Application of response surface methodology and fuzzy logic based system for determining metal cutting temperature

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Abstract. The heat produced in metal cutting process has negative influence on the cutting tool and the machined part in many aspects. This paper deals with measurement of cutting temperature during single-point dry machining of the AISI 4140 steel, using an infrared camera. Various combinations of cutting parameters, i.e. cutting speed, feed rate and depth of cut lead to different values of the measured cutting temperature. Analysis of the measured data should explain the trends in temperature changes depending on changes in the cutting regimes. Furthermore, the temperature data is modelled using response surface methodology and fuzzy logic. The models obtained should determine the influence of cutting regimes on cutting temperature. The main objective is the reduction of cutting temperature, i.e. enabling metal cutting process in optimum conditions.

Key words: machining, cutting temperature, infrared thermography, response surface methodology, fuzzy logic.

1. Introduction

Cutting temperature is a primary factor affecting the cutting tool wear. It can also induce thermal damage to the machined surface since high temperature causes oxidation of the machined surface. Not only do intensive temperatures during the machining limit the tool life, but they also impair the machined surface by inducing tensile residual stresses, micro-cracks and thermal damage [1]. The affected layer has worse mechanical properties than the base material, and it also causes dimensional errors in the machined surface. The cutting tool elongates because of increased temperature, and the position of cutting tool edge shifts toward the machined surface. The result of this process is a dimensional error. The cutting fluid improves the tool life, surface conditions of the work-piece and the process as a whole. It also helps in carrying away the heat and the debris produced during the machining [2].

One of the most commonly analyzed modes of cutting is the orthogonal cutting (two dimensional cutting), in which the depth of the cut is constant and the cutting edge is a straight line, perpendicular to the direction of the relative motion between the edge and the specimen [3]. Nearly all the energy consumed in plastic deformation is converted into heat, which raises the temperature in the cutting area. Three main sources of heat (Fig. 1) can be defined in metal cutting process:

- plastic deformation by shearing in the cutting zone (heat source $S_1$),
- plastic deformation by shearing and friction between the cutting tool and the chip (heat source $S_2$),
- friction between the cutting tool and the machined surface (heat source $S_3$).

![Fig. 1. Heat generation zones during metal cutting process](image)

Total amount of heat produced in metal cutting process can be calculated as a sum of heat produced by the three mentioned sources:

$$Q = Q_{S_1} + Q_{S_2} + Q_{S_3},$$

where: $Q$ – total amount of generated heat, $Q_{S_1}$ – amount of heat generated by heat source $S_1$, $Q_{S_2}$ – amount of heat generated by heat source $S_2$, $Q_{S_3}$ – amount of heat generated by heat source $S_3$.

The heat balance during metal cutting process can be expressed as follows:

$$Q = Q_1 + Q_2 + Q_3 + Q_4,$$

where: $Q_1$ – amount of heat carried away in the chips, $Q_2$ – amount of heat remaining in the cutting tool, $Q_3$ – amount of heat passing into the work-piece, $Q_4$ – amount of heat radiated to the surrounding air.
According to empirical investigations, 60–86% of the heat is carried away in the chips and the percentage is growing with increasing of the cutting speed. For turning operations, this proportion is as follows: 50–86% of the heat is removed in the chip, 10–40% remains in the cutting tool, 3–9% heats up the work-piece and about 1% radiates into the surrounding air. If a coolant fluid is used during cutting, the heat carried away by the chip can even reach 90% of the total heat [4]. Experimental results show that in high speed machining, the heat transferred onto the work-piece is low.

Taking into consideration the thermo-viso-plastic law [5], in the case of large plastic deformations the temperature $T$, along with the effective plastic strain $\varepsilon$ and the effective plastic strain rate $\dot{\varepsilon}$ are the main factors in defining the flow stress $\sigma_{eq}$:

$$
\sigma_{eq} = \left(A + Be^n\right) \left[1 + Cln\left(\frac{\dot{\varepsilon}}{\dot{\varepsilon}_0}\right)\right] \left[1 - \left(\frac{T - T_{amb}}{T_f - T_{amb}}\right)^m\right], (3)
$$

where: $\dot{\varepsilon}_0$ – reference strain rate, $T_{amb}$ – room temperature, $T_f$ – melting temperature, $A$, $B$, $C$, $n$ and $m$ – rheological parameters to be identified.

One of the most frequently used methods for measuring temperature in metal cutting process is measuring with thermocouples. Basically, there are two types of thermocouples: embedded thermocouples and dynamic thermocouples [6]. The embedded thermocouple is a relatively cheap method which can be used to establish the distribution of temperature at different points in a cutting tool. The dynamic thermocouple can measure the temperature at the interface surface between the tool and the work-piece. In that way, two bodies in relative motion are used as the two elements of thermocouples.

The cutting temperature and the temperature distribution along the rake face of the cutting tool and the work-piece are essential factors in the study of machining processes due to their effect on the quality of the surface, the tool life, tolerances, metallurgical behavior and chip-removing rate. Davoodi and Hosseinzadeh [7] used an infrared high-speed sensor with specially designed software to measure the transferred heat to the work-piece during the high speed machining (HSM) of bronze alloys.

Sutter et al. [8] presented an experimental setup which is able to determine the temperature field in the cutting zone during an orthogonal machining operation, performed with a gas gun developed on the principle of pyrometry in the visible spectral range by using an intensified CCD camera.

The infrared method can be utilized to measure the temperature which occurs on the rake face of the cutting insert in a transient cooling process after the feed motion is halted [9].

Measuring techniques based on the projection of the infrared radiation from the tool chip interface onto the scanner of a high resolution thermographic camera allow for the determination of absolute temperature and its distribution in the contact zone between the tool and the chip flow ‘in-situ’, whilst avoiding real time mechanical contact [10].

The most important temperature from metal cutting process point of view is the maximum temperature of the cutting tool. This temperature directly affects cutting characteristics of the tool, deformation of the tool and work-piece, as well as the quality of the machined surface. It is obvious that the measuring of the rake face of the cutting insert in which maximum temperature occurs is not possible to achieve using an infrared camera because of continual presence of the chip covering the area of interest. When the values of the chip’s top temperature, the cutting depth and the physical properties of the work-piece are known, it is then possible (using finite-difference model, FEM analyses, or some other method) to calculate the maximum cutting tool temperature [11–14].

In this work, the chip’s top temperature was adopted as a relevant output parameter. This temperature was measured using the infrared camera during a turning operation, without using any coolant.

The methodology proposed was applied to a specific testing stand and the specific material used in the cutting process (steel). Hence, various material characteristics, such as yield stress, heat conductivity, specific heat, etc. are not included in this demonstration analysis. However, the methodology presented can be expanded onto a wider data set; it also offers the possibility of including various materials along with their characteristics.

The results of the experimental investigations have been presented and analyzed, leading to conclusions about the dependence between cutting regimes and corresponding temperatures. Next, cutting temperature data was modelled using response surface methodology (RSM) and fuzzy logic (FL). The RSM and FL models are able to predict cutting temperature for unknown cutting regimes in experiment space.

### 2. Experiment design, tool and material

The lathe used for examining and measuring is located in the Laboratory for Production Engineering, at the Mechanical Engineering Faculty in Niš, Serbia. The work-piece material is steel, with AISI designation 4140. The basic characteristics of this steel are shown in Table 1. This steel belongs to the group of doped, decent cold drown steels. The work-piece is in the form of a metal rod, with dimensions Φ45 x 250 mm.

<table>
<thead>
<tr>
<th>Chemical composition [%]</th>
<th>Tensile strength $R_m$ [N/mm²]</th>
<th>Hardness HB</th>
<th>Thermal conductivity $K$ [W/mK]</th>
<th>Specific heat $c$ [J/kgK]</th>
</tr>
</thead>
<tbody>
<tr>
<td>C</td>
<td>Cr</td>
<td>Mo</td>
<td>Mn</td>
<td>Si</td>
</tr>
<tr>
<td>0.40</td>
<td>1.00</td>
<td>0.20</td>
<td>0.90</td>
<td>0.25</td>
</tr>
<tr>
<td>1050</td>
<td>205</td>
<td>41.9</td>
<td>460.5</td>
<td></td>
</tr>
</tbody>
</table>
The component relations, information flow of material handling system and the linked information system for processing the data obtained are shown in Fig. 2.

SANDVIK Coromant cutting tool has been chosen: tool holder PCLNR 32 25 P12 in combination with the cutting insert CNMG 12 04 08 (grade 235), according to the recommendations of the manufacturer and empirical knowledge.

Jenoptik Varioscan 3021-ST infrared camera has been used for temperature measuring. Varioscan high resolution is a scanning thermovisics measured system for wave lengths outside of the vision spectrum – from 8 µm to 12 µm, i.e. in the area of infrared emission. Signal from this spectrum is amplified, digitalized with 16 bites and visualized. All colors on the thermogram represent particular temperatures. Temperature resolution of this system is 0.03°C, while the operating range of this camera is –40 to +1200°C.

The experiment setup and the sample of the obtained thermogram are shown in Fig. 3.

3. Results of temperature measuring experiments

Cutting temperature depends on the cutting environment conditions, as well as on cutting regimes. A large number of experiments were performed in order to model this parameter. At the beginning of the metal cutting process, the temperature rises until it reaches the maximum value. That is why the measurement should be done very shortly after the beginning of the process [15]. Following the rising and the arrangement of the temperature at the beginning of the process with an infrared camera, it is concluded that a period of about 60 seconds is enough for stabilizing the temperature measured. The thermograms are submitted to a PC memory card and analyzed. The maximum cutting temperature which occurs on the top of the chip is relevant to the measurement included in this experiment.

The ambient temperature during experimental investigations was 25°C. According to the literature, manufacturer recommendations and empirical knowledge, the following input parameters were adopted:

- cutting speed \( V \) (takes values of 80; 95; 110; 125 and 140 m/min),
- feed rate \( s \) (takes values of 0.071; 0.098; 0.196 and 0.321 mm/rev),
- depth of cut \( a \) (takes values of 0.5; 1; 1.5 and 2 mm).

The measured values of the cutting temperature – \( T \) has been divided into four groups, as it is the only way to present it in a three-dimensional diagram. The measured values of the cutting temperature – \( T \) has been divided into four groups, as it is the only way to present it in a three-dimensional diagram.

I group – temperature data obtained during cutting with feed rate of 0.071 mm/rev;
II group – temperature data obtained during cutting with feed rate of 0.098 mm/rev;
III group – temperature data obtained during cutting with feed rate of 0.196 mm/rev;
IV group – temperature data obtained during cutting with feed rate of 0.321 mm/rev.

3.1. Cubic polynomial interpolation of measured data. In order to model the temperature data, a surface fitting tool with
Table 2
Coefficients of the cubic polynomial interpolation equations

<table>
<thead>
<tr>
<th></th>
<th>V</th>
<th>a</th>
<th>V^2</th>
<th>V·a</th>
<th>a^2</th>
<th>V^3</th>
<th>V^2·a</th>
<th>V·a^2</th>
<th>a^4</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>-0.58</td>
<td>19.98</td>
<td>-2.79</td>
<td>-127.70</td>
<td>-37.98</td>
<td>50.49</td>
<td>299.80</td>
<td>86.58</td>
<td>108.10</td>
</tr>
<tr>
<td>III</td>
<td>-1.33</td>
<td>45.49</td>
<td>-7.06</td>
<td>-403.30</td>
<td>12.70</td>
<td>61.26</td>
<td>1189.00</td>
<td>-53.25</td>
<td>37.93</td>
</tr>
<tr>
<td>IV</td>
<td>0.59</td>
<td>0.56</td>
<td>-7.20</td>
<td>-20.29</td>
<td>23.99</td>
<td>56.57</td>
<td>128.90</td>
<td>-78.95</td>
<td>-11.63</td>
</tr>
</tbody>
</table>

Fig. 4. a) Measured values of cutting temperature, I group of data b) Polynomial interpolation of the measured values

Fig. 5. a) Measured values of cutting temperature, II group of data b) Polynomial interpolation of the measured values

Fig. 6. a) Measured values of cutting temperature, III group of data b) Polynomial interpolation of the measured values
A cubic polynomial interpolation has been used. The interpolation equation is

\[ T = b_0 + b_1 \cdot V + b_2 \cdot a + b_{11} \cdot V^2 + b_{22} \cdot a^2 + b_{111} \cdot V^3 + b_{112} \cdot V \cdot a + b_{222} \cdot a^3 \]  \hspace{1cm} (4)

The coefficients of the cubic polynomial interpolation equations for all groups of data are presented in Table 2.

The measured data, along with the interpolated values of the cutting temperatures are presented in Figs. 4–7. In order to evaluate the goodness of the fit, four statistical characteristics have been analyzed:

- sum of squares due to error (SSE),
- \( R \) – square,
- adjusted \( R \) – square,
- root mean squared error (RMSE).

The value of statistic characteristics, i.e. goodness of fit, for all groups of data is presented in Table 3.

### Table 3

<table>
<thead>
<tr>
<th>Group</th>
<th>SSE</th>
<th>( R ) – square</th>
<th>Adjusted ( R ) – square</th>
<th>RMSE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.001911</td>
<td>0.9869</td>
<td>0.9751</td>
<td>0.01383</td>
</tr>
<tr>
<td>II</td>
<td>0.002157</td>
<td>0.9831</td>
<td>0.9678</td>
<td>0.01469</td>
</tr>
<tr>
<td>III</td>
<td>0.002205</td>
<td>0.9662</td>
<td>0.9358</td>
<td>0.01485</td>
</tr>
<tr>
<td>IV</td>
<td>0.001581</td>
<td>0.9760</td>
<td>0.9545</td>
<td>0.01257</td>
</tr>
</tbody>
</table>

### 3.2. Main remarks

The lowest measured cutting temperature during experimental investigations was 214.82°C, obtained during cutting with the following regime: cutting speed 80 m/min, feed rate 0.071 mm/rev and depth of cut 0.5 mm (expected data, since the values of the input parameters are on minimum level). The highest measured cutting temperature was 594.02°C, obtained for the cutting regime: cutting speed 140 m/min and increasing feed rate in the range from 0.071 to 0.321 mm/rev, the cutting temperature increases by about 29%. In all experiments the depth of cut was kept at a constant level, \( a = 0.5 \) mm.

The rising of the feed rate also increases cutting temperature, which can be concluded from Fig. 8. This phenomenon is especially expressed at low cutting speeds. For example, during the machining with constant cutting speed of 80 m/min, and increasing the feed rate in the range from 0.071 to 0.321 mm/rev, cutting temperature increases by about 89%, while, in the same conditions during machining with the constant cutting speed of 140 m/min and increasing feed rate in the range from 0.071 to 0.321 mm/rev, the cutting temperature increases by about 29%.

Larger values of the depth of cut cause larger values of the resistant forces, which leads to the increase in the cutting temperature. Nevertheless, in this case, the percentage of the cutting temperature increase does not depend very much on the...
4. Modelling of metal cutting temperature

The extraordinary complexity of the mechanical, tribological, and thermodynamic phenomena in cutting zone does not allow for determination of a reliable and comprehensive theoretical model which could explain the essence and the mechanism of the chip formation and the shaping of the surface roughness. A theoretical approach is always based on simplifications and idealizations. It does not take into account any imperfections of the cutting process and neglects the effects of many process factors, as well as the environment factors (noise). Therefore, various theoretical models that have been proposed are not accurate enough and can be applied only to a limited range of processes and cutting conditions. For these reasons, most researchers mainly use empirical research [17–20].

As mentioned before, cutting temperature depends on a large number of factors, and it is almost impossible to include all of them in an integral mathematical model. Taking into consideration the fact that the most influential factors which affect cutting temperature are cutting speed, feed rate and depth of cut, cutting temperature can be presented in the following way:

\[ T = f(V, s, a) = K \cdot V^k_V \cdot s^k_s \cdot a^k_a, \]

Fig. 9. Cutting temperature increase depending on the depth of cut, \( V = 125 \ [\text{m/min}] \)

<table>
<thead>
<tr>
<th>Feed rate ( s ) [mm/rev]</th>
<th>Cutting temperature increase ( a = 0.5 \div 2 ) [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.071</td>
<td>44%</td>
</tr>
<tr>
<td>0.098</td>
<td>41%</td>
</tr>
<tr>
<td>0.196</td>
<td>27%</td>
</tr>
<tr>
<td>0.321</td>
<td>32%</td>
</tr>
</tbody>
</table>

Fig. 10. Cutting temperature increase depending on the cutting speed, \( a = 0.5 \) [mm]

<table>
<thead>
<tr>
<th>Feed rate ( V ) [m/min]</th>
<th>Cutting temperature increase</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>70%</td>
</tr>
<tr>
<td>95</td>
<td>42%</td>
</tr>
<tr>
<td>110</td>
<td>15%</td>
</tr>
<tr>
<td>125</td>
<td>16%</td>
</tr>
</tbody>
</table>

Feed rate. This is shown in Fig. 9. Cutting temperature increase is relatively uniform. For example, during the machining with the constant cutting speed of 125 m/min, with the feed rate of 0.071 mm/rev, increasing in the depth of the cut from 0.5 to 2 mm the cutting temperature increases by about 44%, while, at the feed rate of 0.321 mm/rev, the increase in the cutting temperature is about 32%. Thus, the conclusion is that the cutting temperature increase directly depends on the depth of cut, regardless of cutting speed.

Cutting temperature also depends on the cutting speed, and cutting speed increase causes an increase in the cutting temperature. This phenomenon is obvious at low values of feed rate. According to the Saglam et al. [16], when cutting speed is raised, the cutting forces are reduced but the temperature is increased. At the increased positive rake angle, the cutting forces are decreased, which means less force is required for the machining. However, the increase in cutting temperature does not depend much on the depth of cut. While cutting with the constant depth of cut \( a = 0.5 \) mm, increase in cutting speed from 80 to 140 m/min with feed rate of 0.071 mm/rev causes an increase of cutting temperature by about 70%, while cutting with feed rate of 0.321 mm/rev causes an increase in cutting temperature by about 16%. This occurrence is shown in Fig. 10.
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4.2. Modelling of the cutting temperature using fuzzy logic.

Mathematical and empirical modelling are tools very often used for predicting machining parameters. However, these techniques could be very complicated, demanding and time-consuming. Thus, some artificial intelligence based methods such as fuzzy logic, artificial neural networks, genetic algorithms, etc. can be employed for modelling tasks. Fuzzy logic is a mathematical theory of imprecise reasoning, which allows modelling of the human thinking by using linguistic terms. Detailed explanation of fuzzy logic theory is available in the literature [24–27]. There are many attempts to use fuzzy logic in optimizing metal cutting parameters in milling [28, 29], drilling [30, 31] and tapping processes [32]. Another goal of the FL based systems could be supporting decisions made by a human expert using dedicated fuzzy controllers [33, 34].

The correlations among cutting regimes and unknown cutting parameters (cutting forces, temperatures, etc.) are modelled using fuzzy rules extracted from the machinists’ handbooks, as well as the skills and knowledge of the experienced machinists. In this case, the model has three input values: \( x_1 \) – depth of cut, \( x_2 \) – feed rate \((s)\) and \( x_3 \) – cutting speed \((V)\). The fuzzy system has one output value: \( y \) – cutting temperature \((T)\), as shown on Fig. 12.

\[
y = a_0 + \sum_{i=1}^{n} a_i x_i + \sum_{i=1}^{n} a_{ij} x_i^2 + \sum_{i=1}^{n} a_{ij} x_i x_j + \ldots + \epsilon, \quad i < j,
\]

where \( y \) – response (output variable), \( a_0, a_i, a_{ij}, \ldots \) – unknown (adjustable) coefficients, \( n \) – number of model parameters, \( x_i \) – model parameters (input variables), \( \epsilon \) – random error.

The response surface model is a multivariate polynomial model which can be used to determine the correlation among different indicators of the metal cutting process on one side and the different cutting regimes on the other side [21]. In that way, it is possible to understand the quantitative relationship among the large number of input and output parameters [22].

In order to model the data using the RSM modelling tool, it is necessary to first perform the following transformations:

\[
\ln T = \ln (K \cdot V^{k_1} \cdot a^{k_2}) = \ln K + k_1 \ln V + k_2 \ln a.
\]

Cutting temperature’s dependence on cutting speed, feed rate and depth of cut should be presented with a linear mathematical model. According to the previous equation, the first step in the RSM modelling should be the calculation of the values: \( \ln T \), \( \ln V \) and \( \ln a \). From the obtained linear mathematical model, the coefficients from (12) can be calculated.

In this consideration, after the RSM modelling, the following coefficients are obtained: \( K = 42.93214 \), \( k_1 = 0.5509 \), \( k_2 = 0.1687 \) and \( k_3 = 0.2242 \). Finally, the RSM model for the considered experiment setup can be expressed in the following form:

\[
T = 42.93214 \cdot V^{0.5509} \cdot a^{0.1687} \cdot \ln 0.2242.
\]

The validation of the obtained model was performed over the whole data set, i.e. all four groups of data. The minimum error between the measured and the modelled data is 0.255%, the maximum error is 0.415%, while the average error is 6.14%. The conclusion is that the measured and the modelled values are in good accordance.

As it is well known, the design of experiment (DOE) involves a small sample size, i.e. a small number of experimental units. Using the two-level full factorial design, the number of runs can be reduced to \( 2^n \) (in this case \( 2^3 = 8 \)) experimental units [23]. This data represents geometrically the vertices of a cube in a chosen experimental space. The data used for modelling can be found in Table 4, while the experimental space is shown in Fig. 11.

<table>
<thead>
<tr>
<th>Run no.</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>Experimental results</th>
<th>( y )</th>
<th>( \ln y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>+1</td>
<td>+1</td>
<td>+1</td>
<td>594.02</td>
<td>6.38691</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-1</td>
<td>+1</td>
<td>+1</td>
<td>499.41</td>
<td>6.21343</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>+1</td>
<td>-1</td>
<td>+1</td>
<td>556.14</td>
<td>6.32102</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>-1</td>
<td>-1</td>
<td>+1</td>
<td>350.58</td>
<td>5.85959</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>+1</td>
<td>-1</td>
<td>-1</td>
<td>472.56</td>
<td>6.13816</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>-1</td>
<td>+1</td>
<td>-1</td>
<td>407.02</td>
<td>6.00886</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>+1</td>
<td>+1</td>
<td>-1</td>
<td>366.03</td>
<td>5.90272</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-1</td>
<td>-1</td>
<td>-1</td>
<td>214.82</td>
<td>5.36980</td>
<td></td>
</tr>
</tbody>
</table>

The reduced experiment setup gives the following model:

\[
T = 40.29591 \cdot V^{0.5884} \cdot a^{0.2178} \cdot \ln 0.2419.
\]

The validation of the reduced model was also performed on the whole data set (data obtained from 80 experiments). Minimum error in this case is 0.12%, while maximum error is 20.51%. The average error for this model is 7.14%.

The maximum and minimum errors of the reduced model are even smaller, but the average error has a slightly larger value than the average error of the model which uses the whole data set. Taking into consideration the fact that the reduced model requires only 10% of the overall number of experiments, it can be concluded that the reduced model is very promising. The time needed for data acquisition and data modelling is reduced in that way, thus reducing the overall cost of the experimental investigations.

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![Fig. 11. Experiment space with extreme values of the cutting parameters](image-url)
The ranges of the input and the output values are defined first. The minimum, the middle and the maximum values of the process parameters are given in Table 5.

<table>
<thead>
<tr>
<th>Input parameter</th>
<th>Minimum value</th>
<th>Middle value</th>
<th>Maximum value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Depth of cut a [mm]</td>
<td>0.5</td>
<td>1.25</td>
<td>2</td>
</tr>
<tr>
<td>Feed rate s [mm/rev]</td>
<td>0.071</td>
<td>0.196</td>
<td>0.321</td>
</tr>
<tr>
<td>Cutting speed V [m/min]</td>
<td>80</td>
<td>110</td>
<td>140</td>
</tr>
<tr>
<td>Cutting temperature T [°C]</td>
<td>214</td>
<td>404</td>
<td>594</td>
</tr>
</tbody>
</table>

Interface for fuzzification converts the crisp input value in the number between 0-1 using the fuzzy membership function, which describes the degree of membership of input value to fuzzy set. Fuzzy membership functions can have various shapes, e.g., triangular, trapezoidal, Gaussian etc. Each fuzzy set is defined by a different membership function. Membership functions used in this work are of Gaussian type, depending on two parameters: \( c \) and \( s \).

\[
\mu(x) = e^{-\frac{(x-c)^2}{2s^2}}, \quad (10)
\]

where: \( c \) – mean value (center) and \( s \) – standard deviation (width of the Gaussian curve).

Figure 13 shows the fuzzification of input variable “cutting depth”. Let us assume that the cutting depth is 1.5 mm. Fuzzification in the fuzzy set “cutting depth is large” gives the result 0.15, while the fuzzification in the fuzzy set “cutting depth is small” gives the result 0. All of the input and output variables must be fuzzified in different fuzzy sets before applying fuzzy rules.

The input and the output variables of the fuzzy system have different number of fuzzy membership functions. Each of the input parameters is defined with three fuzzy membership functions: low, medium and high for the cutting speed, as well as: small, medium and big for the depth of cut and the feed rate. The output variable (cutting temperature) has five membership functions: extremely low, low, medium, high and extremely high, as shown in Fig. 14.

![Fuzzy inference system](image1)

![Fuzzification of the input value in different fuzzy sets](image2)

![Fuzzy membership functions of input and output variables](image3)

![Graphical interpretation of one fuzzy rule](image4)

Human knowledge used by the fuzzy system is captured in the fuzzy IF-THEN rules. The IF part of the fuzzy rule is called the antecedent, while the THEN part of the rule is called the consequent. If the antecedent of the fuzzy rule has more than one part, fuzzy operators must be applied. The result is a unique value which represents the whole antecedent. Hence, inputs in fuzzy operator are different values than the membership functions \( \mu \) of the fuzzified input values. AND type fuzzy operator based on the extracting minimum value is applied subsequently. Graphical interpretation of one fuzzy rule is shown in Fig. 15.
The rule base for determining cutting temperature consists of 27 fuzzy rules, which are presented in Table 6.

Fuzzy inference is based on the common sense and it is very easy to understand. Using the proposed base of rules, the input values are mapped into the corresponding output value. The consequent is first reshaped using a number given by the antecedent. The input for the implication process is a single number, while the output is a fuzzy set. Implication is implemented for each rule. The common implication methods are: min (minimum), which truncates the output fuzzy set, and prod (product), which scales the output fuzzy set. In this case, the min implication method is employed.

The fuzzy sets obtained by the implication of each rule are combined into a single fuzzy set. This process is known as the aggregation and it occurs once for each output variable. The input of the aggregation process is truncated output function returned by the implication process. The aggregation process produces one fuzzy set for each output variable. The most frequently used aggregation methods are: max (maximum), probor (probabilistic OR) and sum (the sum of each rule’s output set). The max aggregation method, which calculates the union of the combined fuzzy sets, was used in this study.

The final step is defuzzification, i.e. converting the fuzzy set obtained by the aggregation into a single number. Fuzziness helps the rule evaluation during the intermediate steps, but the final output is a single number. There are several methods for defuzzification, such as: centroid, bisector, middle of maximum (the average of the maximum value of the output set), largest of maximum, smallest of maximum... The defuzzification method used in this paper is the centroid, which calculates the center of the area enclosed by the resulted curve, and this number represents the defuzzified value.

![Fig. 15. Graphic interpretation of one fuzzy rule](image)

**Table 6**

<table>
<thead>
<tr>
<th>Feed rate</th>
<th>Cutting speed</th>
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<tbody>
<tr>
<td></td>
<td>L</td>
</tr>
<tr>
<td>Depth of cut</td>
<td>S</td>
</tr>
<tr>
<td>S</td>
<td>EL L M L M M M M H</td>
</tr>
<tr>
<td>M</td>
<td>L M M M M H M H H</td>
</tr>
<tr>
<td>B</td>
<td>M M H H H EH EH EH</td>
</tr>
</tbody>
</table>

L – low, M – medium, H – high, S – small, B – big, EL – extremely low, EH – extremely high
The same set of data which was used for testing the RSM model is used for testing the FL model. The surface that represents modelled values of the cutting temperature is shown in Fig. 16.

Measured and modelled values of the cutting temperature are shown in Fig. 17. The minimum error obtained by the FL system is 0.243%, while the maximum error is 18.372%. The average error of the whole data set is 6.258%. This result is slightly better than the result obtained by the RSM for a model using extreme values of cutting parameters, proving in that way that the systems based on fuzzy logic can be successfully used in modelling of metal cutting parameters.

5. Conclusion

In this work, a large number of experiments have been performed in order to model cutting temperature. Measured values have been presented and interpolated using cubic polynomial interpolation. The indicators of the quality of fit show good interpolation and prediction capabilities of the temperature models. The main remarks about the dependence of cutting temperature on input cutting parameters have been made. The response surface model was created after that as a multivariate polynomial model of cutting temperature in order to establish the correlation among cutting temperature and cutting regimes. Finally, the system based on fuzzy logic was presented in order to model temperature data according to cutting regimes changes. The results are in good accordance with the experimentally obtained data, confirming the effectiveness of the proposed models in the modelling of cutting temperature. Comparing experimentally obtained data with the RSM and FL modelled data proves that the fuzzy system shows a slightly better performance.
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References