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Gain scheduled controller design for thermo-optical plant

VOJTECH VESELÝ, JAKUB OSUSKÝ and IVAN SEKAJ

This paper presents a gain scheduled controller design for MIMO and SISO systems in the frequency domain using the genetic algorithms approach. The proposed method is derived from the M-delta structure of closed loop MIMO (SISO) systems and the small gain theory is exploited to obtain the stability condition. An example of real system illustrates the effectiveness of the proposed output feedback gain scheduled controller design method and also the possibility to improve its performance using the genetic algorithm.

Key words: gain scheduled control, small gain theorem, frequency domain, genetic algorithm, control performance

1. Introduction

Gain scheduling is a very popular approach to nonlinear control design and has been widely and successfully applied to power systems, aerospace, etc. The gain scheduling design typically employs such an approach that a nonlinear design task is decomposed into a number of linear sub-tasks. These results fall into two main subclasses [9]. First, stability results establishing a relationship between the stability of the nonlinear system and an associated family of linear systems. Second, approximation results which establish a direct relationship between the solution of the nonlinear system and a family of associated linear systems. Gain scheduling is based on a linear parameter varying plant model. Many researchers have therefore tackled the design problem of gain scheduled controllers for linear parameter varying systems using LMI (Linear Matrix Inequality) and the Lyapunov function approach [1, 2, 3, 6, 8, 12, 13, 15, 18, 19, 20, 21, 22]. For the gain scheduling controller design the following models are used [9]:

- Jacobian linearization approach and transform of the results to a linear parameter varying (LPV) system.

The Authors are with Faculty of Electrical Engineering and Information Technology, Slovak University of Technology in Bratislava, Slovak Republic. E-mails: {vojtech.vesely; jakub.osusky, ivan.sekaj}@stuba.sk.

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- Velocity based linearization approach and transform of the results to LPV systems.
- Linear parameter varying systems [15].
- Linear fractional transformation [2].

For all of the above models with a gain scheduled controller mainly the Lyapunov function and LMI are used to guarantee the closed-loop stability of the family of linear parameter varying systems [9, 14]. A gain scheduled controller design in the frequency domain can be found rarely. The frequency domain is better understood in the control engineering community because it gives invaluable insight into simple frequency dependent plots and important concepts for feedback can be defined such as the bandwidth and peaks of closed-loop transfer functions, and so on. Obviously, in the time domain, the gain scheduled controller design procedure is obtained in the form of a bilinear matrix inequality, which does not allow controller design for high order plants. This problem does not occur in the frequency domain. The main motivation for our research is to design a gain scheduled controller in the frequency domain.

In this paper in the frequency domain a new gain scheduled controller design procedure for MIMO and SISO linear parameter-varying systems in combination with genetic algorithms is proposed. Genetic algorithm (GA) is a powerful search/optimization approach [5, 7, 11]. It can be used in the controller design area for various purposes. A survey of evolutionary-based control system designs can be found in [10] or [16]. Note that also other types of evolutionary algorithms can be used instead of the GA [5]. In the presented approach the GA procedure combines the analytical way, which is based on stability conditions formulated in the frequency domain with the simulation-based way, where a selected integral performance index is minimized.

The paper is organized as follows: Problem statement and preliminaries are in Section 2, Section 3 contains the main results, control design on a real example is in Section 4, and conclusions are summarized in Section 5.

2. Problem statement and preliminaries

The linear parameter varying system is a linear time-varying plant in which the system matrices are fixed functions of some vector of varying parameters $\theta(t)$ [14].

LPV systems have at least two interpretations:

- they can be viewed as linear time-invariant plants subject to time-varying vector parameter $\theta(t)$,
- they can be models of linear time-varying plants or result from linearization of nonlinear plants trajectories of the parameter $\theta(t)$.

Consider a MIMO (SISO) system described by the square matrix $\overline{G}(s) \in \Re^{m\times m}$ with m outputs and inputs, and a controller $\overline{R}(s) \in \Re^{m\times m}$ in the form:

$$\overline{G}(s) = G_0(s) + \sum_{i=1}^{p} G_i(s)\theta_i \qquad (1)$$

$$\overline{R}(s) = R_0(s) + \sum_{i=1}^{p} R_i(s)\theta_i \qquad (2)$$

where

$\theta_i \in \Omega,\ \ i = 1,2,...,p$ is a known and frozen (constant) parameter;
$\{G_j(s), R_j(s)\},\ \ j = 1,2,...,p$ are known plant and controller transfer function matrices with constant entries, for their calculation see example (27).

We assume that in time domain $\theta_i(t),\ \ i = 1,2,...,p$ is known and lower and upper bounds are available. Specially, each parameter $\theta_i(t)$ ranges between known extreme values $\underline{\theta}_i$ and $\overline{\theta}_i$, that is $\theta_i(t) \in \Omega$, where

$$\Omega = \{\theta \in \Re^p\ :\ \underline{\theta}_i < \theta_i(t) < \overline{\theta}_i,\ \ i = 1,2,...,p\}. \qquad (3)$$

For the rate of $\theta_i(t)$ change we have $\dot{\theta}_i(t) \in \langle \underline{\dot{\theta}}_i,\ \overline{\dot{\theta}}_i \rangle$. In the frequency domain the situation is different. From the stability point of view, for the closed-loop system for all $\theta_i \in \Omega,\ \ i = 1,2,...,p$ when $\underline{\theta}_i = -\overline{\theta}_i$ (symmetric case) it is sufficient to know θ_i in extreme point (15) thus equation (1) and (2) are written for frozen θ in point $\theta_i = \overline{\theta}_i,\ \ i = 1,2,...,p$.

Note:

1. Simulation of the closed-loop system with (1) and (2) can be made only for frozen θ.

2. For $\theta_i(t) \in \Omega$ the simulation of closed loop system needs to be done only in the time domain.

3. For the next development only frozen θ will be taken account.

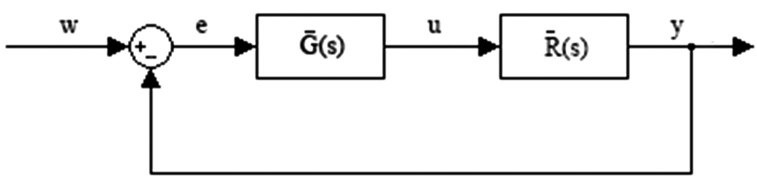

Figure 1. Standard feedback configuration.

The necessary and sufficient closed-loop stability condition for the system in Fig. 1 and frozen θ is formulated in the Generalized Nyquist Stability Theorem based upon the concept of the system return difference.

$$F(s) = I + \overline{G}(s)\overline{R}(s). \tag{4}$$

Theorem 8 *The feedback system in Fig.* 1 *is stable if and only if*

$$\det(F(s)) \neq 0 \tag{5}$$

$$N[0, \det(F(s))] = n_0 \tag{6}$$

where n_0 *is the number of unstable poles of the open loop system* $\overline{G}(s)\overline{R}(s)$ *and* $N[0, \det(F(s))]$ *denotes the number of anti clock-wise encirclements of the origin by the Nyquist plot of* $\det(F(s))$.

In this paper we tackle the following problem. For a given plant described by transfer function matrix (1) design a gain scheduled controller (2) which stabilizes the closed-loop system (Fig. 1) for all $\theta_i \in \Omega$, $i = 1, 2, \ldots, p$. The gain scheduled controller may be structured, thus one can use a centralized, decentralized, PID or other controller.

3. Main results

3.1. Gain scheduling stability condition

Consider substitution of transfer function matrices (1) and (2) into Fig. 1. The obtained structure is shown in Fig. 2 with the following notation:

$$\begin{aligned} R(s) &= [R_1 \ldots R_p], \\ R_i(s) &= \operatorname{diag}\{R_{ik}\}_{m\times m}, \quad i = 1, 2, \ldots p; k = 1, \ldots, m \end{aligned} \tag{7}$$

$$\begin{aligned} \theta^T &= [\theta_{d1}, \ldots, \theta_{dp}] \\ \theta_{di} &= \operatorname{diag}\{\theta_i\}_{m\times m}, \quad i = 1, 2, \ldots, p \end{aligned} \tag{8}$$

$$G(s) = [G_1(s) \ldots G_p(s)] \in \Re^{m\times mp}, G_i(s) \in \Re^{m\times m}, \quad i = 1, 2, \ldots, p \tag{9}$$

$$y \in \Re^m, \quad y_i \in \Re^{mp}, \quad u_i \in \Re^m, \quad i = 1, 2. \tag{10}$$

A particularity of the frequency domain gain scheduling problem is that frozen parameters enter both the plant and the controller. To come round this problem with the small gain theory we must first gather all parameters dependent components into a single block. The gain scheduled feedback configuration (Fig. 2) can be rearranged into $M - \theta$ structure in the following way.

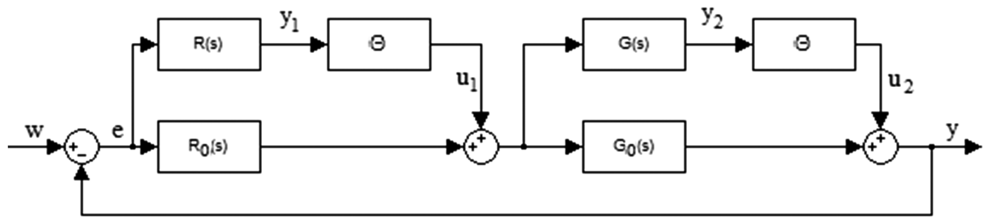

Figure 2. Gain scheduling configuration, closed-loop.

For $w(s) = 0$ and output $y(s)$ one obtains for

$$\begin{aligned} &y(s) = u_2(s) + G_0(s)\,(u_1(s) - R_0(s)y(s)) \Rightarrow y(s) = \\ &= (I + G_0(s)R_0(s))^{-1}\,(u_2(s) + G_0(s)u_1(s))\,. \end{aligned} \tag{11}$$

Let us define the output vector $y_g{}^T = [y_1{}^T, \; y_2{}^T]$ and input vector $u_g{}^T = [u_1{}^T, \; u_2{}^T]$, Fig. 2. After small manipulations one obtains the following matrix form.

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} -R^T(I + G_0R_0)^{-1}G_0 & -R^T(I + G_0R_0)^{-1} \\ G^T(I - R_0(I + G_0R_0)^{-1}G_0) & -G^T R_0(I + G_0R_0)^{-1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix}. \tag{12}$$

Let us denote

$$M_g(s) = \begin{bmatrix} -R^T(I + G_0R_0)^{-1}G_0 & -R^T(I + G_0R_0)^{-1} \\ G^T(I - R_0(I + G_0R_0)^{-1}G_0) & -G^T R_0(I + G_0R_0)^{-1} \end{bmatrix} \tag{13}$$

$$\theta_g = \begin{bmatrix} \theta^T & 0 \\ 0 & \theta^T \end{bmatrix} \in \Re^{2mp \times 2mp} \tag{14}$$

then we obtain an $M - \theta_g$ structure in the form shown in Fig. 3.

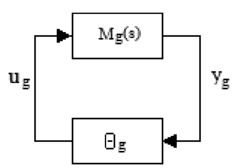

Figure 3. $M_g - \theta_g$ structure of gain scheduling closed-loop system.

By inspection of (Fig. 3) we see that the original problem can be viewed as a classical problem, where necessary and sufficient stability condition is provided by the small gain theory. Next we introduce the following small gain theorem [23].

Theorem 9 *Suppose $M_g(s) \in RH_\infty$ and let $\gamma > 0$. Then the interconnected system shown in Fig. 3 is well-posed and internally stable for all $\theta_g \in RH_\infty$ with*

$$\begin{aligned} ||\theta_g|| \leqslant 1/\gamma \ \textit{if and only if} \ ||M_g(s)||_\infty < \gamma \\ ||\theta_g|| < 1/\gamma \ \textit{if and only if} \ ||M_g(s)||_\infty \leqslant \gamma. \end{aligned} \tag{15}$$

Remark 4 RH_∞ is a closed subspace with functions that are analytic and bounded in the open right half-plane. For a stable real rational transfer function matrix $G(s)$, the H_∞ norm is defined in the usual way: $||G(s)||_\infty = \sup\limits_{\omega \in R} \sigma_M(G(j\omega))$, where $\sigma_M(\cdot)$ stands for the largest singular value of matrix $G(s)$.

Remark 5 It can be shown that the small gain stability condition is sufficient to guarantee internal stability even if $\theta_i \in \Omega,\ i = 1, 2, ..., p$ is a nonlinear and time varying 'stable' operator with an appropriately defined stability notion [4].

Remark 6 Due to note 2, stability of the closed-loop system (Fig. 3) is guaranteed even for the case when θ is nonlinear and time varying stable operator [23].

Remark 7 Let us recall that $\theta_i,\ i = 1, 2, ..., p$ plays the role of a known scheduling variable and gives the rule for controller and plant model updating in simulation or practical realization of closed-loop systems with a gain scheduled controller.

From theorem 2 the following corollary follows.

Corollary 2 *The gain scheduled closed-loop system in Fig. 2 or Fig. 3 is stable if and only if*

$$\begin{aligned} &\text{a) matrix } M_g(s) \text{ is stable,} \\ &\text{b) } \sigma_M(M_g(s)) < \frac{1}{\max\limits_{\theta \in \Omega} \sigma_M(\theta_g)} \end{aligned} \tag{16}$$

where the maximum is taken from all 2^p polytopic vertices of the gain scheduled system substituting for $\theta_i,\ i = 1, 2, ..., p$ its minimal and maximal values. If (16) *holds, the closed-loop system (Fig. 3) is internally stable for all $\theta_i \in \Omega$.*

Rewrite (13) in the following form:

$$M_g(s) = M_{rg}(s) M_{pg}(s) \tag{17}$$

where

$$M_{rg}(s) = \begin{bmatrix} R^T & 0 \\ 0 & I \end{bmatrix} \tag{18}$$

$$M_{pg}(s)=\begin{bmatrix} -(I+G_0R_0)^{-1}G_0 & -(I+G_0R_0)^{-1} \\ G^T\left(I-R_0(I+G_0R_0)^{-1}G_0\right) & -G^TR_0(I+G_0R_0)^{-1}\end{bmatrix} \tag{19}$$

then for the left side of (16) it holds:

$$\sigma_M\left(M_g(s)\right)=\sigma_M\left(M_{rg}(s)\right)\sigma_M\left(M_{pg}(s)\right). \tag{20}$$

After some manipulations for $\sigma_M(M_{rg}(s))$ one obtains

$$\sigma_M(M_{rg}(s))=\max\left(1,\left(\sum_{i=1}^{p}R_{i1}^2(s)\right)^{\frac{1}{2}}\ldots\left(\sum_{i=1}^{p}R_{im}^2(s)\right)^{\frac{1}{2}}\right). \tag{21}$$

By inspection of (19), (21) we see that the structure and parameters of controller $R_0(s)$, (Fig. 2) manipulate with $\sigma_M\left(M_{pg}(s)\right)$ and other parts of the gain scheduled controller $R_1(s)\ldots R_p(s)$ affect the value of $\sigma_M\left(M_{pg}(s)\right)$. When $\sigma_M\left(\theta_g\right)$ is known, that is

$$\max_{\theta\in\Omega}\sigma_M\left(\theta_g\right)\leqslant\frac{1}{\gamma}. \tag{22}$$

Equations (16), (20) and (22) imply that condition of corollary 1 holds if

$$\sigma_M\left(M_{pg}(s)\right)<\gamma_2 \tag{23}$$

$$\sigma_M\left(M_{rg}(s)\right)<\gamma_1 \tag{24}$$

provided that $\gamma=\gamma_1\gamma_2$.

Condition a) of corollary 1 holds if the Nyquist plot of (25)

$$\det\left(I+G_0(s)R_0(s)\right) \tag{25}$$

is stable and matrices $G_i(s), R_i(s),\quad i=1,2,\ldots,p$ are stable as well. Using the Nyquist stability conditions theorem, the necessary and sufficient conditions of (25) can be determined as follows:

Corollary 3 [17] *System* (25) *will be internally stable and well-posed if and only if*

$$N[0,\det\left(I+G_0(s)R_0(s)\right)]=n_q \tag{26}$$

where n_q is the number of unstable poles of $R_0(s)G_0(s)$.

Inequalities (22-24) give one of the ways how one can design the gain scheduled controller. In the next section, another interesting way of designing a gain scheduled controller will be presented on an example.

3.2. Gain scheduling controller parameters searching by genetic algorithm

The essential part of the optimized problem is the cost function definition. It represents the kernel of the solved problem and it is a performance measure of each individual of the population, which has to be maximized or minimized. Cost function may have the form of a function, which is simple to evaluate, but it also may contain complex procedures of modeling, simulation and performance measure evaluation.

In gain scheduling controller design procedure the cost function evaluation consists of the following steps:

1. Calculation of $F_1 = \max\left(\sigma_M\left(M_g(s)\right)\right)$.

2. Penalty calculation according to:

$$\gamma = \frac{1}{\max\limits_{\theta\in\Omega} \sigma_M(\theta_g)}$$
$$\mu = \gamma - F_1$$
$$\text{if } \mu < 0 \quad penalty = 10\left(1+\mu^2\right), \quad \text{else} \, penalty = 0.$$

3. Simulation of the closed-loop with actual controller parameters and the controlled system.

4. Evaluation of the performance index in the form of the 'Absolute control error'

$$F_2 = \int (|e|)\, dt \tag{27}$$

where e is the of control error obtained by simulation.

5. Stability evaluation of the following polynomial (25) $\det(I + G_0R_0)$:

$$\text{if } \det(I+G_0R_0) \text{is stable polynomial} \quad - \quad q = 0$$
$$\text{if } \det(I+G_0R_0) \text{is unstable polynomial} \quad - \quad q = 1000.$$

6. Cost function evaluation in the form

$$\begin{aligned} Cost &= F_1 + penalty + \alpha F_2 + q \\ \alpha &= 0.001 \end{aligned} \tag{28}$$

That means that the cost function, which is to be minimized using the genetic algorithm considers concurrently three independent aspects:

a) the value of $\sigma_M\left(M_g(s)\right)$ – which represents a stability measure in the frequency domain,

b) an integral form of the performance index – which represents the simulation-based control performance measure and

c) stability of polynomial $\det(I + G_0R_0)$ – which represents nominal closed-loop system.

Note: alternatively to performance index (27) other indices can be used. The evolutionary algorithm based controller design is described in more detail in [16].

4. Control design, example

The gain scheduled approach was applied to a real thermo-optical plant. The aim of the control design was to control the light intensity in the whole range, using the bulb voltage [0-5V] as a manipulated input. As a measured disturbance, the led voltage was used for increasing the light intensity. At first, two static characteristics were measured for the whole bulb voltage range and led voltages 0 and 2V, Fig. 4.

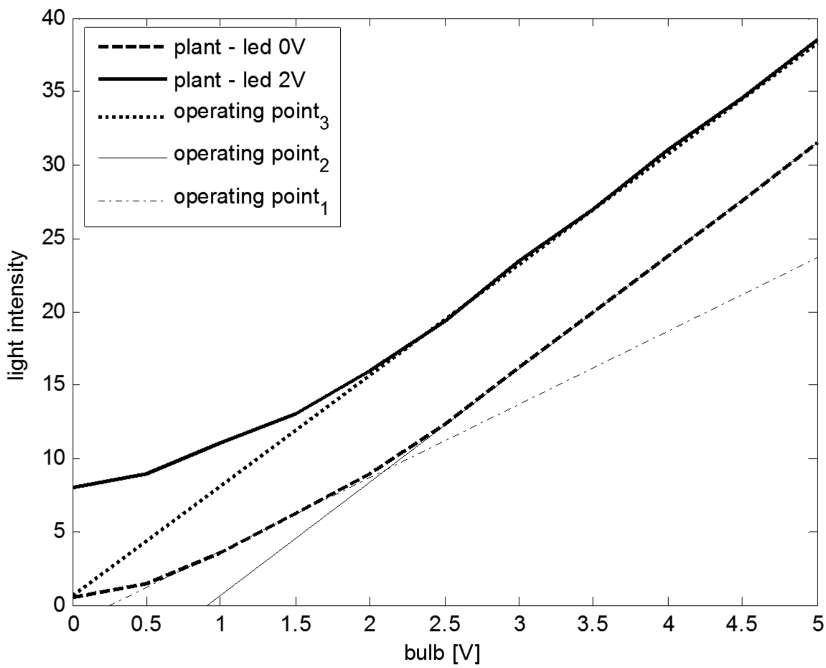

Figure 4. Static characteristics of plant and identified linear models.

4.1. Gain scheduling control – standard design

According to static characteristics the system was identified in three operating points defined by the led and bulb voltages [L; B] and described by linear transfer functions.

Operating point 1: $[0; 1.3]$ V

$$\overline{G}_1(s) = \frac{-0.03156s + 1}{0.1307s^2 + 4.288s + 0.196}$$

Operating point 2: $[0; 4]$ V

$$\overline{G}_2(s) = \frac{-0.03608s + 1}{0.04315s^2 + 2.621s + 0.1286}$$

Operating point 3: $[2; 3.5]$ V

$$\overline{G}_3(s) = \frac{-0.03203s + 1}{0.04036s^2 + 2.537s + 0.1311} \tag{29}$$

According to the number of operating points, two θ parameters (θ_1, θ_2) were defined with working range $\theta_{1,2} \in \langle \underline{\theta}, \overline{\theta} \rangle$, in this case $\theta_{1,2} \in \langle -1, 1 \rangle$.

Bulb voltage u in interval $u \in \langle 1.7; 2.3 \rangle$ is recalculated to $\theta_1 \in \langle -1, 1 \rangle$. Out of this interval θ_1 reaches marginal values.

Led voltage d in interval $d \in \langle 0; 2 \rangle$ is recalculated to $\theta_2 \in \langle -1, 1 \rangle$. Out of this interval θ_2 reaches marginal values.

Transfer functions $G_0(s)$ and $G_i(s)$, $i = 1, 2$ were calculated from identified transfer functions $\overline{G}_1(s)$, $\overline{G}_2(s)$, $\overline{G}_3(s)$ according to (1).

$$G_0(s) = \frac{-0.3722\mathrm{s}^2 + 0.3919\mathrm{s} + 646.1}{\mathrm{s}^3 + 95.62\mathrm{s}^2 + 2063\mathrm{s} + 106.4}$$

$$G_1(s) = \frac{-0.2973\mathrm{s}^2 + 1.391\mathrm{s} + 147.5}{\mathrm{s}^3 + 93.51\mathrm{s}^2 + 1993\mathrm{s} + 102.8}$$

$$G_2(s) = \frac{0.167\mathrm{s}^2 + 0.361\mathrm{s} - -0.0497}{\mathrm{s}^3 + 60.79\mathrm{s}^2 + 5.92\mathrm{s} + 0.144}. \tag{30}$$

For each identified transfer function $\overline{G}_k(s)$, $k = 1, 2, 3$ a local PI controller $\overline{R}_k(s)$, $k = 1, 2, 3$ ensuring a phase margin $PM = 60^\circ$ was designed. The aim of the local controllers design was to keep overshoot less than 20%. Parts of gain scheduled controller $R_0(s)$ and $R_i(s)$, $i = 1, 2$ were calculated in the same way as $G_0(s)$ and $G_i(s)$, $i = 1, 2$.

$$R_0(s) = \frac{1.668\mathrm{s} + 0.685}{s}, \quad R_1(s) = \frac{0.1445\mathrm{s} + 0.227}{s}, \quad R_2(s) = \frac{0.0125\mathrm{s} + 0.0425}{s}. \tag{31}$$

Condition a) of corollary 1 holds if G_0, G_1, G_2 are stable and $\det(I+G_0R_0)$ is a stable polynomial. Controllers R_0, R_1, R_2 according to PI structure are considered as stable. The following roots proof the stability of the mentioned transfer functions:

$$\Lambda(G_0(s)) = \{-62.79; -32.77; -0.052\}$$
$$\Lambda(G_1(s)) = \{-60.71; -32.75; -0.052\}$$
$$\Lambda(G_2(s)) = \{-60.69; -0.052; -0.046\}$$
$$\Lambda(\det(I+G_0(s)R_0(s))) = \{-62.07; -32.35; -0.29 \pm 0.37j\}. \tag{32}$$

Condition b) (equation (16)) of corollary 1 is depicted in Fig. 5. The right side of inequality γ is a scalar number equal to $\gamma = 1/\sqrt{2}$ according to symmetric maximum and minimal values of θ_1, θ_2.

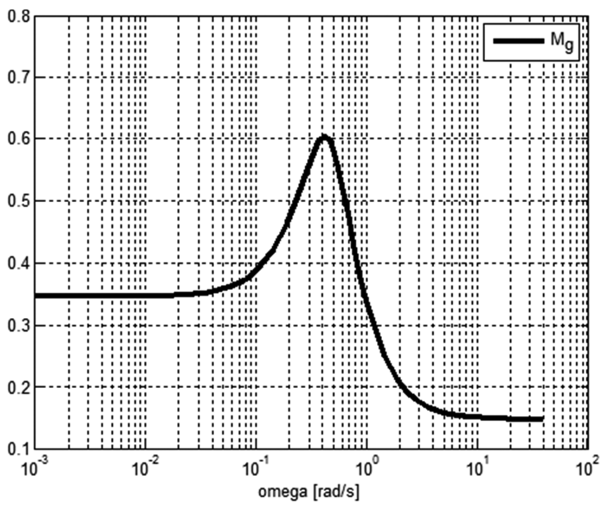

Figure 5. Stability condition of system with gain scheduled controller ($M_g < 0.707$).

For identified transfer functions (29) one robust PI controller was designed for comparison with the gain scheduled control. The parameters of the robust PI controller are following:

$$R_{robust}(s) = \frac{1.754s + 0.765}{s}. \tag{33}$$

The robust PI controller and gain scheduled controller were applied to the real system in the whole operating range, Fig. 6. Graphical results from Fig. 6 were quantified using IAE criterion (34) and compared in Tab. 1. ($T_s = 0.1$ is the sample time)

$$IAE = T_s \sum_{i=1}^{6200} (y_i(s) - w_i(s))^2 \tag{34}$$

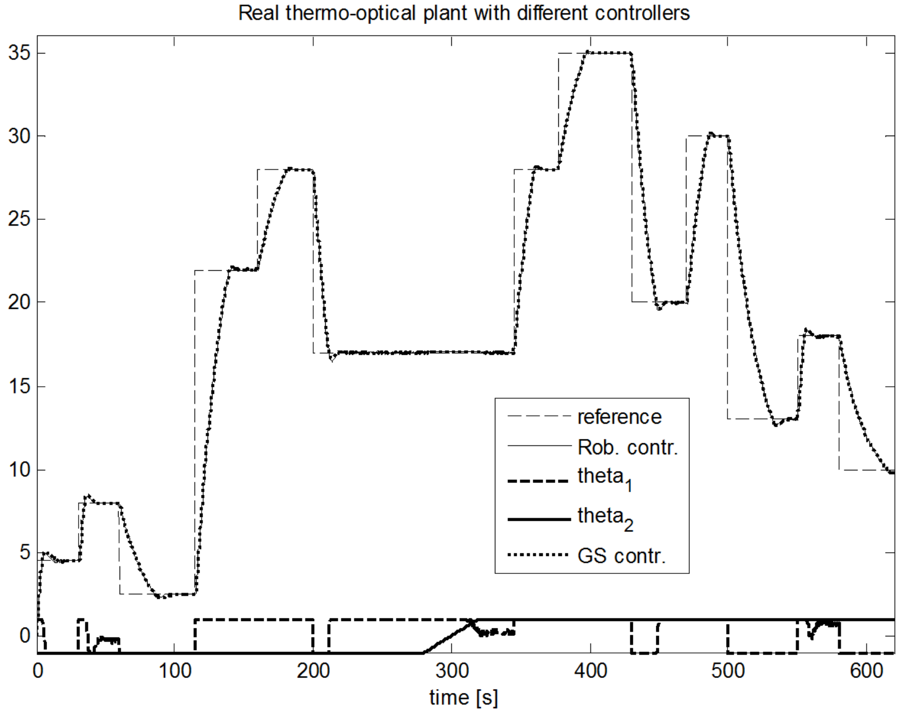

Figure 6. Gain scheduling and robust control comparison in experiment on thermo-optical plant.

Table 6. Numerical comparison of gain scheduling and robust control

	IAE criterion
Gain scheduling control	1068.11
Robust control	1068.43

4.2. Gain scheduling control – genetic algorithm

The GA searches for the six parameters of the gain scheduling controller (two parameters of each controller R_0, R_1, R_2). A part of cost function (28) calculating absolute control error (27) due to possibility measure process values only in sample time, was modified as follows:

$$F_2 = T_s\left(\sum_{t=0}^{N}|e(t)|\right) \tag{35}$$

where N is number of samples in real experiment, what in this example is 6200.

The GA searches directly for the parameters of the gain scheduling controller so the recalculation as in the previous controller design can be omitted. Cost function behavior is depicted with stability condition (16) in graphical form in Fig. 7

$$R_{0\_gen}(s) = \frac{2.689s + 1.313}{s},$$
$$R_{1\_gen}(s) = \frac{- - 0.19s - -0.118}{s}, \quad R_{2\_gen}(s) = \frac{0.532s + 0.245}{s}. \tag{36}$$

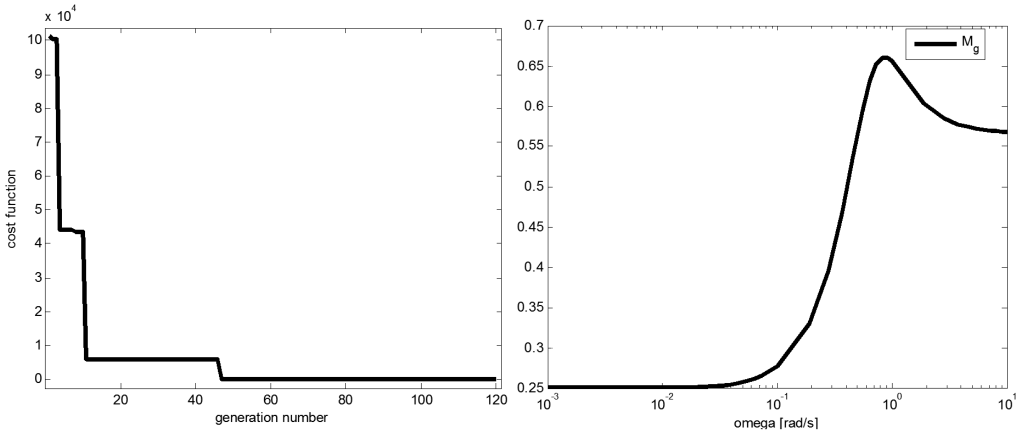

Figure 7. Cost function behavior by optimization process (left) and stability condition (right).

Stability was verified also according to (25) roots of $\det(I + G_0R_0)^{-1}$ has following values:

$$\Lambda(\det(I + G_0(s)R_0(s))^{-1}) = \{-42.35 \pm 13.71j; -0.45 \pm 0.47j\}. \tag{37}$$

The gain scheduled controller, with parameters designed using the genetic algorithm, was verified on a real plant and the measured values were compared with a previously designed gain scheduling controller. Graphical results are depicted in Fig. 8, (more detail in Fig. 9) and numerical results are written in Tab. 2.

Experimental results confirmed that closed-loop system is stable even for the case when $\theta_i \in \Omega$ is a function of time.

Table 7. Numerical comparison of gain scheduling and gain scheduling control – genetic

	IAE criterion
Gain scheduling control	1068.11
Gain scheduling control - genetic	1056.87

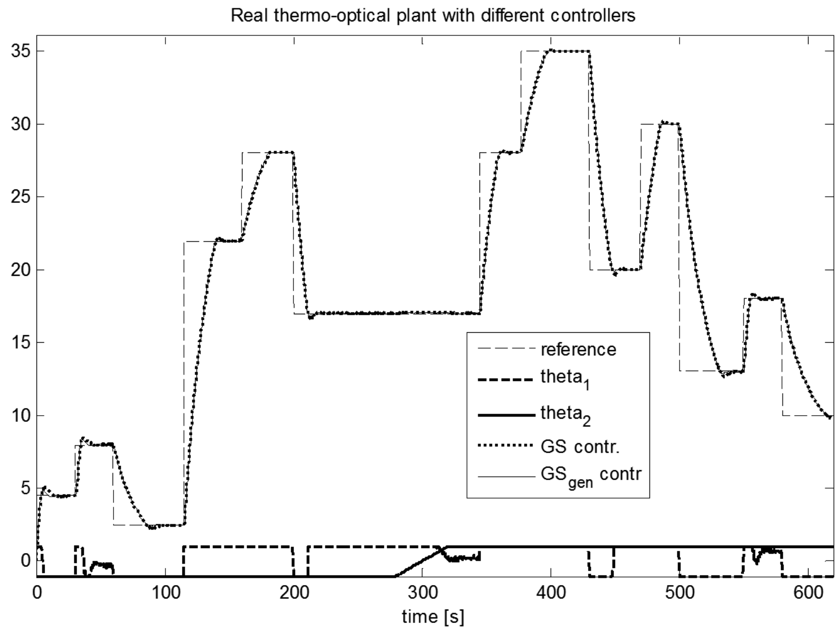

Figure 8. Comparison of gain scheduling and gain scheduling control designed by genetic algorithm on thermo-optical plant.

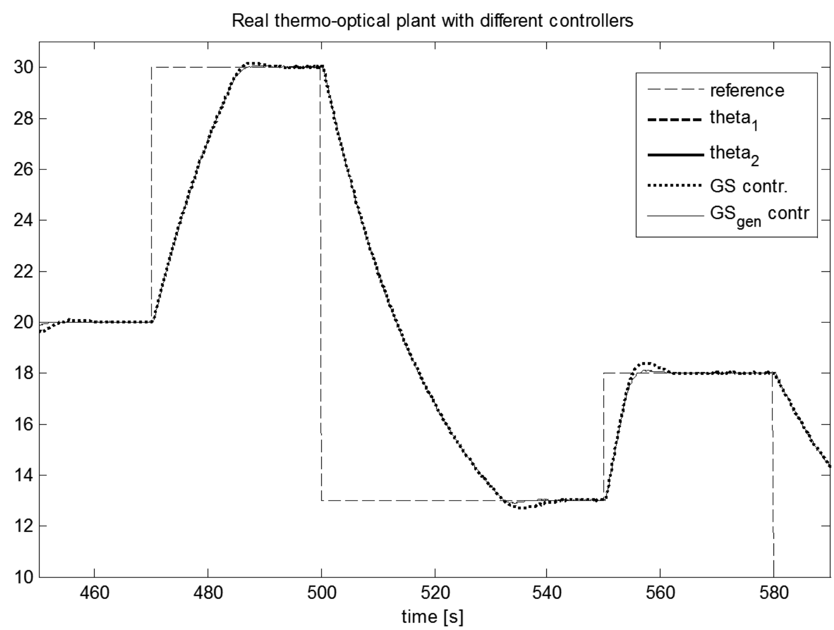

Figure 9. Comparison of gain scheduling and gain scheduling control designed by genetic algorithm on thermo-optical plant - detail.

5. Conclusions

In this paper a new gain scheduled controller design procedure for MIMO and SISO linear parameter-varying systems is proposed. The proposed design procedure is based on the $M-\Delta$ structure and the small gain theory. The possibility to improve the performance using genetic algorithm was included. The controller design procedure was demonstrated in detail on a real example and compared with a classical robust control. The comparison with the classical control approach shows that the gain scheduling control structure gives a better performance also for slightly nonlinear systems. Using the genetic algorithm the parameters of the gain scheduled controller were changed and the performance was improved. The proposed design procedure in the frequency domain brings new results for the stability conditions of gain scheduled closed-loop systems.

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