www.czasopisma.pan.pl PAN www.journals.pan.pl

A R C H I V E O F M E C H A N I C A L E N G I N E E R I N G

VOL. LXII 2015 Number 3

10.1515/meceng-2015-0018

Key words: three-layered plate, thick core, dynamic stability

DOROTA PAWLUS[*]

DYNAMIC RESPONSE OF THREE-LAYERED, ANNULAR PLATE WITH THICK CORE

The paper presents the solutions, calculation results and dynamic observations of three-layers, annular plate with thick core subjected to increasing in time load. The presented solutions use approximate methods: orthogonalization method and finite difference method in analytical and numerical solution of the problem, and finite element method. The observed phenomenon of the reduction of critical load values of the plates, in which the buckling mode is not global and there are different additional deflections of respective plate layers was comprehensively analysed in order to evaluate the critical state and supercritical plate behaviour. The critical deformation could have a form with strong deformation in the region of the loaded plate edge. The observation of the dynamic behaviour of plates, which buckling modes have circumferential waves is an important element of the analysis. Presented in this work the analytical and numerical solution to the problem of dynamic plate deflection was generalized on the case of plates with buckling waves in circumferential direction.

1. Introduction

The evaluation of the critical state of three-layered plates with thick core requires that plate material and geometrical parameters are taken into account in buckling behaviour problem. The thickness of plate core has the important meaning. With the increase in core thickness, a significant change in the mode of plate stability loss could be expected. The mode changes from a quasi-Eulerian form appearing in layered plates with cores of middle thickness to a form characterized with strong symmetrical or unsymmetrical transverse deformations particularly in the region of loaded edge of the plate. It is observed in plates with thick core. Then, the values of critical loads are lower than the corresponding ones, obtained for the global mode buckling. This phenomenon

[*] *Faculty of Mechanical Engineering and Computer Science, University of Bielsko-Biała, Willowa 2, 43-309 Bielsko-Biała, Poland; e-mail: doro@ath.bielsko.pl*

was considered by the Authors of example works [1], [2], [3]. The subject of the analysis presented in this work are observations of buckling behaviour of three-layered plates with the annular shape, which confirm these regularities.

The problem of dynamic stability of layered structure of annular plate is undertaken in numerous works, for example in works [4], [5], [6]. The annular sandwich plate with thin elastic or viscoelastic core is the subject of the analysis.

The problem of the local stability analysis of facing of plates, also ones with thick core, is presented in work [7]. The analytical, numerical solutions with the observations of very seldom undertaken experimental investigations are presented. The problem of the plate transverse loading is analysed. Critical loads and buckling shapes have been obtained. The values of critical loads were compared using two methods. In these loads, one could observe some interesting difference in values distribution depending on plate core thickness.

The dynamic problems presented in this paper are rooted in observations described in work [8]. The calculated values of critical static loads and buckling behaviour analysis indicate differences in stability loss between plates with middle and thick core. In this work, this problem is elaborated into dynamic analyses of annular, three-layered plates with thick core.

2. Problem formulation

The subject of the consideration is the three-layered, annular plate with the core multiple thicker than the thickness of facing. The plate is loaded with uniformly distributed forces that cause the radial compression of outer layers. The main observation is focused on the dynamic response to the loads linearly increasing in time at high rate, expressed by the formula:

$$p = st \tag{1}$$

where: p – compressive stress, s – rate of plate loading growth, t – time.

The scheme of such a plate model is presented in Fig. 1. The subject of examination was a plate with elastic properties of the polyurethane foam of the core material. The calculations were carried out using two methods: finite difference method (FDM) and finite element method (FEM). The expected, quasi-Eulerian form of the loss of plate dynamic stability has been denoted at the moment of time of plate loading, which was determined using the criterion presented in Volmir's work [9]. According to this criterion, the loss of plate stability occurs at the moment when the displacement speed at the point of maximum deflection reaches the first maximum value. Fig. 2 explains the determination of critical state parameters: critical time with calculated dynamic critical load and critical deflection according to the criterion presented above.

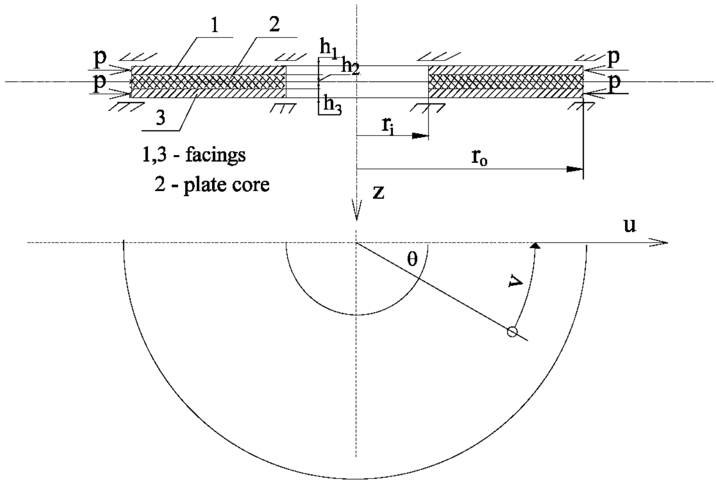


Fig. 1. Scheme of the plate

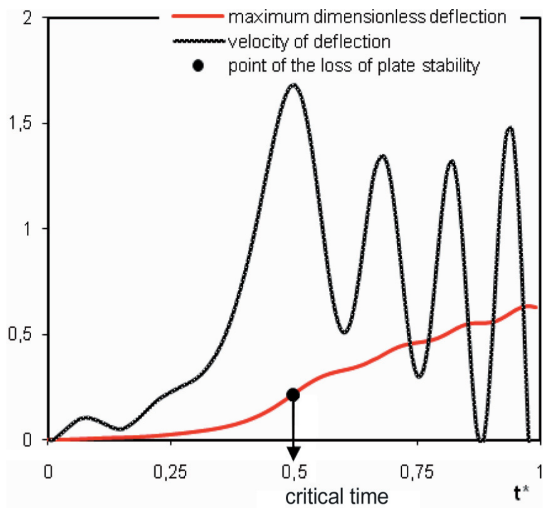


Fig. 2. Exemplary lines of time histories of deflection and velocity of deflection of three-layered plate loaded on inner edge

3. The solution to the dynamic problem using the finite difference method

The analytical and numerical solution uses the approximate methods: orthogonalization method and finite difference method. The detailed description presenting the basic equations for annular, three-layered plate in nonlinear geometrical problem is presented in monograph [10] an in paper [11], too. The solution is based on the assumption of classical theory of sandwich plate using the broken line hypothesis and the participation of plate layers in carrying the plate load: the facings are loaded with normal stresses, but the core is subjected to shear stresses [12]. One assumes equal values of transverse additional deflections of three symmetrically arranged layers in the plate structure. The plate layers are tied. One uses linear physical relationships characterizing the materials of layers and nonlinear geometrical equations expressed by Kármán's equations in mathematical description of facing deformations. In solving the problem, one excludes the impact loading.

The essential elements of the solution are the following:

– formulation of dynamic equilibrium equations for each layer of plate,
– description of core deformation and formulation of physical relations,
– determination of relations between sectional forces and moments with stresses and determination of the resultant forces expressed by the assumed stress function,
– determination of the initial and loading conditions, clamped boundary conditions expressed by the following equations:

$$w_d\,|_{t=0}=0,\quad w_{d,t}\,|_{t=0}=0,$$
$$\sigma_r|_{r=r_i}=-p(t)d_1,\quad \sigma_r\,|_{r=r_o}=-p(t)d_2,\quad \tau_{r\theta}\,|_{r=r_i(r_o)}=0, \tag{2}$$
$$w\,|_{r=r_i(r_o)}=0,\quad w,_r\,|_{r=r_i(r_o)}=0,\quad \delta=\gamma\,|_{r=r_i(r_o)}=0,\quad \delta,_r\,|_{r=r_i(r_o)}=0,$$

where:
w – total deflection, w_d – additional deflection,
σ_r – radial stress, $\tau_{r\theta}$ – shear stress,
d_1, d_2 – quantities, equalled to 0 or 1, determining the loading of the inner or/and outer plate perimeter,
δ, γ – differences of radial and circumferential displacements of the points on middle surfaces of facings $\delta = u_3 - u_1, \gamma = v_3 - v_1$, respectively,
– introducing the dimensionless quantities:

$$\zeta_1=\frac{w_d}{h},\quad \zeta_o=\frac{w_o}{h},\quad F=\frac{\Phi}{Eh^2},\quad \bar{\delta}=\frac{\delta}{h},\quad \bar{\gamma}=\frac{\gamma}{h},\quad \rho=\frac{r}{r_o}, \tag{3}$$

where:
Φ – stress function, E –Young's modulus of facing material,
h – total plate thickness,
ζ_o – function of pre-deflection form w_o [13]:
$\zeta_o(\rho,\theta)=\xi_1(\rho)\eta(\rho)+\xi_2(\rho)\eta(\rho)\cos(m\theta)$,
ξ_1, ξ_2 – calibrating numbers,
$\eta(\rho)$ – function: $\eta(\rho) = \rho^4 + A_1\rho^2 + A_2\rho^2\ln\rho + A_3\ln\rho + A_4$,
A_i – quantities fulfilling the conditions of clamped edges,
expressions and shape functions for the additional plate deflection ζ_1:

$$\xi_1(\rho,\theta,t) = X_1(\rho,t)\cos(m\theta), \tag{4}$$

stress function F [13]:

$$F(\rho,\theta,t) = F_a(\rho,t) + F_b(\rho,t)\cos(m\theta) + F_c(\rho,t)\cos(2m\theta), \tag{5}$$

and differences $\bar{\delta}$, $\bar{\gamma}$:

$$\begin{aligned} &\bar{\delta}(\rho,\theta,t) = \bar{\delta}(\rho,t)\cos(m\theta), \\ &\bar{\gamma}(\rho,\theta,t) = \bar{\gamma}(\rho,t)\sin(m\theta), \end{aligned} \tag{6}$$

– determination of the basic differential equation that expresses the deflections of the analysed sandwich plate after the use of the orthogonalization method for elimination of the angular variable θ:

$$\begin{aligned} &-\mathrm{WK1}\cdot X_{1,\rho\rho\rho\rho} - \frac{2\mathrm{WK1}}{\rho}X_{1,\rho\rho\rho} + \frac{\mathrm{WK3}}{\rho^2}X_{1,\rho\rho} - \frac{\mathrm{WK3}}{\rho^3}X_{1,\rho} - \frac{\mathrm{WK4}}{\rho^4}X_1 + \\ &+\frac{2\mathrm{WK1}}{\rho^4}m^2X_1 + \frac{\mathrm{WK5}}{\rho}H'\left(m\bar{\gamma} + \bar{\delta}_{,\rho} + \bar{\delta} - \frac{m^2}{\rho}\frac{H'}{r_o}X_1 + \frac{H'}{r_o}X_{1,\rho} + \frac{H'}{r_o}\rho X_{1,\rho\rho}\right) + \\ &+\frac{2\mathrm{WK5}^2\,\mathrm{WK2}}{\rho}\left(X_{,\rho}Y_{o,\rho} + Y_oX_{,\rho\rho} - \frac{m^2}{\rho}X\,Y_{o_{,\rho}}\right) + \\ &\left(1-\delta_m^0\right)\left\{\frac{4\mathrm{WK5}^2\,\mathrm{WK2}}{\rho^2}\left(\frac{1}{\rho}m^2F_cX_{,\rho} - F_{c_{,\rho}}m^2X_{,\rho} + \frac{1}{\rho}F_{c_{,\rho}}m^2X - \frac{1}{\rho^2}F_cm^2X\right) + \right. \\ &+\frac{\mathrm{WK5}^2\,\mathrm{WK2}}{\rho}\left(2X_{a_{,\rho}}F_{b_{,\rho\rho}} + X_{,\rho}F_{c_{,\rho\rho}} + 2X_{a_{,\rho\rho}}F_{b_{,\rho}} + X_{,\rho\rho}F_{c_{,\rho}} + \right. \\ &\left.\left.-\frac{2}{\rho}m^2F_bX_{a_{,\rho\rho}} - \frac{4}{\rho}m^2F_cX_{,\rho\rho} - \frac{1}{\rho}m^2F_{c_{,\rho\rho}}X\right)\right\} = \mathrm{K7}^2\cdot\mathrm{WK5}\cdot\mathrm{WK8}\cdot X_{1_{,t^*t^*}} \end{aligned} \tag{7}$$

where:

δ_m^0 – Kronecker's symbol,

$X(\rho,t) = X_1(\rho,t) + X_b(\rho)$, $X_b(\rho) = \xi_2\eta(\rho)$, $Y_o = F_{a,\rho}$

$t^* = t\cdot\mathrm{K7}$, $\mathrm{K7} = \dfrac{s}{p_{cr}}$ (p_{cr} – critical, static stress), WK1, WK2, WK3, WK4, WK5, WK8 – the quantities expressed by the geometric and material parameters of plate layers and by the number m,

- calculation of the quantities $\overline{\delta}$ and $\overline{\gamma}$ in Eq. (7) using the additional equilibrium equations for forces acting on the undeformed outer plate layers in the u and v direction (Fig. 1),
- determination of Eq. (8) using the relations for radial and circumferential normal forces and shearing forces in facings and after the elimination of the quantities described by the sums of the radial ($u_3 + u_1$) and the circumferential ($v_3 + v_1$) facing displacements:

$$\Phi_{,rrrr} + \frac{2}{r}\Phi_{,rrr} - \frac{1}{r^2}\Phi_{,rr} + \frac{1}{r^3}\Phi_{,r} + \frac{1}{r^4}\Phi_{,\theta\theta\theta\theta} + \frac{4}{r^4}\Phi_{,\theta\theta} - \frac{2}{r^3}\Phi_{,r\theta\theta} + \frac{2}{r^2}\Phi_{,rr\theta\theta} =$$
$$= E\left[\frac{1}{r}w_{o,rr}\left(w_{o,r} + \frac{1}{r}w_{o,\theta\theta}\right) - \frac{1}{r^2}\left(\frac{1}{r}w_{o,\theta} - w_{o,r\theta}\right)^2 - \frac{1}{r}w_{,rr}\left(w_{,r} + \frac{1}{r}w_{,\theta\theta}\right) + \right. \tag{8}$$
$$\left. + \frac{1}{r^2}\left(\frac{1}{r}w_{,\theta} - w_{,r\theta}\right)^2\right].$$

The basic system of differential equations, determined using the finite difference method approximating the derivatives with respect to ρ by the central differences in the discrete points has the following form:

$$\mathbf{P}_L\mathbf{U} + \mathbf{Q}_L = \mathrm{K}\cdot\ddot{\mathbf{U}}, \tag{9}$$

$$\mathbf{M}_D\mathbf{D} = \mathbf{M}_U\mathbf{U} + \mathbf{M}_G\mathbf{G}, \tag{10}$$

$$\mathbf{M}_{GG}\mathrm{G} = \mathbf{M}_{GU}\mathbf{U} + \mathbf{M}_{GD}\mathrm{D}, \tag{11}$$

$$\mathbf{M}_Y\ \mathbf{Y} = \mathbf{Q}_Y, \tag{12}$$

$$\mathbf{M}_V\mathbf{V} = \mathbf{Q}_V, \tag{13}$$

$$\mathbf{M}_Z\mathbf{Z} = \mathbf{Q}_Z, \tag{14}$$

where:

K=K7^2 ·WK5·WK8,

U, **Y**, **V**, **Z** – vectors of plate additional deflections and components F_a, F_b, F_c of the stress function $F_{a,\rho} = y$, $F_b = v$, $F_c = z$, respectively,

$\mathbf{Q}_L$, $\mathbf{Q}_Y$, $\mathbf{Q}_V$, $\mathbf{Q}_Z$ – vectors of expressions composed of the initial and additional deflections, geometric and material parameters, components of the stress func-

tion, radius ρ, quantity b (b – length of the interval in the finite difference method), coefficients δ, γ and number m,
$\mathbf{P}_L$, $\mathbf{M}_D$, $\mathbf{M}_U$, $\mathbf{M}_G$, $\mathbf{M}_{GG}$, $\mathbf{M}_{GU}$, $\mathbf{M}_{GD}$, $\mathbf{M}_Y$, $\mathbf{M}_V$, $\mathbf{M}_Z$ – matrices of elements composed of plate parameters, quantity b and number m,
D, **G** – vectors of expressions composed of coefficients δ and γ, respectively.

The Runge-Kutta's integration method for the initial state of plate has been used in the solution of the presented system of equations.

Critical static stress p_{cr} has been calculated solving the eigenproblem for the problem of the disk state neglecting the inertial components and nonlinear expressions.

The critical, static load $p_{cr}{}^*$ as the minimal value of stress p^* is the solution to the eigenvalue problem from the following general equation:

$$\det\left(\mathbf{M}_{ADG} - p^*\mathbf{M}_{AC}\right) = 0, \tag{15}$$

where:
$\mathbf{M}_{ADG}$, $\mathbf{M}_{AC}$ – matrices of elements composed of plate parameters, quantity b and number m.

4. Plate models in finite element method

Two models have been used in analysis:

- basic model in the form of a complete model of annular plate, circularly symmetrical – Fig. 4,
- simplistic model built of axisymmetrical elements – Fig. 5.

Geometry of plate structure is symmetric, composed of thin facings and soft, thicker core. The facings are built of shell elements, but the core mesh is built of solid elements. The grids of facings elements are tied with the grid of core elements using the surface contact interaction. Plate edges are slidably clamped. The calculations were carried out at the Academic Computer Center CYFRONET-CRACOW (KBN/SGI_ORIGIN_2000/PŁódzka/030/1999) using the ABAQUS system.

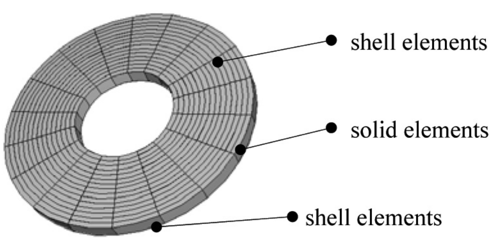


Fig. 4. Basic model

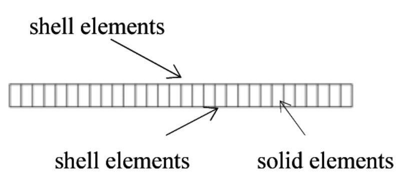


Fig. 5. Simplistic model

5. Examples of numerical calculations

The exemplary numerical calculations were carried out for a plate with the following material, geometrical and load parameters:

– inner radius: r_i = 0.2 m, outer radius: r_o = 0.5 m, facing thickness: h' = 0.005, 0.001 m, core thickness: h_2 = 0.005, 0.01, 0.02, 0.04, 0.06 m,
– steel facing material: Young's modulus $E = 2.1 \cdot 10^5$ MPa, Poisson's ratio $\nu = 0.3$, mass density $\mu = 7.85 \cdot 10^3$ kg/m^3,
– polyurethane foam core material treated as isotropic: Kirchhoff's modulus G_2 = 5 MPa [14], Young's modulus for isotropic material E_2 = 13 MPa, Poisson's ratio $\nu = 0.3$, mass density μ_2 = 64 kg/m^3,
– the rate of loading growth on the plate outer and inner edge is equal to $s \approx 931$ MPa/s, and $s \approx 4346$ MPa/s, respectively.

The form of plate preliminary deflection in dynamic problem is the rotational axially-symmetrical or circumferentially waved with the number of waves consistent with the number m corresponding to the analysed buckling form.

The critical static loads evaluation is the important starting-point of analyses. The exemplary values of the minimal critical static loads of waved plates supported in slidably clamped edges and radially compressed on outer edge are presented in Fig. 6. Line 1 shows the results obtained for the basic FEM model of the plate. Line 2 also shows the results for the basic model, but the layers are connected with the condition of equal deflection. Line 3 presents the results calculated for the plate model solved using the finite difference method (FDM). The point marked by the cross is for the plate case, in which the critical deformation has a form other than the quasi-Eulerian, global buckling form.

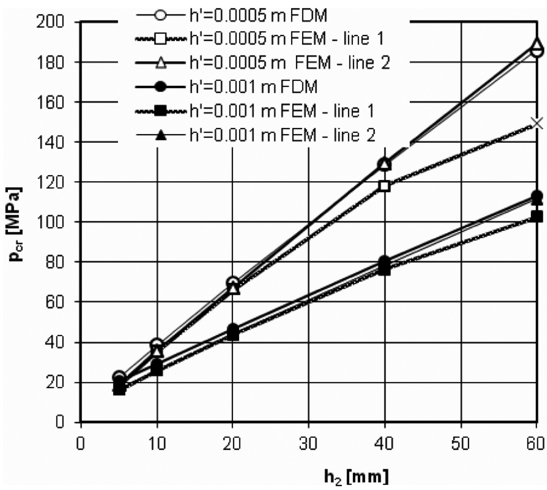


Fig. 6. Critical static loads distribution versus core thickness for facing thickness equal to: h' = 0.0005 m and h' = 0.001 m

The results show the good consistence between the values of p_{cr} for plates with the middle core thickness ($h_2 = 0.005 \div 0.02$ m), the compatibility of p_{cr} values calculated using FEM (line1) an FDM methods and significant differences between p_{cr} values for plates with thick core $h_2 = 0.06$ m. Here, we observe relevant decrease in p_{cr} values of FEM (line 2) basic plate model without the connected layers with the condition of the equal deflection and tendency to the change of global buckling form to a stronger deflected one.

The dynamic plate behaviours are presented for plate model with thick core: $h_2 = 0.06$ m and facing thickness: $h' = 0.001$ m. Fig. 7 shows the time histories in deflections of axially-symmetrical buckling form ($m = 0$) of plates loaded on inner and outer edges. Marked points denote the moment of the loss of plate stability determined according to the accepted criterion (see, Fig. 2). The accurate values of dynamic, critical loads p_{crdyn} for FEM simplistic plate model and FDM plate model are presented in Table 1. The p_{crdyn} loads of FEM and FDM plate models are comparable with one another.

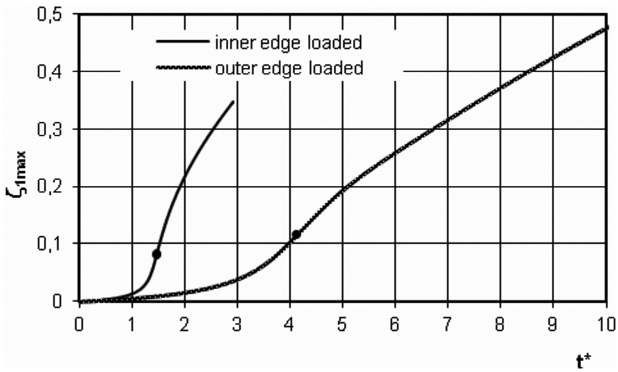


Fig. 7. Time histories of deflection of FDM model of axisymmetric plates ($m = 0$) with thick core ($h_2 = 0.06$ m) loaded on inner or outer edge

Table 1.

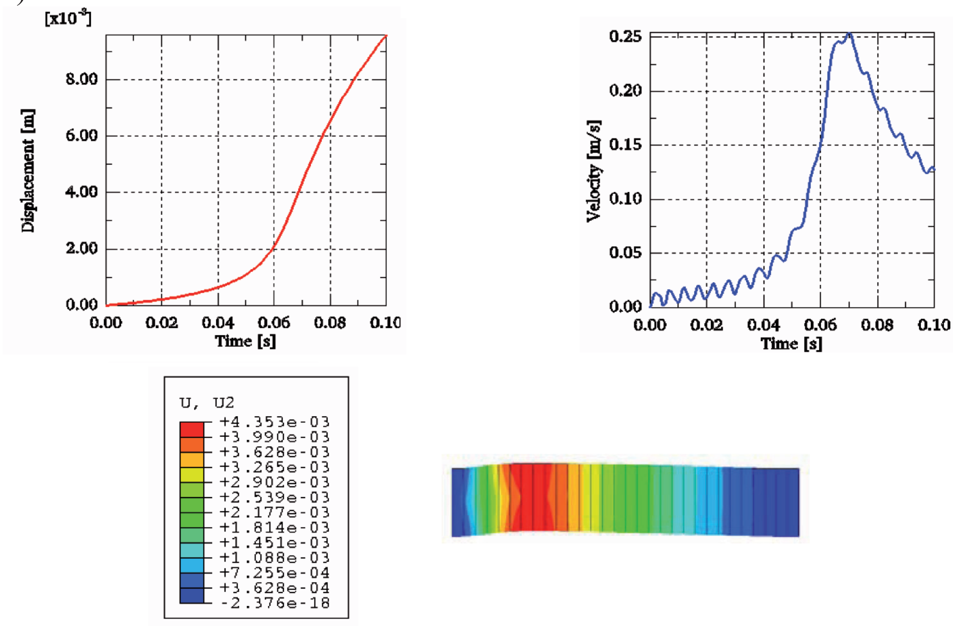
Values of critical dynamic loads of FEM and FDM models of axisymmetric plates loaded on inner or outer edge

Loaded edge	p_{crdyn} [MPa]	
	FEM	FDM
inner	304.22	319.24
outer	183.37	192.06

Figure 8 shows the time histories of deflection and velocity of deflection and buckling modes of simplistic and basic FEM models of plates loaded on inner edge. The values of p_{crdyn} for simplistic plate model are equal to $p_{crdyn} = 304.22$ MPa (see, Table 1) and slightly lower for basic plate model, equal to:

p_{crdyn} = 278.14 MPa. The form of the loss of dynamic stability is quasi-Eulerian, global for these two FEM plate models.

a)



b)

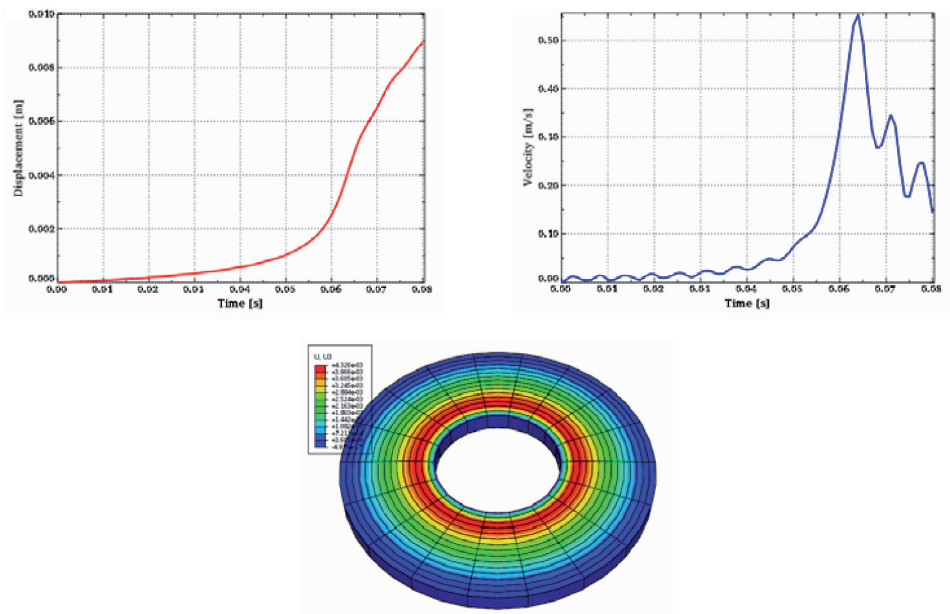


Fig. 8. Time histories of deflection and velocity of deflection and buckling forms (m = 0) of plate FEM models: a) simplistic, b) basic loaded on inner edge

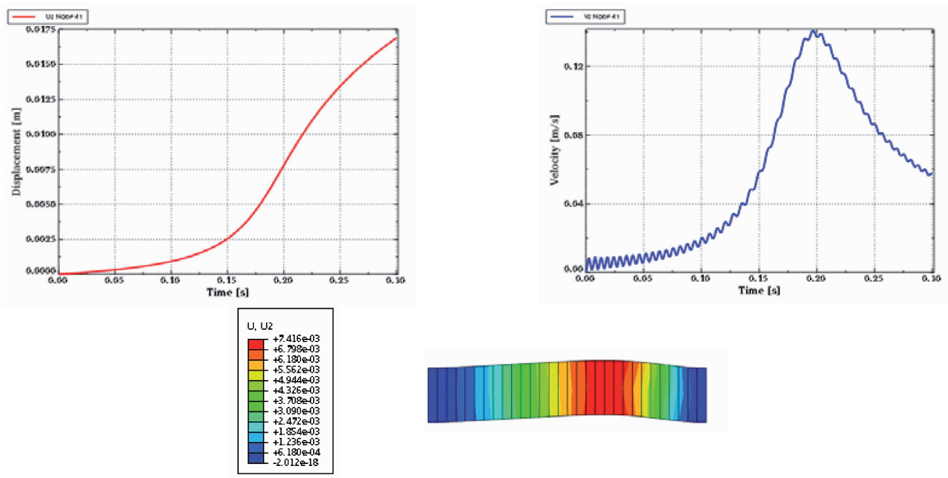
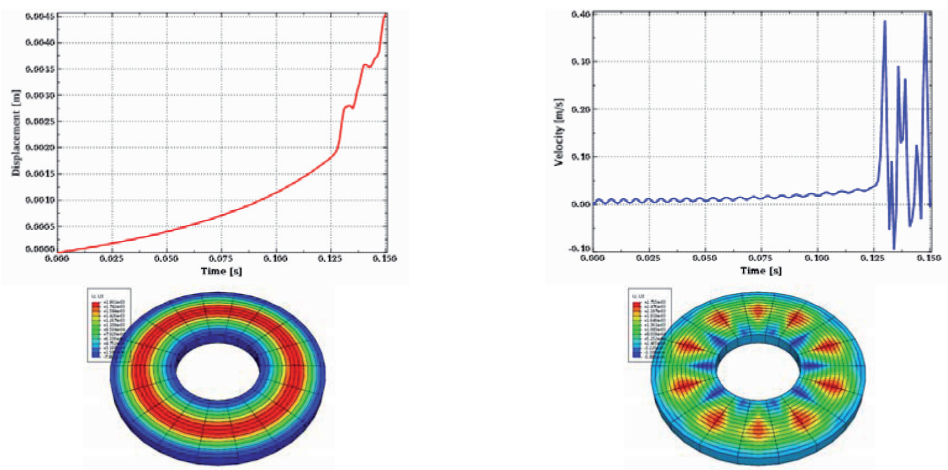


Fig. 9. Time histories of deflection and velocity of deflection and buckling form of plate FEM simplistic model of plate loaded on outer edge

Figure 9 shows the time histories of deflection and velocity of deflection of the simplistic plate model loaded on outer edge. The values of critical dynamic loads are equal to p_{crdyn} = 183.37 MPa and 192.06 MPa for FEM and FDM plate models, respectively. The calculation results of simplistic FEM model and FDM model are consistent for axially-symmetrical form of plate pre-deflection. The dynamic buckling mode is global. The value of critical load obtained for basic FEM model is lower, p_{crdyn} = 121.93 MPa. In the region of the critical state one observes the tendency to the change of the buckling form to a circumferentially waved one. The results are presented in Fig. 10.



t = 0.127 s t = 0.131 s

Fig. 10. Time histories of deflection and velocity of deflection and critical deformation forms of FEM basic model of plate loaded on outer edge

The comparison between dynamic, critical loads distribution depending on the mode number for FEM basic plate model and FDM model is presented in Fig. 11. The presented results are for plates loaded on outer edge. The differences are clear. The FDM plate model includes the accepted assumption of equal layers deflections. The layers are connected. The global form of the stability loss is imposed. The character of curve $\zeta_{1max} = f(t^*)$ is similar for each plate mode for the case of plate loaded on outer edge. It is presented in Fig. 7 for plate mode $m = 0$. This example corresponds with point denoted by $m = 0$, shown in Fig. 11.

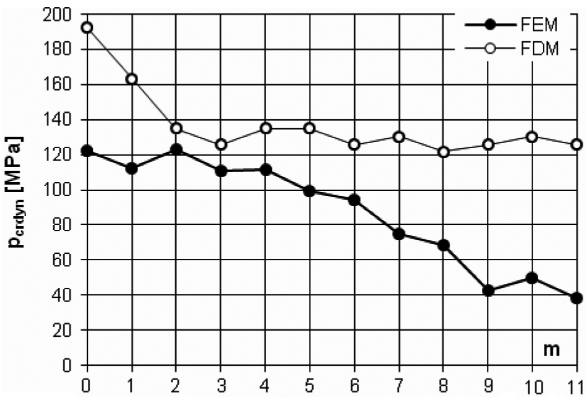


Fig. 11. Dynamic critical loads distribution in the function of mode number for FEM and FDM plate models loaded on outer edge

The examinations of the FEM basic model of plates with waved forms of pre-deflection show a relevant decrease in values of dynamic, critical loads of plates with number m equal to: $m = 9 \div 11$. The critical dynamic load for the plate with $m = 9$ is equal to $p_{crdyn} = 42.76$ MPa. The minimal critical static loads calculated for FEM and FDM models correspond to buckling mode with number $m = 9$ circumferentially waves, too. Additionally, the p_{crdyn} distribution for the FEM basic model of the plate with facing thickness equal to $h' = 0.005$ m is presented in Fig. 12. The comparison of p_{crdyn} distributions in the function of mode number for two kinds of sandwich plates differed with facing thickness shows similar distribution of p_{crdyn} values and the minimal ones for strongly waved plates ($m = 9$ or 11). Of course, the values of p_{crdyn} for plates with thinner facings are greater.

Figure 13 shows the time histories of deflection and velocity of deflection and critical deformation of plate with number of buckling waves $m = 9$. The presented characteristics are obtained for the plate nodes which belong to the transverse line of FEM grid: node 9031 to facing grid, adjacent node 13031 to core grid, node 5031 to second facing grid and adjacent node 31 to core grid

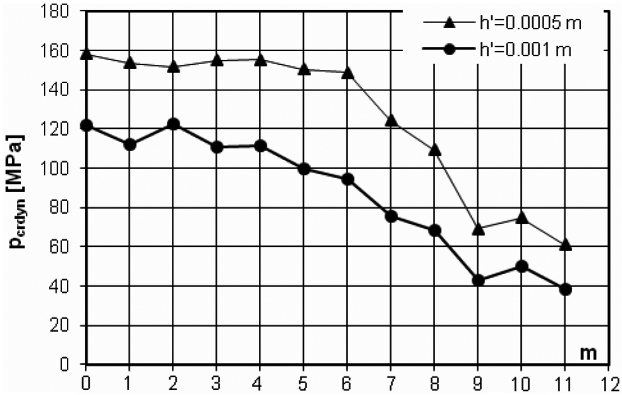


Fig. 12. Dynamic critical loads distribution versus mode number
for FEM models of plate with different facing thickness

and node 17031 to core grid situated in the middle of side line of the solid element. The characteristics show the character of transverse nonlinear deformation of plate structure with thick core. The loss of plate stability has not quite global form characterized by the equal additional deflections of each plate layers. This deformation of structure shows that classical broken line hypothesis in description of the geometrical relations for plates with thick core could not be proper. The hyperbolic deformation should approximate the transverse structure deformation better. This important comment was presented in work [2] and was indicated in earlier investigations (work [15]) concluding analysed

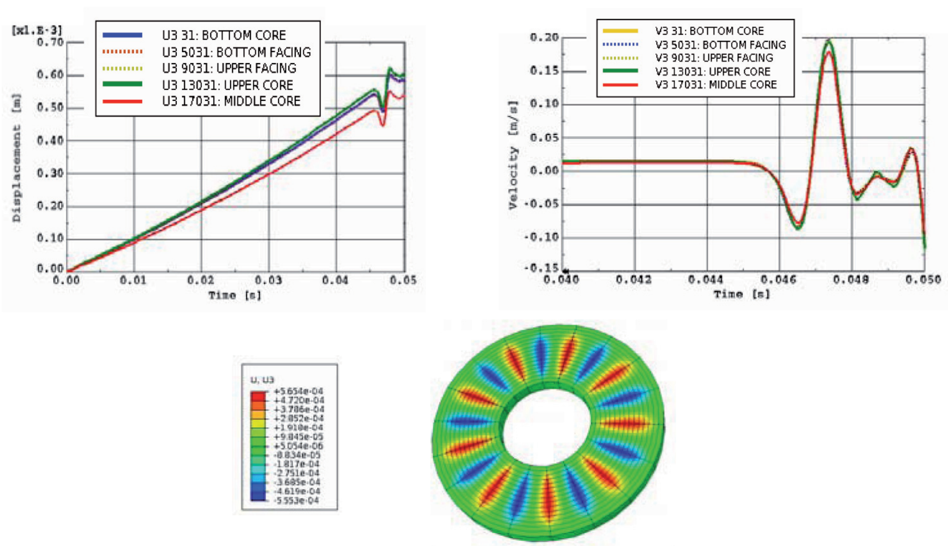


Fig. 13. Time histories of deflection and velocity of deflection in critical zone and buckling form
of FEM basic plate model with mode number: $m = 9$

three-layered, annular plate. The nonlinear core displacement field for sandwich structure of shells including wrinkling local failure mode is presented in work [16].

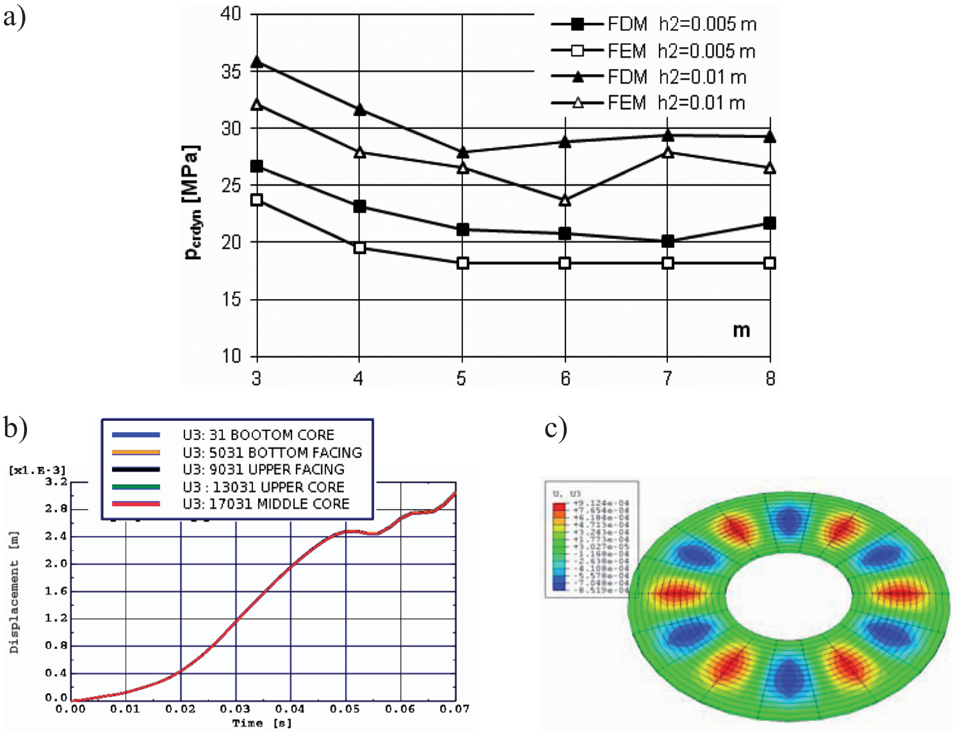
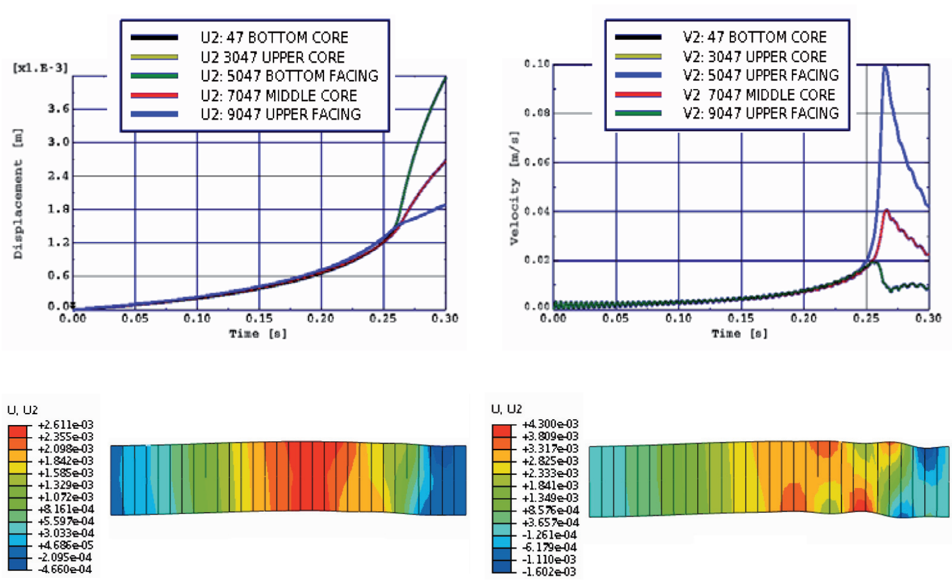


Fig. 14. Results for FEM basic and FDM models of plates with middle ($h_2 = 0.005$, 0.01 m) core thickness: a) dynamic critical loads distribution depended on mode number, b) time histories of deflection of layer nodes from grid transverse line, c) critical deformation form of plate with parameters: $m = 6$, $h_2 = 0.005$ m

The results shown in Fig. 14 could be the background for the presented analysis. There are p_{crdyn} distribution depending on plate mode number, time histories of deflection of layer nodes that belong to the transverse line of FEM grid with buckling deformation for plate with parameters: $m = 6$, $h_2 = 0.005$ m. The loss of plate dynamic stability is global. The deflections of each of the plate layer are equal. The values of p_{crdyn} calculated for two FEM basic and FDM plate models are comparable.

The critical deformation state with the tendency to the change from the regular, global form to the strongly deformed in the region of loaded plate outer edge is presented in Fig. 15. Figure 15 shows the loss of dynamic stability of the plate with the thick core ($h_2 = 0.06$ m) and very thin facings

($h' = 0.0005$ m) built of the axisymmetric elements (FEM simplistic model). The form of the plate preliminary deflection corresponds to the expected buckling mode with m equal to: $m = 0$. The curves show the time histories of the deflections and velocity of deflections of the selected grid nodes. From the moment of the loss of plate dynamic stability, the additional deflections of the nodes of upper and bottom plate layers and the nodes located in the middle core surface take different values. The critical deformation is not global. Additionally, one observes a relevant decrease in the value of p_{crdyn} ($p_{crdyn} = 245.75$ MPa) for FEM simplistic model in comparison with value calculated for FDM model equal to $p_{crdyn} = 347.65$ MPa, where the global buckling exists.



$t = 0.264$ s $t = 0.3$ s

Fig. 15. Time histories of deflections and velocity of deflections and buckling forms of FEM simplistic model of plate loaded on outer edge with parameters: $h_2 = 0.06$ m, $h' = 0.0005$ m

Figure 16 shows the critical behaviour of FEM simplistic plate model loaded on inner edge. Time histories of deflections and velocity of deflections for two grid nodes: the first one (node 4) located near the loaded edge and the second one (node 20) in the middle area of plate (usually, strongly deflected) present a form of plate buckling different from the global one. The values of p_{crdyn} calculated for FEM and FDM models do not differ significantly. They are equal to: $p_{crdyn} = 360.72$ MPa and $p_{crdyn} = 382.70$ MPa, respectively.

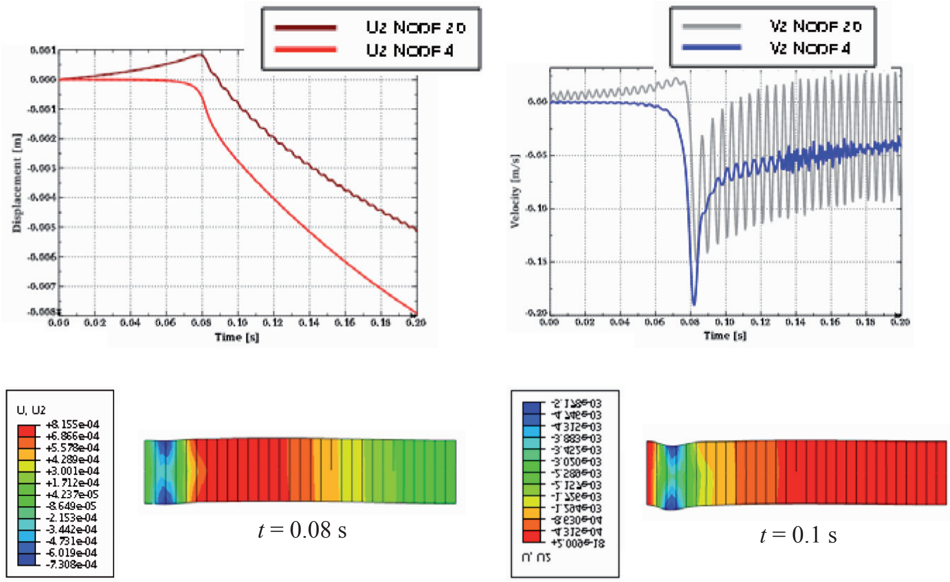


Fig. 16. Time histories of deflections and velocity of deflections and buckling forms of FEM simplistic model of plate loaded on inner edge with parameters: $h_2 = 0.06$ m, $h' = 0.0005$ m

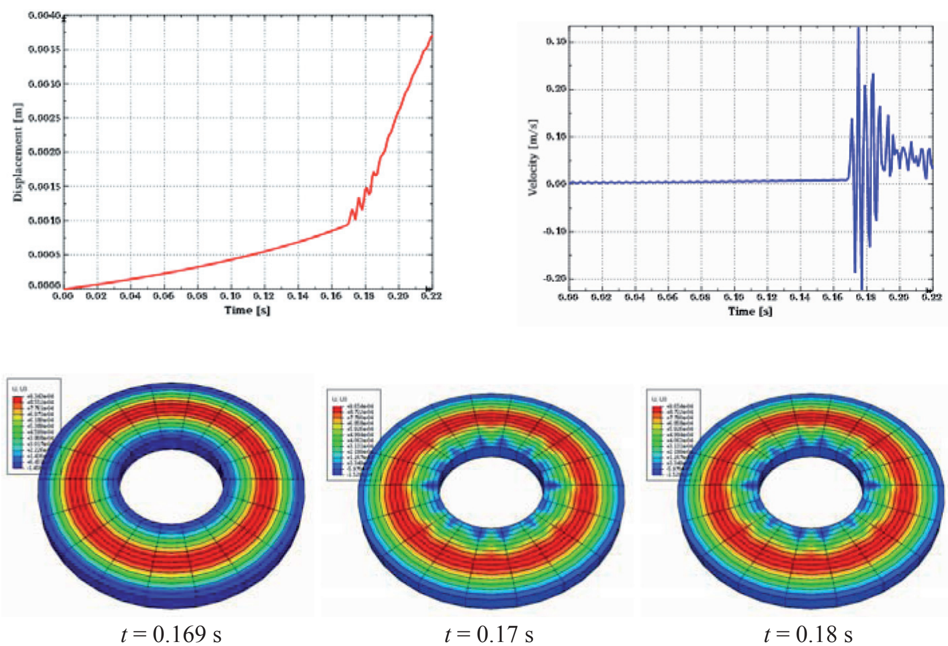


Fig. 17. Time histories of deflection and velocity of deflection and buckling forms of FEM basic model of plate loaded on outer edge with parameters: $h_2 = 0.06$ m, $h' = 0.0005$ m

The illustration of plate dynamic, critical behaviour presented in Figs. 15, 16 and also in Fig. 17 concerns the plate examples with thin facings equal to $h' = 0.0005$ m. The tendency to changes of the regular axisymmetric plate deformation in the critical zone, shown in Fig. 17, confirms earlier observations. The presented results concern the FEM basic model of plate loaded on outer edge with the same parameters as the plate example shown in Fig. 16. The value of p_{crdyn} is much lower ($p_{crdyn} = 158.24$ MPa) than the value calculated for the FDM model (mentioned earlier, $p_{crdyn} = 347.65$ MPa) and than that obtained for FEM simplistic model shown in Fig. 16.

6. Conclusions

The results presented in this work show the importance of circumferentially waved forms of buckling included in the solution to the problem. In general, the loss of static and dynamic stability of three-layered plates with soft thick core has not a global form. The form of critical deformation could be different. The transverse deformation of plates with thick core does not fulfil the condition of equal deflections of plate layers occurring for the global buckling form. This behaviour is particularly characteristic for plates with very thin facings. It could be assumed that the nonlinear, geometrical relations in description of the transverse deformation of such a layered structure should be used. The limitation of the plate examinations to the analysis for quasi-Eulerian buckling forms leads to the calculations of the inflated values of critical loads. Such a conclusion was also formulated and emphasised in the mentioned work [2].

Manuscript received by Editorial Board, December 18, 2014;
final version, May 10, 2015

REFERENCES

[1] Rammerstorfer F. G., Dorninger K., Starlinger A.: Composite and sandwich shells in nonlinear analysis of shells by finite elements (Ed. F.G. Rammerstorfer). Springler-Verlag, Wien, 1992, pp.131-194.
[2] Romanów F.: Wytrzymałość konstrukcji warstwowych (Strength of sandwich constructions). WSI in Zielona Góra, Poland 1995, (in Polish).
[3] Leissa A.W.: A review of laminated composite plate buckling. Appl. Mech. Rev., 1987, 40, 5, 575-591.
[4] Chen YR., Chen, LW.: Axisymmetric parametric resonance of polar orthotropic sandwich annular plates. Composite Structures, 2004, 65, 269-277.
[5] Chen YR., Chen LW., Wang CC.: Axisymmetric dynamic instability of rotating polar orthotropic sandwich annular plates with a constrained damping layer. Composite Structures, 2006, 73, (2), 290-302.
[6] Chen YR., Chen LW.: Vibration and stability of rotating polar orthotropic sandwich annular plates with a viscoelastic core layer. Composite Structures, 2007, 78, 45-57.

[7] Jasion P., Magnucki K.: Modelowanie wyboczenia lokalnego okładziny kołowej płyty sandwiczowej (Modelling of local buckling of the face of the circular sandwich plate). Modelowanie Inżynierskie, 2012, 45, 14, 171-176, (in Polish).
[8] Pawlus D.: Approach to evaluation of critical static loads of annular three-layered plates with various core thickness. Journal of Theoretical and Applied Mechanics, 2008, 46, 1, 85-107.
[9] Volmir C.: Nonlinear dynamic of plates and shells. Moskwa: Science 1972, (in Russian).
[10] Pawlus D.: Stateczność dynamiczna trójwarstwowych płyt pierścieniowych z rdzeniem lepkosprężystym (Dynamic stability of three-layered annular plates with viscoelastic core). Politechnika Łódzka, Zeszyty Naukowe, nr 1075, Łódź 2010 (in Polish).
[11] Pawlus D.: Dynamic stability of three-layered annular plates with wavy forms of buckling. Acta Mechanica, 2011, 216, 123-138.
[12] Volmir C.: Stability of deformed system. Moskwa: Science 1967, (in Russian).
[13] Wojciech S.: Numeryczne rozwiązanie zagadnienia stateczności dynamicznej płyt pierścieniowych (Numerical solution of the problem of dynamic stability of annular plates). Journal of Theoretical and Applied Mechanics, 1979, 17, 2, 247-262, (in Polish).
[14] Majewski S., Maćkowski R.: Pełzanie spienionych tworzyw sztucznych stosowanych jako rdzeń płyt warstwowych (Creep of foamed plastics used as the core of sandwich plate). Inżynieria i Budownictwo, 1975, 3, pp.127-131, (in Polish).
[15] Pawlus D.: Critical static loads calculations in finite element method of three-layered annular plates. Archives of Civil and Mechanical Engineering, 2007, VII, 1, 21-33.
[16] Kühborn A., Schoop H.: A nonlinear theory for sandwich shells including the wrinkling phenomenon. Archive of Applied Mechanics, 1992, 62, 413-427.

Dynamiczna odpowiedź pierścieniowej płyty trójwarstwowej z rdzeniem grubym

Streszczenie

W pracy przedstawiono sposób rozwiązania zagadnienia ugięć dynamicznych trójwarstwowych płyt pierścieniowych oraz liczne wyniki obliczeń numerycznych. Przedmiotem obserwacji są płyty, których rdzeń traktowany jest jako środkowa gruba warstwa. Płyta poddana jest obciążeniom liniowo narastającym w czasie działającym w płaszczyźnie okładzin na brzeg zewnętrzny lub wewnętrzny. Rozwiązanie zagadnienia dynamicznych ugięć płyty wykorzystuje przybliżone metody: ortogonalizacji i różnic skończonych. Problem rozwiązano także wykorzystując metodę elementów skończonych budując modele płyt w programie ABAQUS. Rozwiązanie umożliwia obserwację zarówno osiowosymetrycznych, jak i obwodowo pofalowanych postaci utraty stateczności płyty. Obserwacjom poddano zachowanie płyt w obszarze krytycznym i pokrytycznym. Szczegółowa analiza dotyczy tych przypadków płyt, których utrata stateczności nie ma formy globalnej, a wartość krytycznych obciążeń dynamicznych jest znacznie niższa od wyznaczonej dla globalnej postaci wyboczenia. Liczne obrazy dynamicznych zachowań płyt przedstawione postaci wykresów warstwicowych oraz wiele sformułowanych uwag i spostrzeżeń mogą mieć istotne znaczenie zarówno poznawcze, jak i praktyczne w dziedzinie dynamiki płyt kompozytowych.