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MODELING OF DAMAGE EVOLUTION AND MARTENSITIC TRANSFORMATION IN AUSTENITIC STEEL AT CRYOGENIC TEMPERATURE

In the present work, a constitutive model of materials undergoing the plastic strain induced phase transformation and damage evolution has been developed. The model is based on the linearized transformation kinetics. Moreover, isotropic damage evolution is considered. The constitutive model has been implemented in the finite element software Abaqus/Explicit by means of the external user subroutine VUMAT. A uniaxial tension test was simulated in Abaqus/Explicit to compare experimental and numerical results. Expansion bellows was also modelled and computed as a real structural element, commonly used at cryogenic conditions.

1. Introduction

The present paper is focused on the constitutive description of FCC *(face centered cubic)* materials applied at very low temperatures. FCC metals and alloys are often applied in cryogenic conditions, down to the temperature in the proximity of absolute zero, because of their remarkable properties including ductility [1]. The theoretical description addresses the following two phenomena: damage evolution and austenite to martensite ($\gamma \rightarrow \alpha'$) phase transformation. Phase transformation can be defined as a change in macroscopic configuration of atoms or molecules caused by change of thermodynamic variables characterizing the system, such as temperature, pressure or magnetic field. A phase is understood here as a homogeneous microstructure, having identical properties and defined boundaries. It is assumed here that $\gamma \rightarrow \alpha'$ transformation is the change of crystallographic configuration but

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without the diffusion mechanism. The plastic strain induced phase transformation causes a considerable evolution of material properties (strong hardening).

The classical model of plastic strain induced $\gamma \rightarrow \alpha'$ phase transformation at low temperatures was developed by Olson and Cohen [2]. The authors postulated a three parameter model capable of describing the experimentally verified sigmoidal curve that represents the volume fraction of martensite as a function of plastic strain. However, at very low temperatures the rate of phase transformation becomes less temperature dependent and can be described by a simplified linearized model proposed by Garion and Skoczeń [3]. Since the α'-martensite behaves in the flow range of austenite-martensite composite mostly in elastic way (yield point of α'-martensite is much higher than the yield point of γ-austenite, [4]) its presence in the lattice affects the plastic flow and the process of hardening.

As was mentioned, FCC metals are often applied in cryogenic conditions. As an example, Fe-Cr-Ni austenitic stainless steels are commonly used to manufacture components of superconducting magnets and cryogenic transfer lines since they preserve ductility practically down to 0 K. Such materials also are used to manufacture thin-walled bellows of the Large Hadron Collider interconnections. Failure of these components is usually associated with ductile damage propagation and fatigue crack initiation [3, 5]. Ductile materials strained in cryogenic conditions develop micro-damage fields in a similar way like at room or enhanced temperatures. Evolution of ductile damage fields (micro-cracks and micro-voids) is also driven by plastic strains and similar kinetic laws can be used [7]. In the present work, the model proposed by Chaboche and Lemaitre is applied. This kinetic law of damage evolution is based on irreversible thermodynamics framework where the damage potential function is assumed as a square function of the elastic strain energy release rate. This model is valid for ductile materials, so it is applied only to austenitic phase. Moreover, it is assumed that the damage state in already created martensitic phase is inherited from the parent phase, but there is no further damage evolution in martensitic phase. The lack of damage evolution in martensite makes it possible to include in the model the effect of damage deceleration when martensite fraction appears. This effect is confirmed by the experimental test of uniaxial tension with frequent unloading (see Fig. 1) [6].

The constitutive model was implemented in the well-known FEM program Abaqus/Explicit with the use of the user-defined procedure VUMAT. The correlation between numerical and experimental results was preformed to prove the validity of the proposed model.

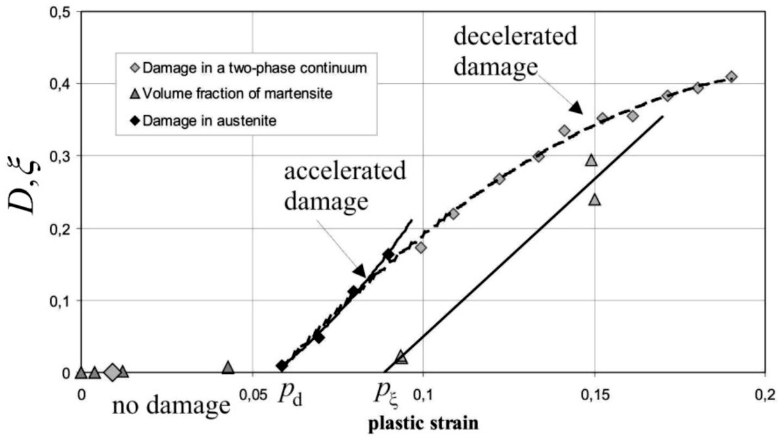

Fig. 1. Evolution of damage and martensite content versus plastic strain for 316L stainless steel subjected to uniaxial tension at 4.2 K (after Egner and Skoczeń [6])

2. Constitutive description of the material

The constitutive model presented in the paper is based on the following assumptions [6, 7]:

- The two-phase material is composed of austenitic matrix and martensitic inclusions. The martensitic platelets are randomly distributed and randomly oriented in the austenitic matrix.
- The austenitic matrix is elastic-plastic-damage, whereas the inclusions show elastic response (the yield stress of martensite fraction is at cryogenic temperatures much higher than the yield stress of austenite).
- Current damage state, in ductile austenitic matrix, is described by the use of the scalar damage parameter. The state of damage in already created martensitic phase is inherited from the parent phase and then the damage state is frozen, there is no further evolution of damage in martensite.
- Small strains are assumed [8, 9], rate independent plasticity is applied and mixed isotropic/kinematic plastic hardening affected by the presence of martensite fraction is included. Additionally, the two-phase material obeys the associated flow rule.
- Isothermal conditions are considered (no fluctuations of temperature are taken into account).

Applying infinitesimal deformation theory to elastic – plastic – two phase material with damage evolution, the total strain ε_{ij} can be expressed as a sum of the elastic part ε_{ij}^{e}, plastic ε_{ij}^{p}, and bain strain $\varepsilon^{bs}=\frac{1}{3}\Delta v\mathbf{I}$, denoting the additional strain caused by phase transformation:

$$\varepsilon_{ij}=\varepsilon_{ij}^{e}+\varepsilon_{ij}^{p}+\xi\varepsilon_{ij}^{bs}\,. \tag{1}$$

The presented model is based on the framework of thermodynamics of irreversible processes with internal state variables, where Helmholtz free energy ψ is postulated as a state potential. The state potential depends on the elastic part of the total strain, and the set of internal state variables, α_{ij}^{p}, r^{p}, ξ, D, which define the current state of the material [10, 11]:

$$\psi=\psi(\varepsilon_{ij}^{e},\alpha_{ij}^{p},r^{p},\xi,D), \tag{2}$$

where α_{ij}^{p}, r^{p}, ξ, D are variables cojugated to the kinematic hardening, isotropic hardening, volume fraction of martensite and damage parameter, respectively.

The Helmholtz free energy of the material can be written as a sum of elastic (E), inelastic (I) and chemical (CH) terms [10]:

$$\rho\psi=\rho\psi^{E}+\rho\psi^{I}+\rho\psi^{CH}\,. \tag{3}$$

In the present model the following classical functions for $\rho\psi^{E}$ and $\rho\psi^{I}$ are assumed:

$$\rho\psi^{E}=\frac{1}{2}\varepsilon_{ij}^{E}E_{ijkl}\varepsilon_{kl}^{E}, \tag{4}$$

$$\rho\psi^{I}=\frac{1}{3}C^{p}\alpha_{ij}^{p}\alpha_{ij}^{p}+R_{\infty}^{p}\left[r^{p}+\frac{1}{b^{p}}\exp\left(-b^{p}r^{p}\right)\right]. \tag{5}$$

Term $\rho\psi^{CH}$ in Eq. (3) represents the chemical free energy:

$$\rho\psi^{CH}=\left(1-\xi\right)\rho\psi_{\gamma}^{CH}+\xi\rho\psi_{\alpha'}^{CH}\,. \tag{6}$$

The terms $\rho\psi_{\gamma}^{CH}$ and $\rho\psi_{\alpha'}^{CH}$ are the chemical energies of the respective phases [12, 13].

Since the $\gamma\rightarrow\alpha'$ phase transformation does not affect the elastic properties of the material, $E_{ijkl}\left(D\right)$, in Eq. (4), stands for the current elastic stiffness tensor affected only by damage. Adopting the strain equivalence principle [14, 15, 16], according to which the strain associated with damaged state under the applied stress σ_{ij} is equivalent to the strain associated with undamaged state under the effective stress $\tilde{\sigma}_{ij}$, the current elastic stiffness is related to the initial stiffness of the undamaged material, E^0_{ijkl}, by the use of the scalar damage parameter D in the following way:

$$E_{ijkl} = (1-D)E^0_{ijkl}\,. \tag{7}$$

Using the Clausius-Duhem inequality for isothermal case, one obtains:

$$\Pi^{mech} = \sigma_{ij}\dot{\varepsilon}_{ij} - \rho\dot{\psi} \geq 0\,, \tag{8}$$

where Π^{mech} is defined as mechanical dissipation.

Taking time derivative of Eq. (3) and using Clausius'a-Duhem (Eq. 8) inequality, the following equations of thermodynamical forces are obtained:

$$\sigma_{ij} = \rho\frac{\partial\psi}{\partial\varepsilon^e_{ij}} = E_{ijkl}\varepsilon^e_{kl} = E_{ijkl}\left(\varepsilon_{kl} - \varepsilon^p_{kl} - \xi\varepsilon^{bs}_{kl}\right), \tag{9}$$

$$X^p_{ij} = \rho\frac{\partial\psi}{\partial\alpha^p_{ij}} = \frac{2}{3}C^p\alpha^p_{ij}\,, \tag{10}$$

$$R^p = \rho\frac{\partial\psi}{\partial r^p} = R^p_\infty\left[1-\exp\left(-b^p r^p\right)\right], \tag{11}$$

$$Z = \rho\frac{\partial\psi}{\partial\xi} = \rho\frac{\partial\psi^I}{\partial\xi} + \frac{dn}{d\xi}\left(\rho\psi^{CH}_{\alpha'} - \rho\psi^{CH}_{\gamma}\right), \tag{12}$$

$$-Y = \rho\frac{\partial\psi}{\partial D} = -\frac{1}{2}\varepsilon^E_{ij}E^0_{ijkl}\varepsilon^E_{kl}\,, \tag{13}$$

where X^p_{ij}, R^p, Z and $-Y$ are the thermodynamic forces conjugated to the state variables α^p_{ij}, r^p, ξ and D, respectively. Thermodynamic forces conjugated to the damage parameter, $-Y$, is called the strain energy density release rate. In the case of isotropic material, it can be expressed as a function of the von Mises equivalent stress σ_{eq} and the triaxiality rate (defined as a ratio of the hydrostatic stress and von Mises stress) [14, 15, 16]:

$$Y = \frac{\sigma^2_{eq}}{2E}\left(\frac{2}{3}(1+\nu) + 3(1-2\nu)\left(\frac{\sigma_h}{\sigma_{eq}}\right)^2\right). \tag{14}$$

The hydrostatic stress σ_h is given by:

$$\sigma_h = \frac{1}{3}\sigma_{ii} \,. \tag{15}$$

It is assumed here that all dissipative mechanisms are governed by plasticity with a single dissipation potential F [10, 16]:

$$F\left(J_{cf}, N_k\right) = F^p\left(\sigma_{ij}, X_{ij}^p, R^p, \xi, D\right) + F^{tr}\left(Q, \xi, D\right) + F^D\left(Y, D\right). \tag{16}$$

Plastic potential F^p is here equal to von Mises type yield surface:

$$F^p = f^p = J_2\left(\tilde{\sigma}_{ij} - \tilde{X}_{ij}^p\right) - \sigma_y - \tilde{R}^p = 0\,, \tag{17}$$

where, with respect to the adopted strain equivalence principle, the effective variables are introduced as follows:

$$\tilde{\sigma}_{ij} = \frac{\sigma_{ij}}{1-D}, \qquad \tilde{X}_{ij}^p = \frac{X_{ij}^p}{1-D}, \qquad \tilde{R}^p = \frac{R^p}{1-D}, \tag{18}$$

and the phase transformation dissipation potential is assumed here in a simple form:

$$F^{tr} = A\frac{Q}{1-D} - B^{tr} = 0\,. \tag{19}$$

The quantity $Q = \sigma_{ij}\varepsilon_{ij}^{bs} - Z$ is conjugated to the transformation rate ξ and can be treated as a thermodynamic force that drives the phase front through the material [12, 17], $A(\theta, \sigma_{ij}, \dot{\varepsilon}_{ij}^p)$, in general, is a function of temperature, stress state and strain rate, and B^{tr} is the barrier force for phase transformation [13, 18]. For rate independent plasticity, isothermal process and small stress variations function A may be treated as a constant value.

The damage potential function F^D is assumed as a square function of the elastic strain energy release rate Y, in the following way [17]:

$$F^D = \frac{Y^2}{2S(1-D)} H(p - p_D)\,. \tag{20}$$

Normality rule involves only one plastic multiplier, determined from the consistency condition. The equations involving the dissipation potentials take the form:

$$\dot{\varepsilon}_{ij}^{p} = \dot{\lambda}^{p}\frac{\partial F^{p}}{\partial \sigma_{ij}} = \dot{\lambda}^{p}\frac{\partial f^{p}\left(\tilde{\sigma}_{kl},\tilde{X}_{kl}^{p},\tilde{R}^{p}\right)}{\partial \tilde{\sigma}_{ij}}\frac{\partial \tilde{\sigma}_{ij}}{\partial \sigma_{ij}} = \frac{\dot{\lambda}^{p}}{1-D}\frac{\frac{3}{2}\left(\tilde{s}_{ij}-\tilde{X}_{ij}^{p}\right)}{\sqrt{\frac{3}{2}\left(\tilde{s}_{pq}-\tilde{X}_{pq}^{p}\right)\left(\tilde{s}_{pq}-\tilde{X}_{pq}^{p}\right)}}, \quad (21)$$

$$\dot{\alpha}_{ij}^{p} = -\dot{\lambda}^{p}\frac{\partial F^{p}}{\partial X_{ij}^{p}} = \dot{\varepsilon}_{ij}^{p}, \quad (22)$$

$$\dot{r}^{p} = -\dot{\lambda}^{p}\frac{\partial F^{p}}{\partial R^{p}} = \frac{\dot{\lambda}^{p}}{1-D}, \quad (23)$$

$$\dot{\xi} = \dot{\lambda}^{p}\frac{\partial F^{tr}}{\partial Q} = A\,\dot{p}H\left[\left(p-p_{\xi}\right)\left(\xi_{L}-\xi\right)\right], \quad (24)$$

$$\dot{D} = \dot{\lambda}^{p}\frac{\partial F^{d}}{\partial Y} = \frac{\sigma_{eq}^{2}}{2ES}\left(\frac{2}{3}(1+\nu)+3(1-2\nu)\left(\frac{\sigma_{h}}{\sigma_{eq}}\right)^{2}\right)H(p-p_{D})\dot{p}, \quad (25)$$

where $p = \int_{0}^{\varepsilon^{p}}\sqrt{\frac{2}{3}d\varepsilon_{ij}^{p}d\varepsilon_{ij}^{p}}$ is the accumulated plastic strain, $\dot{p}$ is the accumulated plastic strain rate that can be derived by means Eq. 21:

$$\dot{p} = \sqrt{\frac{2}{3}d\varepsilon_{ij}^{p}d\varepsilon_{ij}^{p}} = \frac{\dot{\lambda}^{p}}{1-D}. \quad (26)$$

It should be mentioned here that, in order to fulfil the assumption that new martensitic phase inherits damage state from parent austenitic phase but there is no further damage evolution in martensite, the following relation is applied:

$$\dot{D}^{\gamma} = (1-\xi)\dot{D}. \quad (27)$$

The consistency multiplier $\dot{\lambda}^{p}$ is obtained from the consistency condition:

$$\dot{f}^{p} = \frac{\partial f^{p}}{\partial \sigma_{ij}}\left(\dot{\sigma}_{ij}-\dot{X}_{ij}^{p}\right)+\frac{\partial f^{p}}{\partial R}\dot{R}^{p}+\frac{\partial f^{p}}{\partial D}\dot{D}+\frac{\partial f^{p}}{\partial \xi}\dot{\xi}=0. \quad (28)$$

The evolution equations for thermodynamic conjugated forces are obtained by taking time derivatives of quantities defined by equations 9-11. In particular, the force rates appearing in consistency condition (Eq. 28) are given by the following formulae:

$$\dot{\sigma}_{ij} = E_{ijkl}\left[\left(\dot{\varepsilon}_{kl} - \dot{\varepsilon}^{p}_{kl} - \dot{\xi}\varepsilon^{bs}_{kl}\right) - \dot{D}\varepsilon^{E}_{kl}\right], \tag{29}$$

$$\dot{X}^{p}_{ij} = \frac{2}{3}C^{p}\dot{\varepsilon}^{p}_{ij}, \tag{30}$$

$$\dot{R}^{p} = b^{p}\left(R^{p}_{\infty} - R^{p}\right)\dot{r}^{p}, \tag{31}$$

where R^{p}_{∞}, b^{p}, C^{p} are functions of ξ and, in the present paper, are assumed in the following way: $R^{p}_{\infty}(\xi) = R^{p}_{\infty,0}(1 + h_{R}\xi)$, $b^{p}(\xi) = b^{p}_{0}(1 + h_{b}\xi)$, $C^{p}(\xi) = C^{p}_{0}(1 + h_{C}\xi)$.

3. Numerical application of the model

The derived constitutive model was implemented into the Abaqus/Explicit by the use of VUMAT procedure and used to numerically simulate the behaviour of steel structural elements at cryogenic temperatures. At first, the procedure in Wolfram Mathematica program was built to find proper parameters of the model with the use of the least squares method. After obtaining a good agreement between the results, the procedure has been adopted to the explicit finite element code Abaqus/Explicit by means of the user subroutine VUMAT written in FORTRAN. The VUMAT subroutine was adopted with the use of AceGen program which exports a procedure written in Mathematica to FORTRAN automatically.

Applying the explicit dynamic procedure to quasi-static problems requires some special care. The goal is to model the process in the shortest time period in which inertial forces remain insignificant. Time increments of the order 10^{-9} s were used to satisfy the stability criteria. The algorithm, where Newton-Raphson scheme is used to solve the set of nonlinear equations (Eq. 24, 25, 29-31), is shown in Table 1.

Table 1.

Numerical alghorithm

1. Start with stored known variables: $\left\{X^{n}_{ij}, R^{n}, \sigma^{n}_{ij}, \varepsilon^{p,n}_{ij}, \varepsilon^{E,n}_{ij}, \xi^{n}, D^{n}\right\}$
2. An increment of strain gives $\varepsilon^{n+1}_{ij} = \varepsilon^{E,n}_{ij} + \Delta\varepsilon_{ij}$.
3. Compute the elastic trial stress, the trial value for the yield function and test for plastic loading. $\sigma^{trial}_{ij} = \sigma^{n}_{ij} + E_{ijkl}\Delta\varepsilon_{kl}$ $f = f^{trial}(\sigma^{trial}_{ij}, X^{n}_{ij}, R^{n}, \xi^{n}, D^{n})$

4. IF $f^{trial} \leq 0$ then the load step is elastic

$$\sigma_{ij}^{n+1} = \sigma_{ij}^{trial}$$

$$\varepsilon_{ij}^{E,n+1} = \varepsilon_{ij}^{E,n} + \Delta\varepsilon_{ij}$$

EXIT the algorithm

5. IF $f^{trial} > 0$ then the load step is inelastic

The residual vector is defined as:

$R = \left[R_{(\sigma)ij}, R_{(X)ij}, R_R, R_f, R_\xi, R_D \right]^T$, where

$$R_{(\sigma)ij} = \sigma_{ij} - \sigma_{ij}^{old} - E_{ijkl}^0 (1-D)\left[\left(\Delta\varepsilon_{kl} - \frac{\partial f^{p,new}}{\partial \sigma_{kl}}\Delta\lambda^p - \Delta\xi\varepsilon_{kl}^{bs}\right) + \Delta D\varepsilon_{kl}^E\right]$$

$$R_{(X)ij} = X_{ij}^p - X_{ij}^{p,old} - \frac{2}{3}C^{p,new}\frac{\partial f^{p,new}}{\partial \sigma_{ij}}\Delta\lambda^p$$

$$R_{(R)} = R^p - R^{p,old} - b^{p,new}\left(R_\infty^{p,new} - R^{p,old}\right)\Delta p$$

$$R_f = f^p$$

$$R_\xi = \xi - \xi^{old} - \Delta\xi$$

$$R_D = D - D^{old} - (1-\xi)\Delta D$$

and the vector of unknowns is defined as

$$\mathbf{U} = \left[\sigma_{ij}, X_{ij}^p, R^p, \Delta\lambda^p, \xi, D\right]^T$$

The condition $\mathbf{R(U) = 0}$ defines the solution of the problem. The solution can be reached with the use of the following iteration procedure with condition $\mathbf{R(U)} = error$, where error is defined by user.

1. Initialize

$$\mathbf{U}_{n+1}^{(0)} = \mathbf{U}_n$$

$$\mathbf{U}^{new(0)} = \mathbf{U}^{old}$$

2. Iterate

DO UNTIL $\left\|\mathbf{R}(\mathbf{U}^{(k)})\right\| < TOL$

$k \leftarrow k+1$

2.1. Compute iteration $\mathbf{U}^{(k+1)}$

$$\mathbf{U}^{(k+1)} = \mathbf{U}^{(k)} - \left[\frac{\partial \mathbf{R}^{(k)}}{\partial \mathbf{U}}\right]^{-1}\mathbf{R}(\mathbf{U}^{(k)})$$

2.2. Update $\mathbf{U}$

$$\mathbf{U}_{n+1}^{new} = \mathbf{U}^{(k+1)}$$

EXIT

At first, numerical simulation with use of one element with one integration point was performed to obtain stress-strain relation with the use of the considered model. To present coupling of dissipative phenomena, three cases were considered: (a) only softening effect of damage was accounted for (no phase transformation); (b) only hardening effect of phase transformation was considered (damage development was neglected) and (c) both effects and interaction between them were included.

Accounting for two dissipative phenomena: damage evolution and phase transformation in the present constitutive model allows one to obtain a satisfactory reproduction of the experimental stress-strain curve for 316L stainless steel subjected to uniaxial tension at cryogenic temperatures (see Fig. 2).

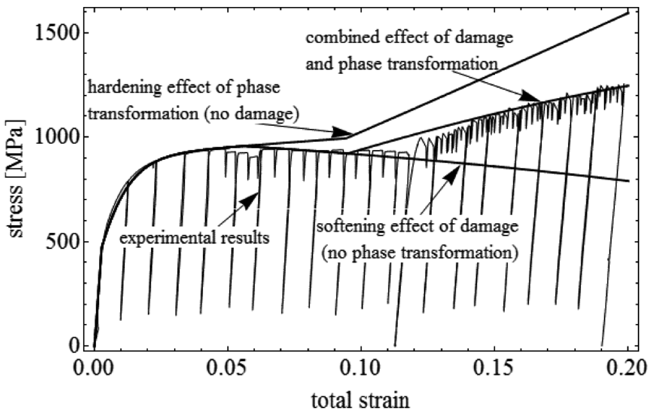

Fig. 2. Experimental (after Egner and Skoczeń, 2010) and numerical stress-strain curve for 316L stainless steel at 4.2 K

Thanks to the implementation of the constitutive model of a material undergoing the plastic strain-induced phase transformation in the finite element software, the mechanical behaviour of different structures made of this material can be easily computed and the evolution of two-phase continuum created during the transformation can be investigated. As an example, the finite element analysis of an expansion bellows is presented.

Bellows expansion joints belong to thin-walled structures of high flexibility. They are used to compensate for the relative motion of two adjacent assemblies subjected to the loads. The bellows expansion joints are crucial elements for systems working at cryogenic temperatures, where all structures contract significantly during cool-down process and the emerging displacement of components needs to be compensated. The choice of material is a crucial point for design of the bellows that are subjected to severe conditions. They have to resist cryogenic temperatures (down to 1.9 K), radiation and mechanical loading (pressure, axial and transversal displacement). Commonly, austenitic stainless steels are used for cryogenic applications because of their ductility at law temperatures, their magnetic and vacuum properties. In order to avoid the ductile-fragile transition, high nickel content is used [3, 5, 19]. As an example, half convolution of a typical U-type bellows has been subjected to mechanical loading at 4.2 K. Precisely, the subjected axial displacements were of –16/+42 mm, where "–" denotes compression and "+" tension, 300 cycles were performed. Basic geometrical parameters of the expansion bellows used in the simulation are listed in the Table 2 [19].

The finite element model has been built by means of CAX4R (4-node bilinear axisimmetric quadrilateral) elements assuming the axial symmetry, seven elements are used thtough the thickness of a ply. The implemented geometry with mesh is shown in Fig. 3. The structural element shown in Fig. 3 is fixed in the vertical direction at point A and the displacemnt is subjected to the edge at point B.

Table 2.

Basic geometrical parameters of the expansion bellows

Material	Thickness of ply t, mm	Number of convultions	Outer diameter Do, mm	Inner diameter Db, mm	Convoluted lenght, mm
SS 316L	0,15	15	90,15	82	78

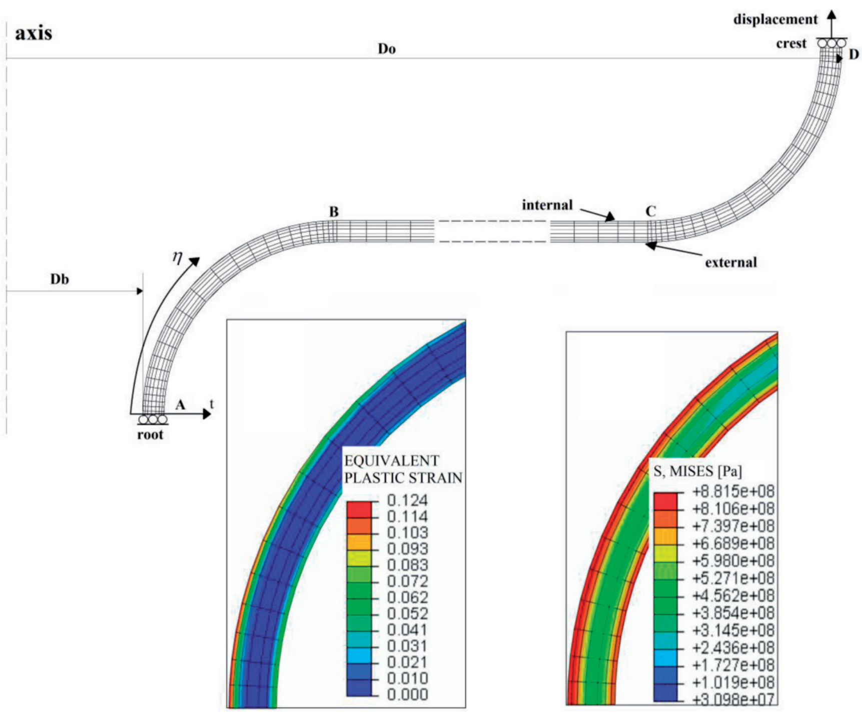

Fig. 3. Boundary conditions and finite element mesh of expansion, distribution of equivalent plastic strain and von Mises stress in the most deteriorated region after 300 cycles

The intensity of the martensitic transformation is maximum at root and at crest of the bellows, due to the localisation of the plastic strains (Fig. 3).

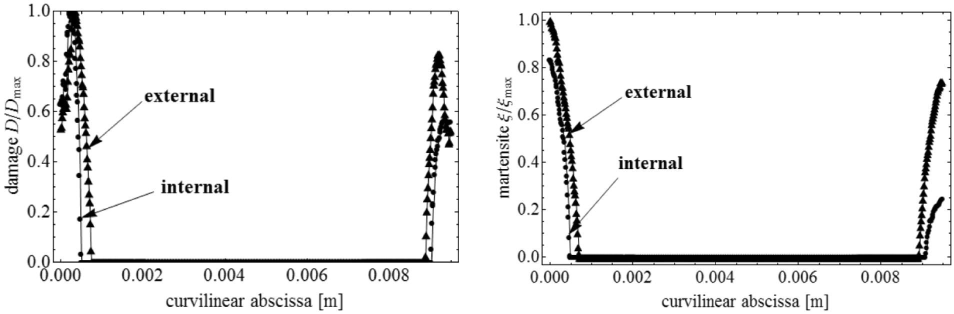

Fig. 4. Distribution of damage (left) and martensite content (right) along the curvilinear abscissa η of external and internal side (ABCD path (compare with Fig. 3)

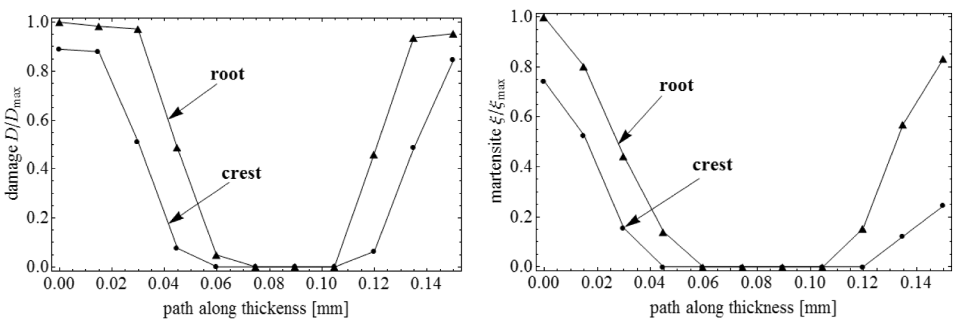

Fig. 5. Distribution of damage (left) and martensite content (right) through thickness of the root and crest (compare Fig. 3)

We can see the drop of damage state at root and crest, which is caused by the assumption that there is no damage evolution in the martensitic phase. As was mentioned before, this effect allows us to model deceleration of damage evolution when martensite appears, which is proved by experiments (Fig. 1). However, large amount of martensite can lead to the fracture, so the model can be used to compute construction only in the case of small strains.

4. Conclusions

The constitutive model presented in the paper includes two dissipative phenomena: damage evolution and plastic strain-induced phase transformation. Both martensitic transformation and damage are of dissipative nature and lead to irreversible rearrangements in the material lattice. The curves that reflect martensitic volume fraction in the austenitic matrix in the course of plastic deformation are usually sigmoidally shaped, and the whole process can be divided into three stages: 1) very low rate of phase transformation; 2) rapid growth of secondary phase content with constant transformation rate; 3) the phase transformation slows down and the volume fraction of martensite approaches asymptotically the saturation level. In the present paper, the linear model of Garion and Skoczeń [3] was used to simulate the secondary phase content in the austenitic matrix, which is simplification of the well-known Olson and Cohen [2] model. This linear model is referred only to the second stage of phase transformation process. However, it gives very good results and also lessens the amount of parameters needed to be defined, what is important if experiments at cryogenic temperatures are concerned [20].

The austenitic phase behaves in a ductile way practically over the whole range of cryogenic temperatures, thus the ductile damage model of Chaboche and Lemaitre was employed to compute damage content in the matrix. It was assumed that martensite inherit the state of damage from previous phase, but there is no further damage development. However, the martensite is a very hard phase and shows rather brittle behaviour. Thus, in the future the model should be improved by introducing separate damage variable for reflecting damage state in martensite and different kinetic law of micro-damage evolution from that one in ductile phase should be introduced.

The model has been successfully tested against the experimental data for 316L stainless steel, subjected to simple tension at 4.2 K. The plastic fields and the zones of phase transformation have been computed at root and at crest of convolutions of thin-walled axi-symmetric corrugated shells. The phase transformation process is highly localised and – as a function of the local bending stresses – can penetrate the whole thickness of the shell. This can create favourable conditions for the embrittlement of the material and fast propagation of a macro-crack across the shell wall. For these reasons, the chemical composition of the material has to be thoroughly controlled in order to reduce the saturation level to the necessary minimum.

It is worth to point it out that the combined model is attractive in view of its simplicity and a relatively small number of parameters to be identified at cryogenic temperatures. The experiments carried out in liquid helium or liquid nitrogen are laborious, expensive and usually require complex cryogenic installations to maintain stable conditions (constant or variable temperature). Therefore, any justified simplification leading to reduction of the number of parameters to be determined is of great importance.

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Modelowanie rozwoju uszkodzeń i przemiany martenzytycznej w stali austenitycznej

Streszczenie

W artykule przedstawiono konstytutywny model materiału podlegającemu przemianie fazowej wywołanej odkształceniami plastycznymi oraz rozwojowi uszkodzeń. Przemiana fazowa opisana jest modelem liniowym. Ponadto, w pracy uwzględniono izotropowy rozwój uszkodzeń. Opis konstytutywny został zaimplementowany w komercyjnym programie Abaqus/Explicit z wykorzystaniem zewnętrznej procedury użytkownika VUMAT. Dokonano symulacji testu jednoosiowego rozciągania w celu porównania wyników eksperymentalnych z numerycznymi. Jako przykład rzeczywistego elementu konstrukcyjnego, pracującego w warunkach temperatur kriogenicznych, dokonano symulacji pracy kompensatora.