorting Genetic Algorithm (NSGA-II) [4]. Similar sophisticated techniques in artificial intelligence, such as SA, other evolutionary algorithms (GA [35], Differential Evolution (DE) and Particle Swarm Optimization (PSO)) can be used in their pure and hybrid form to enhance the effectiveness of the optimization of TECs.

2. Problem formulation

The requirement of TECs as a cooling mechanism for instruments in extreme environment specifically in thermal energy and gas drilling operations is indeed crucial. However, setbacks such as the relatively low energy conversion efficiency and ability to dissipate only a limited amount of heat flux may seriously injure the performance and efficiency of these devices. Multi-objective intelligent techniques as done using NSGA-II can be used to enhance further the optimization of the TECs. The optimal geometrical and physical properties of the thermo-electric elements that maximize the *ROR* and the *COP* for the application of TECs in MWD instruments need to be identified.

3. Mathematical modeling of TECs

Operation of a TECs is based on the Peltier effect. When a DC current passes through a pair of p- and n-type semiconductor materials, one side of the junctions is cooled and the other side is heated. Therefore, TECs acts as a solid state cooling device that can pump heat from one junction to the other junction when a DC current is applied. The energy balance equations at the hot junction and the cold junction for TECs can be described as below. The heat flows αIT_h and αIT_c caused by the Peltier effect are absorbed at the cold junction and released from the hot junction, respectively. Joule heating $1/2I^2(\rho rL/A + 2r_c/A)$ due to the flow of electrical current through the material is generated both inside the TECs legs and at the contact surfaces between the TECs legs and the two substrates [2]. TEC is operated between temperatures T_c and T_h, so heat conduction $kA(T_h - T_c)$ occurs through the TEC legs.

$$Q_c = ROR = N\left[\alpha IT_c - \frac{1}{2}I^2(\rho_r\frac{L}{A} + \frac{2r_c}{A}) - \frac{kA(T_h - T_c)}{L}\right] \tag{1}$$

$$Q_h = N\left[aIT_h + \frac{1}{2}I^2(r_r\frac{L}{A} + \frac{2r_c}{A}) - \frac{kA(T_h - T_c)}{L}\right]. \tag{2}$$

The input electrical power and coefficient of performance can be calculated using following relations:

$$P = Q_h - Q_c \tag{3}$$

$$COP = \frac{Q_c}{Q_h - Q_c} \tag{4}$$

where α, ρ and k are the Seebeck coefficient, electrical resistivity and thermal conductivity, respectively. They represent thermo-electric material properties. A, L, N are geometric properties of TECs model. The performance of TECs (*COP* and *ROR*) strongly depends on thermo-electric materials. A good thermo-electric material should have a large Seebeck coefficient, low thermal conductivity to retain the heat at the junction and maintain a large temperature gradient and low electrical resistance to minimize the Joule heating [6]. The performance evaluation index of thermo-electric materials is the figure of merit Z or dimensionless figure of merit $ZT = \alpha^2 T/\rho k$, which combines the above properties. The increase in Z or ZT leads directly to the improvement in the cooling efficiency of Peltier modules. For TECs with a specific geometry, *ROR* and *COP* are all dependent on its operating conditions which are the temperature difference ΔT and applied current. With a fixed ΔT, *ROR* and *COP* are first increased and then decreased as I is increased [24]. Unfortunately, the optimal applied currents corresponding to $Q_{c,\max}$ and $COP_{\max}$ are not the same, which means that $Q_{c,\max}$ and $COP_{\max}$ always cannot be reached simultaneously. Similarly, with the same operating conditions, as the TECs geometry is varied, *ROR* and *COP* are all varied, but maybe cannot reach the maximums simultaneously [8].

The material properties are considered to be dependent on the average temperature of the cold side and hot side temperatures of each stage and their values can be calculated from the following equations [6]:

$$\alpha_p = -\alpha_n = (-263.38 + 2.78T_{ave} - 0.00406T_{ave}^2)10^{-6} \tag{5}$$

$$\rho_p = \rho_n = (22.39 - 0.13T_{ave} + 0.00030625T_{ave}^2)10^{-6} \tag{6}$$

$$k_n = k_p = 3.95 - 0.014T_{ave} + 0.00001875T_{ave}^2. \tag{7}$$

From the equations (1) and (2), the geometric structure has remarkable effect on the TECs. The maximum Q_c increases with the decrease of leg length until it reaches a maximum and then decrease with a further reduction in the thermo-element length [2]. The *COP* increases with an increase in thermo-element length. As the *COP* increases with the leg area, the Q_c may decrease because the total available volume is limited. As the leg area is reduced, the Q_c generally increases. A smaller leg area and a greater number of legs yield greater cooling capacity. When the leg length is below than this lower bound, the cooling capacity declines enormously [1]. Other elements, like electrical contact resistance (r_c), have affection on the performance of TECs and very small in some calculation and can be neglected.

4. Optimization issues in designing TECs

Parameters of the equation of TECs performance can be grouped into three categories which are specifications, material properties and design parameters [1]. The specification is the operating temperature T_c and T_h, the required output voltage V, current I. The specifications are usually provided by customers depending on the requirement of a particular application. The material parameters are restricted by currently materials and module fabricating technologies. Consequently, the main objective of the TECs design is to determine a set of design parameters which meet the required specifications or create the best performance at minimum cost. In two subsections below, some literature review about optimization issues of TECs is presented.

4.1. Material properties optimization issue

With the effectiveness of material properties on the performance of TECs, there has been many research were conducted during the past ten years in finding a new material and structure for use in green, highly efficient cooling and energy conversion system. Bismuth-Telluride (Bi_2Te_3) is one of the best thermo-electric materials with the highest value figure of merit [31]. Much effort has been made to raise ZT of bulk materials based on Bi_2Te_3 by doping or alloying other elements in various fabricating processes. However, ZT was not much more than one and are not sufficient to improve dramatically the cooling efficiency. The reason is due to the difficulty to increase the electrical conductivity or Seeback coefficient without increasing the thermal conductivity as mentioned by Rodgers [7].

4.2. Geometry properties optimization issue

Some research was conducted for single-objective optimization problems (SOP). Cheng [2] combined a TEC model and a genetic algorithm to optimize the geometry and performance of the STECs [2]. The geometric properties of STECs were considered as the search variables and were optimized simultaneously to reach the maximum cooling rate under the requirement of minimum *COP*, the confined volume of STECs and the restriction on the maximum cost of the material. The optimal search uses GA and converges rapidly (around 20 iterations).

Huang [8] developed an optimization approach which integrates a complete multi-physics TEC model and a simplified conjugate-gradient method [8]. Under a wide range of operating conditions of temperature difference and applied current, the geometric properties of STECs as search variables are optimized to reach the maximum TEC cooling rate. The effects of applied current and temperature difference in the optimal geometry are also discussed.

For TTECs, Cheng and Shih [3] used GA for maximizing separately the cooling capacity and *COP* [3]. The same work was done with Yi-Hsiang [4] . The authors considered the effect of thermal resistance and determined the optimum value of input current and number of legs for two different design configurations of TEC. The optimal search was done by GA and converged rapidly with over 30 runs. These results were not different with those obtained by Xuan's work [24] and showed that GA have a robust behavior and effective search ability.

Table 9. Previous techniques applied in optimizing geometric properties of TECs.

Type of OT	Type of TECs	Technique used	Author/Year
SOP	STECs	GA	Cheng/2005
SOP	STECs	Conjugate-gradient method	Huang/2005
SOP	STECs	GA	Nain/2010
SOP	TTECs	GA	Cheng/2006
MOP	STECs	NSGA-II	Nain/2010
MOP	TTECs	TLBO-II	Rao/2013

For multi-objective optimization problems (MOP), STECs will have a better design if we can find the optimal point of Q_c and *COP* simultaneously. Nain used NSGA-II for multi-objective optimization of STECs [7]. NSGA-II was able to maintain a better spread of solutions and converged better in the obtained non-dominated front compared to two other elitist MOEAs-PAES and SPEA [2]. The value of geometric properties of STECs was optimized to achieve Pareto-optimal solutions at different values of thermal resistance. The authors point out the adverse effects of thermal resistance in obtaining the optimum value of cooling rate or *COP*.

For TTECs, Rao used modified teaching-learning-based optimization (TLBO) in optimizing he dimensional structure of TTECs [10]. TLBO based on the effect of the influence of a teacher on the output learners in a class. The algorithm mimics the teaching-learning ability of teacher and learners in a classroom. Teacher and learners are the two vital components of the algorithm. TLBO was modified and applied successfully to the multi-objective optimization of TTECs and compared with GA. The determination of the number of TE module in hot stage and cold stage as well as the supply current to the hot stage and the cold stage are considered as search variables. Two different configurations of TTECs, electrically separated and electrically connected in series were investigated for the optimization.

5. Meta-heuristic optimization algorithm

In the thermal energy sector, meta-heuristics have been used recently, to solve industrial problems as well as enhance current processes, equipments and field operations. Tab. 1 lists down some applications of meta-heuristic in thermal energy system such as Genetic Algotithm, Simulated Annealing, Particle Swarm Optimization, etc.

Meta-heuristics are widely recognized as efficient approaches for many hard optimization problems. A meta-heuristic is an algorithm designed to solve approximately a wide range of hard optimization problems without having to be deeply adapted to each problem. Almost all meta-heuristics share the following characteristic: they are nature-inspired (based on some principles from physic, biology or ethology) [23]. Categories of meta-heuristic are introduced in Fig. 3. SA is a point based meta-heuristics which is normally started single initial solution and move away from it. GA are population based meta-heuristics which can deal with a set of solutions rather than with a single solution [36]. This research mainly focuses on SA and GA meta-heuristic techniques.

GA can be used to solve a constrained optimization problem and can find good local optimization [29]. GA can be effectively applied in highly nonlinear problem [17]. GA could solve a variety of optimization problems by searching a larger solution space [23]. But GA requires determination of optimum controlling parameters such as crossover rate and mutation rate. GA has the poor global search capability what happens very often when the populations have a lot of subjects. As a result, it is difficult to determine the stopping criterion of the algorithm.

SA has the ability to escape from local optima [32], flexibility and ability to approach global optimality. SA can be applied to large problem regardless of the conditions of differentiability, continuity, and convexity that is normally required in conventional optimization methods [19]. SA is easy to code, even for complex system and can deal with highly nonlinear models with many constraints. But SA has some disadvantages. Well known disadvantage of SA is the difficulty in defining a good cooling schedule (both SOP and MOP). SA is very greedy regarding computation time requirements [21].

Some research was conducted to compare GA with SA. Babak Sohrabi, in a comparison between GA and SA performance in preventive part replacement (2006) [20] showed that SA is easy to understand and also easier than GA to code. SA can easily handle a change in the objective function, but in GA we need to have fitness function, which sometimes make problems. SA can be simply stated and that lend themselves more readily to analysis. Franconi and Jennison (1997) applied GA and SA to the optimization problem arising in finding the maximum of a posteriori (MAP) [36]. They

Table 10. Recent application of intelligent strategies in the thermal energy system.

Author	**Year**	**Application**	**Technique**
Xinhua Xu and Shengwei Wang [9]	2007	Optimal thermal models of building envelope based on frequency domain	GA
Haluk Gozde and M. Cengiz Taplamacio-glu [13]	2011	Automatic generation control application in a thermal power system	PSO
C. W. Chen, J. J. Lee and H. S. Kou [11]	2008	Optimum thermal design of microchannel heat sinks	SA
P. Pezzini, O. G. Bellmunt and A. S. Andreu [14]	2011	Optimize energy efficiency	ACO, PSO, GA, ES, EP
Siddharta, N. Sharma and Varun [15]	2012	Optimization of thermal performance of a smooth flat solar air heater	PSO
J. Eynard, S. Grieu and M. Polit [12]	2011	Forecasting temperature and thermal power consumption	W, ANN
GA – genetic algorithm SA – simulated annealing ES – evolutionary strategy W – wavelets		PSO – particle swarm optimization ACO – ant colony optimization EP – evolutionary programming ANN – artificial neural networks	

found that GA is very sensitive to the choice of mutation probability and fitness function, whereas SA is quite robust with regards to the choice of cooling schedule. The result obtained using GA were disappointing, especially in the larger problems [16]. SA does not converge as fast as the GA in the initial phase [27].

6. System setting

6.1. Parameters Setting for STECs

Base on the previous research, we set up the design parameters for optimizing the design of TECs and divide them into four group as follows:

(1) Objective functions: Maximum *ROR* and/or maximum *COP*.

(2) Fixed parameters:

- Total volume in which thermo electrics elements can be placed.
- Temperature at hot and cold side of TECs.
- Supplied current to TECs.
- Material properties of TECs.

(3) Bounds of variables:

- $L_{\text{min}} \leqslant L \leqslant L_{\text{max}}$
- $A_{\text{min}} \leqslant A \leqslant A_{\text{max}}$
- $1 \leqslant N \leqslant S/A_{\text{min}}$

(4) Constraints:

- Confined volume of TECs (inequality constraint).
- Limitation of *COP* (inequality constraint).
- Defined value of *COP* (equality constraint).

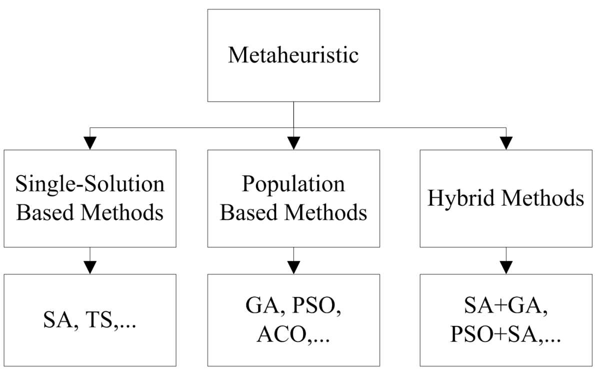


Figure 3. Categories of meta-heuristic.

6.2. Parameters setting for Genetic Algorithm

In this research, GA toolbox in MATLAB will be used. Some parameters setting inside GA can be listed down as follows and the remain parameters will be set as default values:

(1) Objective function: '@Coolingrate'. Objective function together with variables and fixed parameters of TECs will be taken into account in this function.

(2) Constraint function: '@fun_constraint'. Equality constraint and/or inequality constraint of TECs will be contained in this function.

(3) Mutation: '@mutationadaptfeasible'. This function will be used for solving problems with constraints.

(4) Generation: specifies the maximum number of iterations which the genetic algorithm performs.

(5) Population size: specifies how many individuals are contained in each generation. Typical population sizes vary between 30 and 200. The population size is usually set as a function of the chromosome length [13].

(6) Stopping criteria: 'Stallgenlimit'. In Matlab, the algorithm stops if there is no improvement in the objective function for a sequence of consecutive generations of length stall generation [28].

6.3. Parameters setting for Simulated Annealing

SA code for programming will be used to modify the parameters setting so that it can be used for finding the optimal design of TECs. Choosing the good algorithm's parameters is very important because they affect the whole optimization process.

(1) Initial temperature: T_o. The temperature is the control parameter in simulated annealing and it is decreased gradually as the algorithm proceeds. If the initial temperature of the system is too low or cooling is done insufficiently slowly the system may become quenched forming defects or freezing out in meta-stable states (get trapped in a local minimum energy state). To is obtained as a function of maximum possible deterioration of the objective function that can be accepted in a current solution.

(2) Temperature reduction: α. Temperature decrease is $T_n = \alpha T_{n-1}$. Experimentation was done with different α value: 0.70, 0.75, 0.85, 0.90 and 0.95.

(3) Boltzmann constant: k_B. k_B will be used in the Metropolis algorithm to calculate the acceptance probability of the points [20].

(4) Stopping criteria: Final stopping temperature. This value can be obtained as a function of minimum possible deterioration the objective function.

7. Simulation results and analysis

7.1. Tests of Genetic Algorithm

Preliminary tests for finding the optimal value of geometric properties of STECs will be conducted by using the GA toolbox in MATLAB. Single objective optimization

Table 11. Parameter setting of STECs.

Parameters setting	Specific values
Objective function	Maximize ROR
Variables	$0.03\text{mm} < L < 1\text{mm}$
	$0.09\text{mm}^2 < A < 100\text{mm}^2$
	$1 < N < 1000$
Fixed parameters	$S = 100\text{mm}^2$
	$T_h = T_c = 323^o\text{K}$
	$r_c = 10^{-8}\Omega\text{m}^2$
Constraint	- Inequality constraint: $AN < 100\text{mm}^2$
	- Equality constraint: $COP = 1$

Table 12. Parameters setting of GA.

Parameters setting	MATLAB
Objective function	@Coolingrate
Constraint function	@fun_constraint
Generation	200
Population size	50
Stopping criteria	'Stallgenlimit', 50

will be used under constraints. The purpose of this test is to understand the operation and benefits of GA in the optimization of TECs design. Some comments for these results will be helpful for future work. Parameters setting in TECs module are taken from from [2].

Tab. 3 lists down the parameters of STECs. One objective namely maximizing ROR is considered for single objective optimization. The objective of the optimization calculation is to determine the optimal leg length, the leg area and the number of legs. STECs were placed in a confined volume with total area 100mm^2 and a height of 1mm; therefore dimensions of TECs legs are bounded. The temperature of the cold side of the colder stage and the hot side of the hotter stage were both fixed to 323^oK. The effect of electrical resistance was considered with value $10^{-8}\Omega\text{m}^2$. Because optimization of TECs geometry may cause the reduction in the COP, the COP is used as a constraint condition during the optimization in order to guarantee that TECs with the optimal geometry has a relatively high COP.

Parameters setting of GA were listed down in Tab. 4. Using this strategy, GA was run 30 trials randomly on TECs system. The first case is for equality constraint $COP = 1$

(Fig. 4), the second case is for inequality constraint $AN < 100\text{mm}^2$ (Fig. 5) and the last case is for combining of the two cases above (Fig. 6). The best value, average value and lowest value of 30 trials will be noted down to evaluate the performance of the algorithm such as reliability.

After testing, GA converged so quickly in several seconds and satisfied the constraints in all cases. The reason for stopping the GA operation was the average change in the fitness value less than the defined value of tolerance in stopping condition criteria. As shown in Tab. 5 the obtained results of 30 trials are not exactly same. Therefore, the optimal results of GA seem to be not reliable and can easily stand by local optimum.

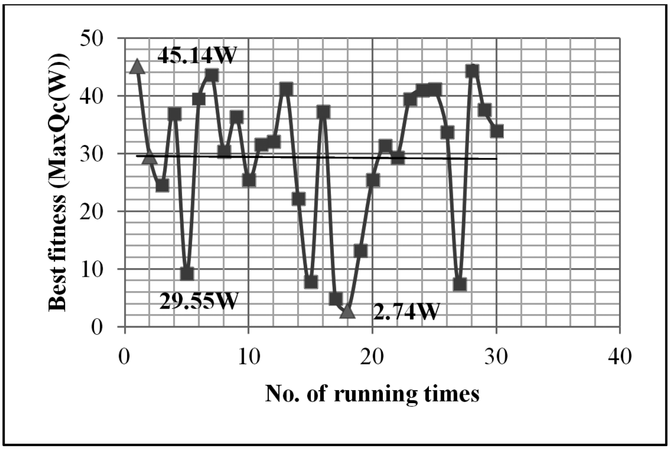


Figure 4. Run GA with equality constraint.

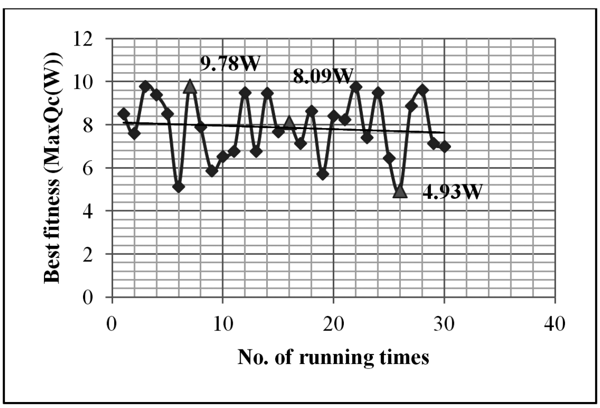


Figure 5. Run GA with inequality constraint.

In the following step, TECs system will be run with GA under various input current the constraint of total area $AN < 100\text{mm}^2$, COP requirement is neglected. Tab. 6 shows

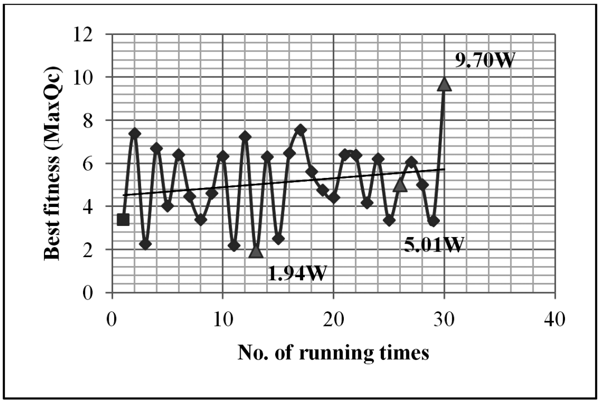


Figure 6. Run GA with inequality and equality constraints.

Table 13. Collected data after running GA.

	Case 1	**Case 2**	**Case 3**
Max. best fitness	45.14W	9.78W	9.70W
[N, A, L]	993, 0.78, 0.99	299, 0.33, 0.3	147, 0.63, 0.69
Ave. best fitness	29.55W	8.09W	5.01W
[N, A, L]	649, 0.71, 0.98	168, 0.59, 0.3	110, 0.71, 0.99
Min. best fitness	2.74W	4.93W	1.94W
[N, A, L]	60, 0.71, 1	85, 1.18, 0.30	43, 0.63, 0.69

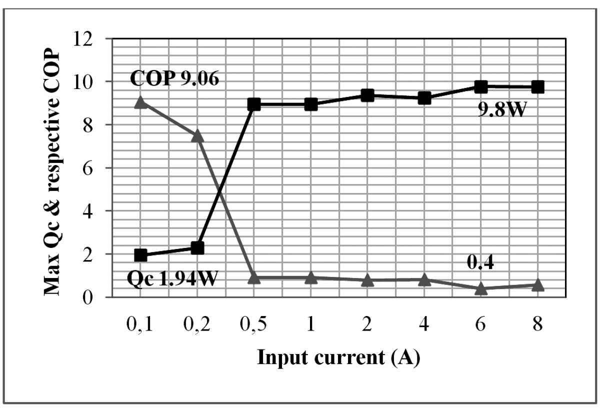


Figure 7. Performance curve of STECs with various input current.

Table 14. Optimal dimension of STECs legs for various input currents.

I (A)	N	$A(\mathrm{mm}^2)$	L (mm)	Q_c (W)	COP
0.1	300	0.3326	0.3	1.94	9.06
0.2	180	0.5570	0.3	2.29	7.51
0.5	404	0.2473	0.3001	8.93	0.92
1	202	0.4931	0.3	8.94	0.92
2	113	0.8828	0.3009	9.35	0.79
4	55	1.8242	0.3	9.24	0.81
6	53	1.8874	0.3	9.76	0.4
8	53	2.9984	0.3	9.74	0.58

the main result. The optimal result of cooling capacity seems unchanged when input current is larger than 0.5A. Tab. 6 demonstrates that TECs can reach its maximum cooling capacity even under various inputs current.

Optimal results of Q_c versus input currents in Tab. 6 were plotted in Fig. 7. Respective COP was calculated based on the equation (4). As shown in Fig. 7, COP and Q_c cannot get maximum value simultaneously.

7.2. Tests of Simulated Annealing

Parameters setting of SA are presented in Tab. 7

Table 15. Parameters setting of SA.

I **Parameters setting**	**MATLAB**
Initial temperature	100
Final temperature	10^{-10}
Temperature reduction	0.95
Boltzmann constant	1

To make sure the consistency, SA should be tested in the same conditions as GA. SA will also be run in 30 times independently for 3 cases (case 1 is nonlinear equality constraint, case 2 is nonlinear inequality constraint and case 3 is the combination of the two cases above). The comparison between the obtained results between SA and GA will be pointed out to understand the benefits and limitations.

SA often converges after 250000 evaluations and the time being consumed in every running time is last longer than GA. Fig. 8, 9 and 10 show that SA can satisfy the constraint and gives the stable results in every case of simulation. SA shows to be more robust than GA and in case 1. SA can find a better optimal design of TECs than GA. However, case 2 and case 3 of simulation show the same results which are a very small value of maximum Q_c when compare to previous research [2] and is not reliable. Code programming for TECs cannot find a good results for nonlinear equality constraint and need to be analyzed more deeply in the further work.

8. Discussion about future work

Today, researchers are combining one meta-heuristic with another meta-heuristic for optimization. Hybrid methods include the integration of various meta-heuristic algorithms which are efficient in finding good solutions that cannot be obtained by any complete method within a feasible time [22]. In the previous papers for optimization the geometric properties of TEC, NSGA-II was a technique that was used for MOP [18] and until now we don't have any update about this issue. Therefore, in the future work hybrid technique which combines GA with SA will be investigated and can be applied to achieve the Pareto optimal point of *ROR* and *COP*. Li (2005) proposed hybrid adaptive GA which is based on the population diversity of GA and the directional research of SA for multi-objective optimization of external beam radiation [25]. The crossover rate and mutation rate of GA are adaptively selected, and niched tournament selection based Pareto dominance is used. The results of test calculation with two objectives and five variables demonstrate high convergence speed. Lei (2008) presented a multi-objective hybrid genetic algorithm (MHGA) to solve the power-performance problem of chip multi-processor (CMP) system [26]. MHGA used Multi-objective Optimization GA with SA to enhance the search ability. The results show that this technique can find the optimal value of two objectives which increasing the efficiency of task scheduling on CMP and decreasing the execution time and energy consumption of the system. Sun Hui (2010) proposed Adaptive Simulated Annealing Genetic Algorithm (ASAGA) to find design parameters for maximum fuel economy [27]. Different objectives were investigated to find the optimal results, such as performance, energy regenerative ability, fuel economy, etc. The simulation results showed that ASAGA is effective and takes advantage of the GA as well as the SA to substantially improving the performance and fuel economy of hydraulic hybrid vehicle (HHV). Nasser Shahsavari (2013) introduced hybrid genetic algorithm and simulated annealing in solving multi-objective Flexible Job Shop scheduling problem (FJSP) [30]. Pareto approach is used in novel hybrid genetic algorithm and simulated annealing (NHGASA) for solving FJSP and compared with other methods such as GA+HA, PSO+SA and so on. The experiment results show that this technique gave the higher quality with less computational time.

9. Conclusion

In this research, mathematical modeling of single-stage thermoelectric coolers was investigated and genetic algorithm and simulated annealing were tested to solve single objective optimization problems with some constraints which are mainly nonlinear constraints. The preliminary results show that genetic algorithm is not stable and easily get stuck at local optima. Simulated annealing shows more robust than genetic algorithm when using the same condition of thermo-electric coolers module, but for nonlinear equality constraint cases, the obtained results are not reliable. Therefore, genetic algorithm and simulated annealing need to be researched deeper to clarify more advantages.

In the future work, hybrid genetic algorithm with another meta-heuristic technique such as simulated annealing [28,34] can be investigated and multi-objective optimization problems need to be studied to find the optimal value of rate of refrigeration (*ROR*) and coefficient of performance (*COP*) simultaneously in not only single-stage thermoelectric coolers but also two-stages or multi-stages thermo-electric coolers. The consequences of the design onto control aspects (control algorithms, difficulty, reachability, or so) will be considered in the future research work.

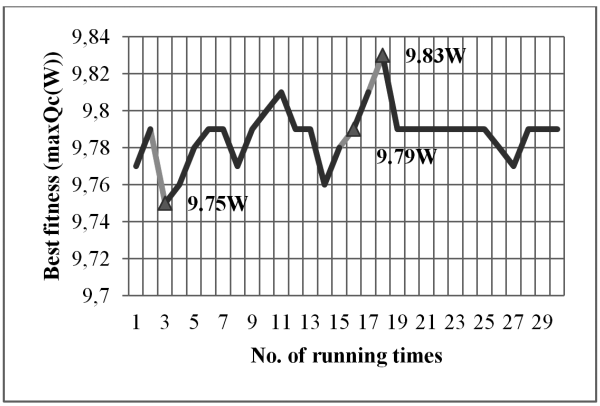


Figure 8. Run SA with inequality constraint.

Appendix A. Nomenclature

A	–	Cross-sectional area of the TECs leg (mm^2)
L	–	Height of the confined volume (mm)
N	–	Number of legs
S	–	Total volume in which TECs can be placed (mm^2)
Q_c	–	Cooling rate (W)

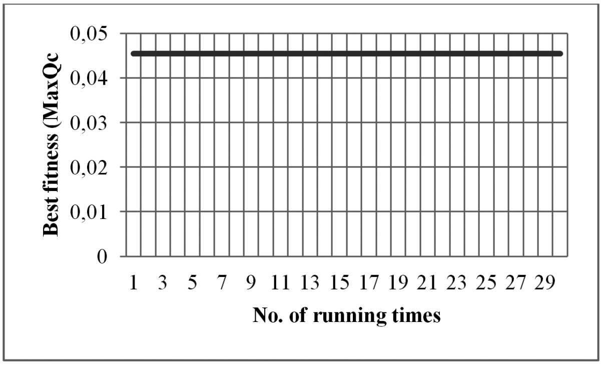


Figure 9. Run SA with equality constraint.

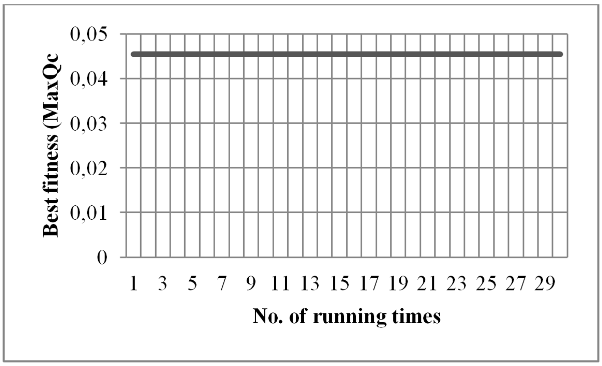
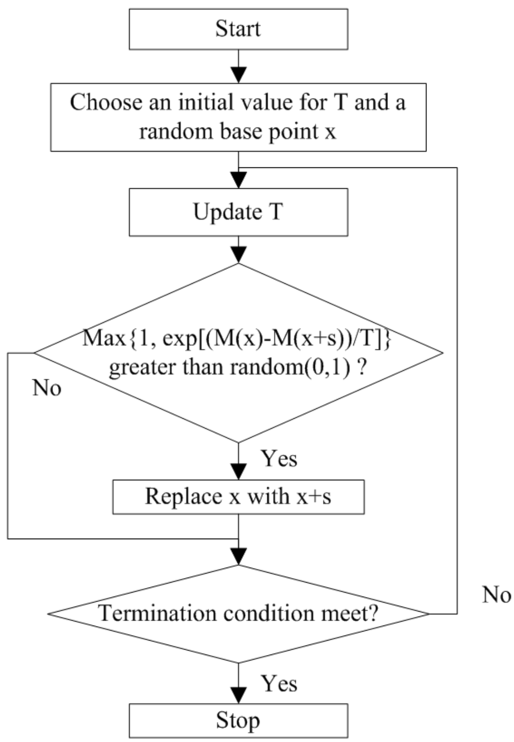


Figure 10. Run SA inequality and equality constraints.

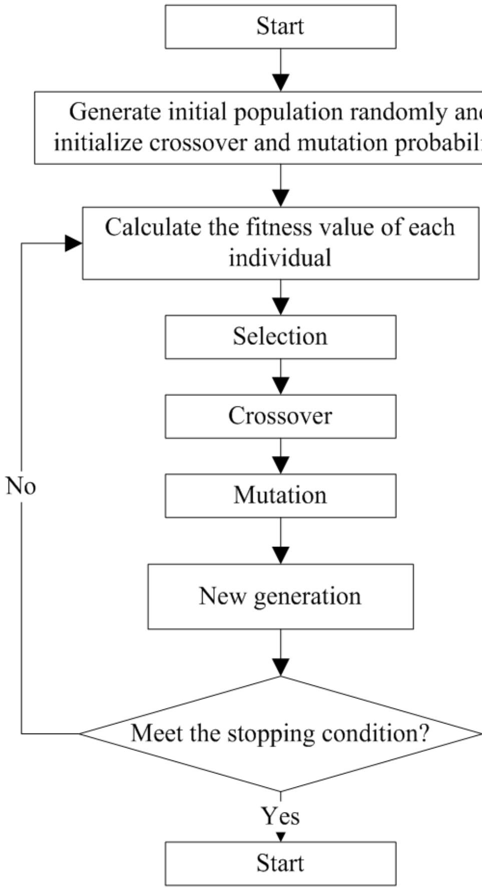
ROR	–	Rate of Refrigeration (W)
Q_h	–	Heat rejected from the hot junction, (W)
COP	–	Coefficient of Performance
I	–	Input current of TEC in Ampere, (A)
T_c	–	Temperature of hot junction (K)
T_h	–	Temperature of cold junction (K)
T_{ave}	–	Average temperature of hot side and cold side (K)
Z	–	Figure of merit
ZT	–	Dimensionless figure of merit
SOP	–	Single-objective Optimization Problem
MOP	–	Multi-objective Optimization Problem
GA	–	Genetic Algorithm
SA	–	Simulated Annealing
PSO	–	Particle Swarm Optimization
ACO	–	Ant Colony Optimization

DE	–	Differential Evolution
TS	–	Tabu Search
NSGA-II	–	Non-dominated Sorting Genetic Algorithm
TLBO	–	Teaching-learning Based Optimization
α	–	Seebeck coefficient, (V/K)
ρ_k	–	Electrical resistivity, (Ωm)
k	–	Thermal conductivity, (W/mK)
r_c	–	Electrical contact resistance, (Ωm^2)

Appendix B. Simulated Annealing flow chart



Appendix C. Genetic Algorithm flow chart



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