

A novel 3-D jerk chaotic system with three quadratic nonlinearities and its adaptive control

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This paper announces an eight-term novel 3-D jerk chaotic system with three quadratic nonlinearities. The phase portraits of the novel jerk chaotic system are displayed and the qualitative properties of the jerk system are described. The novel jerk chaotic system has two equilibrium points, which are saddle-foci and unstable. The Lyapunov exponents of the novel jerk chaotic system are obtained as $L_1 = 0.20572, L_2 = 0$ and $L_3 = -1.20824$. Since the sum of the Lyapunov exponents of the jerk chaotic system is negative, we conclude that the chaotic system is dissipative. The Kaplan-Yorke dimension of the novel jerk chaotic system is derived as $D_{KY} = 2.17026$. Next, an adaptive controller is designed via backstepping control method to globally stabilize the novel jerk chaotic system with unknown parameters. Moreover, an adaptive controller is also designed via backstepping control method to achieve global chaos synchronization of the identical jerk chaotic systems with unknown parameters. The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict feedback systems. MATLAB simulations have been depicted to illustrate the phase portraits of the novel jerk chaotic system and also the adaptive backstepping control results.

Key words: chaos, chaotic system, dissipative chaotic system, adaptive control, backstepping control, synchronization.

1. Introduction

Chaos theory describes the qualitative study of unstable aperiodic behavior in deterministic nonlinear dynamical systems. A dynamical system is called *chaotic* if it satisfies the three properties: boundedness, infinite recurrence and sensitive dependence on initial conditions [1].

A significant development in chaos theory occurred when Lorenz discovered a 3-D chaotic system of a weather model [2]. Subsequently, Rössler discovered a 3-D chaotic system in 1976 [3], which is algebraically simpler than the Lorenz system. Indeed, Lorenz's system is a seven-term chaotic system with two quadratic nonlinearities, while Rössler's system is a seven-term chaotic system with just one quadratic nonlinearity.

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Some well-known paradigms of 3-D chaotic systems are Arneodo system [4], Sprott systems [5], Chen system [6], Hénon-Heiles system [7], Lü-Chen system [8], Liu system [9], Cai system [10], T-system [11], etc. Many new chaotic systems have been also discovered like Li system [12], Sundarapandian systems [13, 14], Vaidyanathan systems [15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30], Pehlivan system [31], Tacha system [32], Jafari system [33], Sampath system [34], Pham systems [35, 36, 37, 38], etc.

Chaos theory has applications in several fields of science and engineering such as oscillators [39, 40, 41, 42, 43, 44, 45, 46], dynamos [47, 48, 49, 50], Tokamak systems [51, 52], chemical reactions [53, 54, 55, 56, 57, 58, 59, 60, 61, 62], neural networks [63, 64, 65, 66, 67, 68], neurology [69, 70, 71, 72, 73, 74], biology [75, 76, 77, 78, 79, 80, 81, 82, 83], electrical circuits [84, 85, 86], cryptosystems [87, 88], memristors [89, 90, 91], random bit generator [92], etc.

In this paper, we announce an eight-term novel 3-D jerk chaotic system with three quadratic nonlinearities. The phase portraits of the novel jerk chaotic system are displayed and the mathematical properties are discussed. The novel jerk chaotic system has two equilibrium points, which are saddle-foci and unstable.

The Lyapunov exponents of the novel jerk chaotic system are obtained as $L_1 = 0.20572$, $L_2 = 0$ and $L_3 = -1.20824$. Since the sum of the Lyapunov exponents of the jerk chaotic system is negative, we conclude that the chaotic system is dissipative. The Kaplan-Yorke dimension of the novel jerk chaotic system is derived as $D_{KY} = 2.17026$.

Next, using backstepping control method, we derive an adaptive control law that stabilizes the novel conservative chaotic system, when the system parameters are unknown. Using backstepping control method, we also derive an adaptive control law that achieves global chaos synchronization of the identical novel conservative systems with unknown parameters. The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict feedback systems.

Synchronization of chaotic systems is a phenomenon that may occur when a chaotic oscillator drives another chaotic oscillator. Because of the butterfly effect which causes the exponential divergence of the trajectories of two identical chaotic systems started with nearly the same initial conditions, synchronizing two chaotic systems is seemingly a very challenging problem.

In most of the synchronization approaches, the *master-slave* or *drive-response* formalism is used. If a particular chaotic system is called the *master* or *drive* system and another chaotic system is called the *slave* or *response* system, then the idea of synchronization is to use the output of the master system to control the response of the slave system so that the slave system tracks the output of the master system asymptotically.

In the chaos literature, an impressive variety of techniques have been proposed for chaos synchronization such as active control method [93, 94, 95, 96, 97], adaptive control method [98, 99, 100, 101, 102, 103, 104, 105, 106, 107], backstepping control method [108, 109, 110, 111, 112, 113], sliding mode control method [114, 115, 116, 117, 118, 119], etc.

All the main adaptive backstepping control results in this paper are proved using Lyapunov stability theory [120]. MATLAB simulations are depicted to illustrate the phase portraits of the novel jerk chaotic system, adaptive stabilization and synchronization results for the novel 3-D jerk chaotic system.

2. A 3-D novel jerk chaotic system

In this section, we describe an eight-term novel 3-D jerk chaotic system with three quadratic nonlinearities, which is described by the dynamics

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ \dot{x}_3 &= ax_1 - bx_2 - x_3 + cx_1x_2 - p(x_1^2 + x_2^2) \end{aligned} \quad (1)$$

where x_1, x_2, x_3 are the states and a, b, c, p are constant, positive, parameters of the system.

The system (1) exhibits a *strange chaotic attractor* for the values

$$a = 7.5, \quad b = 4, \quad c = 0.03, \quad p = 0.9 \quad (2)$$

For numerical simulations, we take the initial conditions of the state $x(t)$ as

$$x_1(0) = 1.8, \quad x_2(0) = 1.3, \quad x_3 = 1.6 \quad (3)$$

Figure 1 shows the 3-D phase portrait of the strange chaotic attractor of the system (1). Figures 2–4 show the 2-D projection of the strange chaotic attractor of the system (1) on (x_1, x_2) , (x_2, x_3) and (x_1, x_3) planes, respectively.

3. Analysis of the 3-D novel jerk chaotic system

3.1. Dissipativity

In vector notation, the new jerk system (1) can be expressed as

$$\dot{\mathbf{x}} = f(\mathbf{x}) = \begin{bmatrix} f_1(x_1, x_2, x_3) \\ f_2(x_1, x_2, x_3) \\ f_3(x_1, x_2, x_3) \end{bmatrix}, \quad (4)$$

where

$$\begin{cases} f_1(x_1, x_2, x_3) = x_2 \\ f_2(x_1, x_2, x_3) = x_3 \\ f_3(x_1, x_2, x_3) = ax_1 - bx_2 - x_3 + cx_1x_2 - p(x_1^2 + x_2^2) \end{cases} \quad (5)$$

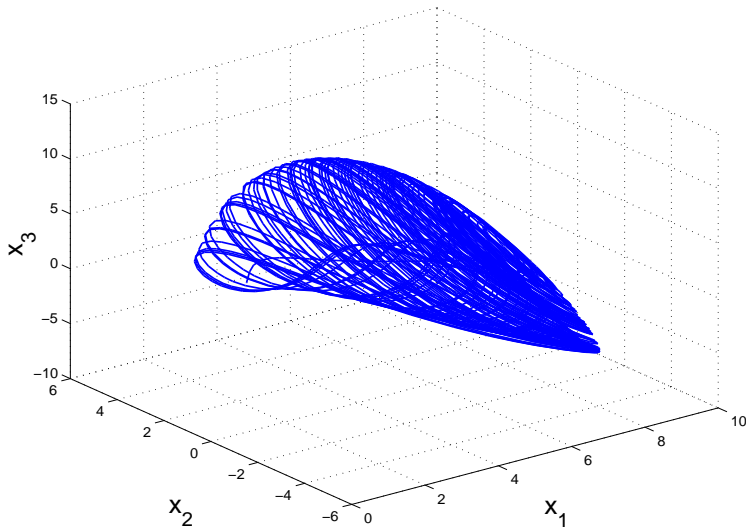


Figure 1. Strange attractor of the 3-D novel jerk chaotic System

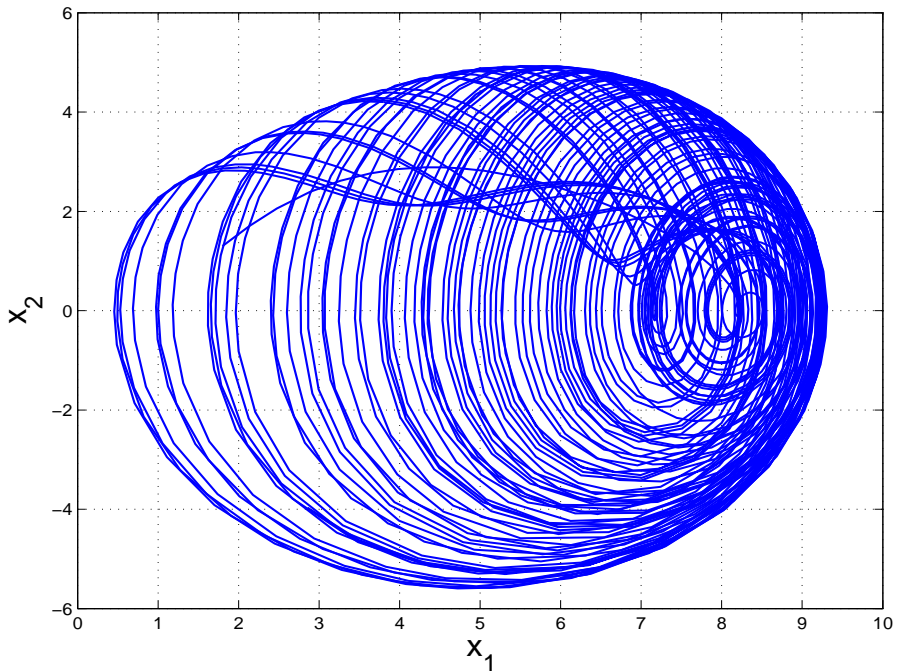


Figure 2. 2-D projection of the novel jerk chaotic system on the (x_1, x_2) plane

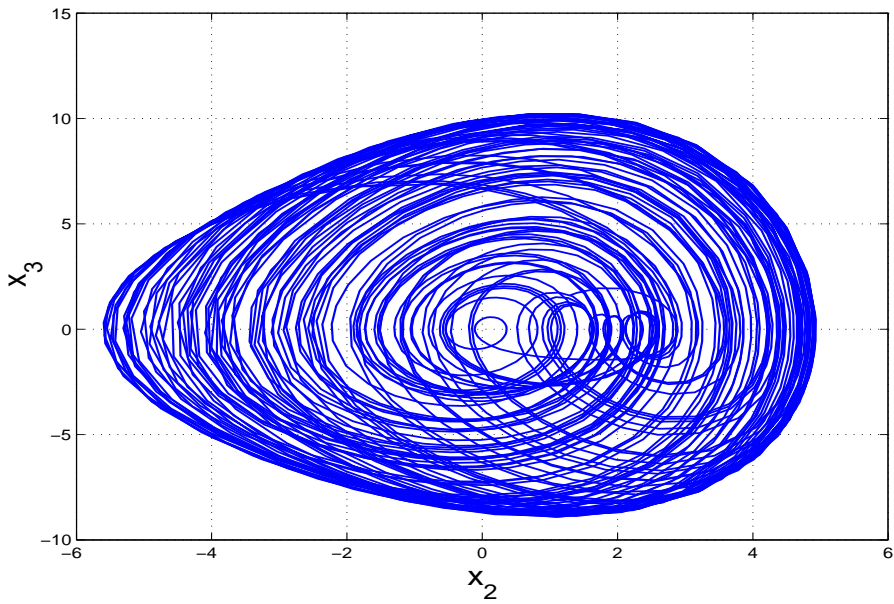


Figure 3. 2-D projection of the novel jerk chaotic system on the (x_2, x_3) plane

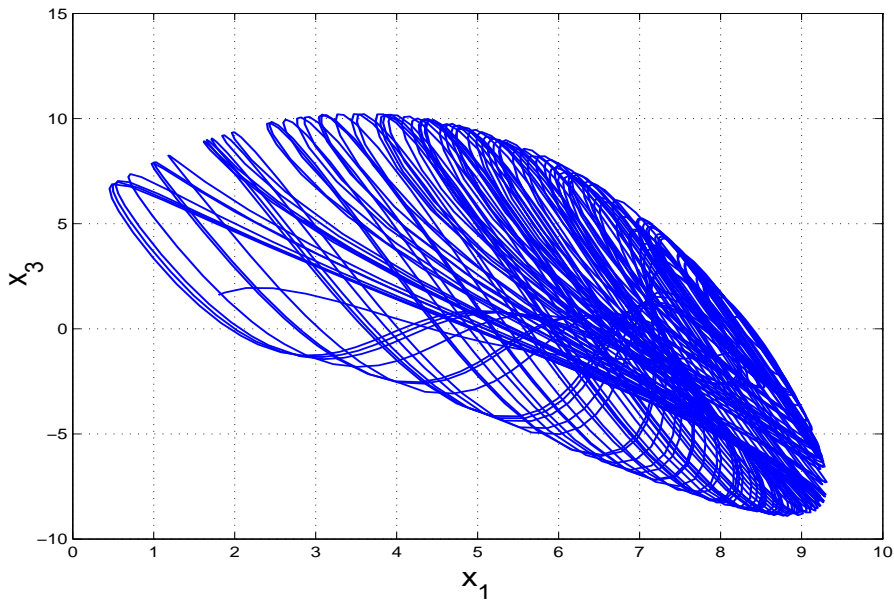


Figure 4. 2-D projection of the novel jerk chaotic system on the (x_1, x_3) plane

Let Ω be any region in \mathfrak{R}^3 with a smooth boundary and also, $\Omega(t) = \Phi_t(\Omega)$, where Φ_t is the flow of f . Furthermore, let $V(t)$ denote the volume of $\Omega(t)$.

By Liouville's theorem, we know that

$$\dot{V}(t) = \int_{\Omega(t)} (\nabla \cdot f) dx_1 dx_2 dx_3 \quad (6)$$

The divergence of the novel jerk system (4) is found as:

$$\nabla \cdot f = \frac{\partial f_1}{\partial x_1} + \frac{\partial f_2}{\partial x_2} + \frac{\partial f_3}{\partial x_3} = -1 < 0 \quad (7)$$

Inserting the value of $\nabla \cdot f$ from (7) into (6), we get

$$\dot{V}(t) = \int_{\Omega(t)} (-1) dx_1 dx_2 dx_3 = -V(t) \quad (8)$$

Integrating the first order linear differential equation (8), we get

$$V(t) = \exp(-t)V(0) \quad (9)$$

From Eq. (9), it is clear that $V(t) \rightarrow 0$ exponentially as $t \rightarrow \infty$. This shows that the novel 3-D jerk chaotic system (1) is dissipative. Hence, the system limit sets are ultimately confined into a specific limit set of zero volume, and the asymptotic motion of the novel jerk chaotic system (1) settles onto a strange attractor of the system.

3.2. Equilibrium Points

The equilibrium points of the 3-D novel jerk chaotic system (1) are obtained by solving the equations

$$\left. \begin{aligned} f_1(x_1, x_2, x_3) &= x_2 &= 0 \\ f_2(x_1, x_2, x_3) &= x_3 &= 0 \\ f_3(x_1, x_2, x_3) &= ax_1 - bx_2 - x_3 + cx_1x_2 - p(x_1^2 + x_2^2) &= 0 \end{aligned} \right\} \quad (10)$$

We take the parameter values as in the chaotic case (2), *i.e.*

$$a = 7.5, \quad b = 4, \quad c = 0.03, \quad p = 0.9 \quad (11)$$

Thus, the equilibrium points of the system (1) are characterized by the equations

$$x_1(a - px_1) = 0, \quad x_2 = 0, \quad x_3 = 0 \quad (12)$$

Solving the system (12), we get the equilibrium points of the system (1) as

$$E_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad E_1 = \begin{bmatrix} 8.3333 \\ 0 \\ 0 \end{bmatrix} \quad (13)$$

To test the stability type of the equilibrium points E_0 and E_1 , we calculate the Jacobian matrix of the novel jerk chaotic system (1) at any point \mathbf{x} :

$$J(\mathbf{x}) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 7.5 + 0.03x_2 - 1.8x_1 & -4 + 0.03x_1 - 1.8x_2 & -1 \end{bmatrix} \quad (14)$$

We note that

$$J_0 \triangleq J(E_0) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 7.5 & -4 & -1 \end{bmatrix} \quad (15)$$

which has the eigenvalues

$$\lambda_1 = 1.1555, \quad \lambda_{2,3} = -1.0778 \pm 2.3085 i \quad (16)$$

This shows that the equilibrium point E_0 is a saddle-focus point.

Next, we note that

$$J_1 \triangleq J(E_1) = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7.4999 & -3.7500 & -1 \end{bmatrix} \quad (17)$$

which has the eigenvalues

$$\lambda_1 = -1.5956, \quad \lambda_{2,3} = 0.2978 \pm 2.1475 i \quad (18)$$

This shows that the equilibrium point E_1 is also a saddle-focus point.

Hence, the novel jerk chaotic system (1) has two equilibrium points E_0, E_1 defined by (13), which are saddle-foci and unstable.

3.3. Lyapunov exponents and Kaplan-Yorke dimension

We take the parameter values of the novel jerk system (1) as

$$a = 7.5, \quad b = 4, \quad c = 0.03, \quad p = 0.9 \quad (19)$$

Then the Lyapunov exponents are numerically obtained using MATLAB as

$$L_1 = 0.20572, \quad L_2 = 0, \quad L_3 = -1.20824 \quad (20)$$

Thus, the maximal Lyapunov exponent (MLE) of the novel jerk system (1) is positive, which means that the system has a chaotic behavior.

Since $L_1 + L_2 + L_3 = -1.00252 < 0$, it follows that the novel jerk chaotic system (1) is dissipative.

Also, the Kaplan-Yorke dimension of the novel jerk chaotic system (1) is obtained as

$$D_{KY} = 2 + \frac{L_1 + L_2}{|L_3|} = 2.17026, \quad (21)$$

which is fractional.

4. Adaptive control of the 3-D novel jerk chaotic system

In this section, we use backstepping control method to derive an adaptive feedback control law for globally stabilizing the 3-D novel jerk chaotic system with unknown parameters.

Thus, we consider the 3-D novel jerk chaotic system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = ax_1 - bx_2 - x_3 + cx_1x_2 - p(x_1^2 + x_2^2) + u \end{cases} \quad (22)$$

where a, b, c, p are unknown constant parameters, and u is a backstepping control law to be determined using estimates of the unknown system parameters.

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \end{cases} \quad (23)$$

Differentiating (23) with respect to t , we obtain the following equations:

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \end{cases} \quad (24)$$

Next, we shall state and prove the main result of this section.

Theorem 1 *The 3-D novel jerk chaotic system (22), with unknown parameters a and b , is globally and exponentially stabilized by the adaptive feedback control law,*

$$u(t) = -[3 + \hat{a}(t)]x_1 - [5 - \hat{b}(t)]x_2 - 2x_3 - \hat{c}(t)x_1x_2 + \hat{p}(t)(x_1^2 + x_2^2) - kz_3 \quad (25)$$

where $k > 0$ is a gain constant,

$$z_3 = 2x_1 + 2x_2 + x_3, \quad (26)$$

and the update law for the parameter estimates $\hat{a}(t), \hat{b}(t), \hat{c}(t), \hat{p}(t)$ is given by

$$\begin{cases} \dot{\hat{a}}(t) = x_1 z_3 \\ \dot{\hat{b}}(t) = -x_2 z_3 \\ \dot{\hat{c}}(t) = x_1 x_2 z_3 \\ \dot{\hat{p}}(t) = -(x_1^2 + x_2^2) z_3 \end{cases} \quad (27)$$

Proof We prove this result via backstepping control method and Lyapunov stability theory.

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \quad (28)$$

where

$$z_1 = x_1 \quad (29)$$

Differentiating V_1 along the dynamics (22), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = x_1 x_2 = -z_1^2 + z_1(x_1 + x_2) \quad (30)$$

Now, we define

$$z_2 = x_1 + x_2 \quad (31)$$

Using (31), we can simplify the equation (30) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \quad (32)$$

Secondly, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2} (z_1^2 + z_2^2) \quad (33)$$

Differentiating V_2 along the dynamics (22), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2x_1 + 2x_2 + x_3) \quad (34)$$

Now, we define

$$z_3 = 2x_1 + 2x_2 + x_3 \quad (35)$$

Using (35), we can simplify the equation (34) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \quad (36)$$

Finally, we define a quadratic Lyapunov function

$$V(z_1, z_2, z_3, e_a, e_b, e_c, e_p) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 + \frac{1}{2} (e_a^2 + e_b^2 + e_c^2 + e_p^2) \quad (37)$$

which is a positive definite function on \mathfrak{R}^7 .

Differentiating V along the dynamics (22), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3(z_3 + z_2 + \dot{z}_3) - e_a \dot{a} - e_b \dot{b} - e_c \dot{c} - e_p \dot{p} \quad (38)$$

Eq. (38) can be written compactly as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a \dot{a} - e_b \dot{b} - e_c \dot{c} - e_p \dot{p} \quad (39)$$

where

$$S = z_3 + z_2 + \dot{z}_3 = z_3 + z_2 + 2\dot{x}_1 + 2\dot{x}_2 + \dot{x}_3 \quad (40)$$

A simple calculation gives

$$S = (3+a)x_1 + (5-b)x_2 + 2x_3 + cx_1x_2 - p(x_1^2 + x_2^2) + u \quad (41)$$

Substituting the adaptive control law (25) into (41), we obtain

$$S = [a - \hat{a}(t)]x_1 - [b - \hat{b}(t)]x_2 + [c - \hat{c}(t)]x_1x_2 - [p - \hat{p}(t)](x_1^2 + x_2^2) - kz_3 \quad (42)$$

Using the definitions (24), we can simplify (42) as

$$S = e_ax_1 - e_bx_2 + e_cx_1x_2 - e_p(x_1^2 + x_2^2) - kz_3 \quad (43)$$

Substituting the value of S from (43) into (39), we obtain

$$\begin{cases} \dot{V} = -z_1 - z_2 - (1+k)z_3^2 + e_a [x_1z_3 - \hat{a}] + e_b [-x_2z_3 - \hat{b}] \\ \quad + e_c [x_1x_2z_3 - \hat{c}] + e_p [-(x_1^2 + x_2^2)z_3 - \hat{p}] \end{cases} \quad (44)$$

Substituting the update law (27) into (44), we get

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2, \quad (45)$$

which is a negative semi-definite function on \mathfrak{R}^7 .

From (45), it follows that the vector $\mathbf{z}(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t), e_c(t), e_p(t))$ are globally bounded, i.e.

$$\begin{bmatrix} z_1(t) & z_2(t) & z_3(t) & e_a(t) & e_b(t) & e_c(t) & e_p(t) \end{bmatrix} \in L_\infty \quad (46)$$

Also, it follows from (45) that

$$\dot{V} \leq -z_1^2 - z_2^2 - z_3^2 = -\|\mathbf{z}\|^2 \quad (47)$$

That is,

$$\|\mathbf{z}\|^2 \leq -\dot{V} \quad (48)$$

Integrating the inequality (48) from 0 to t , we get

$$\int_0^t |\mathbf{z}(\tau)|^2 d\tau \leq V(0) - V(t) \quad (49)$$

From (49), it follows that $\mathbf{z}(t) \in L_2$.

From Eq. (22), it can be deduced that $\dot{\mathbf{z}}(t) \in L_\infty$.

Thus, using Barbalat's lemma, we conclude that $\mathbf{z}(t) \rightarrow \mathbf{0}$ exponentially as $t \rightarrow \infty$ for all initial conditions $\mathbf{z}(0) \in \mathfrak{R}^3$.

Hence, it is immediate that $\mathbf{x}(t) \rightarrow \mathbf{0}$ exponentially as $t \rightarrow \infty$ for all initial conditions $\mathbf{x}(0) \in \mathfrak{R}^3$.

This completes the proof. \square

For the numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (22) and (27), when the adaptive control law (25) is applied.

The parameter values of the novel jerk chaotic system (22) are taken as in the chaotic case (2), *i.e.*

$$a = 7.5, \quad b = 4, \quad c = 0.03, \quad p = 0.9 \quad (50)$$

The positive gain constant k is taken as $k = 10$.

As initial conditions of the novel jerk chaotic system (22), we take

$$x_1(0) = 6.2, \quad x_2(0) = 15.9, \quad x_3(0) = 9.7 \quad (51)$$

Also, as initial conditions of the parameter estimates, we take

$$\hat{a}(0) = 2.1, \quad \hat{b}(0) = 7.3, \quad \hat{c}(0) = 5.4, \quad \hat{p}(0) = 8.6 \quad (52)$$

In Figure 5, the exponential convergence of the controlled states is depicted, when the adaptive control law (25) and parameter update law (27) are implemented.

5. Adaptive synchronization of the identical 3-D novel jerk chaotic systems

In this section, we use backstepping control method to derive an adaptive control law for globally and exponentially synchronizing the identical 3-D novel jerk chaotic systems with unknown parameters.

As the master system, we consider the 3-D novel jerk chaotic system given by

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = x_3 \\ \dot{x}_3 = ax_1 - bx_2 - x_3 + cx_1x_2 - p(x_1^2 + x_2^2) \end{cases} \quad (53)$$

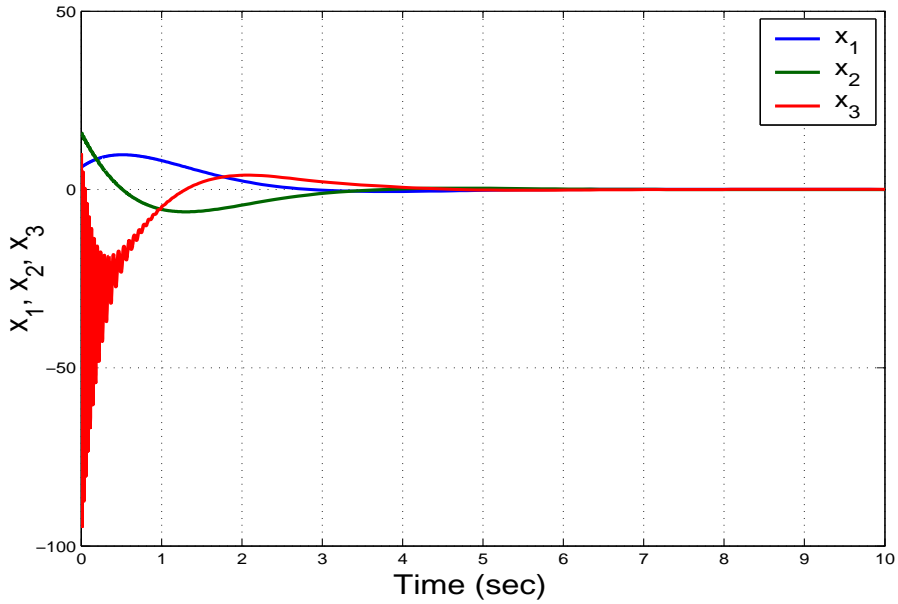


Figure 5. Time-history of the controlled states $x_1(t), x_2(t), x_3(t)$

where x_1, x_2, x_3 are the states of the system, and a, b, c, p are unknown constant parameters.

As the slave system, we consider the 3-D novel jerk chaotic system given by

$$\begin{cases} \dot{y}_1 = y_2 \\ \dot{y}_2 = y_3 \\ \dot{y}_3 = ay_1 - by_2 - y_3 + cy_1y_2 - p(y_1^2 + y_2^2) + u \end{cases} \quad (54)$$

where y_1, y_2, y_3 are the states of the system, and u is a backstepping control to be determined using estimates of the unknown system parameters.

We define the synchronization errors between the states of the master system (53) and the slave system (54) as

$$\begin{cases} e_1 = y_1 - x_1 \\ e_2 = y_2 - x_2 \\ e_3 = y_3 - x_3 \end{cases} \quad (55)$$

Then the error dynamics is easily obtained as

$$\begin{cases} \dot{e}_1 = e_2 \\ \dot{e}_2 = e_3 \\ \dot{e}_3 = ae_1 - be_2 - e_3 + c(y_1y_2 - x_1x_2) \\ \quad - p(y_1^2 - x_1^2 + y_2^2 - x_2^2) + u \end{cases} \quad (56)$$

The parameter estimation errors are defined as:

$$\begin{cases} e_a(t) = a - \hat{a}(t) \\ e_b(t) = b - \hat{b}(t) \\ e_c(t) = c - \hat{c}(t) \\ e_p(t) = p - \hat{p}(t) \end{cases} \quad (57)$$

Differentiating (57) with respect to t , we obtain the following equations:

$$\begin{cases} \dot{e}_a(t) = -\dot{\hat{a}}(t) \\ \dot{e}_b(t) = -\dot{\hat{b}}(t) \\ \dot{e}_c(t) = -\dot{\hat{c}}(t) \\ \dot{e}_p(t) = -\dot{\hat{p}}(t) \end{cases} \quad (58)$$

Next, we shall state and prove the main result of this section.

Theorem 2 *The identical 3-D novel jerk chaotic systems (53) and (54) with unknown parameters a and b are globally and exponentially synchronized by the adaptive control law*

$$\begin{cases} u(t) = -[3 + \hat{a}(t)]e_1 - [5 - \hat{b}(t)]e_2 - 2e_3 - \hat{c}(t)[y_1y_2 - x_1x_2] \\ \quad + \hat{p}(t)[y_1^2 - x_1^2 + y_2^2 - x_2^2] - kz_3 \end{cases} \quad (59)$$

where $k > 0$ is a gain constant,

$$z_3 = 2e_1 + 2e_2 + e_3, \quad (60)$$

and the update law for the parameter estimates $\hat{a}(t), \hat{b}(t)$ is given by

$$\begin{cases} \dot{\hat{a}}(t) = e_1z_3 \\ \dot{\hat{b}}(t) = -e_2z_3 \\ \dot{\hat{c}}(t) = (y_1y_2 - x_1x_2)z_3 \\ \dot{\hat{p}}(t) = -(y_1^2 - x_1^2 + y_2^2 - x_2^2)z_3 \end{cases} \quad (61)$$

Proof We prove this result via backstepping control method and Lyapunov stability theory.

First, we define a quadratic Lyapunov function

$$V_1(z_1) = \frac{1}{2} z_1^2 \quad (62)$$

where

$$z_1 = e_1 \quad (63)$$

Differentiating V_1 along the error dynamics (56), we get

$$\dot{V}_1 = z_1 \dot{z}_1 = e_1 e_2 = -z_1^2 + z_1(e_1 + e_2) \quad (64)$$

Now, we define

$$z_2 = e_1 + e_2 \quad (65)$$

Using (65), we can simplify the equation (64) as

$$\dot{V}_1 = -z_1^2 + z_1 z_2 \quad (66)$$

Secondly, we define a quadratic Lyapunov function

$$V_2(z_1, z_2) = V_1(z_1) + \frac{1}{2} z_2^2 = \frac{1}{2} (z_1^2 + z_2^2) \quad (67)$$

Differentiating V_2 along the error dynamics (56), we get

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2(2e_1 + 2e_2 + e_3) \quad (68)$$

Now, we define

$$z_3 = 2e_1 + 2e_2 + e_3 \quad (69)$$

Using (69), we can simplify the equation (68) as

$$\dot{V}_2 = -z_1^2 - z_2^2 + z_2 z_3 \quad (70)$$

Finally, we define a quadratic Lyapunov function

$$V(z_1, z_2, z_3, e_a, e_b, e_c, e_p) = V_2(z_1, z_2) + \frac{1}{2} z_3^2 + \frac{1}{2} (e_a^2 + e_b^2 + e_c^2 + e_p^2) \quad (71)$$

which is a positive definite function on \mathfrak{R}^7 .

Differentiating V along the error dynamics (56), we get

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3(z_3 + z_2 + \dot{z}_3) - e_a \dot{a} - e_b \dot{b} - e_c \dot{c} - e_p \dot{p} \quad (72)$$

Eq. (72) can be written compactly as

$$\dot{V} = -z_1^2 - z_2^2 - z_3^2 + z_3 S - e_a \dot{a} - e_b \dot{b} - e_c \dot{c} - e_p \dot{p} \quad (73)$$

where

$$S = z_3 + z_2 + \dot{z}_3 = z_3 + z_2 + 2\dot{e}_1 + 2\dot{e}_2 + \dot{e}_3 \quad (74)$$

A simple calculation gives

$$S = (3+a)e_1 + (5-b)e_2 + 2e_3 + c(y_1y_2 - x_1x_2) - p(y_1^2 - x_1^2 + y_2^2 - x_2^2) + u \quad (75)$$

Substituting the adaptive control law (59) into (41), we obtain

$$\begin{cases} S = [a - \hat{a}(t)]e_1 - [b - \hat{b}(t)]e_2 + [c - \hat{c}(t)](y_1y_2 - x_1x_2) \\ \quad - [p - \hat{p}(t)](y_1^2 - x_1^2 + y_2^2 - x_2^2) - kz_3 \end{cases} \quad (76)$$

Using the definitions (58), we can simplify (76) as

$$S = e_a e_1 - e_b e_2 + e_c (y_1 y_2 - x_1 x_2) - e_p (y_1^2 - x_1^2 + y_2^2 - x_2^2) - k z_3 \quad (77)$$

Substituting the value of S from (77) into (73), we obtain

$$\begin{cases} \dot{V} = -z_1 - z_2 - (1+k)z_3^2 + e_a [e_1 z_3 - \hat{a}] + e_b [-e_2 z_3 - \hat{b}] \\ \quad + e_c [(y_1 y_2 - x_1 x_2) z_3 - \hat{c}] + e_p [-(y_1^2 - x_1^2 + y_2^2 - x_2^2) z_3 - \hat{p}] \end{cases} \quad (78)$$

Substituting the update law (61) into (78), we get

$$\dot{V} = -z_1^2 - z_2^2 - (1+k)z_3^2, \quad (79)$$

which is a negative semi-definite function on \mathfrak{R}^7 .

From (79), it follows that the vector $\mathbf{z}(t) = (z_1(t), z_2(t), z_3(t))$ and the parameter estimation error $(e_a(t), e_b(t), e_c(t), e_p(t))$ are globally bounded, i.e.

$$\begin{bmatrix} z_1(t) & z_2(t) & z_3(t) & e_a(t) & e_b(t) & e_c(t) & e_p(t) \end{bmatrix} \in L_\infty \quad (80)$$

Also, it follows from (79) that

$$\dot{V} \leq -z_1^2 - z_2^2 - z_3^2 = -\|\mathbf{z}\|^2 \quad (81)$$

That is,

$$\|\mathbf{z}\|^2 \leq -\dot{V} \quad (82)$$

Integrating the inequality (82) from 0 to t , we get

$$\int_0^t |\mathbf{z}(\tau)|^2 d\tau \leq V(0) - V(t) \quad (83)$$

From (83), it follows that $\mathbf{z}(t) \in L_2$.

From Eq. (56), it can be deduced that $\dot{\mathbf{z}}(t) \in L_\infty$.

Thus, using Barbalat's lemma, we conclude that $\mathbf{z}(t) \rightarrow \mathbf{0}$ exponentially as $t \rightarrow \infty$ for all initial conditions $\mathbf{z}(0) \in \mathfrak{R}^3$.

Hence, it is immediate that $\mathbf{e}(t) \rightarrow \mathbf{0}$ exponentially as $t \rightarrow \infty$ for all initial conditions $\mathbf{e}(0) \in \mathfrak{R}^3$.

This completes the proof. \square

For the numerical simulations, the classical fourth-order Runge-Kutta method with step size $h = 10^{-8}$ is used to solve the system of differential equations (53) and (54).

The parameter values of the novel jerk chaotic systems are taken as

$$a = 7.5, \quad b = 4, \quad c = 0.03, \quad p = 0.9 \quad (84)$$

The positive gain constant is taken as $k = 10$.

As initial conditions of the master chaotic system (53), we take

$$x_1(0) = 5.2, \quad x_2(0) = 6.7, \quad x_3(0) = -8.1 \quad (85)$$

As initial conditions of the slave chaotic system (54), we take

$$y_1(0) = -3.7, \quad y_2(0) = 12.4, \quad y_3(0) = 7.5 \quad (86)$$

Also, as initial conditions of the parameter estimates, we take

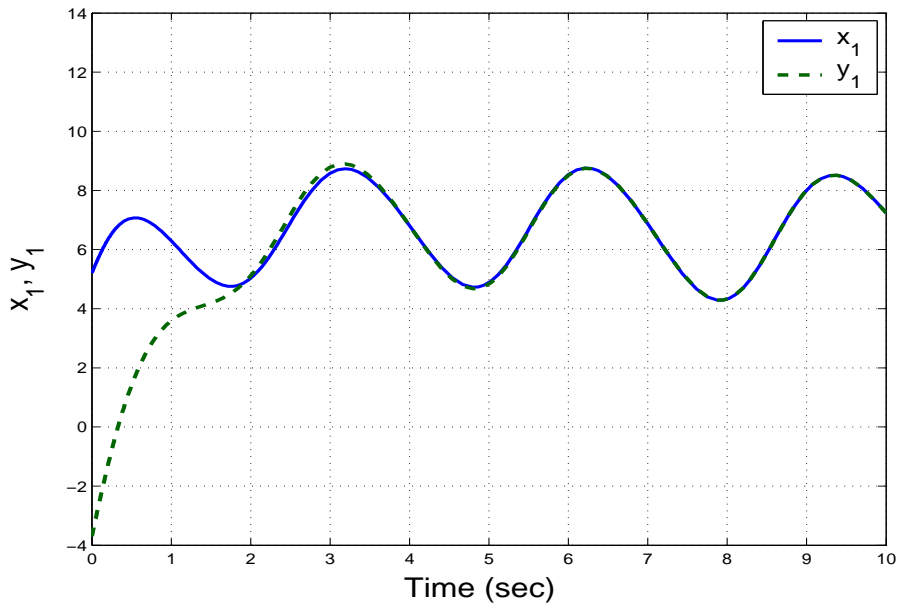
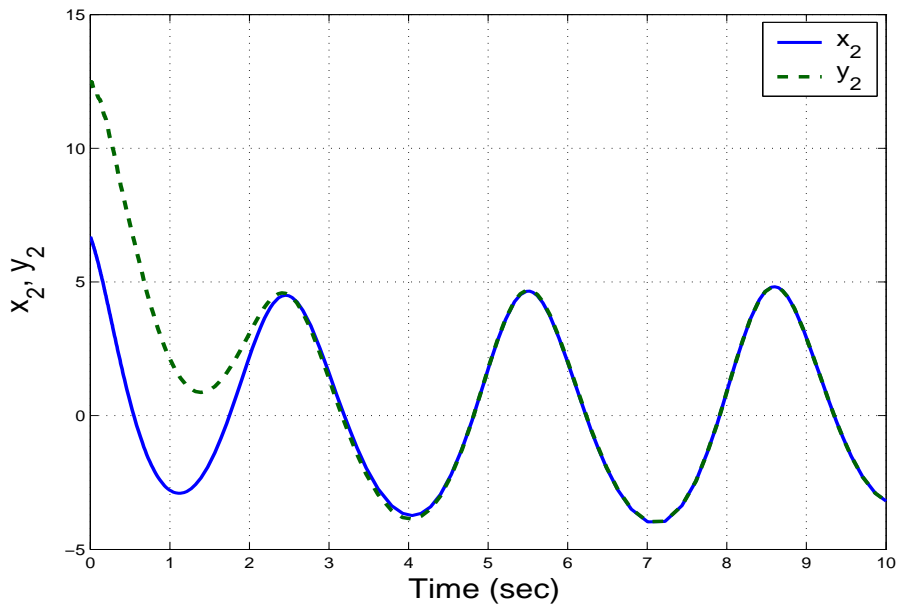
$$\hat{a}(0) = 8.2, \quad \hat{b}(0) = 10.1, \quad \hat{c}(0) = 5.6, \quad \hat{p}(0) = 2.3 \quad (87)$$

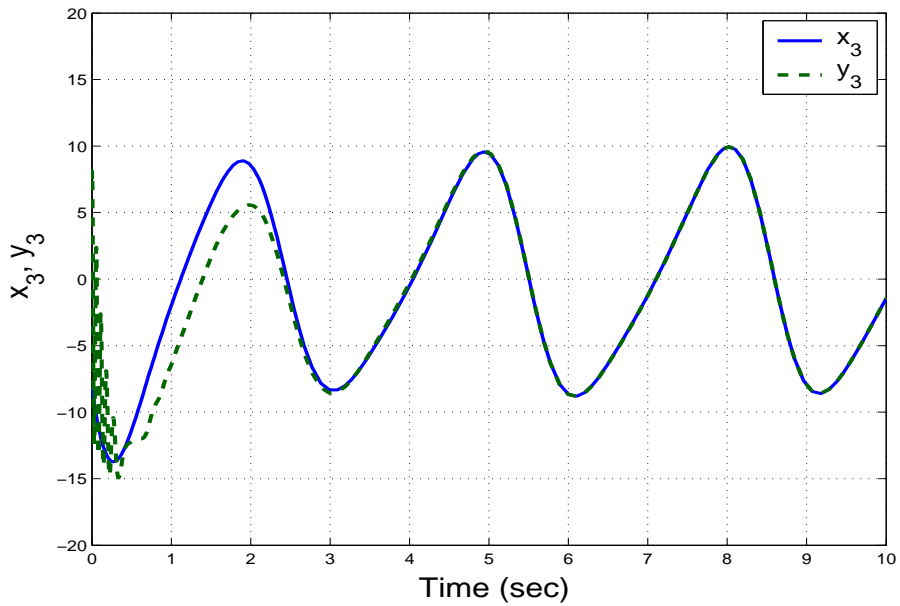
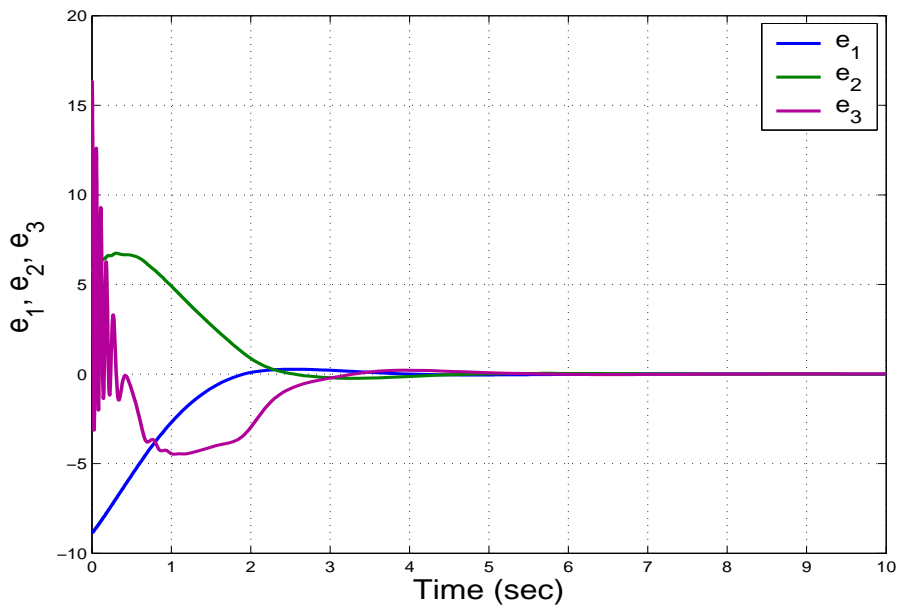
In Figs. 6-8, the complete synchronization of the identical 3-D jerk chaotic systems (53) and (54) is shown, when the adaptive control law and the parameter update law are implemented.

Also, in Fig. 9, the time-history of the synchronization errors $e_1(t), e_2(t), e_3(t)$, is shown.

6. Conclusions

In this paper, we announced an eight-term novel 3-D jerk chaotic system with three quadratic nonlinearities. The phase portraits of the novel jerk chaotic system were displayed and the mathematical properties were discussed. We showed that the novel jerk chaotic system has two equilibrium points, which are saddle-foci and unstable. The Lyapunov exponents of the novel jerk chaotic system have been obtained as $L_1 = 0.20572, L_2 = 0$ and $L_3 = -1.20824$. Since the sum of the Lyapunov exponents of the jerk chaotic system is negative, we conclude that the chaotic system is dissipative. The Kaplan-Yorke dimension of the novel jerk chaotic system has been derived as $D_{KY} = 2.17026$.

Figure 6. Synchronization of the states $x_1(t)$ and $y_1(t)$ Figure 7. Synchronization of the states $x_2(t)$ and $y_2(t)$

Figure 8. Synchronization of the states $x_3(t)$ and $y_3(t)$ Figure 9. Time-history of the synchronization errors $e_1(t)$, $e_2(t)$, $e_3(t)$

Next, an adaptive controller was designed via backstepping control method to globally stabilize the novel jerk chaotic system with unknown parameters. Moreover, an adaptive controller was also designed via backstepping control method to achieve global chaos synchronization of the identical jerk chaotic systems with unknown parameters. The backstepping control method is a recursive procedure that links the choice of a Lyapunov function with the design of a controller and guarantees global asymptotic stability of strict feedback systems. MATLAB simulations were depicted to illustrate the phase portraits of the novel jerk chaotic system and also the adaptive backstepping control results.

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