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N-Body Potential Interaction as a Cost Function in the Elastic Model for SANET Cloud Computing

Zenon Chaczko, Germano Resconi, Christopher Chiu, and Shahrazad Aslanzadeh

***Abstract*—Given a connection graph of entities that send and receive a flow of data controlled by effort and given the parameters, the metric tensor is computed that is in the elastic relational flow to effort. The metric tensor can be represented by the Hessian of the interaction potential. Now the interaction potential or cost function can be among two entities: 3 entities or 'N' entities and can be separated into two main parts. The first part is the repulsion potential the entities move further from the others to obtain minimum cost, the second part is the attraction potential for which the entities move near to others to obtain the minimum cost. For Pauli's model [1], the attraction potential is a functional set of parameters given from the environment (all the elements that have an influence in the module can be the attraction of one entity to another). Now the cost function can be created in a space of macro-variables or macro-states that is less of all possible variables. Any macro-variable collect a set of micro-variables or microstates. Now from the hessian of the macro-variables, the Hessian is computed of the micro-variables in the singular points as stable or unstable only by matrix calculus without any analytical computation – possible when the macro-states are distant among entities. Trivially, the same method can be obtained by a general definition of the macro-variable or macro-states and micro-states or variables. As cloud computing for Sensor-Actor Networks (SANETS) is based on the bonding concept for complex interrelated systems; the bond valence or couple corresponds to the minimum of the interaction potential V and in the SANET cloud as the minimum cost.**

***Keywords*—Elastic netwrok model, cloud computing, Sensor-Actor networks, matrix calculus, N-Body interactions, cost functions.**

I. Introduction

SENSOR Actor Network (SANET) systems are envisioned to become the next generation of large-scale, distributed, ad-hoc and autonomous software intensive systems. These network systems made of collaborating sensors and actors can be deployed in the Cloud Computing environments which can include many ubiquitous, concurrently running applications often supported by several different infrastructure providers that collaboratively deliver services to both ad-hoc and stationary users [2]–[4]. The term Cloud Computing has been evolving for some time; and on its way it has absorbed several paradigms that have been used in different situations and various contexts. This has made it a multi-faceted concept that takes on various meanings. Essentially, Cloud Computing for SANETS refers to the software, data or/and computer processing technology made into a service for distributed actors in the network [3]–[5]. The 'cloud' relates to interconnected actors in the environment; of which the actors may be networked computer devices that are organized into individual servers, clusters of servers, farms of clusters or virtual machines.

Typically, virtualization is performed using some form of hypervisor technology on which a host operating-system runs multiple operating environments. A cloud usually is associated with something intangible, expansive and elastic. These metaphoric impressions mainly relate to the availability of resources. However, it is with the term computing is where the real confusion can actually occur. This is due to the fact that cloud computing can be perceived and used in various ways (i.e. as an application, as a platform, a server infrastructure or as a service delivery mode). The most critical factors that constitute a barrier to a wider adaptation of cloud computing is the cost of data transfer, as well as difficulties associated with accurate measurement of the cost of service usage [3], [5].

In cloud computing for SANETS, due to non-locale and multi-tenancy of services and resources, there is a need for sophisticated mechanisms of service and resource utilization as well as their metering per user and application on an hourly, daily, weekly, monthly or yearly basis [3]. Currently there is a lack of effective mechanisms and tools that deal with these problems. This paper describes how the elastic network model [3], [6] concept can be adopted to plan, monitor (meter), manage and mitigate usage of the cloud computing actors and services; as seen as a grid of entities where effort control the flow of the messages in the grid in a best way. The connection between effort and flow is an 'N' ports matrix which form is given by the oriented connection graph of the entities and of the edge's self and mutual weighted couple.

The connection matrix is the Laplacian of the weighted graph or the Hessian of the interaction potential or cost function. Any entity (*atom*) in the cloud computing has a cost or potential, which value is a function of the conceptual position of all the other entities as atoms [7]. Now all the entities (*atoms*) can be in special states where the cost assumes the minimum conceptual value. The aim of this paper is to show that is possible to have an algorithm that computes from cluster variables. At any cluster of atoms or cloud servers, a macro-variable is associated as a cost function of this macro-variable. At the beginning, the stability condition is computed (*minimum cost*) for the macro-space of the macro-variables, after which the stability is computed for all the other variables of the SANET network.

Now, cloud computing by analogical system in a dynamical way serves the SANET actor (*constraint*) in a best possible

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manner with the minimum cost. The elastic network model approach method [3], [6] to the computation is different from the Turing machine method, because the solution of the problem is found without a step-by-step procedural algorithm; but with a grid of elements (*atoms*) guided and controlled by one function (*interaction potential* or *cost function*) that the form is defined by the N-body interaction which is constrained by the SANET actor's needs and limitations.

Elasticity and utility are interconnected in the elastic network model of computation, because the entire cloud system can be activated by external sources (*actor*). The activation or actor sources move dynamically throughout the cloud system to different stable conditions (*minimum cost*), which is compatible with the sources and the internal constraints. Changes in the cloud system grid means a change of the constraint and change of the interaction cost of the service. In conclusion, two main parts is identified in cloud computing [3].

The first is from the interaction potential, bonds, minimum cost and so forth;

- The second is the actor's action that can change the dynamic of the system to move all the states to different stable points.

At the maximum level, the action by a SANET actor can activate a more drastic change of the cloud by the changing of fundamental constraints represented by the interaction potential form. To have a deeper idea of the cloud computing based on the elasticity model we should be able to view a metaphorical image of the atoms and macromolecules (*proteins*) as clouds atomic interaction by one interaction potential, that can be of two-body or many-body potential interactions [8]. Many different types of macromolecules compete in a super-interaction potential to obtain the minimum cost of the right functionality of the body.

It is argued that the elastic network model can be applied as a general guide to biology, cloud computing, bio-computing and other systems that contain interacting entities in a grid of interactions. Initially, the biological model in the physical 2-body was captured in the Gaussian Network Model (*GNM*) that was used to study the dynamics of proteins at the atomic level [7], [9]. Later the technique was adapted to represent the amino-acid level interaction system; after which the Anisotropic Network Model (*ANM*) and the Elastic Network Model (*ENM*) approach was used to study the dynamics of complex, large scale networks where computer simulations [7] using detailed all-node models (including atoms, proteins and elements) are not feasible – due to the exponential complexity of the system with an increase in size.

II. Connection Matrix And Hessian Matrix

A. Laplacian, Green Matrix, Potentials and Flows

Given the graph in Fig. 1, the incidence edges, nodes relation matrix M, is done as:

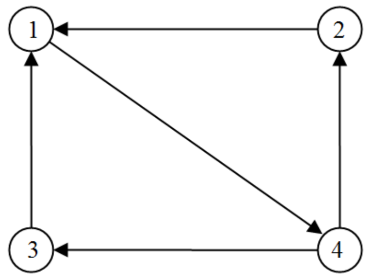

Fig. 1. Graph with cloud nodes and oriented edges.

$$M = \begin{bmatrix} & V_1 & V_2 & V_3 & V_4 \\ e_{1,2} & 1 & -1 & 0 & 0 \\ e_{1,3} & 1 & 0 & -1 & 0 \\ e_{1,4} & -1 & 0 & 0 & 1 \\ e_{2,4} & 0 & 1 & 0 & -1 \\ e_{3,4} & 0 & 0 & 1 & -1 \end{bmatrix} \tag{1}$$

Now we compute the node, with the node relation as this way:

$$MV = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & -1 & 0 \\ -1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_1 - V_2 \\ V_1 - V_3 \\ V_4 - V_1 \\ V_2 - V_4 \\ V_3 - V_4 \end{bmatrix} \tag{2}$$

Now we remark that for the two cycles we have the identities:

$$\begin{cases} (V_1 - V_2) + (V_4 - V_1) + (V_2 - V_4) = 0 \\ (V_1 - V_3) + (V_4 - V_1) + (V_3 - V_4) = 0 \end{cases} \tag{3}$$

Now we transform the identities in equation by the sources:

$$\begin{cases} V_3 - V_1 = E_1 \\ V_4 - V_2 = E_2 \end{cases} \tag{4}$$

and we have the system:

$$\begin{cases} (V_1 - V_2) + (V_4 - V_1) = E_2 \\ (V_4 - V_1) + (V_3 - V_4) = E_1 \end{cases} \tag{5}$$

And the solution is:

$$\begin{cases} V_1 = V_3 - E_1 \\ V_2 = V_4 - E_2 \end{cases} \tag{6}$$

In a graphical way, we have the following structure:

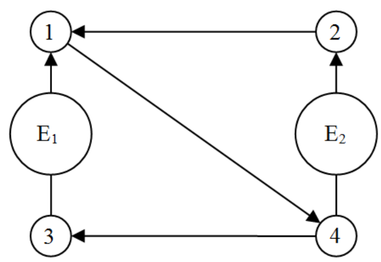

Fig. 2. Transformed graph with above-mentioned solution.

So we have only two free variables that are V3 and V4. The edge node relation can be reduced into this form:

$$M = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ -1 & 1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} \tag{7}$$

Now the node-to-node relation is:

$$N = M^T M = \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \tag{8}$$

That is the Laplacian of the graph:

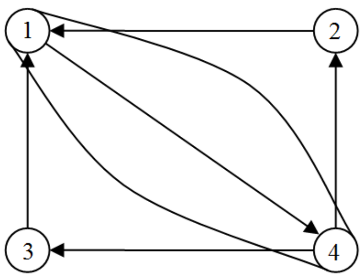

Fig. 3. Laplacian of the graph solution.

In Figure 2 the nodes 1 and 4 are the independent nodes; the others are nodes that connect 1 with 4.

We remark that the sources E_1 and E_2 are connected with the flows of the two cycles in this way:

$$\begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \frac{1}{8} \begin{bmatrix} 3 & -1 \\ -1 & 3 \end{bmatrix} \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} \tag{9}$$

Now for the flow edge relation we have:

$$P = \begin{bmatrix} & f_1 & f_2 \\ e_{1,2} & 1 & 0 \\ e_{1,3} & 0 & 1 \\ e_{1,4} & 1 & 1 \\ e_{2,4} & 1 & 0 \\ e_{3,4} & 0 & 1 \end{bmatrix} \tag{10}$$

The flow-to-flow matrix is

$$F = P^T P = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ -1 & 1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix}^T \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ -1 & 1 \\ 0 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \tag{11}$$

and we have:

$$\begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = \begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} \tag{12}$$

That is the inverse of the previous matrix. The matrix is the Green function of the Laplacian

$$\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix} \tag{13}$$

B. Edge Coupling Weight

Given the couple matrix of the edges:

$$C = \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} \\ k_{3,1} & k_{3,2} & k_{3,3} & k_{3,4} & k_{3,5} \\ k_{4,1} & k_{4,2} & k_{4,3} & k_{4,4} & k_{4,5} \\ k_{5,1} & k_{5,2} & k_{5,3} & k_{5,4} & k_{5,5} \end{bmatrix} \tag{14}$$

in a graphical way we have:

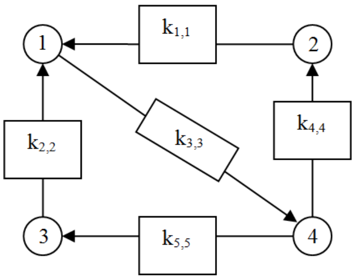

Fig. 4. Self-weight edge elements.

We have the weight between edge (1,2) and (1,4) that is:

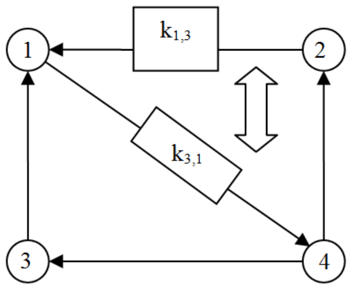

Fig. 5. Couple between edges (1,2) and (1,4).

We can see that the couple is inside a cycle, but the influence is also the two cycles together. Now given the flowedges relation we have the 2-port matrix:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}^T \begin{bmatrix} k_{1,1} & k_{1,2} & k_{1,3} & k_{1,4} & k_{1,5} \\ k_{2,1} & k_{2,2} & k_{2,3} & k_{2,4} & k_{2,5} \\ k_{3,1} & k_{3,2} & k_{3,3} & k_{3,4} & k_{3,5} \\ k_{4,1} & k_{4,2} & k_{4,3} & k_{4,4} & k_{4,5} \\ k_{5,1} & k_{5,2} & k_{5,3} & k_{5,4} & k_{5,5} \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{bmatrix} = $$

$$= \begin{bmatrix} K_{1,1} & K_{1,2} \\ K_{2,1} & K_{2,2} \end{bmatrix} \tag{15}$$

where:

$$K_{1,1} = k_{1,1} + k_{3,3} + k_{4,4} + (k_{1,3} + k_{3,1}) + (k_{3,4} + k_{4,3}) + \\ + (k_{1,4} + k_{4,1})$$

$$K_{1,2} = k_{1,2} + k_{1,3} + k_{1,5} + k_{3,2} + k_{3,3} + k_{3,4} + k_{4,2} + k_{4,3} + k_{4,5}$$

$$K_{2,1} = k_{2,1} + k_{3,1} + k_{5,1} + k_{2,3} + k_{3,3} + k_{4,3} + k_{2,4} + k_{3,4} + k_{5,4}$$

$$K_{2,2} = k_{2,2} + k_{3,3} + k_{5,5} + (k_{2,3} + k_{3,2}) + (k_{3,5} + k_{5,3}) + \\ + (k_{2,5} + k_{5,2}) \tag{16}$$

are the weights of the edge coupling elements.

C. Hessian and Lagrangian

Given the graph:

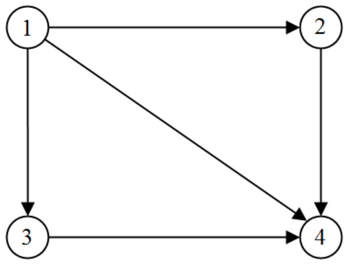

Fig. 6. Graph with edge-point relation.

we have the edge-to-point relation:

$$M = \begin{bmatrix} & V_1 & V_2 & V_3 & V_4 \\ e_{1,2} & -1 & 1 & 0 & 0 \\ e_{1,3} & -1 & 0 & 1 & 0 \\ e_{1,4} & -1 & 0 & 0 & 1 \\ e_{2,4} & 0 & -1 & 0 & 1 \\ e_{3,4} & 0 & 0 & -1 & 1 \end{bmatrix} \tag{17}$$

The Laplacian is:

$$G = M^T M = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} \tag{18}$$

Lets now establish a connection between the Hessian and the Laplacian in this way by referring to the Anisotropic Network Model (*ANM*) and also the Elastic Network Model (*ENM*):

$$G = \begin{bmatrix} 3 & -1 & -1 & -1 \\ -1 & 2 & 0 & -1 \\ -1 & 0 & 2 & -1 \\ -1 & -1 & -1 & 3 \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{\partial^2 V}{\partial x_1^2} & \frac{\partial^2 V}{\partial x_1 \partial x_2} & \frac{\partial^2 V}{\partial x_1 \partial x_3} & \frac{\partial^2 V}{\partial x_1 \partial x_4} \\ \frac{\partial^2 V}{\partial x_2 \partial x_1} & \frac{\partial^2 V}{\partial x_2^2} & \frac{\partial^2 V}{\partial x_2 \partial x_3} & \frac{\partial^2 V}{\partial x_2 \partial x_4} \\ \frac{\partial^2 V}{\partial x_3 \partial x_1} & \frac{\partial^2 V}{\partial x_3 \partial x_2} & \frac{\partial^2 V}{\partial x_3^2} & \frac{\partial^2 V}{\partial x_3 \partial x_4} \\ \frac{\partial^2 V}{\partial x_4 \partial x_1} & \frac{\partial^2 V}{\partial x_4 \partial x_2} & \frac{\partial^2 V}{\partial x_4 \partial x_3} & \frac{\partial^2 V}{\partial x_4^2} \end{bmatrix} = \text{Hessian} \tag{19}$$

Now when V is unknown, we can use the previous equation and thus found the solution for V in this way:

$$V = 3x_1^2 + 2x_2^2 + 2x_3^2 + 3x_4^2 - 2x_1x_2 - 2x_1x_3 - \\ -2x_1x_4 - 2x_2x_4 - 2x_3x_4 \tag{20}$$

or

$$V = 3x_1^2 + 2x_2^2 + 2x_3^2 + 3x_4^2 - 2x_1x_2 - 2x_1x_3 - \\ -2x_1x_4 - 2x_2x_4 - 2x_3x_4 = (x_1 - x_2)^2 + \\ +(x_1 - x_3)^2 + (x_1 - x_4)^2 + (x_2 - x_4)^2 + (x_3 - x_4)^2 \tag{21}$$

V is a positive definite function (always positive). With the eigenvalues we compute the stability condition for the four points or atoms. In this situation we have the eigenvalues:

$$\lambda = \begin{bmatrix} 0 \\ 2 \\ 2 \\ 4 \end{bmatrix} \tag{22}$$

The stability of the first point is not defined, though the other singular points are stable. Now given the potential:

$$V(R_{1,2}, R_{1,3}, R_{1,4}, R_{2,4}, R_{3,4}) = \\ = (R_{1,2} - R^0_{1,2})^2 + (R_{1,3} - R^0_{1,3})^2 + (R_{1,4} - R^0_{1,4})^2 + \\ +(R_{2,4} - R^0_{2,4})^2 + (R_{3,4} - R^0_{3,4})^2 = \\ = V(x_1, y_1, z_1, \ldots, x_4, y_4, z_4) = \\ = V(q_1, q_2, \ldots, q_{12}) \tag{23}$$

where q are the general coordinates in the configuration space and $R_{i,j}$ are the distance between two points i and j.

For:

$$R_1 = R_{1,2} - R^0_{1,2}, \\ R_2 = R_{1,3} - R^0_{1,3} \\ R_3 = R_{1,4} - R^0_{1,4} \\ R_4 = R_{2,4} - R^0_{2,4} \\ R_5 = R_{3,4} - R^0_{3,4} \tag{24}$$

we have:

$$\frac{\partial V(R_1, R_2, \ldots, R_5)}{\partial x_1} = \frac{\partial V}{\partial R_1}\frac{\partial R_1}{\partial x_1} + \frac{\partial V}{\partial R_2}\frac{\partial R_2}{\partial x_1} + \\ + \frac{\partial V}{\partial R_3}\frac{\partial R_3}{\partial x_1} + \frac{\partial V}{\partial R_4}\frac{\partial R_4}{\partial x_1} + \frac{\partial V}{\partial R_5}\frac{\partial R_5}{\partial x_1}$$

$$\frac{\partial}{\partial x_1}\frac{\partial V}{\partial x_1} = \frac{\partial}{\partial x_1}\Big(\frac{\partial V}{\partial R_1}\frac{\partial R_1}{\partial x_1} + \frac{\partial V}{\partial R_2}\frac{\partial R_2}{\partial x_1} + \frac{\partial V}{\partial R_3}\frac{\partial R_3}{\partial x_1} + \\ + \frac{\partial V}{\partial R_4}\frac{\partial R_4}{\partial x_1} + \frac{\partial V}{\partial R_5}\frac{\partial R_5}{\partial x_1}\Big) = \frac{\partial^2 V}{\partial R_1^2}\frac{\partial R_1}{\partial x_1}\frac{\partial R_1}{\partial x_1} + \frac{\partial^2 V}{\partial R_2^2}\frac{\partial R_2}{\partial x_1}\frac{\partial R_2}{\partial x_1} + \\ + \frac{\partial^2 V}{\partial R_3^2}\frac{\partial R_3}{\partial x_1}\frac{\partial R_3}{\partial x_1} + \frac{\partial^2 V}{\partial R_4^2}\frac{\partial R_4}{\partial x_1}\frac{\partial R_4}{\partial x_1} + \frac{\partial^2 V}{\partial R_5^2}\frac{\partial R_5}{\partial x_1}\frac{\partial R_5}{\partial x_1} + \\ + \frac{\partial V}{\partial R_1}\frac{\partial^2 R_1}{\partial x_1^2} + \frac{\partial V}{\partial R_2}\frac{\partial^2 R_2}{\partial x_1^2} + \frac{\partial V}{\partial R_3}\frac{\partial^2 R_3}{\partial x_1^2} + \frac{\partial V}{\partial R_4}\frac{\partial^2 R_4}{\partial x_1^2} + \\ + \frac{\partial V}{\partial R_5}\frac{\partial^2 R_5}{\partial x_1^2} \tag{25}$$

So the total Hessian for the four points x and the five edges is:

$$G = \frac{\partial^2 V}{\partial q_i \partial q_j} = \sum_{h,k} \frac{\partial^2 V}{\partial R_h \partial R_k}\frac{\partial R_h}{\partial q_i}\frac{\partial R_k}{\partial q_j} + \sum_k \frac{\partial^2 R_k}{\partial q_i \partial q_j}\frac{\partial V}{\partial R_k} + \\ = J^T Z J + H\left[\frac{\partial V}{\partial R_k}\right] = S + L \tag{26}$$

where:

$$Z = \frac{\partial^2 V}{\partial R_h \partial R_k},$$
$$H = \frac{\partial^2 R_k}{\partial q_i \partial q_j},$$
$$J = \frac{\partial^2 R_h}{\partial q_i} \tag{27}$$

Now we can explain the main formula as cluster reduction computation for the Hessian matrix. Now for the first we compute the Hessian for the macro-variables R so we have the macro Hessian:

$$\frac{\partial^2 V}{\partial R_h \partial R_k} \tag{28}$$

After we can compute the micro-Hessian:

$$G = \frac{\partial^2 V}{\partial q_i \partial q_j} \tag{29}$$

For all the states in q: In the critical points where the derivatives of the potential are equal to zero we can simplify a lot the previous expression so the final calculus is:

$$G = \frac{\partial^2 V}{\partial q_i \partial q_j} = \sum_{h,k} \frac{\partial^2 V}{\partial R_h \partial R_k} \frac{\partial R_h}{\partial q_i} \frac{\partial R_k}{\partial q_j} = J_i^T Z J_j = S \tag{30}$$

$$\frac{\partial V}{\partial R_k} = 0$$

where J is the Jacobian of the functions R with the general coordinates of q. Z is the Hessian of the voltage function respect to R.

D. *Example for a Very Simple Example*

Edge matrix for the potential:

$$V(R_1, R_2) = R_1^2 + R_2^2 \tag{31}$$

where:

$$R_1 = R_{1,2} - R^0_{1,2}, \qquad R_2 = R_{2,1} - R^0_{2,1} \tag{32}$$

Now the Z is given by the expression:

$$Z = \frac{\partial V}{\partial R_h \partial R_k} = 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{33}$$

and for:

$$R_1 = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} - R^0_{1,2}$$
$$R_2 = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} - R^0_{2,1} \tag{34}$$

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So for:

$$J=\begin{bmatrix}\frac{\partial R_1}{\partial x_1} & \frac{\partial R_1}{\partial y_1} & \frac{\partial R_1}{\partial x_2} & \frac{\partial R_1}{\partial y_2}\\ \frac{\partial R_2}{\partial x_1} & \frac{\partial R_2}{\partial y_1} & \frac{\partial R_2}{\partial x_2} & \frac{\partial R_2}{\partial y_2}\end{bmatrix}=$$

$$=\begin{bmatrix}\frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\\ \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\end{bmatrix} \tag{35}$$

$$S=J^TZJ=$$

$$=2\begin{bmatrix}\frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\\ \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\\ \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\\ \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\end{bmatrix}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$

$$\begin{bmatrix}\frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\\ \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}}\end{bmatrix}=$$

$$=4\begin{bmatrix}\frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2}\\ \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(y_1-y_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2}\\ \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{2(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2}\\ \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{2(y_1-y_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2}\end{bmatrix} \tag{36}$$

and the linear part L is:

$$L=\begin{bmatrix}\frac{\partial^2 R_1}{\partial x_1^2} & \frac{\partial R_1}{\partial x_1\partial y_1} & \frac{\partial R_1}{\partial x_1\partial x_2} & \frac{\partial R_1}{\partial x_1\partial y_2}\\ \frac{\partial R_1}{\partial y_1\partial x_1} & \frac{\partial^2 R_1}{\partial y_1^2} & \frac{\partial R_1}{\partial y_1\partial x_2} & \frac{\partial R_1}{\partial y_1\partial y_2}\\ \frac{\partial R_1}{\partial x_2\partial x_1} & \frac{\partial R_1}{\partial x_2\partial y_1} & \frac{\partial^2 R_1}{\partial x_2^2} & \frac{\partial R_1}{\partial x_2\partial y_2}\\ \frac{\partial R_1}{\partial y_2\partial x_1} & \frac{\partial R_1}{\partial y_2\partial y_1} & \frac{\partial R_1}{\partial y_2\partial x_2} & \frac{\partial^2 R_1}{\partial y_2^2}\end{bmatrix}\frac{\partial V}{\partial R_1}+$$

$$+\begin{bmatrix}\frac{\partial^2 R_2}{\partial x_1^2} & \frac{\partial R_2}{\partial x_1\partial y_1} & \frac{\partial R_2}{\partial x_1\partial x_2} & \frac{\partial R_2}{\partial x_1\partial y_2}\\ \frac{\partial R_2}{\partial y_1\partial x_1} & \frac{\partial^2 R_2}{\partial y_1^2} & \frac{\partial R_2}{\partial y_1\partial x_2} & \frac{\partial R_2}{\partial y_1\partial y_2}\\ \frac{\partial R_2}{\partial x_2\partial x_1} & \frac{\partial R_2}{\partial x_2\partial y_1} & \frac{\partial^2 R_2}{\partial x_2^2} & \frac{\partial R_2}{\partial x_2\partial y_2}\\ \frac{\partial R_2}{\partial y_2\partial x_1} & \frac{\partial R_2}{\partial y_2\partial y_1} & \frac{\partial R_2}{\partial y_2\partial x_2} & \frac{\partial^2 R_2}{\partial y_2^2}\end{bmatrix}\frac{\partial V}{\partial R_2} \tag{37}$$

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When $R_1 = R_2 = R$ we have:

$$S = J^T Z J =$$

$$= 2 \begin{bmatrix} \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} \\ \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} \\ \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} \\ \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} \end{bmatrix} \begin{bmatrix} \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{x_1-x_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} & \frac{y_1-y_2}{\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}} \end{bmatrix} =$$

$$= 2 \begin{bmatrix} \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} \\ \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(y_1-y_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} \\ \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{2(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} \\ \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{2(y_1-y_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} \end{bmatrix} \tag{38}$$

$$G = \begin{bmatrix} \frac{\partial^2 V}{\partial x_1^2} & \frac{\partial V}{\partial x_1 \partial y_1} & \frac{\partial V}{\partial x_1 \partial x_2} & \frac{\partial V}{\partial x_1 \partial y_2} \\ \frac{\partial V}{\partial y_1 \partial x_1} & \frac{\partial^2 V}{\partial y_1^2} & \frac{\partial V}{\partial y_1 \partial x_2} & \frac{\partial V}{\partial y_1 \partial y_2} \\ \frac{\partial V}{\partial x_2 \partial x_1} & \frac{\partial V}{\partial x_2 \partial y_1} & \frac{\partial^2 V}{\partial x_2^2} & \frac{\partial V}{\partial x_2 \partial y_2} \\ \frac{\partial V}{\partial y_2 \partial x_1} & \frac{\partial V}{\partial y_2 \partial y_1} & \frac{\partial V}{\partial y_2 \partial x_2} & \frac{\partial^2 V}{\partial y_2^2} \end{bmatrix} =$$

$$= 2 \begin{bmatrix} \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} \\ \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(y_1-y_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} \\ \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{2(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} \\ \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{2(y_1-y_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} \end{bmatrix} +$$

$$+ \begin{bmatrix} \frac{\partial^2 R}{\partial x_1^2} & \frac{\partial R}{\partial x_1 \partial y_1} & \frac{\partial R}{\partial x_1 \partial x_2} & \frac{\partial R}{\partial x_1 \partial y_2} \\ \frac{\partial R}{\partial y_1 \partial x_1} & \frac{\partial^2 R}{\partial y_1^2} & \frac{\partial R}{\partial y_1 \partial x_2} & \frac{\partial R}{\partial y_1 \partial y_2} \\ \frac{\partial R}{\partial x_2 \partial x_1} & \frac{\partial R}{\partial x_2 \partial y_1} & \frac{\partial^2 R}{\partial x_2^2} & \frac{\partial R}{\partial x_2 \partial y_2} \\ \frac{\partial R}{\partial y_2 \partial x_1} & \frac{\partial R}{\partial y_2 \partial y_1} & \frac{\partial R}{\partial y_2 \partial x_2} & \frac{\partial^2 R}{\partial y_2^2} \end{bmatrix} \frac{\partial V}{\partial R} = S + L \tag{39}$$

Now we can reduce the previous form to only two-dimensions, so we have:

$$G = \begin{bmatrix} \frac{\partial^2 V}{\partial x_1^2} & \frac{\partial V}{\partial x_1 \partial y_1} \\ \frac{\partial V}{\partial y_1 \partial x_1} & \frac{\partial^2 V}{\partial y_1^2} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} \\ \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(y_1-y_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} \end{bmatrix} +$$

$$+ \begin{bmatrix} \frac{\partial^2 R}{\partial x_1^2} & \frac{\partial R}{\partial x_1 \partial y_1} \\ \frac{\partial R}{\partial x_1 \partial y_1} & \frac{\partial^2 R}{\partial y_1^2} \end{bmatrix} \frac{\partial V}{\partial R} \tag{40}$$

When $\frac{\partial V}{\partial R} = 0$, we have:

$$G = \begin{bmatrix} \frac{\partial^2 V}{\partial x_1^2} & \frac{\partial V}{\partial x_1 \partial y_1} \\ \frac{\partial V}{\partial y_1 \partial x_1} & \frac{\partial^2 V}{\partial y_1^2} \end{bmatrix} =$$

$$= \begin{bmatrix} \frac{(x_1-x_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} \\ \frac{(x_1-x_2)(y_1-y_2)}{(x_1-x_2)^2+(y_1-y_2)^2} & \frac{(y_1-y_2)^2}{(x_1-x_2)^2+(y_1-y_2)^2} \end{bmatrix} \tag{41}$$

Now the eigenvalues of G are:

$$\lambda_1 = R^2, \qquad \lambda_2 = 0 \tag{42}$$

And the eigenvectors are:

$$\psi = \begin{bmatrix} \frac{x_1-x_2}{y_1-y_2} & \frac{y_1-y_2}{x_1-x_2} \\ 1 & 1 \end{bmatrix} \tag{43}$$

III. Conclusion

The development of rapidly mixing Markov chains has intertwined with advances in randomized approximation algorithms, this also coincided with the recent progress on expander graphs and eigenvalues which can be found useful for resolving problems of resource management and task allocation in distributed cloud computing. Spectral graph theory typically had applications related to chemistry, physics and biology, where eigenvalues are associated with the stability of molecules. Graph spectra issues arise naturally in various problems of theoretical physics and quantum mechanics (for instance, in minimizing the energies of Hamiltonian systems). However, in our research, we try to address not as much the issues of energy or resource minimalisation but rather to explore the bounds of the system stability and how movements in availability/usage of resources affect the entire system (the elastic Cloud).

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