A R C H I V E O F M E C H A N I C A L E N G I N E E R I N G

VOL. LXI 2014 Number 4

10.2478/meceng-2014-0032

Key words: slide bearings surfaces, wear prognosis, recurrence equations, computational analysis

ANDRZEJ MISZCZAK [*], *KRZYSZTOF WIERZCHOLSKI* [**]

A METHOD OF THE WEAR PROGNOSIS OF SLIDING BEARINGS

The considerations presented in this paper include a computer analysis of slide bearing wear prognosis using the solutions of recurrence equations complemented with the experimental data values. On the ground of the results obtained from analytical and computational numerical calculations, and taking into account the experimental parameters of bearing material and operation boundary conditions, the control problems of slide bearing wear surfaces have been presented. The obtained results allow us to see a connection between roughness, material properties, the amplitude of vibrations, the kind of the friction forces, the hardness of materials, the sliding speed in one side and the wear increments in succeeding time units of the exploitation process in other side.

1. Introduction

The wear value prognosis and control of two cooperating bearing surfaces during the numerous, particular operation time has a very important significance in the contemporary classical technological and bioengineering processes [1-5, 7, 8, 13]. Ludema K.C., Bhusha M. and other Authors [1, 2, 6, 8] have made many measurements in the field of wear determination. Such interesting practical achievements are up to now not finally illustrated and elaborated in the simple analytical and numerical expressions [6]. Moreover, for many friction pairs the wear values are unknown after numerous time units of operation. Now, we will show some experimental results that have been obtained so far presenting the influence on wear during the operating times.

[*] *Gdynia Maritime University, Morska Street 81-87, 81-225 Gdynia, Poland; E-mail: miszczak@wm.am.gdynia.pl*

[**] *Koszalin University of Technology, Institute of Technology and Education, Śniadeckich Street 2, 75-453 Koszalin, Poland; E-mail: krzysztof.wierzcholski@wp.pl*

Figure 1 shows the wear depth increase versus the succeeding operating times for a constant normal load and various materials of cooperating surfaces, according to the experimental results published according to Bhushan B. and Ludema K.C. [1, 2, 8].

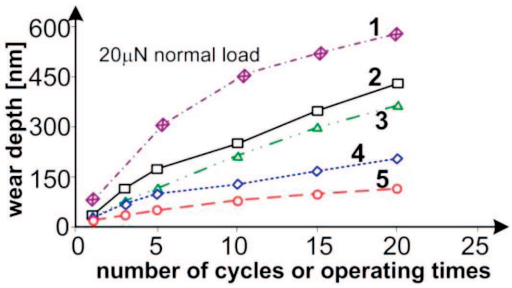

Fig. 1. Wear depths as a function of normal load as a function of cycles or operating times for various samples: 1 – Nylon for normal load 15 μN, 2 – undoped Si(100) (load 20 μN), 3 – undoped polysilicon film(load 20 μN), 4 – n-type polysilicon film (load 20 μN), 5 – SiC film (load 20 μN)

Figure 2 presents the weight loss values in μm^3 for various materials versus time in hours for sliding perpendicular to the lay of roughness and for sliding parallel to the lay of roughness. The values presented have been elaborated by the Authors on the grounds of experimental results found in literature, according to Ludema K.C. and Bhushan B. [1, 2, 8].

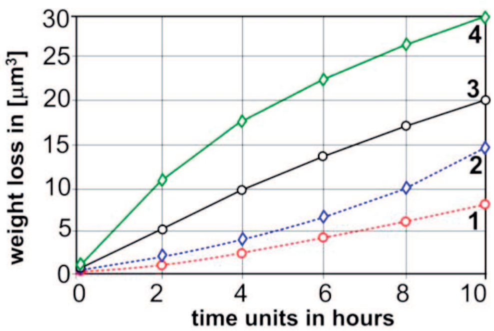

Fig. 2. Influence of surface roughness on wear loss in μm^3 for Nylon 1, 3 and undoped Si(100) 2, 4 taking into account: 1, 2 – sliding parallel to the lay of roughness, 3, 4 – sliding perpendicular to the lay of roughness

The effects of severe amplitude vibration on wear, expressed in μm^3, are elaborated by the Authors on the grounds of the experimental results published by Ludema K.C., Bhushan B., and Kapoor A., Pytko S. [1, 2, 7-9]. Figure 3 presents the increase of wear value versus amplitude of vibration increments.

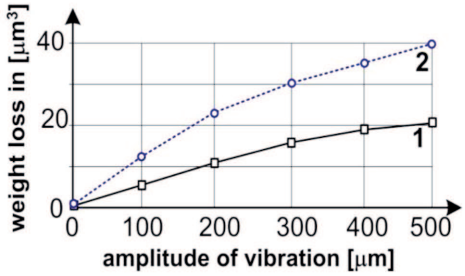

Fig. 3. The effect of amplitude vibration on wear in μm^3 for Nylon – 2 and undoped Si(100) – 1

This paper describes the calculation method for wear anticipation in bearing surfaces taking into account the dynamic behavior of the sliding pair during the operating time and many other data such as: the number of cycles, sliding directions referred to the lay of roughness, the amplitude of vibrations, the kind of bearing materials, the kind of friction forces, continuous or non-continuous mode of the slide bearing operation, the hardness of sliding materials, the sliding speed, and the magnitude of pressure values. The research effort presented herein is aimed at deriving second order non homogeneous recurrence equations and formulae [10, 12] which engineers can use in product design to determine the wear or the sum of wear after arbitrary time periods of slide pair operation.

2. Physical data connected with wear recurrence phenomena

After many experimental AFM measurements, one may find it evident that the wear values of a loss of the weight of a bearing journal and a sleeve are equal to the mean average coefficient $0 < a < 1$ from the weight loss values in two foregoing succeeding time units plus some correction described by exponent dimensional function $g(b, D, n) = bD^n$ for the following time units $n = 1, 2, 3, \ldots$ [1-4, 7, 8, 12]. Mean average coefficient $0 < a < 1$ depends on the kind of the bearing material, and increases in a direct proportion to the temperature increase, the magnitude of the friction force increase, and the magnitude of the amplitude of vibration decrease.

We have two variables of the above-mentioned correction function g. The first variable b increases in a direct proportion to the load, the increase in bearing capacity, the increase in sliding velocity, and the increases of unit time length and the friction coefficient. The first variable b decreases in a direct proportion to the increments of hardness of the bearing material. The second variable takes values only in the interval $0 < D < 1.618$ and it increases almost in a direct proportion to the increments of the angle between the direction of sliding and the lay of roughness. The variable D increases with the increments of the frequency of the non-continuous mode of the bearing operation and with magnitude increments of the changes in standard deviations of the bearing gap height during the operation [8, 12].

Thus, discrete wear values f_{n+2} of the sequence $\{f_n\}$ for $n = 1, 2, 3, \ldots$ i.e. the values of a loss of volume in mm^3 or μm^3 of slide bearing surfaces are equal to the sum $(f_{n+1} + f_n)$ of wear (loss of weight) in two foregoing successive time units (which may be hours or months) multiplied by the dimensionless mean average wear coefficient $0 < a < 1$ plus some exponential dimensional function of bD^n [1-4, 7, 8, 12]. The index n indicates time units and is expressed in natural numbers 1, 2, 3, ...

Table 1 illustrates coefficients $0 < a < 1$, $b \geq 0$, $0 < D < 1.618$ depended on various factors $x_1, x_2, x_3, \ldots, y_1, y_2, y_3, \ldots, k, H, V, L$ obtained experimentally [1-4, 7, 8]. The correction D^n increases in succeeding time units $n = 1, 2, 3, \ldots$ for $1 < D < 1.618$ and it decreases for $0 < D < 1$.

Table 1.

Definition of wear parameters

$a = x_1 x_2 x_3 \ldots$ [-]	x_1 – dimensionless coefficient describing kind of bearing materials and its temperature conditions, x_2 – dimensionless coefficient describing kind of friction or friction coefficient, x_3 – dimensionless coefficient describing magnitude of amplitude and frequency of vibrations,
$D = y_1 y_2 y_3 \ldots$ [-]	y_1 – dimensionless coefficient describing direction of sliding to the lay of roughness, y_2 – dimensionless coefficient describing continuous or non-continuous mode of slide bearing operation, y_3 – dimensionless coefficient describing magnitude of standard deviation changes of bearing gap height,
$b = 10^{-6}\mu \cdot t \dfrac{LV}{H}$ in $[mm^3]$	μ – friction coefficient, t – time unit in [s], L – load applied in [N], H – hardness of sliding materials in $[N/mm^2]$, V – sliding speed in [mm/s],

Now we will describe the values of factors x which are presented in Table 2.

Taking into account experimental results included in literature [7, 8], we can see in Table 2 that:

– the factor x_1 attains the largest values for metals Ag, Cu, Fe, Pb at a high temperature, average values for ceramic metals and the smallest values for metals at a low temperature [2, 3];
– the factor x_2 attains the largest values for dry friction, middle (intermediary) values for boundary friction and the smallest values for friction in hydrodynamic lubrication [8];
– the factor x_3 attains the largest values for a low amplitude, intermediary values for the middle amplitude and the smallest values for the upper amplitude of vibration [8] (see Fig. 3).

Table 2.

Determination of dimensionless factors x

Factors x	Kinds of individual factors	Interval values referred to kinds of individual factors
x_1 – dimensionless factor describing kind of bearing materials	Metals at high temperature	$0.500 < x_1 < 0.999$
	Ceramic and solid lubricants	$0.300 < x_1 < 0.499$
	Metals at low temperature	$0.010 < x_1 < 0.299$
x_2 – dimensionless factor describing kind of friction or friction coefficient	Dry friction	$0.600 < x_1 < 0.990$
	Boundary friction	$0.400 < x_1 < 0.599$
	Hydrodynamic friction	$0.200 < x_1 < 0.399$
x_3 – dimensionless factor describing magnitude of amplitude and frequency of vibrations	Low amplitude values	$0.700 < x_1 < 0.990$
	Average amplitude values	$0.300 < x_1 < 0.699$
	Upper amplitude values of bearing surface vibrations	$0.099 < x_1 < 0.299$

Now we will describe the values of factors y presented in Table 3.

Table 3.

Determination of dimensionless factors y

Factors y	Kinds of individual factors	Interval values referred to kinds of individual factors
y_1 – dimensionless factor describing direction of sliding to the lay of roughness	Perpendicular sliding	$1.000 < y_1 < 1.618$
	Sliding for angle from 0° to 90°	$0.700 < y_1 < 0.999$
	Parallel sliding	$0.010 < y_1 < 0.699$
y_2 – dimensionless factor describing continuous or non continuous mode of slide bearing operation	Non continuous mode	$0.700 < y_1 < 1.600$
	Average continuous mode	$0.500 < y_1 < 0.699$
	Regular continuous mode of slide bearing operation	$0.100 < y_1 < 0.499$
y_3 – dimensionless factor describing magnitude of standard deviation changes of bearing gap height	High values	$0.600 < y_1 < 0.999$
	Average values	$0.400 < y_1 < 0.599$
	Low values of standard deviations	$0.099 < y_1 < 0.399$

Taking into account the experimental results included in literature [7, 8] we can see that:

– the factor y_1 attains the largest values for perpendicular sliding, intermediary values if we have sliding for angle from 0° to 90° and the smallest value for parallel sliding to the lay of the roughness [8] (see Fig. 2);
– the factor y_2 attains the largest values for the non-continuous mode, intermediary values for the average continuous mode and the smallest value for the regular continuous mode of the sliding bearing operation [8];

– the factor y_3 attains the largest values for high, intermediary values for the average and the smallest value for low values of the standard deviation [3, 7, 8].

3. Some remarks on wear recurrence equation and its solutions

• The standard deviation dependent term bD^n as an exponential function, and the average term $a(f_{n+1} + f_n)$ of two foregoing wears, make up the sequence values of the real wear in the succeeding time unit [1-4, 7]. In this case, the wear of slide–bearing surfaces can be described with the following nonhomogeneous recurrence equation [10, 13]:

$$f^*_{\mathrm{n}+2} = a(f^*_{\mathrm{n}+1} + f^*_{\mathrm{n}}) + bD^n \quad \text{for} \quad n = 1, 2, 3, ... \tag{1}$$

Recurrent equation (1) determines the analytical formula $\{f^*_{\mathrm{n}}\}$ presenting a sequence of wear values numbered for n = 1, 2, 3, . . . time units if we know dimensionless values D[1], a [1] and the dimensional value b [mm^3, μm^3]. To solve the abovementioned problem, it is necessary to add boundary conditions [13]. Hence, based on the measurements performed, we assume that in the two first time units (which may be hours or months), the wear obtained in an experimental way attains dimensional values W_1 [mm^3, μm^3], W_2 [mm^3, μm^3].

The general solution of Eq. (1) has the following form [10, 12]:

$$f^*_{\mathrm{n}} = C_1\chi_1^{\mathrm{n}} + C_1\chi_2^{\mathrm{n}} + f^{\mathrm{b}}_{\mathrm{n}}, \quad \text{for} \quad n = 1, 2, 3, \ldots \tag{2}$$

where C_1, C_2 arbitrary constants and sequences $\{f^*_n\}$, $\left\{f^b_n\right\}$ for n = 1, 2, 3, ... denote the total general and particular solutions of the non-homogeneous recurrence equation respectively. The real roots χ_1, χ_2 of the characteristic equation are as follows:

$$\chi_{1,2} = \frac{a}{2} \pm \sqrt{a + \frac{a^2}{4}}, \quad \text{for} \quad 0 < a < 1, \;\; D_2 < \chi_{1,2} < D_1, \quad D_{1,2} = \left(1 \pm \sqrt{5}\right)/2 \tag{3}$$

By imposing the boundary conditions $f^*_1 = W_1$, $f^*_2 = W_2$ on the general solution (2), we obtain for $\Delta\chi \equiv \chi_2 - \chi_1$ the following particular solution [14]:

$$\begin{aligned} f^*_n = {} & \frac{1}{\Delta\chi}\left(\chi_2\chi_1^{n-1} - \chi_1\chi_2^{n-1}\right)W_1 + \frac{1}{\Delta\chi}\left(\chi_2^{n-1} - \chi_1^{n-1}\right)W_2 + \\ & + \frac{1}{\Delta\chi}\left\{\left[\left(f^b_2 - \chi_2 f^b_1\right)\chi_1^{n-1} - \left(f^b_2 - \chi_1 f^b_1\right)\chi_2^{n-1}\right]\right\} + f^b_n. \end{aligned} \tag{4}$$

• Usually, on the basis of measurements and values presented in Tables 1, 2, 3, we obtain the three following regions of wear parameters:

a) The first surface region is illustrated in Fig. 4 without all points lying on the indicated curve including point $(a, D) = (1.00; 0.50)$;

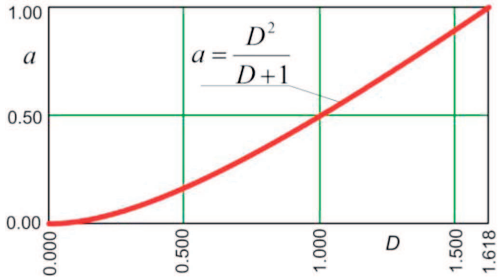

Fig. 4. Surface of the first region of wear parameters for $0 < a < 1$, $0 < D < D_1 = 1.618$ does not include the indicated curve $a = D^2/(D+1)$

For the parameters lying in the first region, the particular solution of non-homogeneous Eq. (1) has the form:

$$f_{\mathrm{n}}^{\mathrm{b}} = \frac{bD^{\mathrm{n}}}{D^2 - aD - a}, \quad \text{for} \quad n = 1, 2, 3, ... \tag{5}$$

b) The second region, in the form of the curve, is described with the following formula:

$$(0 < a < 1) \wedge (0 \leq D \leq 1.618) \wedge \left(a = \frac{D^2}{D+1}\right) \wedge (a \neq 0.50; D \neq 1) \tag{6}$$

and illustrates the curve shown in Fig. 4 without the point $(a, D) = (1/2, 1)$.

For parameters lying in the second region, the particular solution of non-homogeneous Eq. (1) has the form:

$$f_n^b = \frac{bnD^{n-2}(D+1)}{D+2}, \quad \text{for} \quad n = 1, 2, 3, ... \tag{7}$$

c) The third region is depicted in Fig. 5 as one singular point $(a, D) =$ $= (1/2, 1)$:

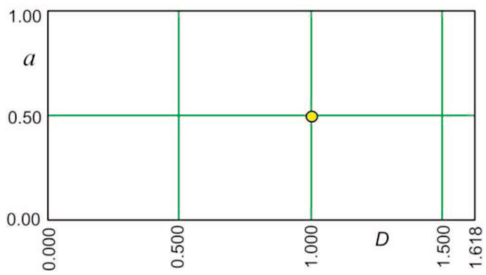

Fig. 5. One point sub-region of divergent wear process for $D = 1.0$, $a = 0.5$

For the parameters $a = 0.5$; $D = 1.0$ lying in the third region, the particular solution of non-homogeneous Eq. (1) has the form:

$$f_n^b = \frac{2}{3} bn, \quad \text{for} \quad n = 1, 2, 3, ... \tag{8}$$

• Taking into account Solution (7) and the roots (3), we obtain, for each point (a, D) lying in the first region, the following dimensional sums of final wear values terms (4) after N time units for $n = 1, 2, 3, \ldots, N$ [12]:

$$\sum_{n=1}^{N} f_n^* = F_{1N}(a)W_1 + F_{2N}(a)W_2 + F_{3N}(a, D)b, \tag{9}$$

where coefficients of sums $F_{1N}(a), F_{2N}(a), F_{3N}(a, D)$ are presented in an analytical form [12].

Now we will consider four sub-regions of the first region.

a) For the convergent wear process, the first sub-region is presented with points lying inside the domain (a, D) defined with the following inequalities:

$$\text{O :} \qquad (0 < a < 0.50) \wedge (0 < D < 1.00) \wedge \left(a \neq \frac{D^2}{D+1}\right), \tag{10}$$

and Fig. 6 shows the location without all points lying on the indicated curve:

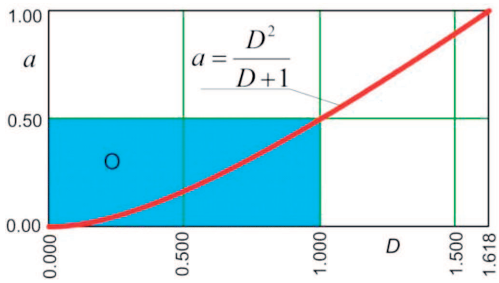

Fig. 6. Sub-region O of the wear convergent process for $a \neq D^2/(D+1)$, $0 < a < 1/2$, $0 < D < 1$

For each point in this region, the considered wear process is convergent for $N \to \infty$. The limits $F_{1\infty}, F_{2\infty}, F_{3\infty}$ take some finite values.

b) Next sub-regions for the divergent wear process are presented with the points (a, D) lying inside domains A, B, C defined by the following inequalities:

$$\text{A:} \qquad a \neq \frac{D^2}{D+1}, \quad \left(\frac{1}{2} \leq a < 1\right) \times (1 \leq D < D_1),$$

$$\text{B:} \qquad \left(0 < a < \frac{1}{2}\right) \times (1 \leq D < D_1), \tag{11}$$

C: $$\left(\frac{1}{2} \leq a < 1\right) \times (0 < D < 1),$$

and Fig. 7 shows the field without all points lying on the indicated curve:

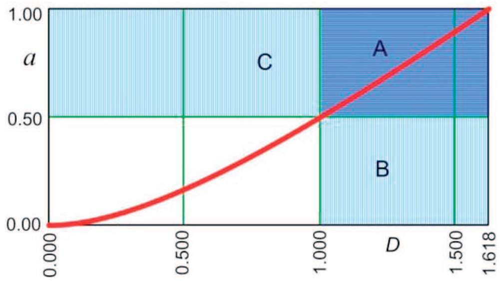

Fig. 7. Sub-regions for divergent wear process of: A) $a \neq D^2/(D+1)$, $1/2 \leq a < 1$, $1 \leq D < 1.618$; B) $0 < a < 1/2$, $1.00 \leq D < 1.618$; C) $0.5 \leq a < 1.0$; $0 < D < 1$

For each point inside and on the boundaries of sub-regions A, B, C, the considered wear process is divergent for $N \to \infty$ i.e. we haven't any infinite limits: $F_{1\infty} = \infty, F_{2\infty} = \infty, F_{3\infty} = \infty$.

■ Now we go to the numerical examination of the summary wear values in the first parameter region. We calculate and show in Fig. 8 depicted numerical values of functions F_{1N}, F_{2N} in time units N = 4, 6, 8, 10, 12, 14 for the points (a) lying in the first region, and its O, A, B, C sub-regions, except the points $a = D^2/(D+1)$, (D =1, a =1/2). To obtain the dimensional wear value, we must multiply corresponding dimensionless values F_{1N}, F_{2N} by the dimension experimental values W_1, W_2, respectively.

Making use of Mathcad Professional Program we calculate numerical values of functions F_{3N}, in time units N = 4, 8, 12, 14 for the points (a, D) lying in the first region, and its sub-regions: O, A, B, C where $a \neq D^2/(D+1)$, $D \neq 1$, $a \neq 1/2$, as shown in Fig. 9a,b,c,d. The left and right column of the pictures presents the distributions in 3D and 2D views, respectively. To obtain the dimensional wear value, we must multiply the dimensionless values F_{3N} by the dimension experimental values b. Finally, we add dimensional values $F_{1N}W_1 + F_{2N}W_2$ and $F_{3N}b$ hence the total wear value after N time units takes the form (9).

• Taking into account Solution (7) and the roots $\chi_1 = D$, $\chi_2 = -D/(D+1)$, we obtain, for each point with coordinate D lying in the second region, the sums of dimensional final wear values (4) after N time units for n = 1, 2, 3, ..., N in the following form:

$$\sum_{n=1}^{N} f_n^* = G_{1N}(D)W_1 + G_{2N}(D)W_2 + G_{3N}(D)b, \tag{12}$$

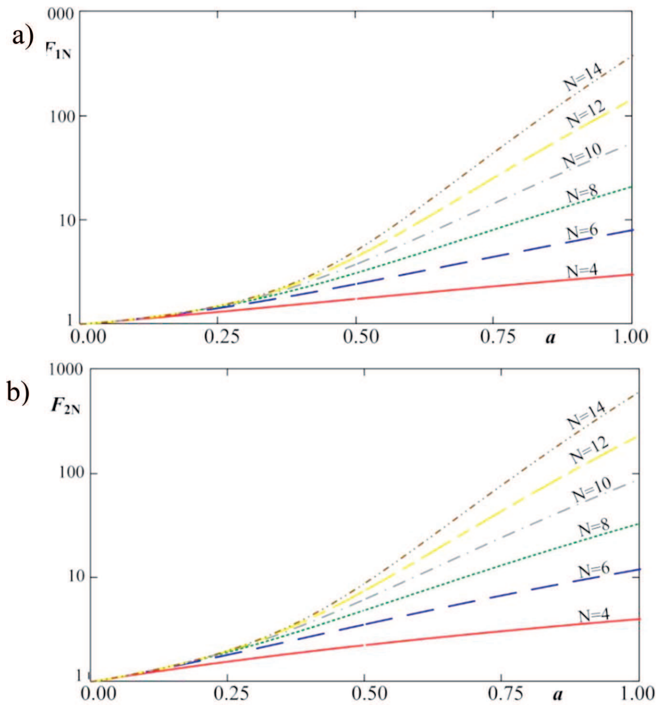

Fig. 8. Dimensionless convergent wear values inside interval $0 < a < 1$, in time units $N = 4, 6, 8, 10, 12, 14$ for the first region and its sub-regions: O, A, B, C excepting points $a = D^2/(D+1)$, ($D = 1$, $a = 1/2$) a) dimensionless values of function $F_{1N}(a)$, b) dimensionless values of function $F_{2N}(a)$

where the sums $G_{1N}(D), G_{2N}(D), G_{3N}(D)$ can be presented in an analytical form [12].

Now we consider two sub-regions of the second region.

a) The first sub-region Γ_1 is presented, for the convergent wear process, with the points (a, D) lying inside the domain defined by the following inequalities:

$$(0 < a < 0.50) \wedge (0 < D < 1.00) \wedge \left(a = \frac{D^2}{D+1}\right), \tag{13}$$

and Fig. 10a illustrates the field of all points lying on the curve without the point (1.00; 0.50).

For each point in this region, the considered wear process is convergent.

b) The second sub-region Γ_2 is presented, for the divergent wear process, with points (a,D) lying inside the domain defined by the following inequalities:

$$(0.50 < a < 1.00) \wedge (1.00 < D < 1.618) \wedge \left(a = \frac{D^2}{D+1}\right), \tag{14}$$

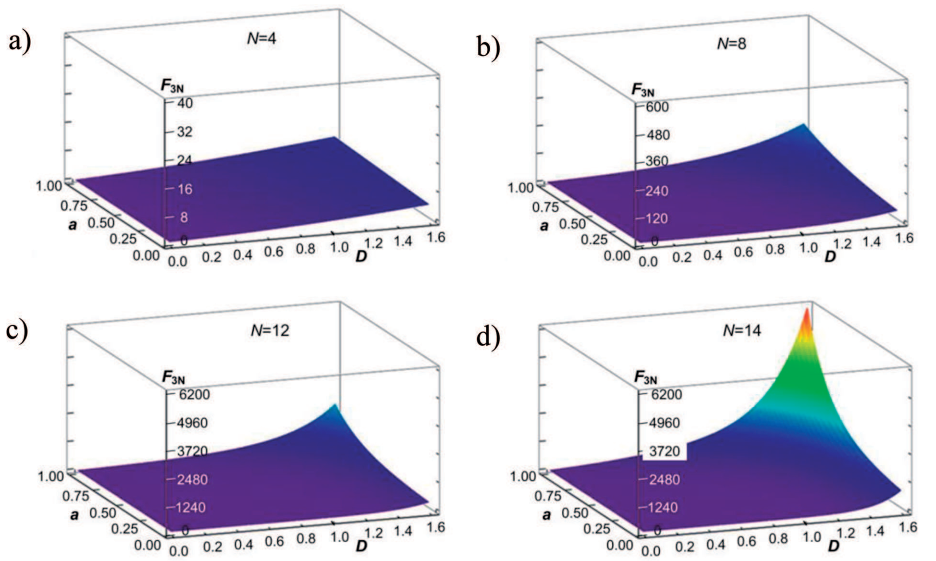

Fig. 9. Left and rig ht column of pictures for 3D and 2D views respectively, present convergent in sub-region O and divergent in sub-regions A, B, C, dimensionless wear values prognosis of functions F_{3N} in region $0 < a < 1.0$; $0 < D < 1.618$ excepting points ($a = 1/2$, $D = 1$), $a = D^2/(D{+}1)$: a) for time units $N = 4$, b) for time units $N = 8$, c) for time units $N = 12$, d) for time units $N = 14$

and Fig. 10b illustrates the field of all points lying on the curve without the point (1.00; 0.50).

For each point in this region, the considered wear process is divergent.

To obtain the total dimensional wear value after N time units, we must multiply the corresponding dimensionless values G_{1N}, G_{2N}, G_{3N} by the dimensional experimental values W_1, W_2, b, respectively, and next add the obtained values as shown in formula (12).

■ We perform calculation in Mathcad Professional Program and show in Fig. 11a,b,c the depicted numerical dimensionless values of functions G_{1N}, G_{2N}, G_{3N}, respectively, for parameters (a, D) lying in the following fields:

$$\begin{aligned} &\Gamma_1 : a = D^2/(D+1), 0<a<1/2; 0<D<1; \text{(convergent wear process)} \\ &\Gamma_2 : a = D^2/(D+1), 1/2<a<1; 1.00<D<1.618; \text{(divergent wear process)} \end{aligned} \tag{15}$$

The curve presenting wear after $N = 30$ time units illustrates the convergences and divergences of the cumulative wear processes in sub-regions Γ_1, and Γ_2 respectively.

To obtain the total dimensional wear value after N time units we must multiply the corresponding dimensionless values G_{1N}, G_{2N}, G_{3N} by the di-

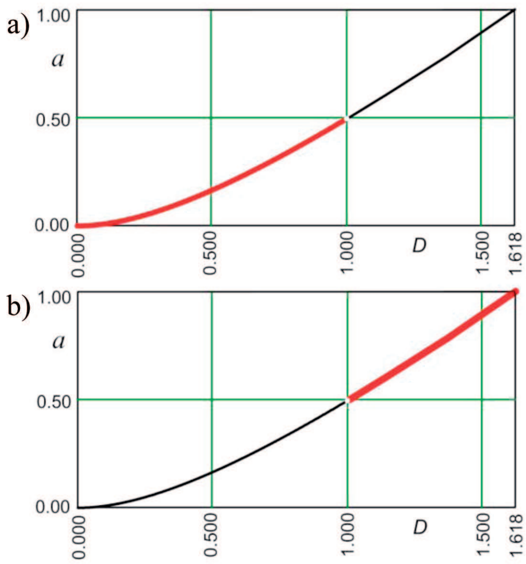

Fig. 10. Sub-regions of the second region: a) sub-region Γ_1 for $a = D^2/(D+1)$, $0 < a < 1/2$, $0 < D < 1$ of the wear convergent process, b) sub-region Γ_2 for $a = D^2/(D+1)$, $1/2 < a < 1$; $1.00 < D < 1.618$ of the wear divergent process

mensional experimental values W_1, W_2, b, respectively, and next add the obtained values, as shown in formula (12).

• Taking into account Solution (8) and the roots $\chi_1 = 1$, $\chi_2 = -0.5$, we obtain for the point ($a = 1/2$, $D = 1.0$) lying in the third region the sums of dimensional final wear values (4) after N time units for $n = 1, 2, \ldots, N$ in the following form:

$$\sum_{n=1}^{N} f_n^* = H_{1N}(N)W_1 + H_{2N}(N)W_2 + H_{3N}(N)b, \tag{16}$$

where coefficients of sums $H_{1N}(D), H_{2N}(D), H_{3N}(D)$ can be presented in an analytical form.

■ Using Eq. (16), we proceed to the numerical examination of the summary wear values in the third parameter region. We calculate and show in Fig. 12 the numerical values of functions H_{1N}, H_{2N}, H_{3N} for ($a = 1/2$, $D = 1.0$), the point covering the third region.

To obtain the total dimensional wear value after N time units, we must multiply the corresponding dimensionless values H_{1N}, H_{2N}, H_{3N} by the dimensional experimental values W_1,W_2, b, respectively, and next add the obtained values, as it was shown in Formula (16).

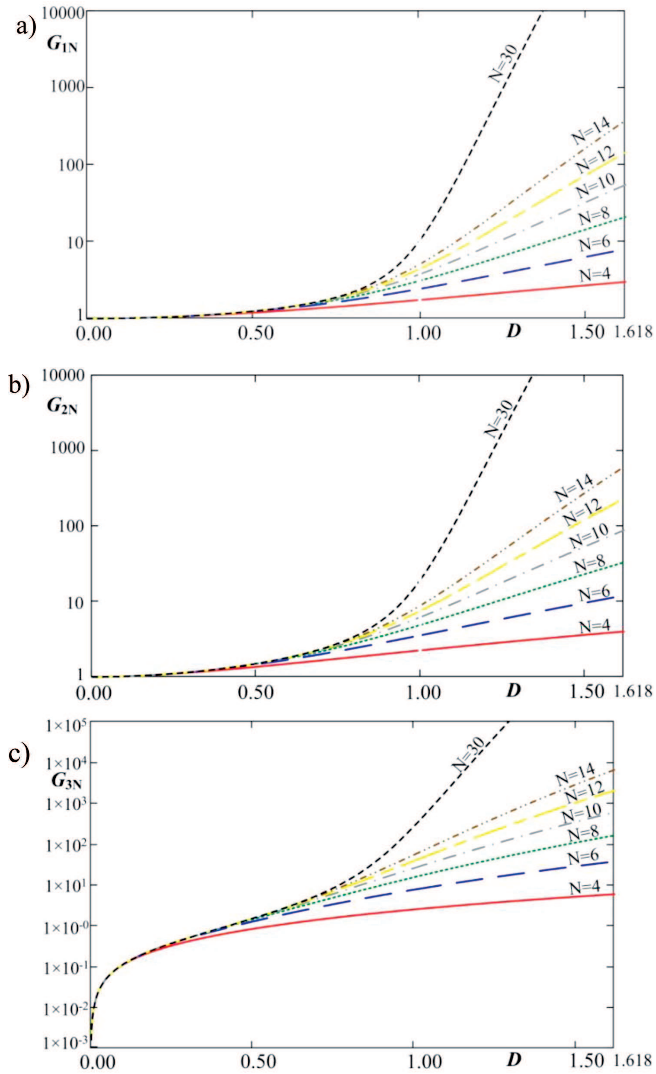

Fig. 11. Convergent in sub-region Γ_1 and divergent in sub-region Γ_2 the dimensionless wear values prognosis of G_{1N}, G_{2N}, G_{3N} in interval $0 < D < 1.618$ in time units $N = 4, 6, 8, 10, 12, 14, ..., 30$: a) for values G_{1N}, b) for values G_{2N}, c) for values G_{3N}

4. Engineering results in the form of calculation example

We may determine the wear after ten time units if we know that in the sliding nod we have metals at an average temperature, hydrodynamic lubrication occurs in the bearing gap and the amplitude of vibration is low. Moreover, we assume that sliding direction is parallel to the lay of roughness, the slide bearing works in an average continuous mode, and the magnitude of the standard deviation is very high.

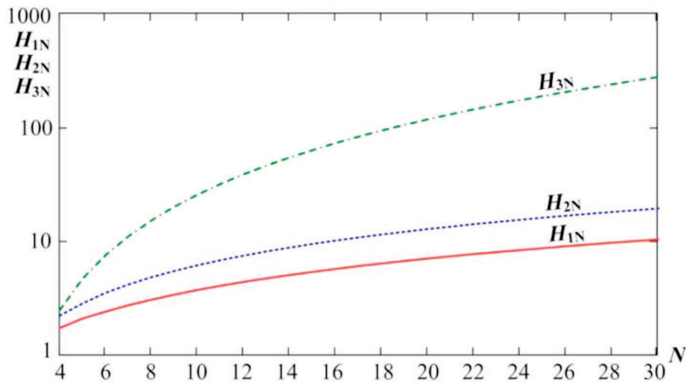

Fig. 12. Dimensionless divergent wear values prognosis of functions H_{1N}, H_{2N}, H_{3N} in the third-region covering point a = 0.5; D =1.0, in time units N = 4, 6, 8, ..., 30

For large slide journal bearings, we anticipate the classical magnitudes of wear values, for example in the first two successive time units, wear in the slide bearing journal decreases to W_1 [mm^3] and W_2 [mm^3], respectively. The load takes the value L =1000 N with the peripheral velocity of the journal V = 10 m/s, the hardness of the bearing material is H = 10 000 MPa, the friction coefficient μ = 0.001, according to [9]. We accept the time unit of 100 days i.e. = 8640 000 s. By measurements we obtain the wear (during 100 days), which after the first time unit is equal to W_1 = 4 mm^3 and the wear during the next time unit W_2 = 6 mm^3. Determine the wear after 1000 days i.e. after 10 time units.

Solution

From Table 2 we have: for metals at the average temperature x_1 = 0.499, for hydrodynamic lubrication x_2 = 0.33, for a low amplitude of vibration x_3 = 0.99, hence $a = x_1x_2x_3 = 0.499 \cdot 0.333 \cdot 0.99 = 0.1630$.

From Table 3 we read: for parallel sliding direction to the lay of roughness y_1 = 0.699, for the average continuous operating mode y_1 = 0.500, for a high magnitude of the standard deviation of the gap the height changes y_3 = = 0.955, hence $D = y_1y_2y_3 = 0.699 \cdot 0.500 \cdot 0.955 = 0.3337$.

The parameter b takes the following value:

$$b = 10^{-6}\mu t\frac{LV}{H} = 10^{-6}\cdot 0.001\cdot 8640000\text{ s}\cdot\frac{1000\text{N}\cdot 10\cdot 1000\ \frac{\text{mm}}{\text{s}}}{10000\ \frac{\text{N}}{\text{mm}^2}} = 8.64\text{ mm}^3. \tag{17}$$

For the accepted values W_1,W_2, b and in accordance with Eq. (9) the wear after N time units assumes the following form:

$$\sum_{n=1}^{N} f_n^* = F_{1N}(a)\cdot 4\text{ mm}^3 + F_{2N}(a)\cdot 6\text{ mm}^3 + F_{3N}(a,D)\cdot 8.64\text{ mm}^3. \tag{18}$$

We put values $a = 0.1630$, $D = 0.3337$ into the functions $F_{1N}(a)$, $F_{2N}(a)$, $F_{3N}(a, D)$, in successive time units $N = 1, 2, 3 \ldots$, or we read values $F_{1N}(a)$, $F_{2N}(a)$, $F_{3N}(a, D)$ for $a = 0.1630$, $D = 0.3337$ directly from distributions which are presented in Fig. 8 and Fig. 9a,b. In Fig. 13, there are presented the charts of values calculated for abovementioned particular data.

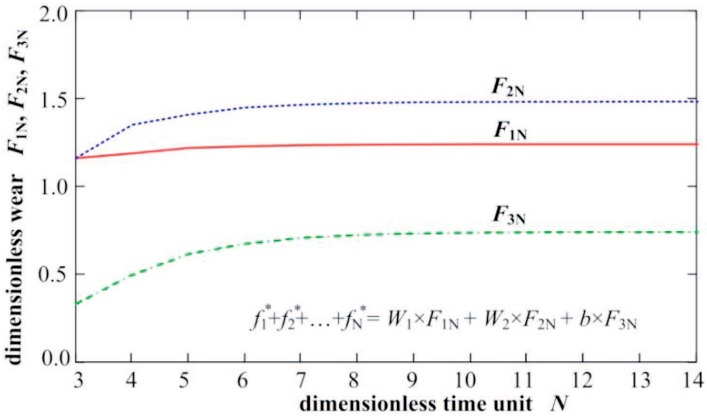

Fig. 13. Convergent process of the sum of wear values in particular point $a = 0.1645$, $D = 0.3337$ versus N time units if N tends to infinity for $a \neq D^2/(D+1)$, $0 < a < 1/2$, $0 < D < 1$

The obtained values F_{1N}, F_{2N}, F_{3N} for $a = 0.1630$, $D = 0.3337$ for $N = 1, 2, 3, \ldots$ are substituted into Equation (18). Here arbitrary dimensional values W_1 [mm^3], W_2 [mm^3], b [mm^3] are assumed. After calculations, we obtain a sequence of the following convergent sums of wear values in successive time units:

$$
\begin{aligned}
&\text{for} \quad N = 1, \quad f_1^* = W_1 = 4\ \text{mm}^3, \\
&\text{for} \quad N = 2, \quad f_1^* + f_2^* = W_1 + W_2 = 4\ \text{mm}^3 + 6\ \text{mm}^3 = 10\ \text{mm}^3, \\
&\text{for} \quad N = 3, \quad f_1^* + f_2^* + f_3^* = 1.1630\ W_1 + 1.1630 \cdot 6\ \text{mm}^3 + 0.3337\ b = \\
&\quad = 1.1630 \cdot 4\ \text{mm}^3 + 1.1630 \cdot 6\ \text{mm}^3 + 0.3337 \cdot 8.64\ \text{mm}^3 = 14.5132\ \text{mm}^3, \\
&\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\text{for} \quad N = 10, \quad \sum_{n=1}^{10} f_n^* = 1.2412\ W_1 + 1.4816\ W_2 + 0.7389\ b = \\
&\quad = 1.2412 \cdot 4\ \text{mm}^3 + 1.4816 \cdot 6\ \text{mm}^3 + 0.7389 \cdot 8.64\ \text{mm}^3 = 20.2382\ \text{mm}^3, \\
&\qquad \cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots \\
&\text{for} \quad N = \infty, \quad \sum_{n=1}^{\infty} f_n^* = 1.2418\ W_1 + 1.4837\ W_2 + 0.7431\ b = \\
&\quad = 1.2418 \cdot 4\ \text{mm}^3 + 1.4837 \cdot 6\ \text{mm}^3 + 0.7431 \cdot 8.64\ \text{mm}^3 = 20.2895\ \text{mm}^3.
\end{aligned}
\tag{19}
$$

5. Discussion

Numerous machine exploitation problems in the field of the experimental and numerical determination of slide bearing wear problems require the knowledge of data referring to the features of the sequence of the existing wear process during the operation time. In particular, the abovementioned information includes the velocity of the increase in wear values during particular time units of the operation. Very important are the phenomena of convergence and divergence of the wear value increase. If we have two divergent wear processes, then very important is information which process diverges more slowly. On the other hand, in comparison between two convergent processes, we must decide which process converges more quickly. This paper allows us to consider such a problem. If the wear process in particular time units of the operating time is decreasing to some wear limit value, then it is possible to obtain stabilization in the bearing wear in a sufficient number of time units of the operation. If the wear process increases in particular time units of the operating time, then we have the phenomena of continuous increments of wear in bearing in a sufficient number of time units of the operation. In both above cases, the sum of wear values after the considered time units of the operation, i.e. the cumulative function of wear values can be divergent. The most important for practical needs is the convergence or divergence of the cumulative values of bearing wear after the considered time units of the operation.

To perform investigations of the abovementioned control problems, the wear value processes are described by non-homogeneous recurrent equations with a variable free term. The results obtained in this paper show that convergence or divergence of the wear processes depends on experimentally determined coefficients which appear in the considered recurrent equations of wear. The coefficients depend on bearing materials, operation conditions and environmental external influences. Therefore, the assumption and knowledge of materials, exploitation conditions and finally external influences enable us to determine the control of wear during the operation time.

The results obtained in this paper show how to anticipate wear in a slide bearing after many time units of the operation if bearing materials, exploitation and environmental conditions are known.

6. Conclusions

1. The bearing wear prognosis presented in an analytical recurrent form was performed on the basis of a hypothesis examined experimentally, where the wear after a sufficiently high number of time units of the exploitation

process depends on the wear value occurring in two succeeding earlier time intervals.

2. This paper presents the influence of classical bearing materials and non-classical bearing bio-material properties, journal vibrations and revolution, and standard deviation of bearing surface deformations on the values of the bearing wear process in the succeeding time periods of the operation.
3. The results obtained in this paper allow one to identify the region of experimental parameter values where a convergent or divergent slide-bearing wear process occurs.
4. The theory presented here may find new application possibility in analytical methods for anticipation of wear slide bearing convergence course in particular time units of operating time. Moreover, the divergence process of the sum of wear, i.e. cumulative values after arbitrary time units of the bearing exploitation were considered.
5. The theory presented here may be applied in preparation of methods for controlling divergence and convergence of slide bearing wear course in particular time units of the operating time.
6. The numerical analysis illustrated by the calculation example and the numerical results depicted in Fig. 14 and 15 make it possible to apply the above-presented recurrence prognosis in practical bearing design.

Manuscript received by Editorial Board, August 11, 2014;
final version, October 19, 2014.

REFERENCES

[1] Bhushan B.: Handbook of Micro/Nano Tribology, second ed. CRC Press, Boca Raton, London, New York, Washington D.C., 1999.
[2] Bhushan B.: Nanotribology and nanomechanics of MEMS/NEMS and BioMEMS/BioNEMS materials and devices, Microelectronic Engineering, 2007, 84, pp. 387-412.
[3] Canter N.: Developments of a Lean, Green Automobile, Tribology and Lubrication Technology, 2004, 60, pp. 15-16.
[4] Chizhik S., Wierzcholski K., Trushko A., Zbytkova M., Miszczak A.: Properties of Cartilage on Micro- and Nanolevel, Advances in Tribology, 2010, Volume 2010, doi:10.1155/2010/243150.
[5] Hiratsuka K., Goto M.: The role of changes in hardness of subsurfaces, transfer particles and wear particles in initial-steady wear transition, Wear, 2000, 238, pp. 70-77.
[6] Jang G.H., Seo C.H., Ho Scong Lee: Finite element model analysis of an HDD considering the flexibility of spinning disc-spindle, head-suspension-actuator and supporting structure, Microsystem Technologies, 2007, 13, pp. 837-847.
[7] Kapoor A. et al.: Automotive Tribology, in: Bharat Bhushan, Modern Tribology Handbook, CRC Press, Boca Raton, London, New York, Washington D.C., 2000, pp. 1187-1229.
[8] Ludema K.C.: Friction, Wear, Lubrication, CRC Press, N.Y., London, Tokyo, 1996.

[9] Pytko S., Wierzcholski K.: Elastohydrodynamic contact between two rollers under conditions of unsteady motion, Wear, 1979, 55, pp. 245-260.
[10] Ralston A., Rabinowitz P.: First Course in Numerical Analysis, Courier Dover Publications, 2001.
[11] Rezaei A., Paepegem W.V., Debaets P., Ost W., Degrieck J.: Adaptive finite element simulation of wear evolution in radial sliding bearings, Wear, 2012, 296, pp. 660-671.
[12] Wierzcholski K.: Unified summation equations and their applications in tribology wear process, Bulletin of the Polish Academy of Sciences Technical Sciences, 2013, Vol. 61 No. 2, pp. 405-417.
[13] Yang L.J.: A test methodology for the determination of wear coefficient, Wear, 2005, 259, pp. 1453-1461.
[14] Yuan C.Q., Peng Z., Yan X.P., Zhou X.C.: Surface roughness evaluation in sliding wear process, Wear, 2008, 265, pp. 341-348.

Metoda prognozowania zużycia łożyska ślizgowego z wykorzystaniem komputera

S t r e s z c z e n i e

Rozważania przedstawione w niniejszej pracy obejmują komputerową analizę prognozy zużycia łożyska ślizgowego z wykorzystaniem rozwiązań równań rekurencyjnych oraz danych eksperymentalnych.

Problem optymalizacji zużycia powierzchni ślizgowych łożyska został rozwiązany przy wykorzystaniu obliczeń analitycznych i numerycznych a także przy uwzględnieniu materiału łożyskowego, parametrów doświadczalnych oraz warunków brzegowych.

Uzyskane wyniki pozwalają nam dostrzec związek między chropowatością, właściwościami materiału, amplitudą drgań, rodzajem sił tarcia, twardością materiałów, prędkością ślizgania z jednej strony, a przyrostem zużycia w kolejnych jednostkach czasowych procesu eksploatacji z drugiej strony.