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# EXPERIMENTAL ANALYSIS OF PROPAGATION OF PLASTIC ZONES IN TWO-DIMENSIONAL PROBLEMS

In the paper, the author presents experimental analysis of propagation of plastic zones in two-dimensional models with different stress concentrators. The experimental tests were carried out by photoelastic coating method on duralumin stripes loaded by tensile stresses. For various levels of loading, the photographs of isochromatic pattern were taken under loading and after removing loading. On the basis of isochromatic pattern recorded for loaded models, the boundaries of plastic zones were determined using the Treska-Coulomb yield condition. The isochromatic pattern taken for the unloaded, but previously partly plastified elements, show the picture of the residual strain remaining in the material. A discussion of the results is presented.

## 1. Introduction

The development of modern technology allows for more economic use of the constructional material. It is connected with common acceptation of small, local plastic deformation in elements working in the range of the limited fatigue strength. That is why the analysis of elastic-plastic strain states has many important applications in engineering design.

In general, the quantitative analysis of strain and stress in the plastified zones concerns any object working in over-elastic range. It is necessary to distinguish, however, some specific elastic-plastic problems, which are important, from the practical point of view, e.g. determining the bearing capacity of elements or the analysis of residual strain.

Determining the bearing capacity of the element means, strictly speaking, finding the value of loading level, for which one of its cross-sections becomes completely plastified, and the element is losing its usefulness as a part of the structure. It is, in many cases, difficult, especially for elements of complicated

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shapes and subjected to complex loading. Therefore, nowadays the extreme principles of the theory of elasticity are applied, which make it possible to find upper and lower estimation of bearing capacity. The lower estimation of bearing capacity can be found by assuming, in the analyzed area, a statically admissible stress field. The upper estimation of bearing capacity can be obtained by accepting any, kinematically admissible mechanism of elements failure.

The acceptance of partial plastifying of material in constructional elements during exploitation causes the necessity of solving nonlinear problems. In such cases, even extensively developed and widely used, numerical methods can not assure full and reliable result due to modeling problems. To verify numerical calculation and to supply information for hybrid methods, experimental testing is very useful [14].

Experimental investigation of bearing capacity concerns solving typical, technical problems, like testing of point welds [18, 17] or railway carriage frames [12]. It may also concern, in more general sense, the analysis of propagation of plastic zones created around stress concentrators (holes and notches) [4, 5, 11]. Their uncontrolled expansion can conduce to elements failure.

Bearing capacity problems are often associated with elastic-plastic investigations from the domain of cracking mechanics or low-cyclic fatigue. They concern, for example, the analysis of plastic zones developing nearby a crack in a slot bottom [13, 16].

Another important problem of elastic-plastic analysis is the assessment of residual (plastic) strain existing in unloaded material after its previous loading over yield point. The experimental investigations in this field concern testing the mechanical properties of constructional materials which can change as the result of the residual strain [3]. They could also be applied to the exploitation investigation of actual structures [1] and the analysis of residual strain cumulation under cyclic loading [2, 20].

## 2. Experimental testing

### 2.1. Method of photoelastic coating

One of the experimental methods, which can be applied to the analysis of elastic-plastic states, is the photoelastic coating method. This method gives information about the deformation of the real object in the whole tested area (not only at several points) and can be used to investigate objects of complex shapes and loaded in different ways, also in the over-elastic range of material.

The photoelastic coating method makes use of the effect of the optical birefringence which occurs in some transparent materials under loading [19]. It is based on the assumption that there is univocal dependence between the strain occurring on the surface of the analyzed element and the deformation of the thin layer of birefringent material integrally bonded to this surface. When the object is loaded, the surface strains are transmitted to the coating, which may be observed through a reflection polariscope. In that case, loaded coating exhibits two families of fringe patterns: isoclinic fringes providing information about directions of principal strains, and isochromatic fringes supplying information about the difference of principal strains.

The method of photoelastic coating can be applied to the elastic-plastic states analysis, because of the linear relation between the photoelastic effect and strain in the birefringent materials, valid in a wide range. In the range of strain where the material of the constructional element is already plastified, the characteristic of photoelastic coating material is still linear.

The information obtained from the photoelastic coating method is not sufficient for determining all strain components in general case of two-dimensional state of stress occurring on the surface of the investigated object. Besides, the measurement of isoclinic parameter is labour-consuming and usually not precise. Therefore, to make the analysis of full strain state, one often uses additional information obtained from other experimental methods or from analytical (or numerical) calculations.

The examples of determining the strain and stress values in elastic-plastic areas around stress concentrators in two-dimensional models can be found in another paper by the author [10]. For strain separation on the basis of isochromatic pattern only, the author applied the analytical method of characteristics, and the strain and stress components were calculated using multi-sectional schematization of the material ($\sigma$-$\varepsilon$) curve [6].

The most important advantage of the method, especially with reference to the investigation of creation and development of plastic zones, is an excellent visualization and possibility of direct observation of the progressing plastifying process.

## 2.2. Models of constructional elements

The investigation of propagation of plastic zones by photoelastic coating method was performed on two-dimensional models of constructional elements weakened by different stress concentrators (holes) and subjected to tensile stresses – Fig. 1.

The elements of this type and loaded in this way are often used in various structures, particularly as construction joints (e.g. in the aerospace industry [15]).

The models were made of duralumin sheet 3 mm thick, from which stripes of 100 mm in width and 450 mm in length were cut out. The length of the stripes was assumed large enough to compensate potential non-uniformity of distribution of tensile stresses applied at their ends.

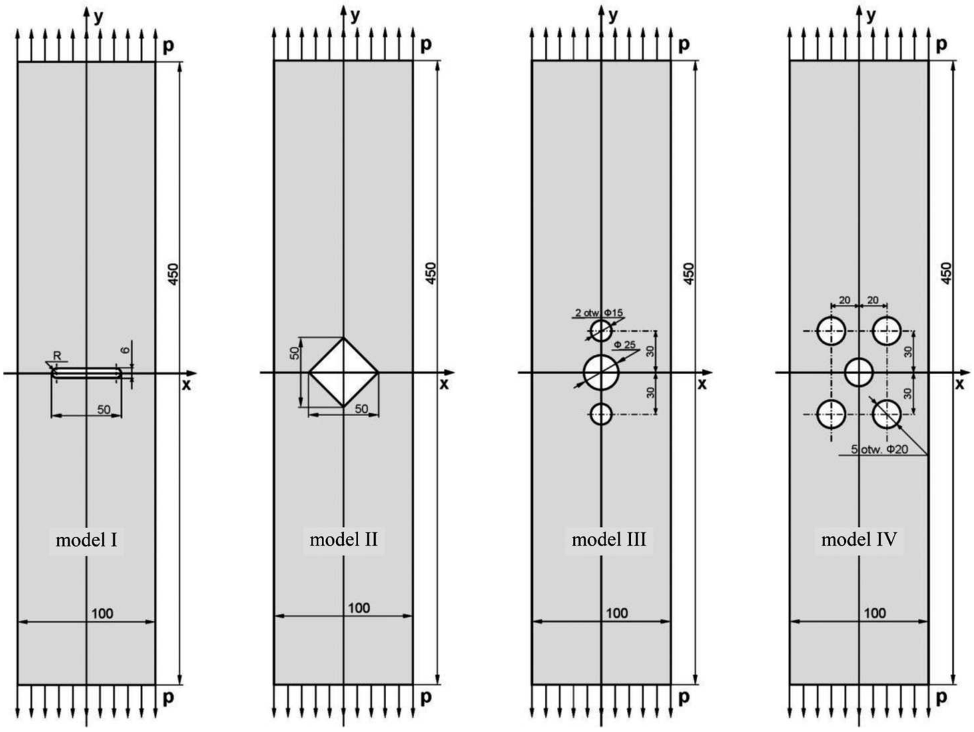

Fig. 1. Models of constructional elements

After mechanical working and special surface preparation (grinding and etching), the models were covered on both sides (to avoid bending effect) with the photoelastic coating made of epoxy resin. Next, different holes were cut out in the way which allowed avoiding the stresses arising as the result of machining. Shapes of the stress concentrators were designed on the basis of engineering practice (single central holes of various shapes and groups of circular holes of various configurations) – Fig. 1.

The characteristic of the material (duralumin alloy EN-AW-2024) was determined experimentally on the basis of standard static uniaxial tensile test (according to PN-EN 10002-1 for the test at ambient temperature). It is shown in Fig. 2.

The strain constant of the photoelastic coating: $f_\varepsilon = 1.114 \cdot 10^{-3}$ 1/fringe order, was determined experimentally.

The models were loaded at their ends with uniformly distributed tensile stresses (p). As the measure of loading intensity, the 'loading factor' (s) was accepted. It was calculated as a ratio of the average tensile stresses at the cross-section weakened by the hole (on the axis of symmetry perpendicular to the stretching direction) in relation to the offset yield strength $R_{0.2} = 182$ MPa (taken from the material characteristic).

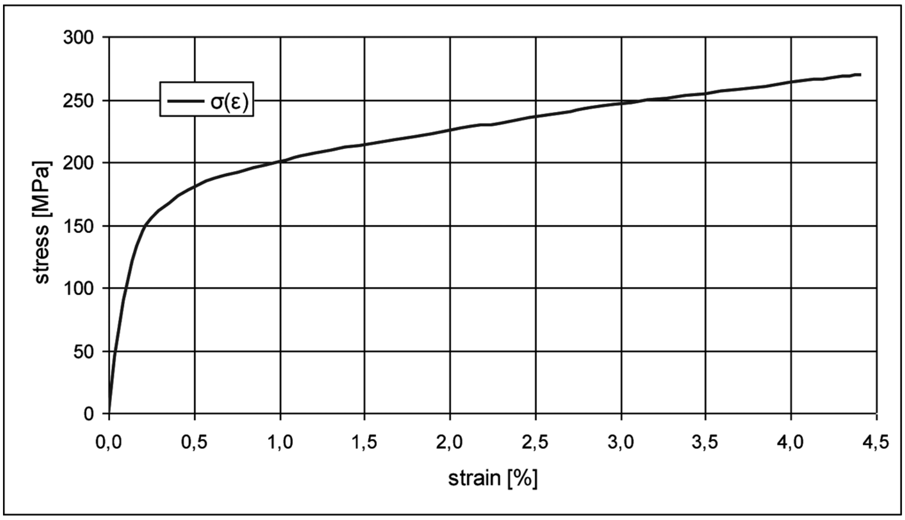

Fig. 2. Material characteristic

The loading of the models was increased step by step within the over-elastic range of material and at selected levels of loading the photographs of isochromatic pattern were taken twice: for both dark- and light-field polariscope. After every successive level of loading, the models were completely relieved of load, and the images of residual isochromatic pattern were also recorded.

## 3. Analysis of propagation of plastic zones under loading – determining the element's bearing capacity

The boundaries of plastic zones can be determined in a simple way using only the isochromatic pattern, when signs of the principal stress components can be predicted before their separation. When the principal stress components have different signs or if they are of the same sign, but one of them is much greater than the other, the order of isochromatic fringe which corresponds to the boundary of the plastic zone can be calculated using the Treska-Coulomb yield criterion [21]. Applying the Hooke's law and Wertheim's law, one obtains

$$m_{gr} = \frac{1+\nu}{E \cdot f_{\varepsilon}} \sigma_{pl} \tag{1}$$

where: $\sigma_{pl}$ – yield point of the model's material;
$\nu$ – Poisson ratio of the model's material;
E – modulus of elasticity of the model's material;
$f_{\varepsilon}$ – strain constant of the photoelastic coating.

On the basis of the above formula and taking into account properties of the photoelastic coating material and the model material, the value of boundary isochromatic fringe order was calculated as: $m_{gr} = 3.04$. For the model material, the offset yield strength $R_{0.2} = 182$ MPa was assumed as the yield point $\sigma_{pl}$.

Exemplary pictures of isochromatic pattern taken for dark-field polariscope at selected levels of loading, at which visible zones of the plastified material near the holes appeared, are shown in Fig. 3, Fig. 4, Fig. 5 and Fig. 6. These pictures were recorded in monochromatic light of wavelength $\lambda = 540$ nm, obtained by transmitting white light through a monochromatic filter. Because of the double symmetry of the model and loading, only one quarter of the image is considered.

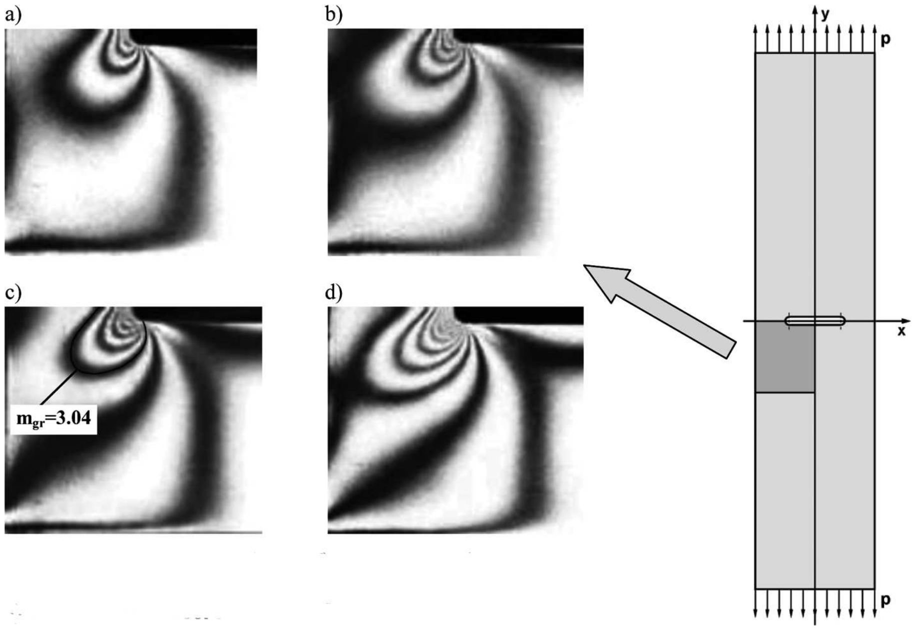

Fig. 3. Isochromatic pattern (model I) at loading level: a) s=0.733, b) s=0.806, c) s=0.879, d) s=0.952

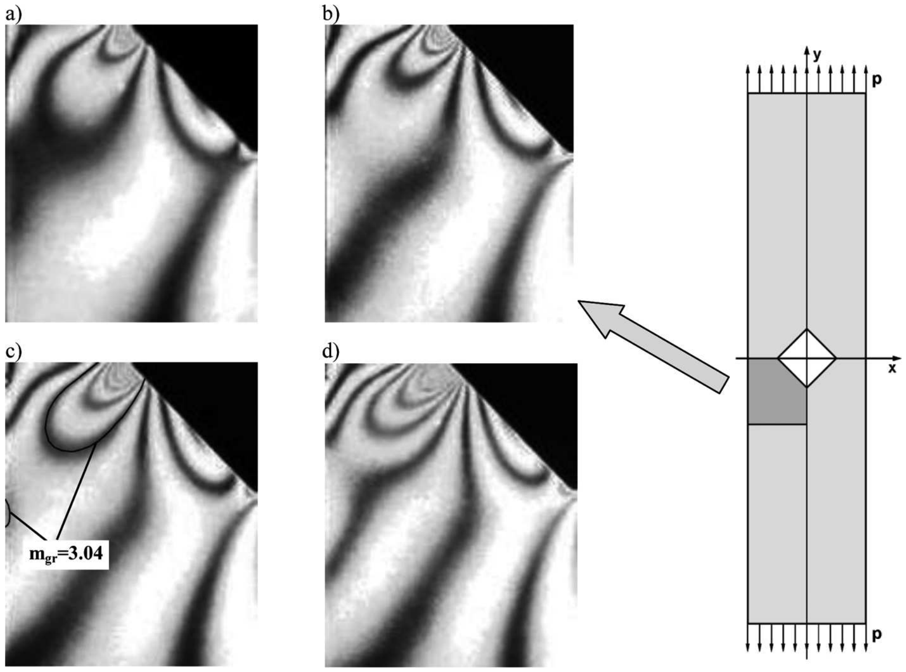

Fig. 4. Isochromatic pattern (model II) at loading level: a) s=0.733, b) s=0.806, c) s=0.879, d) s=0.952

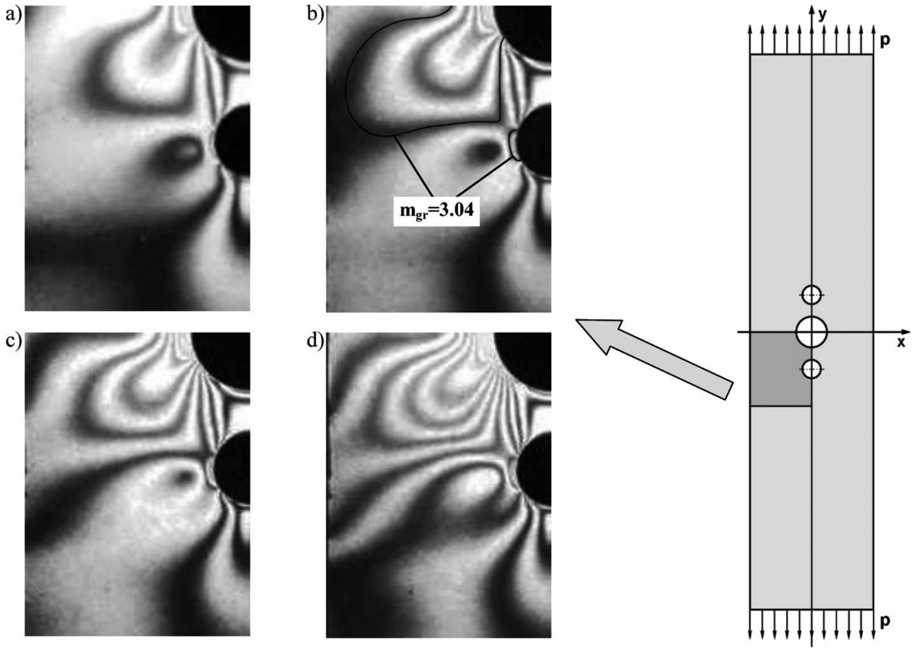

Fig. 5. Isochromatic pattern (model III) at loading level: a) 0.879, b) s=0.928, c) s=0.977, d) s=1.026

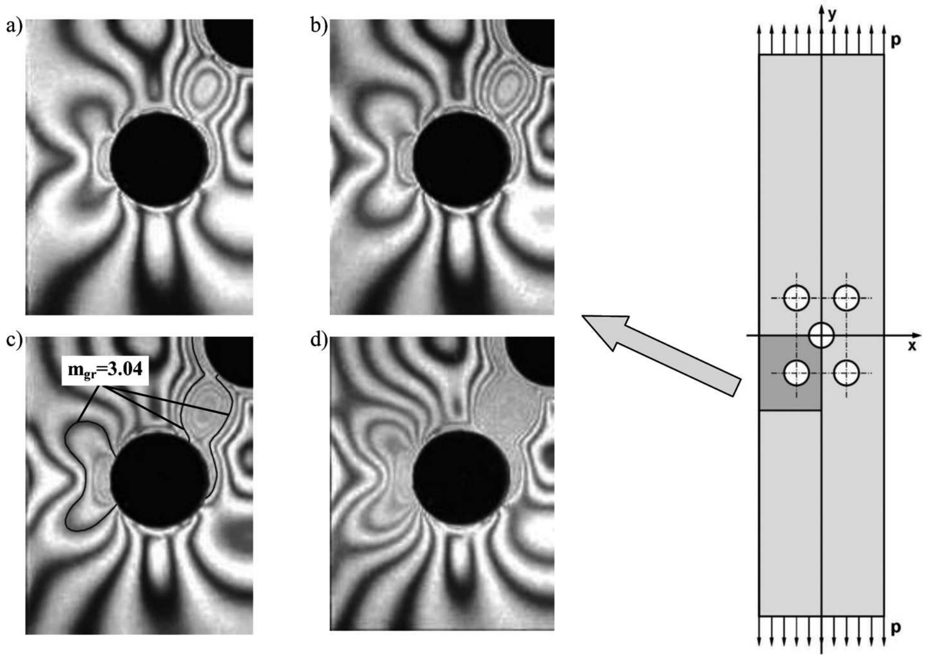

Fig. 6. Isochromatic pattern (model IV) at loading level: a) s=0. 595, b) s=0.641, c) s=0.687, d) s=0.733

In Fig. 3c, Fig. 4c, Fig. 5b and Fig. 6c, there are marked exemplary contours of boundary isochromatic pattern ($m_{gr}$ =3.04) for each model at one of loading levels. The contours of plastic zones determined on the basis of boundary isochromatic pattern for all selected, increasing loading levels are shown in Fig. 7 and Fig. 8.

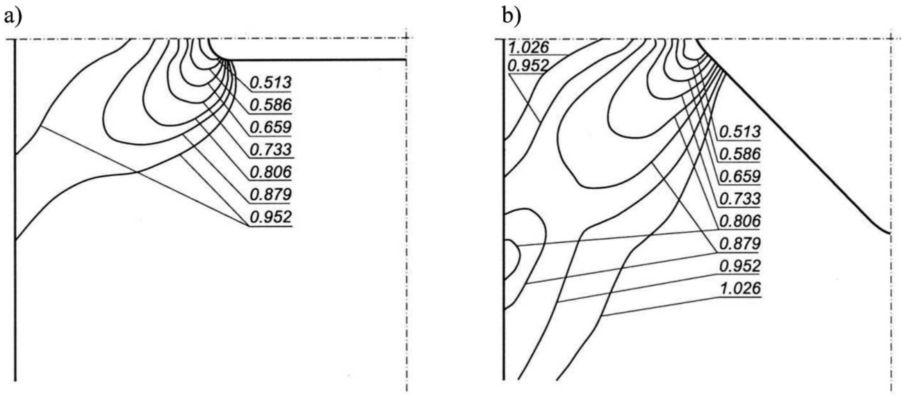

Fig. 7. Boundaries of plastic zones for models with one, central hole: a) model I – with slot, b) model II – with square hole

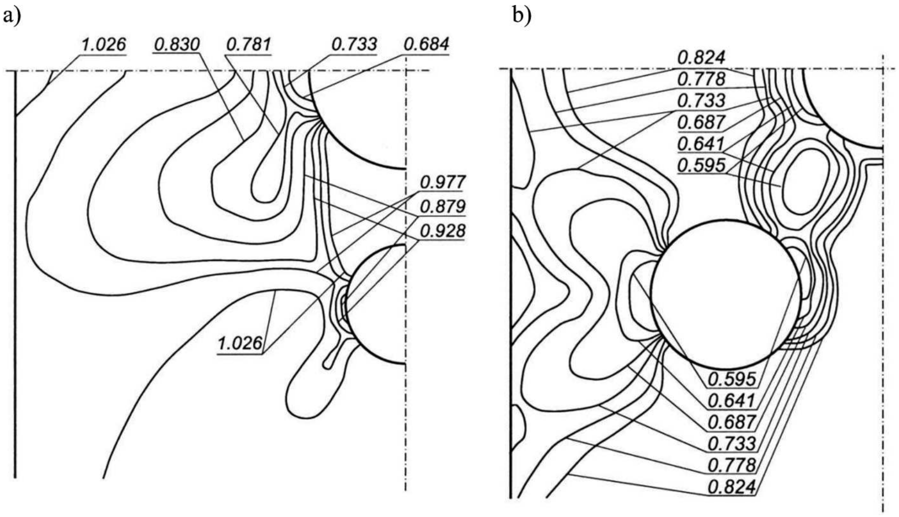

Fig. 8. Boundaries of plastic zones for models with groups of holes: a) model III – with three holes, b) model IV – with five holes

The ‘loading factors’ (s) for loading levels shown in Fig. 3, Fig. 4, Fig. 5 and Fig. 6 correspond to the applied tensile stresses: 67 MPa, 73 MPa, 80 MPa, 87 MPa for model I and model II; 120 MPa, 127 MPa, 133 MPa, 140 MPa for model III and 87 MPa, 93 MPa, 100 MPa, 107 MPa for model IV, respectively.

The distributions of plastic zones created in two dimensional elements with different stress concentrators, presented in Fig. 7, and Fig. 8, show that the process of material plastifying not always develops in a predictable way [7, 9].

In the models with one hole of various shapes, propagation of plastic zones is similar, differing only slightly in the angle of deflection from the axis of symmetry perpendicular to the stretching direction (in both cases, this angle is close to 45°) and in the intensity of plastic strain distribution. It results from differences in rounding radii of the hole bottom. Square hole (with a minimum radius of rounding) causes a greater concentration of plastic strains in the bottom of the notch, while the increase of strains at a distance from the bottom is very mild. In the element with a slot, whose bottom has a radius equal to a half of the slot’s width, the propagation of plastic zones is more uniform – the concentration of plastic strains in the bottom of the notch is lower, but their increase in the whole plastic zone is steeper.

In elements with more complicated schemes of weakening (groups of holes) it is difficult to predict in which direction and how intensively the plastic zones start to propagate at different loading levels, because the initial direction of propagation is not always kept. This depends mostly on the number of holes, their diameter and spacing. For example, in the model III, starting from the load level s=0.879, at the moment when plastic zones around the smaller holes appear, the direction of development of plastic zones around the central hole apparently changes (Fig. 8a). One can get the impression that later formed, smaller plastic zones begin to 'push away' the main zone, which starts to propagate in the direction inclined at a smaller angle to the x axis of symmetry. In the model IV (with five holes) one can observe even more variety concerning the number of plastic zones and directions of their development.

Localization of the areas where first plastic strains are created and directions of their development allow estimating the level of dangerous loading, which can lead to formation of "plastic joints" in the weakest cross-sections of the element.

For theoretical estimation of the limits of bearing capacity, we assume the rigid-perfectly plastic model of material characteristic. Taking into account real material characteristic, the value of loading level accepted as bearing capacity can be conventionally defined as the force at which the yield point of the material is exceeded in the whole element's cross-section, but maximum stresses (or strains) in this cross-section do not exceed a certain assumed value.

## 4. Analysis of propagation of plastic zones in unloaded (released) elements

Another very important problem in experimental investigation of elastic-plastic states is the analysis of residual strain remaining in unloaded element after loading over the yield point.

The photoelastic coating method also provides the largest possibilities for estimating the extensiveness and intensity of the residual (plastic) strain.

Analysis of the development of residual (plastic) strain around the stress concentrators was carried out for the models I, II, III and IV (Fig. 1). During the test, after each successive level of loading in over-elastic range, the models were completely relieved. Then, the images of isochromatic pattern – the result of permanent material plastifying, were recorded again.

Exemplary pictures of isochromatic patterns in the models I, II, III and IV (in monochromatic light for dark-field polariscope) obtained after relieving of load for selected levels of loading are shown in Fig. 9, Fig. 10, Fig. 11 and Fig. 12. Because of the double symmetry of the model and loading, only one quarter of the image is considered.

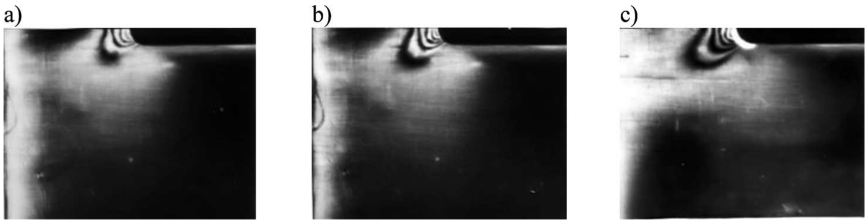

Fig. 9. Isochromatics for model I released of load at the level: a) s=0.806, b) s=0.879, c) s=0.952

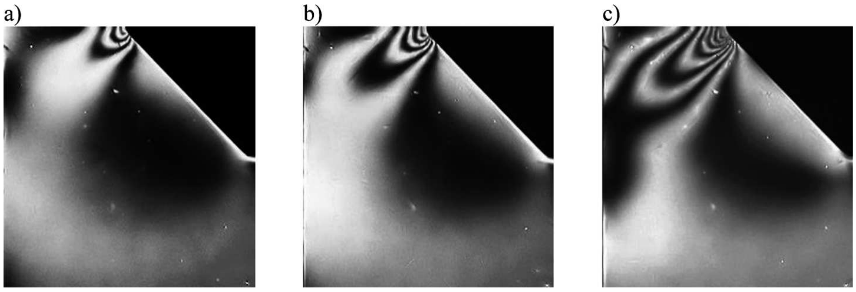

Fig. 10. Isochromatics for model II released of load at the level: a) s=0.879, b) s=0.952, c) s=1.002

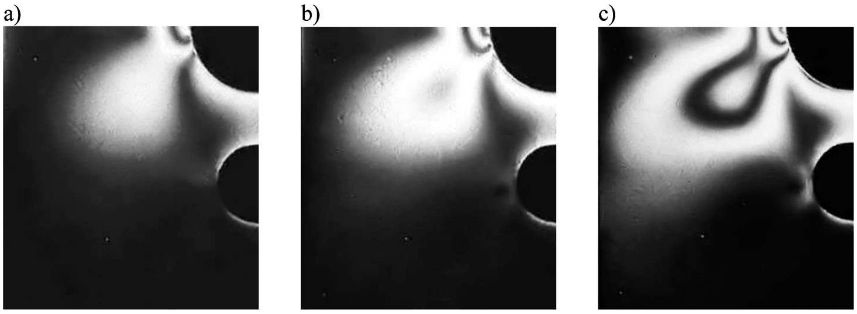

Fig. 11. Isochromatics for model III released of load at the level: a) s=0.830, b) s=0.879, c) s=0.928

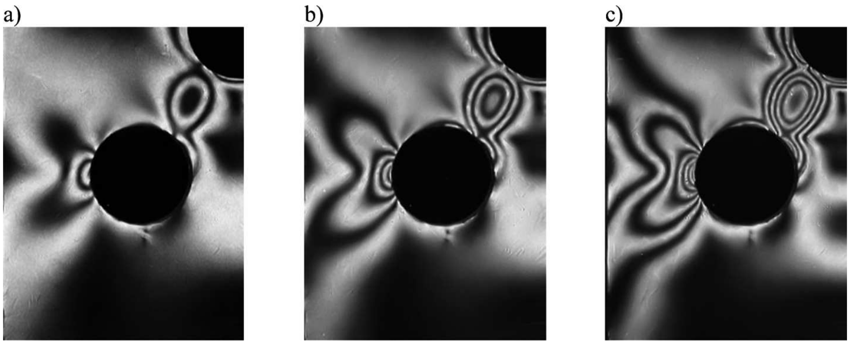

Fig. 12. Isochromatics for model IV released of load at the level: a) s=0.687, b) s=0.733, c) s=0.778

The boundaries of plastic zones determined on the basis of residual isochromatic pattern for the models I and II (with one hole of different shapes but the same cross-section) are shown in Fig. 13. Fig. 14 presents boundaries of plastic zones, obtained in a similar way, in model III and model IV – with groups of circular holes.

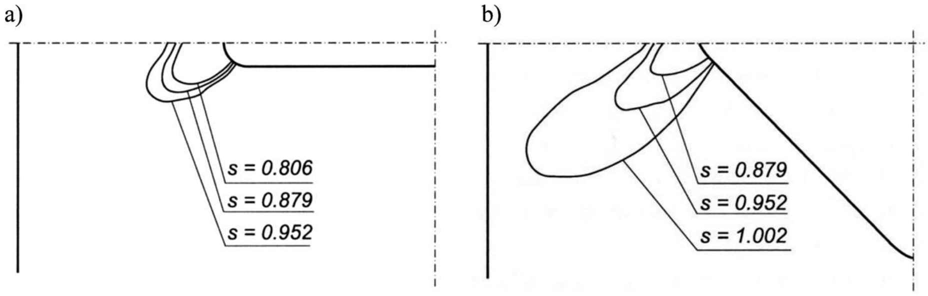

Fig. 13. Boundaries of residual plastic zones for models with one, central hole: a) model I – with slot, b) model II – with square hole

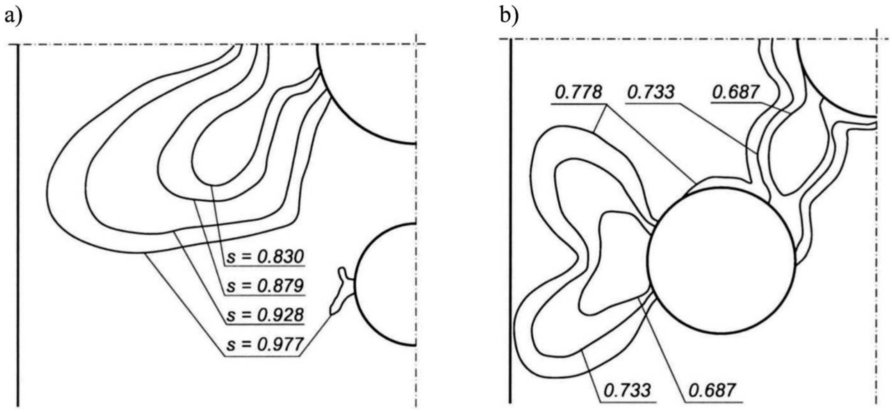

Fig. 14. Boundaries of residual plastic zones for models with groups of holes: a) model III – with three holes, b) model IV – with five holes

Comparing the obtained results with those found under the same loading (Fig. 7, 8), we can conclude that the areas of residual strain in the unloaded models are smaller than the plastic zones in the models under loading, although their shape and direction of propagation is similar ([8, 9]). The fact that the range of plastified material is smaller in the unloaded (released) element can be explained by the reaction of the areas remaining in the elastic state surrounding plastic zones. It is due to the way of the material releasing from the point of characteristic in the over-elastic range which proceeds along a straight line parallel to this defining elastic state. As a result, the residual strain remaining in the material is smaller than the plastic strain in the corresponding point on the $\sigma$-$\varepsilon$ curve.

## 5. Conclusions

Proper assessment of the plastic zones, which form and develop in the constructional elements working in the over-elastic range of material makes it possible to determine the factor of safety for the whole system. As long as these zones have a local character, the structure is not endangered by failure. On the other hand, in the result of development of large plastic deformations the element becomes a pseudo-mechanism, which cannot work exactly as intended.

The process of development of plastic zones in constructional elements may take a variety of forms, depending on the geometry of these elements, the shape of stress concentrators and the way of loading.

The analysis of propagation of plastic zones under increasing loading makes it possible to determine the true value of limiting loading and experimental verification of the numerically-calculated bearing capacity of the constructional elements. It is necessary to mention that bearing capacity is described as extreme loading of the element (in considered cases – the extreme tensile loading).

The analysis of the areas of residual (plastic) strain created in unloaded (released) elements allows us, among other things, to check whether these elements have the possibility of working after a temporary, emergency overload exceeding the working value (after which a residual strain may appear). It also allows us determining the directions of propagation of plastic zones at the cyclic work of the element in over-elastic range.

Comparison of the borders of plastic zones created under loading in the elastic-plastic range with the areas of residual strain in the element released of load at the same level gives additional information about the mechanisms of the material plastifying.

Photoelastic coating method allows relatively easy and quick identification of boundaries of the plastic zones. This gives the possibility of direct observation of the process of forming and developing of plastic strain, its intensity and direction of propagation.

This method also allows us, through the observation of residual isochromatic pattern, to analyze the development of residual (plastic) strain zones that appear in the unloaded constructional elements in the result of previous loading beyond the yield point.

Thus, the method of photoelastic coating is particularly useful for investigating specific elastic-plastic problems, especially in the cases where the study of development of plastic zones is necessary for defining the plastic failure mechanism. It takes place, for example, in the constructional elements with stress concentrators.

Taking into account also the suitability of the photoelastic coating method to the full-field quantitative analysis of strain and stress in over-elastic range of material [6, 10], we may assume that the method is a good tool for universal and complex experimental investigation of processes in which a part of the constructional element is in plastic state.

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## Eksperymentalna analiza propagacji stref plastycznych w zagadnieniach dwuwymiarowych

### Streszczenie

W pracy przedstawione zostały wyniki badań przeprowadzonych metodą elastooptycznej warstwy powierzchniowej na płaskich modelach elementów konstrukcyjnych osłabionych koncentratorami naprężeń (otworami) o różnych kształtach. Modele, wykonane z duraluminium, zostały obciążone równomiernie rozłożonymi na końcach naprężeniami rozciągającymi wywołującymi częściowe uplastycznienie materiału. Na wybranych poziomach obciążenia zostały zarejestrowane obrazy izochrom, które posłużyły do wyznaczenia granic obszarów uplastycznionych metodą izochromy granicznej (przy wykorzystaniu hipotezy Treski-Coulomba). Po każdym etapie obciążenia, modele były całkowicie odciążane i wtedy ponownie rejestrowano obrazy izochrom (resztkowych), które odzwierciedlają stan odkształceń trwałych pozostałych w materiale po obciążaniu modelu powyżej granicy plastyczności. W pracy została przeprowadzona dyskusja otrzymanych wyników.