

# OPTIMALLY CONTROLLED HEATING OF SOLID PARTICLES IN A FLUIDISED BED WITH A DISPERSIVE FLOW OF THE SOLID

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*Dedicated to Prof. Ryszard Pohorecki on the occasion of his 80th birthday*

In this study the authors minimise the total process cost for the heating of solid particles in a horizontal fluidised bed by an optimal choice of the inlet heating gas temperature profile and the total gas flow. Solid particles flowed along the apparatus and were heated by a hot gas entering from the bottom of the fluidised apparatus. The hydrodynamics of the fluidised bed is described by a two-phase Kunii – Levenspiel model. We assumed that the gas was flowing only vertically, whereas solid particles were flowing horizontally and because of dispersion they could be additionally mixed up in the same direction. The mixing rate was described by the axial dispersion coefficient. As any economic values of variables describing analysing process are subject to local and time fluctuations, the accepted objective function describes the total cost of the process expressed in exergy units. The continuous optimisation algorithm of the Maximum Principle was used for calculations. A mathematical model of the process, including boundary conditions in a form convenient for optimisation, was derived and presented. The optimization results are presented as an optimal profile of inlet gas temperature. The influence of heat transfer kinetics and dispersion coefficients on optimal runs of the heating process is discussed. Results of this discussion constitute a novelty in comparison to information presented in current literature.

**Keywords:** optimisation, cost minimization, fluidised heating, fluidization

## 1. INTRODUCTION

Mathematical models of fluidised heating processes can simulate and optimise them, as well as define the influence of various process parameters on the process. However, the use of highly accurate, complex models is very labour-intensive, time-consuming and often requires large computational power. Therefore, simplified models are often used. Specifying the complexity of a model is an important problem that must be thoroughly considered before proceeding to the process simulations. The problem becomes even more significant in optimisation calculations due to their complexity. It involves many aspects, the most important of which are the definition of the objective function, the model selection process, the choice of method (algorithm) of optimisation and the adaptation of the model to the optimisation algorithm. The investigation of optimal processes to determine general relationships in such processes is a very significant issue, and results can help engineers to optimise processes. This paper presents an optimisation method and general optimisation results for the heating of fine solid particles in a horizontal fluidised bed with a dispersed flow of solid particles. Optimisation proceeding in an ideally mixed bed and a bed with a plug flow of solid particles was described in earlier publications (Pościata, 2008; Pościata and Szwast, 2000; Pościata and Szwast, 2010). In this paper optimisation with dispersive flow of solid particles is presented. The dispersive flow of solid

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particles makes optimisation calculations very complex due to the increased dimensionality of the problem and changes of boundary conditions. In the optimisation problem considered here, the factors searched for are: the optimal profile of heating gas temperature and the total gas flow rate, which minimise the total cost of the process.

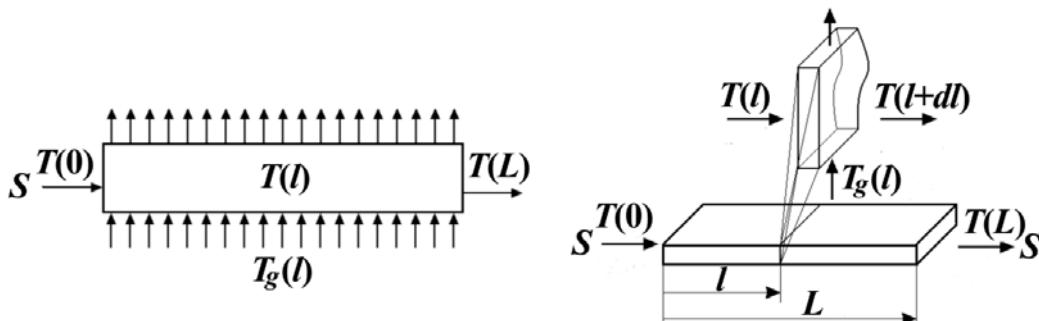


Fig. 1. Schematic diagram of horizontal fluidised heat exchanger process.

The first step to develop a model of heating is to determine the hydrodynamics of a fluidised bed. In a general case, a fluidised bed is not homogeneous. Various structures can form such as channels, plugs and bubbles of pure gas, which are usually accompanied by traces and clouds having different porosity than the rest of the bed. For heating and drying processes, if the height of the bed is not too high, the hydrodynamics of the bed can be described by Kunii' and Levenspiel's model (Kunii and Levenspiel, 1991). A scheme of the two-phase bed is presented in Fig. 1. The main assumptions for this model are as follows: The fluidised bed is composed of two phases: the dense phase and the bubble phase. The dense phase contains gas and solid particles, both of which are ideally mixed; the bubble phase consists of bubbles of pure gas; for the bubble phase the plug flow of gas is assumed, the diameter of bubbles is constant along the bed height and it is a bed parameter. Bubbles are described by a volume fraction of the bubble phase in the total volume of the bed and the coefficient of heat transfer between the bubble and dense phases.

The enthalpy balance for solid particles in the differential layer of the cross-section fluidised bed is a fundamental Eq. of the mathematical model for the heating process (Fig.1):

$$A_f u_s \rho_s c_s T - A_f u_s \rho_s c_s \left( T + \frac{dT}{dl} dl \right) - A_f D \rho_s c_s \frac{dT}{dl} + A_f D \rho_s c_s \left( \frac{dT}{dl} + \frac{d}{dl} \frac{dT}{dl} dl \right) + q(l) dl = 0 \quad (1)$$

The first two terms on the right-hand side of Eq. (1) describe the enthalpy flux related to the convective flow of solid particles, while the next two terms describe the enthalpy flux related to the diffusive flux of solid particles. The last term,  $q(l)$ , describes the heat exchanged between the gas and solid particles in the differential slice of the fluidised bed. A reduction of similar terms gives:

$$A_f u_s \rho_s c_s \frac{dT}{dl} - A_f D \rho_s c_s \frac{d^2 T}{dl^2} - q(l) = 0 \quad (2)$$

The heat transferred from gas to solid in the differential slice of the bed must be determined from the enthalpy balance and kinetics equations describing the rate of heat transfers between bed phases.

The enthalpy balance takes the following form:

$$q(l) dl = c_g [(1 - \sigma_b)(T_g - T_{mf}) + \sigma_b(T_g - T_b)] dG \quad (3)$$

The differential gas flow rate,  $dG$ , and the differential length of the bed,  $dl$ , satisfy the following relationship:

$$dG = y u_g \rho_g dl \quad (4)$$

Two kinetics Eqs. (5) and (6) describe the heat transfer rate between the gas in the dense phase and solid particles and between the bubble phase and the dense phase, respectively:

$$q(l)dl = \alpha_{mf}(T_{mf} - T)(1 - \sigma)yzdl \quad (5)$$

$$T_b - T_g = -\frac{\alpha_b z \sigma}{c_g u_g \rho_g \sigma_g} \frac{T_b - T_g}{\ln\left(\frac{T_b - T_{mf}}{T_g - T_{mf}}\right)} \quad (6)$$

The elimination of unknown temperatures  $T_{mf}$  and  $T_b$  from Eqs. (3), (5) and (6) leads to the determination of the heat transferred from gas to solid:

$$q(l) = c_g u_g \rho_g y \beta(T_g - T) \quad (7)$$

where coefficient  $\beta$  is given by the following expression:

$$\beta = (1 - \sigma_g e^{-1/Z_b}) / [1 + Z_{mf} (1 - \sigma_g e^{-1/Z_b})] \quad (8)$$

while coefficients  $Z_{mf}$  and  $Z_b$  are defined as:

$$Z_{mf} = (c_g u_g \rho_g) / (\alpha_{mf} z (1 - \sigma)) \quad (9)$$

$$Z_b = (c_g u_g \rho_g \sigma_g) / (\alpha_b z \sigma) \quad (10)$$

Dimensionless coefficients  $Z_{mf}$  and  $Z_b$  describe heat transfer resistance, respectively, between gas and solid, and between the bubble and dense phases, whereas the coefficient  $\beta$  can take values from 0 to 1 as it describes the deviation from the ideal bed defined as a bed in which the solid and gas reach a thermodynamic equilibrium ( $\beta = 1$ ).

It is important that equation (8) defining coefficient  $\beta$  results from the solution of balance and kinetic equations given by Kunii and Levenspiel model describing heat transfer in a bubble fluidised bed. Coefficient  $\beta$  includes all parameters of the fluidised bed which affect heat transfer in considered fluidised bed namely superficial velocity and density of the heating gas, bed height, volume fraction of bubble phase and volume fraction of gas flowing through bubble phase, and heat transfer coefficients. Coefficient  $\beta$  can be assigned the role of the substitute kinetic coefficient describing, generally, the heat transfer rate in a specific fluidised bed. For a very good heat transfer, coefficient  $\beta$  reaches values close to one, and for perfect conditions, when thermodynamic equilibrium between gas and solid is achieved, it takes the value of  $\beta = 1$ . The coefficient  $\beta$  is convenient to determine how conditions for heat transfer in the fluidised bed are far from ideal, and to compare heat transfer in different apparatus, and to optimise the process.

Applying Eqs. (4) and (7) to Eq. (2) the enthalpy balance can be written as:

$$A_f u_s \rho_s c_s \frac{dT}{dG} - D A_f \rho_s c_s (u_g \rho_g y) \frac{d^2 T}{dG^2} - c_g \beta (T_g - T) = 0 \quad (11)$$

After an introduction of the dimensionless gas flow rate defined as a ratio of gas mass flow rate to solid one,  $t = G / (A_f u_s \rho_s)$ , and a ratio of specific heat capacity,  $K = c_g / c_s$ , Eq. (11) can be given by:

$$\frac{d^2 T}{dt^2} - \frac{Pe}{t_k} \frac{dT}{dt} + \frac{Pe}{t_k} K \beta (T_g - T) = 0 \quad (12)$$

where  $Pe$  is the Peclet number and  $t_k$  is a dimensionless total gas flow rate. A ratio of Peclet number to total gas flow rate can be treated as process parameter, defined by:

$$P = \frac{Pe}{t_k} = \frac{u_s^2 \rho_s z}{D u_g \rho_g} \quad (13)$$

Differential Eq. (12) must be completed by boundary conditions. In the case considered, Danckwerts' boundary conditions are applied. The first boundary condition results from the enthalpy balance for the initial cross-section of the fluidised bed, for  $l = 0$ . The boundary condition takes the following form:

$$T(0^-) = T(0^+) - \frac{D}{u_s} \frac{dT(0^+)}{dl} \quad (14)$$

In Eq. (14),  $T(0^-)$  and  $T(0^+)$  signify the left-sided and right-sided limits of solid temperature for  $l = 0$ , respectively. The temperature  $T(0^-)$  is the initial temperature of solid particles, which usually equals the ambient temperature. The  $T(0^+)$  is the temperature of the solid in the fluidised bed at the inlet to the apparatus. This temperature is unknown and different from the inlet temperature due to dispersion. Hence, on the surface, for  $l = 0$  there is a discontinuity of the solid temperature. By introducing the dimensionless gas flow rate to the above condition, it takes the form of:

$$T(0^-) = T(0^+) - \frac{1}{P} \frac{dT(0^+)}{dt} \quad (15)$$

The second boundary condition results from the enthalpy balance for the other end of the heat exchanger. Since the temperature of the solid after leaving the heat exchanger does not change, and assuming no dispersion outside the heat exchanger, the temperature derivative for  $l = l_k$  must be equal to zero:

$$\left. \frac{dT}{dt} \right|_{l=l_k} = \left. \frac{dT}{dt} \right|_{t=t_k} = 0 \quad (16)$$

## 2. PERFORMANCE INDEX

The performance index (objective function) describes the total process cost including the investment cost and the operating cost. The investment costs include the expense of buying a new apparatus with a complete instrumentation and installation, as well as maintenance and utilisation costs. In a fluidised apparatus the investment cost is proportional to the area of the perforated bottom in the apparatus. The operating costs for fluidised heating include the cost of heating the gas, which is a function of the gas temperature and flow rate, and the cost of gas pumping which is a function of the total gas flow rate. Hence the total process cost per unit of a heated solid flow rate is given by (Berry et al., 2000):

$$K_e = \left( \frac{\tilde{z}}{\tau_m} + \beta_r \right) \frac{J}{\tau_u S} + \frac{e_e \Delta P G_k}{\eta \rho_g S} + \int_0^{G_k} c_g(T_g(G)) S^{-1} dG \quad (17)$$

The cost of a new heat exchanger,  $J$ , can be expressed as a linear function of fluidised apparatus bottom area,  $A_b$ , and thereafter also as a function of the total gas flow rate:

$$J = j_0 + p A_b = j_0 + \frac{P}{\rho_g u_g} G \quad (18)$$

Combining Eqs. (17) and (18) and introducing dimensionless gas flow rate  $t$  yields:

$$K_e = \left( \frac{\tilde{z}}{\tau_m} + \beta_r \right) \frac{j_0}{\tau_u S} + \int_0^{t_k} [c_g(T_g(t)) + \kappa_e] dt \quad (19)$$

where  $\kappa_e$  is the so-called economic coefficient of the investment and gas pumping cost, defined as:

$$\kappa_e = \left( \frac{\tilde{z}}{\tau_m} + \beta_r \right) \frac{p}{\tau_u \rho_g u_g} + \frac{e_e \Delta P}{\eta \rho_g} \quad (20)$$

The first term on the right-hand side of Eq. (19) can be ignored because it is constant and does not influence optimisation results. Therefore the performance index takes the following form:

$$I' = \int_0^{t_k} [c_g(T_g(t)) + \kappa_e] dt \quad (21)$$

The unit price of gas  $c_g(T_g)$  as a function of solid temperature can be determined using the so-called exergy tariff of prices. According to the tariff, the specific gas price is proportional to the specific gas exergy  $b_g(T_g)$ , where  $e$  is the economic value of the exergy unit:

$$c_g(T_g) = e b_g(T_g) \quad (22)$$

The thermal exergy of gas is described with Eq.:

$$b_g(T_g) = h_g(T_g) - h_g(T_a) - T_a(s_g(T_g) - s_g(T_a)) \quad (23)$$

Using an average specific heat of gas, the difference of the gas enthalpy on the right-hand side of equation (23) can be expressed by the difference of temperature:

$$h_g(T_g) - h_g(T_a) = c_{pg}(T_g - T_a) \quad (24)$$

Otherwise, after the Taylor expansion of function  $s_g(T_g)$  around  $s_g(T_a)$  and neglecting the terms of the third and higher order, the difference of gas entropies can be written as:

$$s_g(T_g) - s_g(T_a) = (T_g - T_a) \frac{ds_g(T_g)}{dT_g} \Big|_{T_a} + \frac{1}{2} (T_g - T_a)^2 \frac{d^2 s_g(T_g)}{dT_g^2} \Big|_{T_a} \quad (25)$$

On the basis of the following definition:

$$ds_g(T_g) = \frac{c_{pg} dT_g}{T_g} \quad (26)$$

one can accept:

$$\frac{ds_g(T_g)}{dT_g} \Big|_{T_a} = \frac{c_g}{T_a} \quad \frac{d^2 s_g(T_g)}{dT_g^2} \Big|_{T_a} = -\frac{c_g}{T_a^2} \quad (27)$$

Combining Eqs. (23), (24), (25) and (27), it is possible to obtain the following simple quadratic equation describing gas thermal exergy as a function of gas temperature:

$$b_g(T_g) = \frac{1}{2} A (T_g - T_a) \quad (28)$$

where  $A = c_{pg}/T_a$ .

This simple Eq. (28) is sufficiently exact for the optimisation of many practical systems. After applying Eq. (28) the performance index (21) takes the following form:

$$I'' = \int_0^{t_k} \left[ \frac{1}{2} A (\tau_g - T_a)^2 + \kappa \right] dt \quad (29)$$

where  $\kappa = \kappa_e/e$  is the so-called exergy coefficient of investment and gas pumping costs.

Outlet gases leaving the fluidised bed can still have high temperature, therefore they have high exergy, which can be used in other processes. The recuperation of the exergy from outlet gas reduces the cost of the considered process, therefore, after the introduction of a degree of exergy recovery,  $\mu$ , the final form of the performance index is given by:

$$I = \int_0^{t_k} \left[ \frac{1}{2} A \tau_g^2 - \frac{1}{2} A \mu \tau_{out}^2 + \kappa \right] dt \quad (30)$$

In Eq. (30) substituted temperature, defined as the difference of the current temperature and ambient temperature, ( $\tau_x = T_x - T_a$ ), is introduced to simplify the notation of the Eqs. (31-43) and optimisation calculation.

### 3. OPTIMISATION

The well-known and often described in literature continuous optimisation algorithm of the Maximum Principle was used in this study (Pontryagin et.al. 1962). The algorithm can be applied in order to resolve optimisation problems in which mathematical models of processes are described with a system of first-order differential equations. Thus, the second-order differential Eq. (12) describing the change of temperature of solid particles must be written by a system of two first-order differential equations:

$$\frac{d\tau}{dt} = \theta \quad (31)$$

$$\frac{d\theta}{dt} = P\theta - PK\beta(\tau_g - \tau) \quad (32)$$

Eqs. (31-32) called state transformations (state functions) describe changes of state variables,  $\theta$  and  $\tau$ . The third state variable is variable  $t$ . In the optimisation theory,  $t$  is called “time”, although it does not need to be physical time, as in the problem considered, in which it is a dimensionless gas flow rate. Hamiltonian for the process is defined by:

$$H(\tau, \theta, \tau_g, z_\tau, z_\theta, t) = \frac{1}{2} A \tau_g^2 - \frac{1}{2} A \mu \tau_{out}^2 + \kappa + z_\tau \theta + z_\theta [P\theta - PK\beta(\tau_g - \tau)] \quad (33)$$

where  $z_\tau$  and  $z_\theta$  are adjoint variables (analogous to Lagrange multipliers). The outlet gas temperature  $\tau_{out}$  depends only on gas inlet temperature, solid temperature and coefficient  $\beta$ , and is given by:

$$\tau_{out} = \tau_g - \beta(\tau_g - \tau) \quad (34)$$

The optimality condition for the gas temperature, which is a decision variable, is given by zeroing the partial derivative of the Hamiltonian Eq. (35):

$$\frac{\partial H(\tau, \theta, \tau_g, z_\tau, z_\theta, t)}{\partial \tau_g} = A \tau_g - A \mu [\tau_g - \beta(\tau_g - \tau)] (1 - \beta) - z_\theta P K \beta = 0 \quad (35)$$

Moreover, the adjoint equations describing changes of adjoint variables must be satisfied along an optimal trajectory:

$$\frac{dz_\tau}{dt} = -\frac{\partial H(\tau, \theta, \tau_g, z_\tau, z_\theta, t)}{\partial \tau} = A\mu[\tau_g - \beta(\tau_g - \tau)]\beta - z_\theta P K \beta \quad (36)$$

$$\frac{dz_\theta}{dt} = -\frac{\partial H(\tau, \theta, \tau_g, z_\tau, z_\theta, t)}{\partial \theta} = -z_\tau - z_\theta P \quad (37)$$

and in addition to the Hamiltonian, the following equation is valid:

$$\frac{dH(\tau, \theta, \tau_g, z_\tau, z_\theta, t)}{dt} = \frac{\partial H(\tau, \theta, \tau_g, z_\tau, z_\theta, t)}{\partial t} = 0 \quad (38)$$

Eq. (38) shows that the Hamiltonian is constant along the optimal trajectory. The actual value of the Hamiltonian results from boundary conditions for the Hamiltonian, which for the performance index given by Eq. (31) can be written as:

$$\begin{aligned} H(t_k) &= 0, & \text{if } t_k - \text{free} \\ H(t_k) &- \text{undetermined} & \text{if } t_k - \text{fixed} \end{aligned} \quad (39)$$

Taking into account the Hamiltonian (Eq. (33)) and Eq. (38) together with the conditions (39) for the free total gas flow rate (free  $t_k$ ) we obtain:

$$H(\tau, \theta, \tau_g, z_\tau, z_\theta, t) = \frac{1}{2} A \tau_g^2 - \frac{1}{2} A \mu \tau_{gout}^2 + \kappa + z_\tau \theta + z_\theta [P \theta - P K \beta (\tau_g - \tau)] = 0 \quad (40)$$

For the adjoint variables, boundary conditions at the end of the optimal trajectory are defined similarly to the conditions (39). Hence, the adjoint variables  $z_\theta(t_k)$  and  $z_\theta(t_k)$  are undetermined because the final solid temperature must be specified, and values of state variable  $\theta(t_k)$  are defined with Eq. (16), so  $\theta(t_k) = dT(t_k)/dt = 0$ . For the beginning of the optimal trajectory, the boundary conditions are more complicated because the boundary condition for  $\tau(0^+)$  and  $\theta(0^+)$  given by Eq. (15) only defines the relationship between these variables but not their values. Therefore, the conditions for beginning of the optimal trajectory define only the relationship between adjoint variables, so the adjoint variables for  $t = 0$  must satisfy the following equation:

$$z_\tau(0) = -P z_\theta(0) \quad (41)$$

After collecting all the boundary conditions we get:

$$\begin{aligned} t = 0 & \quad \tau(0) = \frac{1}{P} \theta(0), \quad z_\tau(0) = -P z_\theta(0) \\ t = t_k - \text{unknown} & \quad \tau(t_k) = \tau_k, \quad \theta(t_k) = 0, \quad H(t_k) = 0, \quad z_\tau(t_k), z_\theta(t_k) - \text{unknown} \end{aligned} \quad (42)$$

Using the boundary condition for variable  $\theta$  for the end of the optimal trajectory and Eq. (41) for the beginning of the optimal trajectory, the Hamiltonian takes the form which is the same as that of processes without dispersion. Thus, for the beginning and the end of the optimal trajectory we can use a solution for these processes (Poświaty and Szwast, 2010). Therefore, the optimal gas temperature in these points is given by a following expression:

$$\hat{\tau}_g = \tau + \sqrt{\frac{(1-\mu)\tau^2 + 2\kappa/A}{[1-\mu(1-\beta)^2]}} \quad (43)$$

Using Eq. (43), we can reduce the number of values which must be assumed during the calculations. Thus, if a calculation starts from the end of the trajectory, the solid temperature is known, so we can

calculate the optimal gas temperature from Eq. (43) and next find the value of  $z_\theta$  from Eq. (35), so the only value that must be assumed is  $z_\tau$ . If a calculation starts from the beginning, the solid temperature must be assumed; next, the value of  $\theta$  can be calculated from the boundary condition and the optimal gas temperature from Eq. (43); then, the adjoint variable  $z_\theta$  from Eq. (35), and finally,  $z_\tau$  from the boundary condition for adjoint variables. After assumption and calculation of the initial values, differential equations (state equations and adjoint equation) must be integrated. In each integration step, the optimal gas temperature is computed by solving Eq. (35). If the integration of differential equations take place from the beginning to the end (i.e. according to the solid particles flow), then calculations are finished when the final solid temperature is achieved. The condition validating the assumption is the value of  $\theta$  which must be 0. For the integration taking place in the opposite direction (i.e. from the end to the beginning), calculations are finished if the solid temperature  $\tau$  and its differential  $\theta$  satisfy the boundary condition,  $\theta = \tau/P$ , and the assumption is checked by the boundary condition for adjoint variables,  $z_\tau = -Pz_\theta$ . The calculation results are the profiles of the solid temperature and the optimum temperature of the gas, as well as the total gas flow rate,  $t_k$ .

#### 4. OPTIMISATION RESULTS

Optimisation calculations were performed for the following assumed values: the inlet temperature of solid particles is equal to the ambient temperature,  $\tau(0^-) = 0$ ; the fixed outlet substitute temperature of solid equals 100,  $\tau(t_k) = 100$ ; and the so-called exergy coefficient of the investment and gas pumping cost equals 3,  $\kappa = 3$ . Only the results for cases where all outlet exergy of gas is lost ( $\mu = 0$ ) are analysed below.

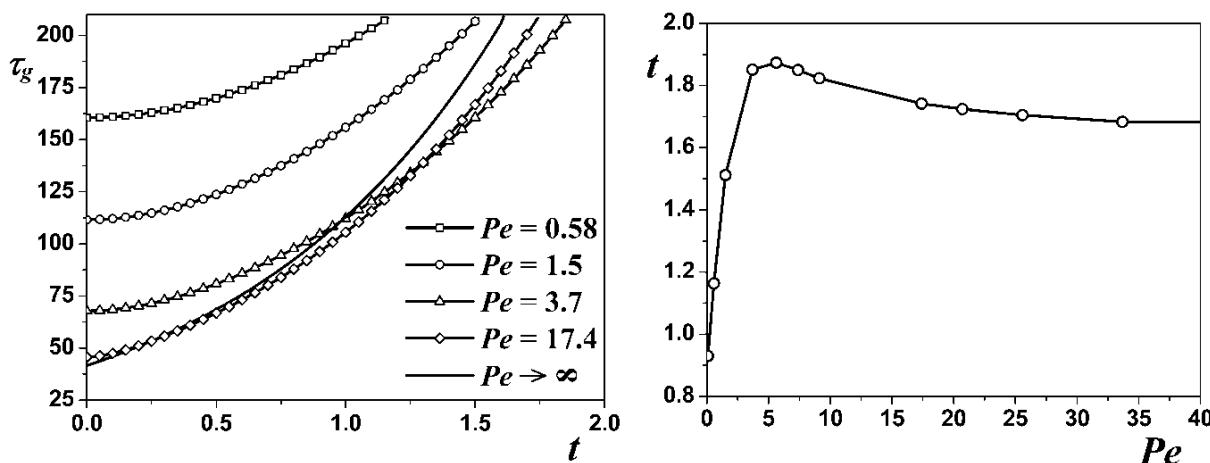


Fig. 2. Profiles of the optimal gas temperature for various values of the Peclet number and the relationship between the Peclet number and optimal total time

Figure 2 shows the optimal profiles of the inlet gas temperature for various values of the Peclet number and the dependence of the optimal dimensionless gas flow rate,  $t_k$ , on the Peclet number. The first graph in Fig. 2 shows the optimal profiles of the inlet gas temperature for various values of the Peclet number. The optimal profiles of gas temperature strongly depend on the Peclet Number. If the Peclet number increases, the profiles become less steep, because the initial temperature increases, whereas the final temperature is constant due to the constant final solid temperature (see Eq. (43)). For the increasing values of Peclet number the optimal profiles of the gas temperature become narrower and intersect because the total flow rate of the gas decreases (look at the solid line and lines with triangles and diamonds). The second graph presents the dependence of the optimal dimensionless gas flow rate,  $t_k$ , on

the Peclet number. This dependence is not monotonic. The optimal process “time”,  $t_k$ , starting from the value valid for the ideal mixing, initially increases with increasing Peclet number and then decreases, approaching finally the value valid for the plug flow. It is an interesting observation which requires a further study.

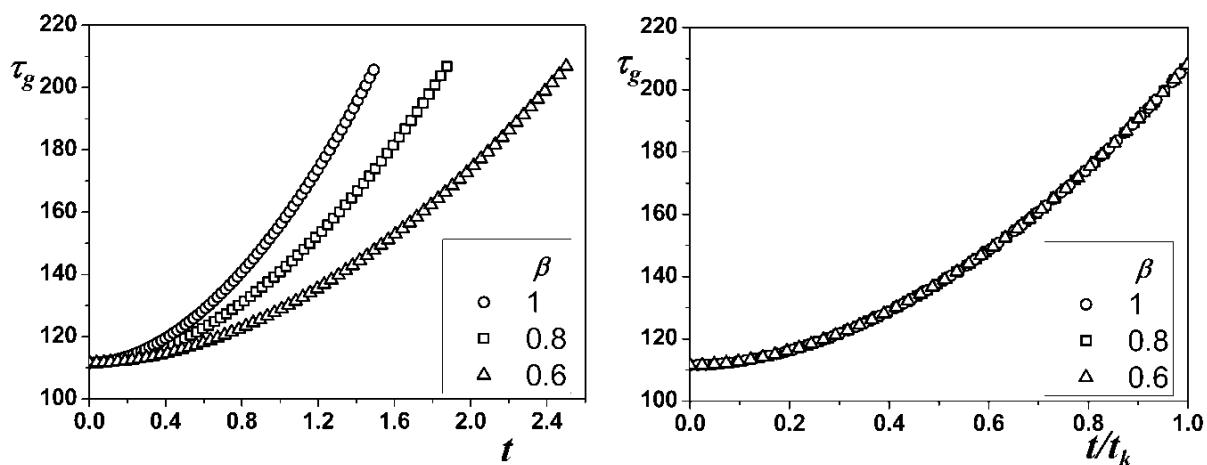


Fig. 3. Profiles of the optimal inlet gas temperature for a constant value of the Peclet number ( $Pe=1.5$ ) and for various values of the coefficient  $\beta$ .

Fig. 3 presents optimisation results for a constant value of the Peclet number. In the first and second charts, the optimal profiles of gas temperature are shown as a function of actual time and normalised time, respectively. All curves in the first chart of Fig. 3 describe the optimal temperature profiles of heating gas along the bed length. All these curves start from the same point whose position results from the boundary condition (15). The starting position depends on the value of Peclet number. As coefficient Beta describes the kinetics of heat transfer, one can observe that for Beta increase the heating process intensity increases whereas the total process time (the lengths of the apparatus,  $t_k$ ) decreases. The second chart of Fig. 3 presents an interesting observation that after time normalisation ( $t/t_k$ ) all curves for different values of  $\beta$  overlap. It means that for a normalised time the optimal temperature profiles of the heating gas do not depend on the hydrodynamics of the bed and the heat transfer kinetics. The hydrodynamics and kinetics are manifested in the gas flow rate.

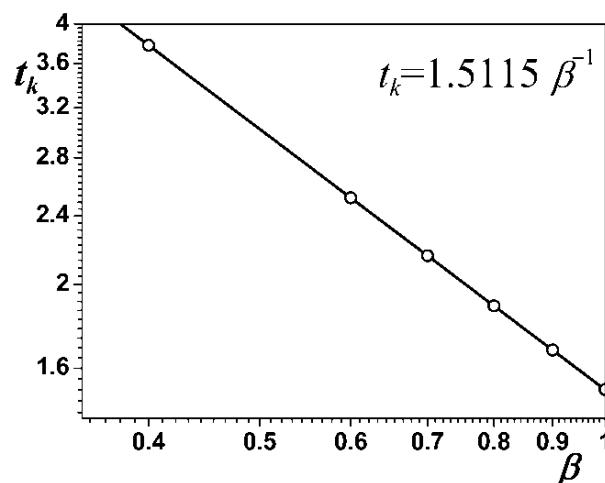


Fig. 4. Profiles of the optimal total gas flow rate for a constant value of the Peclet number ( $Pe=1.5$ ) and for various values of the coefficient  $\beta$

Fig. 4 presents the dependence between the total gas flow rate and the coefficient  $\beta$ . It is shown that this dependence is linear in log – log scale and it is inversely proportional to the coefficient  $\beta$ . As was mentioned above, the coefficient  $\beta$  can take values between 0 and 1 (if  $\beta$  equals 1 the heat transfer is infinitely fast and the thermodynamic equilibrium is achieved – ideal bed), so the proportionality factor is equal to the total gas flow rate for an ideal bed. This relationship together with the optimal profile of gas temperature for normalised time provides an easy way to predict optimal control for different process conditions.

## 5. CONCLUSIONS

The paper presents an optimisation method of a horizontal fluidised heat exchanger. The applied performance index describes the total cost of the process expressed in exergy units. The use of thermodynamic quantities instead of the economic ones enables generalisation of the solution independent of local and time fluctuations of economic parameters. To optimise the problem of a complicated two-phase process kinetics lengthwise dispersion, the continuous algorithm of the Maximum Principle was applied. The derivation and adaptation of the model to the form required by the algorithm and the boundary conditions for adjoint variables are presented. The heat transfer kinetics strongly influences the total gas flow rate,  $t_k$ , and the profile of optimal gas temperature. If the process kinetics deteriorates (value of  $\beta$  decreases), the optimal total gas flow rate increases and, as a consequence, temperature profiles also change. However, if a constant value of Peclet number is maintained and all exergy of outlet gases is lost ( $\mu = 0$ ), the optimal profiles of gas temperature for all values of  $\beta$  overlap for the normalised time,  $t/t_k$ , therefore we can conclude that for the normalised time, the profiles of optimal gas temperature are independent of the process kinetics.

Moreover, for constant values of Peclet number, process duration is inversely proportional to the coefficient  $\beta$  which describes the fluidised bed hydrodynamics and the process kinetics. These conclusions are very important because they allow to reduce the number of computed data for optimisation due to easy prediction of some optimal solutions.

## SYMBOLS

$A$	coefficient in the performance index, $c_g/T_a$ , kJ/(kg K <sup>2</sup> )
$A_f$	area of bed cross-section, m <sup>2</sup>
$b$	specific exergy, kJ/kg
$c$	specific heat capacity, kJ/(kg K)
$c_g(T)$	cost of gas at temperature $T$ , \$/kg
$D$	dispersion coefficient, m <sup>2</sup> /s
$e$	unit price of exergy, \$/kJ
$e_e$	unit price of electrical energy for gas pumping, \$/kJ
$G$	gas mass flow rate, kg/s
$H$	Hamiltonian, kJ/kg
$h$	specific enthalpy, kJ/kg
$j_0$	fixed term of apparatus price, \$
$J$	total cost of new apparatus, \$
$K$	ratio of specific heat,
$l$	actual length of bed, m
$l_k$	total length of bed, m
$p$	proportional term of apparatus price, \$/m <sup>2</sup>
$P$	ratio of the Peclet number to total process “time”

$\Delta P$	the pressure drop in the fluidised layer, Pa
$q(l)$	heat stream in cross-section for $l$ , kJ/(s m)
$S$	solid mass flow rate, kg/s
$s$	specific entropy, kJ/(kg K)
$t$	actual dimensionless gas flow rate, "time",
$T$	temperature, K
$t_k$	total dimensionless gas flow rate, process total "time",
$u$	velocity, m/s
$y$	bed width, m
$z$	adjoint variables, kJ/(kg K)
$z$	bed height, m
$\tilde{z}$	factor describing the freezing of the capital cost

#### Greek symbols

$\theta$	temperature derivation, $d\tau/dt$
$\alpha$	volume coefficient of the heat transfer, kJ/(s m <sup>3</sup> K)
$\beta$	dimensionless coefficient describing bed hydrodynamics and heat transfer kinetics
$\beta_r$	coefficient describing renovations, yr <sup>-1</sup>
$\varepsilon$	bed porosity
$\eta$	the pumping efficiency
$\kappa$	exergy coefficient of the investment and gas pumping cost, kJ/kg
$\kappa_e$	economic coefficient of the investment and gas pumping cost, \$/kg
$\mu$	degree of outlet exergy recuperation,
$\rho$	density, kg/m <sup>3</sup>
$\sigma$	volume fraction of the bubble phase
$\sigma_g$	fraction of gas flowing through the bubble phase
$\tau$	substituted temperature, K, °C
$\tau_m$	the maximum acceptable payout time, yr
$\tau_u$	the utilisation time, s/yr

#### Subscripts

$a$	ambient
$b$	bubble phase
$g$	inlet gas
$mf$	minimum fluidisation, dense phase
$s$	solid particles

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