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Magnetic field of complex helical conductors

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Abstract: Transmission of the electric power is accompanied with generation of low – frequency electromagnetic fields. Electromagnetic compatibility studies require that the fields from sources of electric power be well known. Unfortunately, many of these sources are not defined to the desired degree of accuracy. This applies e.g. to the case of the twisted-wire pair used in telephone communication; already practiced is twisting of insulated high-voltage three phase power cables and single-phase distribution cables as well. The paper presents a theoretical study of the calculation of magnetic fields in vicinity of conductors having helical structure. For the helical conductor with finite length the method is based on the Biot-Savart law. Since the lay-out of the cables is much more similar to a broken line than to strait line, in the paper the magnetic flux densities produced by helical conductor of complex geometry are also derived. The analytical formulas for calculating the 3D magnetic field can be used by a software tool to model the magnetic fields generated by e.g. twisted wires, helical coils, etc.

Key words: Biot-Savart law, magnetic flux density, helical line

1. Magnetic field calculation

An analytical method for calculating the low-frequency magnetic field of an infinitely long helical line current using the magnetic vector potential has been derived in the pioneering work [1] and the problem has afterwards been revisited in [2-6], [8-12]. Few papers take up the problem of finite length helical conductors [13-16]. However the lack of the basic data as regards the parameters of the system considered (e.g. the helix pitch in [15]) precludes the comparison of results presented in these papers with results obtained by the use of the method derived. None of papers published till now raise the subject of magnetic fields of helical conductors formed complex paths.

The realistic model of twisted cables should be based on the theory of a helical line current of finite length. Moreover, the lay-out of the cables is much more similar to a broken line than to strait line. In the paper the magnetic flux density produced by finite length helical conductor of complex geometry is derived. It is assumed, that the helical conductor is replaced by an infinitely thin filament in its axis, so that the mutual interaction on the current density distribution in twisted wires can be neglected. The analytical formulas for calculating the 3D magnetic field with respect to a convenient and unique reference system are derived.

The magnetic field can be obtained using the Biot-Savart law:

$$\vec{B}(x, y, z) = \frac{\mu_0}{4\pi} \int_c \frac{\underline{I}(\vec{dl} \times 1_r)}{r^2}, \tag{1}$$

where $\underline{I}$ is a phasor current, the vector element $\vec{dl}$ coincides with the direction of the current I, 1_r is a unit vector in the direction of vector $\vec{r}$, r is the distance between the source point and the observation point and μ_0 is the magnetic permeability of the vacuum.

Consider the arbitrary configuration of a helical line. For calculation purposes, the helical line can be divided into straight segments. For simplicity consider only the i-th line segment mathematically represented as a helix. It is convenient to define two different Cartesian reference systems: the first one x, y, z is a reference system (external reference system), the second one x', y', z' (local system) is referred to the i-th segment, Figure 1. It should be noted, that the reference coordinate system (unprimed) can be arbitrary located in the space.

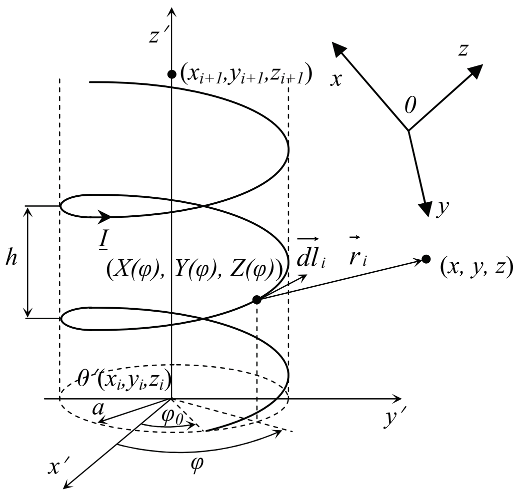


Fig. 1. Reference systems and the i-th segment of the helix

The point of origin of the primed coordinate system $0'$ (outset of the helix axis) have in the external (unprimed) reference system the coordinates (x_i,y_i,z_i), whereas the end point of the i-th segment axis has the coordinates $(x_{i+1},y_{i+1},z_{i+1})$, respectively.

The parametric equations of the helical line with respect to the parameter φ ($\varphi_0 \le \varphi \le$ $\le 2\pi L_i/h + \varphi_0$) indicated on Figure1 are:

$$X'(\varphi) = a\cos\varphi, \quad Y'(\varphi) = a\sin\varphi,$$
$$Z'(\varphi) = \frac{h}{2\pi}(\varphi - \varphi_0), \tag{2}$$

where a is the helix radius, h means the helix pitch, φ_0 is the φ co-ordinate of the point where the helix intersects the plane $z' = 0$, L_i is the length of the i-th helix segment and

$$L_i = \sqrt{(x_i - x_{i+1})^2 + (y_i - y_{i+1})^2 + (z_i - z_{i+1})^2}\ . \tag{3}$$

To obtain the Equation (2) in the reference coordinates system, the roto-translation formulas in the tridimensional space should be applied. Thus:

$$\begin{bmatrix} X(\varphi) \\ Y(\varphi) \\ Z(\varphi) \end{bmatrix} = \begin{bmatrix} \alpha_{1i} & \beta_{1i} & \gamma_{1i} \\ \alpha_{2i} & \beta_{2i} & \gamma_{2i} \\ \alpha_{3i} & \beta_{3i} & \gamma_{3i} \end{bmatrix} \begin{bmatrix} X'(\varphi) \\ Y'(\varphi) \\ Z'(\varphi) \end{bmatrix} + \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix}, \tag{4}$$

where generally: α, β, γ are the direction cosines of the rotated X'-, Y'- and Z'-axis relative to the original X-, Y-, Z-axes, respectively, and

$$\alpha_l \alpha_m + \beta_l \beta_m + \gamma_l \gamma_m = \delta_{lm} \qquad l, m = 1,\ 2,\ 3, \tag{5}$$

where δ_{lm} is the Kronecker delta.

In order to apply the Biot-Savart (1), we have to find suitable expressions $1_{ri}(\varphi)$ and $\overrightarrow{dl_i}(\varphi)$. By looking at Figure 1, if $X(\varphi)$, $Y(\varphi)$, $Z(\varphi)$ are the coordinates of the generic element $\overrightarrow{dl_i}(\varphi)$ and $1_x, 1_y, 1_z$ are rectangular unit vectors, we have

$$1_{ri}(\varphi) = \frac{(x - X(\varphi))1_x + (y - Y(\varphi))1_y + (z - Z(\varphi))1_z}{\sqrt{(x - X(\varphi))^2 + (y - Y(\varphi))^2 + (z - Z(\varphi))^2}}. \tag{6}$$

Taking into account (6), (4) and (2) let us denote:

$$1_{ri}(\varphi) = u_{rxi} 1_x + u_{ryi} 1_y + u_{rzi} 1_z, \tag{7}$$

where with $\varphi_0 = 0$

$$u_{rxi} = \frac{x - \alpha_{1i} a \cos\varphi - \beta_{1i} a \sin\varphi - \gamma_{1i} k\varphi - x_i}{r_i}, \tag{8}$$

$$u_{ryi} = \frac{y - \alpha_{2i} a \cos\varphi - \beta_{2i} a \sin\varphi - \gamma_{2i} k\varphi - y_i}{r_i}, \tag{9}$$

$$u_{rzi} = \frac{z - \alpha_{3i} a \cos\varphi - \beta_{3i} a \sin\varphi - \gamma_{3i} k\varphi - z_i}{r_i} \tag{10}$$

with

$$k = \frac{h}{2\pi}, \tag{11}$$

$$\begin{aligned} r_i = [&(x - \alpha_{1i} a \cos\varphi - \beta_{1i} a \sin\varphi - \gamma_{1i} k\varphi - x_i)^2, \\ &+ (y - \alpha_{2i} a \cos\varphi - \beta_{2i} a \sin\varphi - \gamma_{2i} k\varphi - y_i)^2, \\ &+ (z - \alpha_{3i} a \cos\varphi - \beta_{3i} a \sin\varphi - \gamma_{3i} k\varphi - z_i)^2]^{1/2}. \end{aligned} \tag{12}$$

It follows from (2) and (4) that

$$\overrightarrow{dl}_i(\varphi) = dl_{xi} 1_x + dl_{yi} 1_y + dl_{zi} 1_z, \tag{13}$$

where

$$dl_{xi} = (-\alpha_{1i} a \sin\varphi + \beta_{1i} a \cos\varphi + \gamma_{1i} k) d\varphi, \tag{14}$$

$$dl_{yi} = (-\alpha_{2i} a \sin\varphi + \beta_{2i} a \cos\varphi + \gamma_{2i} k) d\varphi, \tag{15}$$

$$dl_{zi} = (-\alpha_{3i} a \sin\varphi + \beta_{3i} a \cos\varphi + \gamma_{3i} k) d\varphi. \tag{16}$$

The vector product in (1) is found to be:

$$\overrightarrow{dl_i}(\varphi) \times 1_{ri}(\varphi) = (dl_{yi} u_{rzi} - dl_{zi} u_{ryi}) 1_x - (dl_{xi} u_{rzi} - dl_{zi} u_{rxi}) 1_y + (dl_{xi} u_{ryi} - dl_{yi} u_{rxi}) 1_z. \tag{17}$$

Thus the three components of the magnetic flux density according to (1) and (17) can be written in the forms:

$$B_{xi}(x, y, z) = \frac{\mu_0 I}{4\pi} \int_0^{\frac{2\pi L_i}{h}} \frac{dl_{yi} u_{rzi} - dl_{zi} u_{ryi}}{r_i^2}, \tag{18}$$

$$B_{yi}(x, y, z) = \frac{\mu_0 I}{4\pi} \int_0^{\frac{2\pi L_i}{h}} \frac{dl_{zi} u_{rxi} - dl_{xi} u_{rzi}}{r_i^2}, \tag{19}$$

$$B_{zi}(x, y, z) = \frac{\mu_0 I}{4\pi} \int_0^{\frac{2\pi L_i}{h}} \frac{dl_{xi} u_{ryi} - dl_{yi} u_{rxi}}{r_i^2}, \tag{20}$$

whereas integrand terms are given by (8-10), (12) and (14-16).

The integrals in (18-20) have to be solved numerically.

Finally, the modulus of the magnetic flux density field due in the observation point $P(x, y, z)$ to the current in the i-th section of the helical conductor can be obtained from the formula:

$$\left|B_i(x,y,z)\right| = \sqrt{\left|B_{xi}(x,y,z)\right|^2 + \left|B_{yi}(x,y,z)\right|^2 + \left|B_{zi}(x,y,z)\right|^2} \ . \tag{21}$$

The total magnetic field of the helical conductor with complex geometry can be obtained by superposition of the contributions produced by each segment.

It should be noted, that the formulas derived enable to analyze magnetic fields produced by twisted-wire pair as well as by three-core cable considering the conductor twist. The twisted-pair cable can be represented mathematically as a double helix that consists of two helices having the same radius and pitch and carrying currents $\underline{I}$ and $-\underline{I}$; the helices are located 180 spatial degrees from each other. In the three-wire helix structure the current in the i-th conductor (i = 1, 2, 3) is $i_i = I\sqrt{2}\sin(\omega t + \psi_i)$ and the current phase angle $\psi_i = (i-1)2\pi/3$. The location of conductors in the $z'=0$ plane is fixed by angles φ_{0i}, where $\varphi_{0i} = (i-1)2\pi/3$. The total field components are found by summation.

It should be also pointed out, that an alternative way to the direct application of (18)-(20) consists in the discretization of the helix by means of a suitable number of rectilinear segments. Thus, the field is evaluated by superposition of the contributions produced by each segment [7].

2. Examples of calculations

Verification of the method presented lies in comparison of calculation results with calculation and measurement results available in some papers.

In order to verify the correctness of the analytical calculations presented in the paper, first comparison has been made with an analytical solution in form of infinite series containing Bessel functions obtained in [5] for infinitely long helical conductor. It follows from the calculations, that independent of helix length the agreement was excellent, for field components except of the B_φ, what is evident from physical point of view. Figure 2 shows the B_φ component of the magnetic flux density along the axial direction at radial distance from helix axis r = 1 cm. Discrepancy between the results obtained by the use of the method appropriate for the infinitely long helical conductor and by use of the method for conductor finite in the length shows that for short helical conductors the proposed method shall be applied.

Further verification of the method described is made by comparison with results of measurements on an experimental helical arrangement. In the laboratory experiment presented in [6] a rig consisting of plastic coated 50 mm^2 stranded copper wires helically wound on an 11 m long plastic pipe of 0.2 m diameter has been constructed. The wires were fed by a balanced 200 A, 50 Hz 3-phase current. The three components of the magnetic flux density were measured at various positions by use of a one-coil magnetic field meter. For each distance, the field at six points was measured and the average value was calculated and presented in [6] for the 1.0 m pitch. Following Figures present the calculation results obtained according to the proposed method. The calculations have been curried out in the middle part of the three-wire helix structure for each radial distance at six points spaced 0.1 m (analogical to the test [6]) and the average values of field components are shown in Figure 3 as functions of radial distance. The agreement between the experimental results and the theoretical solution is excellent. Figure 4 shows moreover the distribution of the module of the magnetic field density in the *xz*-plane over the three-wire helix structure studied.

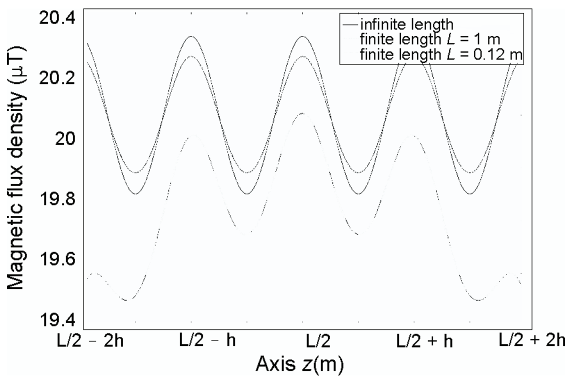


Fig. 2. B_φ component versus z (h = 2 cm)

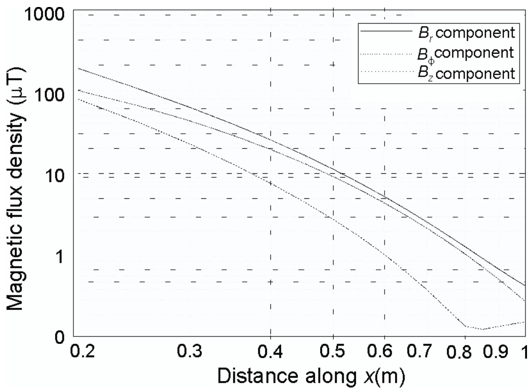


Fig. 3. Components of the magnetic field density as function of radial distance from axis of the 3-phase cable

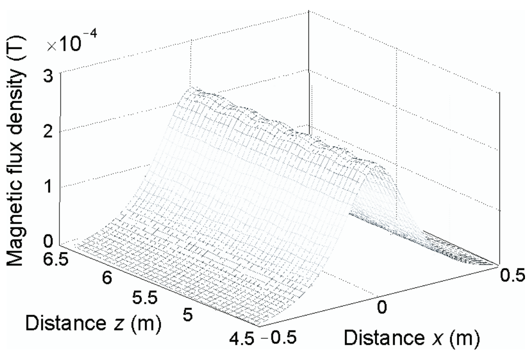
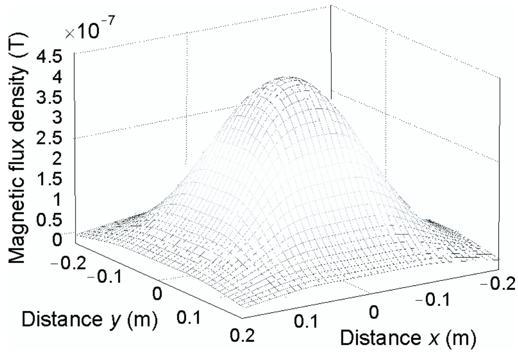


Fig. 4. Distribution of the magnetic field density in the *xz*-plane at radial distance 5 cm from center of the 3-phase cable

The usefulness of the presented method shall be next shown through its application for the calculation of the 3D magnetic field produced in the vicinity of helical conductor - the twisted-wire pair formed a square loop lying in the xy plane ($z = 0$). The calculations have been curried out in the xy plane 10 cm above the loop plane, and refer to $I_1 = 1$ A, $I_2 = -1$ A, $\varphi_{01} = 0$, $\varphi_{02} = \pi$, $a = 1$ cm, $h = 2$ cm, and the side of the square $L = 12$ cm. Fig.5 shows the 3D distribution of the module of the magnetic flux density.

Fig. 5. Distribution of the magnetic field density in the xz-plane above the two-wire pair formed a loop



3. Final remarks

The design of installation generating low-frequency magnetic field requires access to effective analytical and computational tools. The analytical method for calculating magnetic fields produced by currents in helical conductors using the magnetic vector potential can be applied only in a case, when the assumption regarding the infinite length of the helical line can be acceptable. In other cases (short helical line, helical line formed complex path) this method leads to calculation results with considerable errors. Therefore the paper presents procedures of determining the magnetic flux densities intensities produced by currents in helical conductors with finite length basing on the Biot-Savart law. The analytical formulas for calculating the 3D magnetic field are derived and allow also managing cases with any complex geometry of the helical conductor such as changes of direction of a conductor line, changes of burial depth/height of the line and cables with twisted conductors as well.

The formulas allow tackling the magnetic field of the two-wire helix, as well as for the three-wire helix and can be used by a software tool to model the magnetic fields generated by e.g. twisted-wire pairs, twisted three-phase power cables, triplex service cables, helical coils, etc.

The necessary data for magnetic field calculations are: the number of segments the helical line is divided into, the helix radius, the helix pitch, the number of helix conductors, the current in each conductor, the φ_0 co-ordinate, the x, y, z coordinates of the observation point P, and the coordinates (x_i, y_i, z_i) and (x_{i+1}, y_{i+1}, z_{i+1}) of terminating points of each helical line segment. It should be noted that all coordinates refer to the reference system, which can be arbitrary located in the space.

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