

Application of optimal current driver for the torque control of BLDC motor

JAKUB BERNAT, SŁAWOMIR STĘPIEŃ

*Chair of Computer Engineering, Poznan University of Technology
Piotrowo 3a, 60-965 Poznań, Poland
tel. +48(061) 6652504*

e-mail: {Jakub.Bernat/Slawomir.Stepien}@put.poznan.pl

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Abstract: This research presents the novel control strategy of the brushless DC motor. The optimal current driver is designed using Linear Quadratic Regulator and feedback linearization. Additionally, the current reshaping strategy is applied to control the motor torque. Thus, the torque controller is built based on the optimal current driver. The motor is simulated using the FEM analysis.

Key words: current driver, BLDC motor, control law, linear quadratic problem, identification

1. Introduction

The brushless DC motors (BLDC) are widely used in industrial applications, where high precision electromechanic converters are required. The motor performance and its efficiency are important factors during the application design. Thus, many researchers work on new control designs and motor geometries. The motor shape analysis has been presented in previous publications [13]. Although, its relevance for motor efficiency has been proved, the control design is also important factor to achieve performance's goals.

The most popular BLDC control system is based on hallotron feedback, which along with electronic driver provides reliable control design. Researchers have proposed a number of optimal converter designs [9]. This approach enables to build angular position controller or speed controller. However, its drawbacks are torque ripple effect [13].

Another control problem for BLDC is a design of torque controller. In previous works, many solutions have been proposed to achieve reference torque. The adaptive change of a current slope has been presented in [11]. However it assumes a perfect trapezoidal shape of the back emf force. Another method is described in [8] where the BLDC controller is based on observer design.

The alternative to mentioned works is to use current controller presented in [1, 12], where the phase currents are considered as motor inputs. It simplifies a control law because the torque is a function of position and current, therefore it is possible to calculate a current shape, which realizes a reference torque.

This work presents the current driver, which is required, when the current is considered as the motor input. The design methodology is based on feedback linearization to compensate back emf force. Additionally, the Linear Quadratic Regulator (LQR) is applied to the presented current driver to achieve the best performance.

2. The BLDC model

The BLDC model is based on previous works [1, 10]. The analyzed motor has three phases and two pole pairs. Each winding on the stator is defined by equal parameters, therefore the phase equation is given by:

$$u_s = R i_s + \frac{\partial \Phi_s}{\partial t}, \quad (1)$$

where u_s is the phase voltage, i_s is the phase current, Φ_s is the magnetic flux of windings, R is the phase resistance and s is the winding index ($s = 1, 2, 3$). The flux is produced by a permanent magnet and coil's windings. The permanent magnet is modeled as a virtual current, which is assumed to be constant. Thus, the permanent magnet flux is expressed as $\Phi_{m,s} = M_s(\theta) i_m$, where i_m is constant virtual magnet current and $M_s(\theta)$ describes the flux change in motor windings due to different rotor positions. The BLDC motor allows to assume that a self inductance is constant, because the rotor has a uniform shape. Additionally, the mutual inductance has a small influence. Hence, the flux change is defined as $\Phi_{ind,s} = L i_s$. In summary, the windings flux is:

$$\Phi_s = \Phi_{ind,s} + \Phi_{m,s}. \quad (2)$$

Applying the flux model to circuit equation, one has:

$$u_s = R i_s + L \frac{d i_s}{d t} + \frac{\partial M_s}{\partial \theta} \omega i_m. \quad (3)$$

The term $\partial M_s / \partial \theta$ causes that equation (3) is nonlinear. The function $\partial M_s / \partial \theta$ shape is related with motor geometry and for a BLDC motor has more trapezoidal shape and for a PMSM motor has more sinusoidal shape.

The torque acting on the rotor can be calculated using principle of virtual work, presented in [7]:

$$T = \frac{\partial W_{co}}{\partial \theta} = \frac{\partial}{\partial t} \int_0^t \Phi^T \mathbf{d} \mathbf{i}. \quad (4)$$

The W_{co} is the magnetic co-energy, Φ is the vector of the windings flux: $\Phi = [\Phi_1 \ \Phi_2 \ \Phi_3]^T$ and \mathbf{di} is the vector of windings currents: $\mathbf{di} = [di_1 \ di_2 \ di_3]^T$. Considering the BLDC flux model, the magnetic co-energy is defined as:

$$W_{co} = \frac{1}{2} \sum_{s=1}^3 [M_s i_m i_s + L i_s^2]. \quad (5)$$

Hence, the torque T acting on a rotor is as follows:

$$T = \frac{1}{2} \sum_{s=1}^3 \frac{\partial M_s}{\partial \theta} i_m i_s. \quad (6)$$

The motion modeling is required only for simulation, because the control law presented in this paper is independent from motion parameters. Thus, the first order motion equation is defined considering a rotor inertia and a viscous friction.

3. Parameter estimation

The presented controller requires an exact values of the parameter of the electric circuit. To design the identification process for the BLDC motor, we have to cope with the system nonlinearities. From the point of view of control design, the influence of the eddy current or B-H hysteresis can be neglected in this case. Whereas the rotor motion nonlinearity has great impact on the BLDC, hence this phenomena must be included in the motor model. There are well known identification techniques, widely discussed in Astrom and Wittenmark [2] and Astolfi and Ortega [4], which allow to estimate linearly parameterized models. To meet these method requirements, equation (3) is approximated by the Fourier series. Therefore, the $\partial M_s / \partial \theta$ is expressed by the following terms:

$$\frac{\partial M_s}{\partial \theta} = \sum_{k=1}^{N_f} A_k \sin(k N_r \theta + \varphi_s) + B_k \cos(k N_r \theta + \varphi_s). \quad (7)$$

The N_f defines the numbers of Fourier series elements, the A_k and B_k are the harmonics coefficients and N_r is the number of rotor pole pairs. To define the estimation formula, equation (3) is rewritten in a state space form and discretized by the Euler method with the sampling time T_s :

$$i(n) = \left(1 - T_s \frac{R}{L}\right) i(n-1) + \frac{T_s}{L} u(n-1) + \frac{T_s}{L} \omega(n-1) i_m \sum_{k=1}^{N_f} [A_k \sin(k N_k \theta(n-1)) + B_k \cos(k N_k \theta(n-1))]. \quad (8)$$

The new coefficients, which are defined as:

$$\begin{aligned}
 \psi_1(n) &= 1 - T_s \frac{R}{L} & \psi_2(n) &= \frac{T_s}{L} \\
 \psi_3(n) &= \frac{T_s}{L} A_1 i_m & \psi_4(n) &= \frac{T_s}{L} B_1 i_m \\
 & & & \vdots \\
 \psi_p(n) &= \frac{T_s}{L} A_p i_m & \psi_{p+1}(n) &= \frac{T_s}{L} B_p i_m \\
 & & & \vdots \\
 \psi_{2N_f+1}(n) &= \frac{T_s}{L} A_{N_f} i_m & \psi_{2N_f+2}(n) &= \frac{T_s}{L} B_{N_f} i_m,
 \end{aligned} \tag{9}$$

are introduced by replacing the unknown parameters. When the virtual current i_m , without loss of generality, is employed to 1, then all motor parameters are uniquely defined by equation (9). Furthermore, equation (8) can be rewritten in a linear regression form, which is as follows:

$$y(n) = \varphi^T(n) \psi(n), \tag{10}$$

where $y(n) = [i(n)]$ and $\psi(n) = [\psi_1(n) \dots \psi_{2N_f+2}(n)]$. The nonlinear regressor $\varphi(n) = [\varphi_1(n) \dots \varphi_{2N_f+2}(n)]$, which includes the nonlinear parts, is defined by expressions:

$$\begin{aligned}
 \varphi_1(n) &= i(n-1) \\
 \varphi_2(n) &= u(n-1) \\
 \varphi_3(n) &= \omega(n-1) \sin(N_r \theta(n-1)) \\
 \varphi_4(n) &= \omega(n-1) \cos(N_r \theta(n-1)) \\
 & \vdots \\
 \varphi_p(n) &= \omega(n-1) \sin(k N_r \theta(n-1)) \\
 \varphi_{p+1}(n) &= \omega(n-1) \cos(k N_r \theta(n-1)) \\
 & \vdots \\
 \varphi_{2N_f+1}(n) &= \omega(n-1) \sin(N_f N_r \theta(n-1)) \\
 \varphi_{2N_f+2}(n) &= \omega(n-1) \cos(N_f N_r \theta(n-1)).
 \end{aligned} \tag{11}$$

Finally, all motor parameters are iteratively estimated by applying the RLS method, which is described in details in Astrom and Wittenmark [2], to equation (10).

4. The torque controller

The considered BLDC model, along with the identification method described above, allow to design the torque controller presented in Aghili [1]. These controller considers the phase current as a control input and generates a current shape such that an output torque is equal to a reference torque. Thus, the system requires a current driver to excitate the motor's windings. The described control idea is demonstrated in Fig. 1. To define the control law, the current shape function u is introduced, which describes the waveform of the input current. This function is based on a finite Fourier series and it is given by:

$$u_t(\theta) = \sum_{k=-N_f}^{N_f} c_n e^{jn N_r \theta}, \tag{12}$$

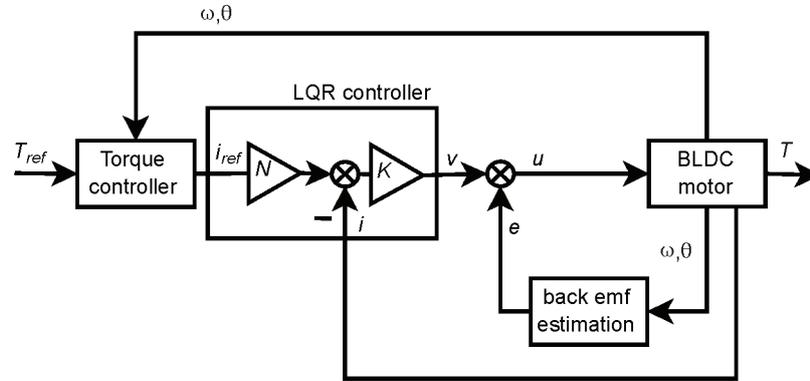


Fig. 1. The brushless DC motor control schema

where $j^2 = -1$ is the imaginary unit, c_n is the complex Fourier series coefficient, n is the harmonics index and N_f is the total number of the harmonics. Although, the right side of equation (12) contains complex numbers, the left side is a real valued function because the conjugate of c_n is equal to \bar{c}_{-n} . The current shape function is applied to calculate the reference current:

$$i_{ref} = T_{ref} \left[u_t(\theta) \quad u_t\left(\theta + \frac{2}{3}\pi\right) \quad u_t\left(\theta + \frac{4}{3}\pi\right) \right]^T, \quad (13)$$

where T_{ref} is the reference torque, which is assumed to be a constant. However, to calculate i_{ref} the current shape function coefficients c_n must be known. Therefore, the electromagnetic torque shape function is also defined based on the approximation by finite Fourier series:

$$w(\theta) = \sum_{m=-N_f}^{N_f} d_m e^{jmN_r\theta}, \quad (14)$$

where d_m is the complex Fourier series coefficient and it also satisfies $d_m = \bar{d}_{-m}$. Considering the model defined in this paper, the torque shape function is equal to:

$$\begin{aligned} w(\theta) &= \frac{\partial M_1}{\partial \theta} \\ w\left(\theta + \frac{2}{3}\pi\right) &= \frac{\partial M_2}{\partial \theta} \\ w\left(\theta + \frac{4}{3}\pi\right) &= \frac{\partial M_3}{\partial \theta}, \end{aligned} \quad (15)$$

where $d_m = 1/2(A_m + jB_m)$ and the coefficients A_m and B_m are defined in (7). Now, the electromagnetic torque with the help of (6), (13) and (15) is given by:

$$T_e(\theta) = T_{ref} \sum_s^3 \sum_{n=1}^{N_f} \sum_{m=1}^{N_f} c_n d_m e^{j(n+m)\left(2\theta + \frac{2\pi}{3}(s-1)\right)}, \quad (16)$$

where s is the phase index, c_n and d_n are the coefficients from (12) and (14) respectively. In equation (16) the influence of current driver dynamics is neglected, hence the reference current is always equal to the windings' current. The control objective will be satisfied if the c_n coefficients are found such that the electromagnetic torque T_e follows the reference torque T_{ref} . Indeed, from (16) it follows that $T_e(\theta) = T_{ref}$, when:

$$\sum_s^3 \sum_{n=1}^{N_f} \sum_{m=1}^{N_f} c_n d_m e^{j(n+m)\left(2\theta + \frac{2\pi}{3}(s-1)\right)} = 1. \quad (17)$$

Since, equation (17) has the infinite number of solutions, many controllers can be produced satisfying the control objective. The Aghili [1] proposed additional constraint, which minimize the norm of vector \mathbf{c} and can be represented as:

$$\text{minimize } \|\mathbf{c}\|, \quad \text{subject to } \sum_s^3 \sum_{n=1}^{N_f} \sum_{m=1}^{N_f} c_n d_m e^{j(n+m)\left(2\theta + \frac{2\pi}{3}(s-1)\right)} = 1. \quad (18)$$

The solution of these problem produces a vector \mathbf{d} , which enables us to calculate the command current \mathbf{i}_{ref} . It is worth to notice that the optimization problem is solved offline and while the driver is running only equation (13) has to be evaluated. This simplifies the control law application in a real device.

5. The optimal current driver

The current driver is crucial for the torque controller presented above. Therefore, to achieve the best performance and efficiency, the LQR technique is chosen to build this driver. Unfortunately, the circuit equation, which describes a current behavior in the windings, is non-linear. To cope with this problem, the back emf is compensated and the circuit equation becomes linear and infinite time optimal controller is applicable. Therefore, the back emf is defined using terms from equation (3) and it is given by:

$$e_s = \frac{\partial M_s}{\partial \theta} \omega i_m. \quad (19)$$

The value of back emf can be calculated using the parameters from identification process presented in the previous section. Hence, the approximation of term $\partial M_s / \partial \theta$ using equation (7) is applied to equation (19). Thus, the back emf is expressed as follows:

$$e_s = \omega i_m \sum_{k=1}^{N_f} A_k \sin(k N_r \theta + \varphi_s) + B_k \cos(k N_r \theta + \varphi_s). \quad (20)$$

To perform the linearization of the circuit equation (3), the new control input v_s is introduced, which is defined as $v_s = u_s - e_s$. Now, the control system could rewritten to:

$$v_s = R i_s + L \frac{d i_s}{d t}. \quad (21)$$

It is worth to mention that the system (21) is linear and LQR regulator can be applied. The objective function is given by:

$$J[u] = \int_0^{\infty} (\mathbf{i}^T \mathbf{Q} \mathbf{i} + \mathbf{v}^T \mathbf{P} \mathbf{v}) dt, \quad (22)$$

where \mathbf{Q} , \mathbf{P} are the weight matrix coefficients, and the vector \mathbf{i} and \mathbf{v} are defined as $\mathbf{i} = [i_1 \ i_2 \ i_3]^T$ and $\mathbf{v} = [v_1 \ v_2 \ v_3]^T$. Therefore, the control law, which works in the optimal point, is given by:

$$\mathbf{v} = -\mathbf{R}^{-1} \mathbf{B}^T \mathbf{G} (\mathbf{i} - \mathbf{N} \mathbf{i}_{\text{ref}}), \quad (23)$$

where \mathbf{G} is a solution of Riccati equation and \mathbf{N} is the weighting matrix. Based on the extended control input v_s , which is the term of the vector \mathbf{v} , the input voltage is defined as $u_s = v_s + e_s$ where s is the phase index.

6. Simulation experiment

In this paper, the control law is built based on the non-stationary model with lumped parameters. Nevertheless, to improve accuracy, the simulation experiment is performed using the FEM model of brushless DC motor. Thus, this model simulates a real device, which is excited by a controller based on the model with the lumped parameters. The FEM model of BLDC consists of three phases in the stator and two pole pairs on the rotor. The motor performance and its geometry optimization has been studied in [5, 6]. The exact motor parameters are given in Table 1.

Table 1. The BLDC motor parameters

Quantity	Value	Unit
Motor length	90	mm
Resistance/phase	15	Ω
Rotor inertia	1.098e-6	gcm ²
Damping	1e-5	Nms
Outer diameter of stator	27.5	mm
Inner diameter of stator	16.25	mm
Diameter of rotor	15	mm
Inner stator teeth width	20	deg
Outer stator teeth width	52	deg
Permanent magnet width	54	deg

Firstly, the identification process is performed, which estimates the BLDC model parameters using the transients of the FEM model. To meet persistence condition, required by RLS

method, the input is square waveform as shown in Fig. 2. As a result of identification, the estimation error is demonstrated in Fig. 3. Since, the error convergences fast to 0, it can be stated that lumped BLDC model fits very well to the FEM motor model.

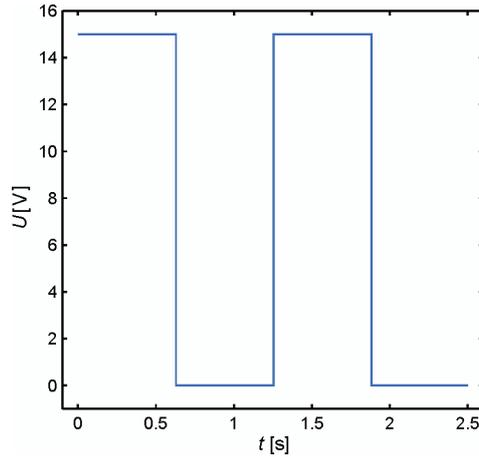


Fig. 2. The command signal for the identification process

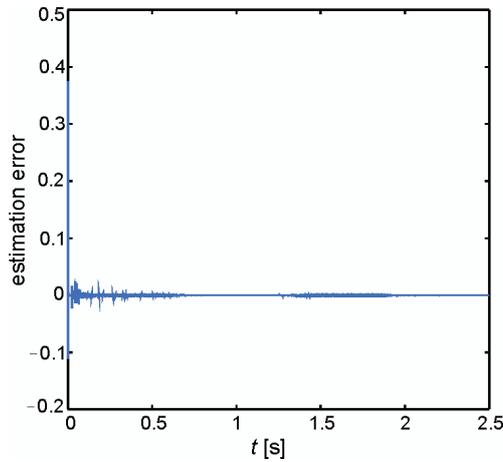


Fig. 3. The estimation error of the BLDC motor

The next step is to analyze the performance of the optimal current driver working with the torque controller. It is interesting to compare the responses of the BLDC motor, which is excited by the presented controller and as an alternative by the hallotron feedback. Referring to Fig. 4, where the both torque transients are shown, the torque controller significantly reduces the oscillations. Thus, the speed of the motor is greater, as demonstrated in Fig. 5, when the BLDC is run by the control law studied in this paper. Nevertheless the influence of the cogging torque, which is not compensated by torque controller, is also visible. Thus, in Fig. 4 the torque response of BLDC model with controller has a small oscillations. Furthermore, it is worth to look at the Fig. 6, where the current responses are demonstrated, because the torque controller introduces different current shape with the comparison to the hallotron feedback regulator.

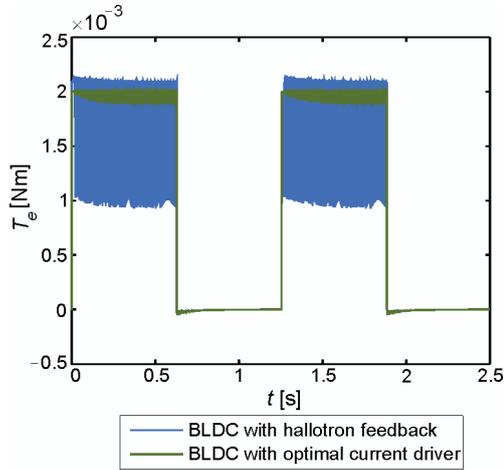


Fig. 4. The comparison of the torque responses

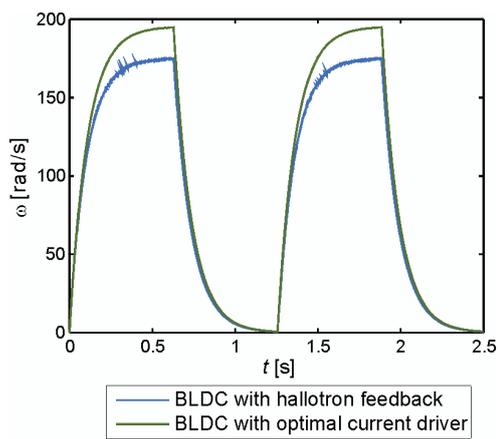


Fig. 5. The comparison of the speed responses

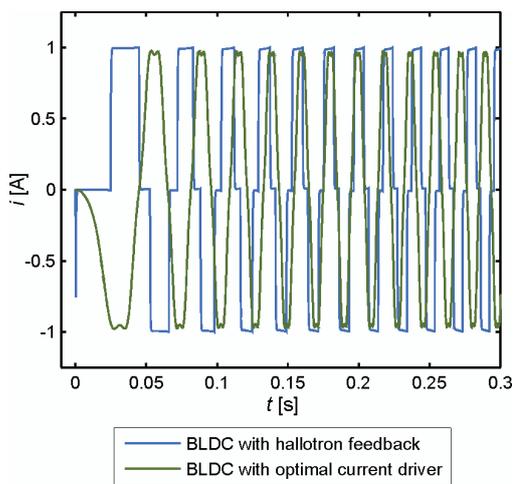


Fig. 6. The comparison of the current responses

7. Conclusions

The new concept of the optimal current driver with the torque controller, which is applied to control the BLDC motor, has been developed. Nevertheless the control law is based on the model with lumped parameters, the efficiency and performance are improved significantly. Unfortunately, the influence of the cogging torque, which causes a small oscillations in a torque response, is not considered by the presented driver. However, this subject is for a further study and it requires the modeling of self flux change of permanent magnets.

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