# The Parameters of Unconventional DoubleCurrent Circuit, their Accuracy Measures and Measurement of Strain and Temperature 

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#### Abstract

The two-voltage-output circuit called a doublecurrent bridge is presented. A single DC current source is switched over and connected in turns to opposite arms of the four-element bridge circuit. The output voltages are dependent on the arm resistance increments and their values are given in absolute and relative units. The simplified signal conditioning formulas of strain and temperature simultaneous measurement of a cantilever beam are featured. The results achieved with the use of the circuit are also published.The accuracy measures (actual errors, limited errors and standard uncertainties) of the bridgetransfer functions are described.


## Keywords-electronics, measurement circuits

## I. Introduction

0NE of basic and well known measurement toolis Wheatstone bridge. This circuit, connected to additional integrated circuits such as analog-to-digital converter (ADC) or microprocessor systems provides great accuracy and speed of conducted measurement [3], [5]. Majority of DAQ systems are based on measuring one quantity. There is also a group of measurement methods which are used to measure several quantities at the same time [1], [4], [8], [9]. As an example it can bea system based on simultaneous measurement of two parameters of resistance increments in a four-terminal (4T) network [2], [7], [8]. Two types of circuits which realize such conditioning and analog signal measurement are presented in literature [9]. One of them is a circuit made up two four-arm classical (Wheatstone) bridges connected in cascade.
The authors of the article have not come across a work describing an accuracy analysis for two simultaneously measured parameters (2D). A description of the measurement accuracy with the use of the classical bridge circuit for one measured parameter (1D)was partly presented in previous work [10].There were shown two methods of describing measurement accuracy of the transfer coefficients of an unbalanced bridges. The general description of accuracy measures(actual errors, limited errors and standard uncertainties)may be useful for measurement systems constructors who use sensors of two different physical quantities (for example strain and temperature RTD sensors).

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## II. Unconventional Double-Current Circuit and the Parameters

The parameters of a double-current bridge are featured.


Fig. 1. Double-current bridge (2J) - idea of circuit
As shown in Fig. 1, power is deliveredby two equal current sources $J=J_{1}=J_{2}\left(R_{\mathrm{G}}=\infty\right)$. The output voltages of the bridge are:

$$
\begin{align*}
& U_{A B}=J \frac{R_{1} R_{4}-R_{2} R_{3}}{\sum R_{i}} \equiv J r_{A B}\left(\varepsilon_{i}\right)  \tag{1}\\
& U_{D C}=J \frac{R_{1} R_{2}-R_{3} R_{4}}{\sum R_{i}} \equiv J r_{D C}\left(\varepsilon_{i}\right) \tag{2}
\end{align*}
$$

where: $\sum R_{i}=R_{1}+R_{2}+R_{3}+R_{4}, r_{D C}, r_{A B}$ - open-circuit voltage to current parameters of $\mathrm{D}-\mathrm{C}$ and A-B outputs.Additionally, it is assumed that resistances $R_{i}$ in the bridge are variables and are represented by equation [6]:

$$
\begin{equation*}
R_{i}=R_{i 0}\left(1+\varepsilon_{i}\right) \tag{3}
\end{equation*}
$$

where $R_{i 0}$ - initial (nominal) resistances, $\varepsilon_{i}$ - relative increments of resistances ( $i=1,2,3,4$ ).

Bridge transfer functions (1), (2) can be simplified to products of their initial sensitivities $t_{0}{ }^{\prime}, t_{0}{ }^{\prime \prime}$ in the balance and normalized unbalance functions $f^{\prime}\left(\varepsilon_{i}\right), f^{\prime \prime}\left(\varepsilon_{i}\right)$. Their formulas can be expressed by initial values $R_{i 0}$ and increments of all resistances, i.e. $R_{i}=R_{i 0}\left(1+\varepsilon_{i}\right)$ and $R_{i 0}$ referencing to one of the first arm, i.e.: $R_{20} \equiv m R_{10}, R_{40} \equiv n R_{10}$ and $R_{30}{ }^{\prime}=(\mathrm{m} / \mathrm{n}) R_{10}$ or $R_{30} "=(n / m) R_{10}$, as is shown in Table 1. It results from two balance states of this bridge-circuit.

If the sensors are situated in all arms of the bridge circuit and their resistances $R_{i 0}$ are equal and their resistance changes are small (thus $\varepsilon_{i} \varepsilon_{j} \ll \varepsilon_{i}+\varepsilon_{j}$ and $\Sigma R_{i 0} \varepsilon_{i} \ll \Sigma R_{i 0}$ ), the simplified version of the equations can be provided as follows:

$$
\begin{align*}
& U_{D C}=t_{0}{ }^{\prime \prime}\left(\varepsilon_{1}+\varepsilon_{2}-\varepsilon_{3}-\varepsilon_{4}\right)  \tag{4}\\
& U_{A B}=t_{0}{ }^{\prime}\left(\varepsilon_{1}-\varepsilon_{2}-\varepsilon_{3}+\varepsilon_{4}\right) \tag{5}
\end{align*}
$$

TABLE 1
Open-Circuit Voltage to Current Parameters in Function of Relative Increments of Resistance

| Circuit with arbitrary $R_{i}$ | Open-circuit voltage to current parameters of A-B and D-C outputs ( $r_{A B}, r_{C D}$ ) |  |
| :---: | :---: | :---: |
| $\begin{gather*} \hline \text { In balance: }  \tag{6}\\ r_{A B 0}=0  \tag{7}\\ R_{10} \cdot R_{40}=R_{20} \cdot R_{30} \\ R_{20}=m R_{10}, R_{40}=n R_{10} \\ R_{30}=(m / n) R_{10} \\ \text { or } \\ r_{C D 0}=0 \\ R_{10} \cdot R_{20}=R_{33} \cdot R_{40} \\ R_{20}=m R_{10,} R_{40}=n R_{10} \\ R_{30}=(n / m) R_{10} \\ \hline \end{gather*}$ | $\begin{gathered} r_{A B} \equiv \frac{U_{A B}}{J}=\frac{R_{1} R_{4}-R_{2} R_{3}}{\sum_{\text {where: }} R_{i}} \equiv t_{0}^{\prime} f^{\prime}\left(\varepsilon_{i}\right) \\ t_{0}^{\prime} \equiv \frac{m n R_{10}}{(m+n)(1+m)} \quad f^{\prime}\left(\varepsilon_{\mathrm{i}}\right)=\frac{\Delta L^{\prime}\left(\varepsilon_{\mathrm{i}}\right)}{1+\varepsilon_{\Sigma R}^{\prime}} \quad \varepsilon_{\Sigma R}^{\prime}=\frac{m\left(m \varepsilon_{2}+\varepsilon_{1}\right)+n\left(m \varepsilon_{4}+\varepsilon_{3}\right)}{(m+n)(1+m)} \\ \Delta L^{\prime}\left(\varepsilon_{i}\right)=\varepsilon_{1}-\varepsilon_{2}-\varepsilon_{3}+\varepsilon_{4}+\varepsilon_{1} \varepsilon_{4}-\varepsilon_{2} \varepsilon_{3} \end{gathered}$ | $\begin{gathered} r_{C D} \equiv \frac{U_{C D}}{J}=\frac{R_{1} R_{2}-R_{3} R_{4}}{\sum R_{i}} \equiv t_{0}{ }^{\prime \prime} f^{\prime \prime}\left(\varepsilon_{i}\right)^{(7)} \\ \text { where: } \\ t_{0}^{\prime \prime \equiv} \frac{m n R_{10}}{(m+n)(1+n)} \quad f^{\prime \prime}\left(\varepsilon_{\mathrm{i}}\right)=\frac{\Delta L^{\prime \prime}\left(\varepsilon_{i}\right)}{1+\varepsilon^{\prime \prime}} \quad \varepsilon_{\Sigma R}^{\prime \prime}=\frac{m\left(n \varepsilon_{2}+\varepsilon_{3}\right)+n\left(n \varepsilon_{4}+\varepsilon_{1}\right)}{(\mathrm{m}+n)(1+n)} \\ \Delta L^{\prime \prime}\left(\varepsilon_{i}\right)=\varepsilon_{1}+\varepsilon_{2}-\varepsilon_{3}-\varepsilon_{4}+\varepsilon_{1} \varepsilon_{2}-\varepsilon_{3} \varepsilon_{4} \end{gathered}$ |

## III. Accuracy Measures

Assume that open-circuit voltage to current parameters $r_{A B}$, $r_{C D}$ consist of two components. The parameters $r_{A B O}, r_{D C 0}$ are initial values and $\Delta_{r A B}, \Delta_{r C D}$ are the increments of $r_{A B}, r_{C D}$ :

$$
\begin{align*}
& r_{A B}=r_{A B 0}+\Delta_{r A B}  \tag{8}\\
& r_{C D}=r_{C D 0}+\Delta_{r C D} \tag{9}
\end{align*}
$$

Then one can modify equations (1) and (2) as follows:

$$
\begin{gather*}
U_{A B}=J\left(r_{A B O}+\Delta_{r A B}\right) \equiv U_{A B O}+J t_{0} '^{\prime} f^{\prime}\left(\varepsilon_{i}\right)  \tag{10}\\
U_{C D}=J\left(r_{C D O}+\Delta_{r C D}\right) \equiv U_{C D O}+J t_{0}^{\prime \prime} f^{\prime \prime}\left(\varepsilon_{i}\right) \tag{11}
\end{gather*}
$$

Actual (absolute) errors of parameters $r_{A B O}$ and $r_{D C O}$ are related to initial sensitivities $t_{0}{ }^{\prime}, t_{0}{ }^{\prime \prime}$. The relative errors $\delta_{\text {rABO }}$, $\delta_{\text {rCDO }}$ are the functions of algebraic sum of the initial (zero) errors of all the resistances [10]:

$$
\begin{gather*}
\delta_{r A B 0} \equiv \frac{\Delta_{r A B 0}}{t_{0}{ }^{\prime}}=\delta_{10}-\delta_{20}-\delta_{30}+\delta_{40}  \tag{12}\\
\delta_{r C D 0} \equiv \frac{\Delta_{r C D 0}}{t_{0}{ }^{\prime \prime}}=\delta_{10}+\delta_{20}-\delta_{30}-\delta_{40} \tag{13}
\end{gather*}
$$

Actual relative errors $\delta_{R i}$ of resistances can beexpressed by initial errors $\delta_{i 0}$ and errors $\delta_{\varepsilon i}$ of their increments

$$
\begin{gather*}
\delta_{i} \equiv \frac{\Delta_{i}}{R_{i 0}}=\delta_{i 0}\left(1+\varepsilon_{i}\right)+\Delta_{\varepsilon i}=\delta_{i 0}\left(1+\varepsilon_{i}\right)+\varepsilon_{i} \delta_{\varepsilon i}  \tag{14}\\
\delta_{R i} \equiv \frac{\Delta_{i}}{R_{i}}=\delta_{i 0}+\frac{\Delta_{\varepsilon i}}{1+\varepsilon_{i}}=\delta_{i 0}+\frac{\varepsilon_{i}}{1+\varepsilon_{i}} \delta_{\varepsilon i} \tag{15}
\end{gather*}
$$

Actual (absolute) errors of parameters $r_{A B}$ and $r_{D C}$ are also related to initial sensitivities $t_{0}{ }^{\prime}, t_{0}{ }^{\prime \prime}$ :

$$
\begin{align*}
& \delta_{r A B} \equiv \frac{\Delta_{r A B}}{t_{0}{ }^{\prime}}  \tag{16}\\
& \delta_{r C D} \equiv \frac{U_{r C D}}{t_{0}{ }^{\prime \prime}} \tag{17}
\end{align*}
$$

Their equations are more complicated. Actual values of measurement errors of bridge transfer functions $r_{A B}$ and $r_{C D}$ result from the total differential of analytical equations (6) and (7) from Table 1. After ordering all components of actual errors $\delta_{R i}$ of resistances $R_{i}$ one can estimateabsolute errors $\Delta r_{A B}(20 \mathrm{a})$ and $\Delta r_{C D}$ (20b). Then with (14-17) the following accuracy measures can be expressed:

- relative actual errors (21a) and (21b),
- relative limited errors (22a) and (22b),
- random (mean square) measures (23a) and (23b).

All the accuracy measures are ordered in Table 2.Additionally, actual errors of increments $r_{A B}-r_{A B O}$ and $r_{C D}-r_{C D 0}$ are defined. They were related to $r_{A B}$ and $r_{C D}$ parameters as follows:

$$
\begin{align*}
& \delta_{r A B r} \equiv \frac{\Delta_{r A B}-\Delta_{r A B 0} 0}{r_{A B}}=\frac{t_{0} \delta_{r A B}-t_{0}^{\prime}\left(\delta_{10}-\delta_{20}-\delta_{30}+\delta_{40}\right)}{t_{0}^{\prime} f^{\prime}\left(\varepsilon_{i}\right)}=  \tag{18}\\
&=\frac{\delta_{r A B}-\left(\delta_{10}-\delta_{20}-\delta_{30}+\delta_{40}\right)}{f^{\prime}\left(\varepsilon_{i}\right)} \\
& \delta_{r C D r} \equiv \frac{\Delta_{r C D}-\Delta_{r C D 0}}{r_{C D}}=\frac{\delta_{r C D}-\left(\delta_{10}+\delta_{20}-\delta_{30}-\delta_{40}\right)}{f^{\prime \prime}\left(\varepsilon_{i}\right)} \tag{19}
\end{align*}
$$

Actual (26a, 26b, 28a, 28b) and limited (27a, 27b, 29a, 29b) relative errors for the circuit with two sensors (with two resistances $R_{1}$ and $R_{2}$ variable) is shown in Table 3. All of errors depend on the two increments $\varepsilon_{A}$ and $\varepsilon_{B}$ (what represent two measured quantities A and B ). They are also the functions of $\delta_{i 0}$ (zero errors) and $\delta_{\varepsilon i}$ (gain errors) of resistors and sensors. This is a novelty in presenting of the accuracy measures in two parameter (2D) measurement.

## IV. Two-Parameter Measurement of Strain and Temperature Change

A prototype version was worked out in order to explain the concept of operation of a system measuring two parameters at the same time. The system shown in Fig. 2 can be applied to measure strain in one axis (e.g. x-axis by strain gauge $R_{1}$ ) and temperature (strain gauge or resistance thermometer $R_{2}$ ). This type of measurement system can be also applied to examine other quantities which can be measured with the use of resistance sensors(parametric).

Great advantage of thiscircuitis possibility of compensation of temperature influence on a measurement strain gauge resistance (without using additional temperature sensors). An indirect method, examining appropriate voltage on the diagonals of a double-current bridge makes possible tomeasure temperature and resistance of a strain gauge simultaneously.

TABLE 2
Accuracy Measures of the Voltage to Current Parameters of 2J Bridge in General Case

| Parameters of circuit | Measure | Accuracy measures $\delta_{r A B},\left\|\delta_{r A B}\right\|, \bar{\delta}_{r A B}$ ofr ${ }_{\text {AB }}$ |  |  | Accuracy measures $\delta_{r \mathrm{CD}},\left\|\delta_{\mathrm{rCD}}\right\|, \bar{\delta}_{r C D}$ of $r_{\mathrm{CD}}$ |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| General <br> case $R_{1} \subset R_{2}$ | Z0000000000000000 | $\Delta_{r A B}=R_{1} \frac{R_{4}-r_{A B}}{\sum R_{i}} \delta_{R 1}-R_{2} \frac{R_{3}+r_{A B}}{\sum R_{i}} \delta_{R 2}-R_{3} \frac{R_{2}+r_{A B}}{\sum R_{i}} \delta_{R 3}+R_{4} \frac{R_{1}-r_{A B}}{\sum R_{i}} \delta_{R 4}$ <br> (20a) |  |  | $\Delta_{r D}=R_{1} \frac{R_{2}-r_{C D}}{\sum R_{i}} \delta_{R 1}+R_{2} \frac{R_{1}-r_{C D}}{\sum R_{i}} \delta_{R 2}-R_{3} \frac{R_{4}+r_{C D}}{\sum R_{i}} \delta_{R 3}-R_{4} \frac{R_{3}+r_{C D}}{\sum R_{i}} \delta_{R 4}$ <br> (20b) |  |  |
|  |  | $\delta_{r A B} \equiv \frac{\Delta_{r A B}}{t_{0}^{\prime}}=\sum_{i=1}^{4} w_{R i}^{\prime} \delta_{R i}=\sum_{i=1}^{4} w_{R i}^{\prime}\left(\delta_{i 0}+\frac{\varepsilon_{i}}{1+\varepsilon_{i}} \delta_{\varepsilon i}\right)$ <br> (21a) <br> where: $w_{R 1}^{\prime}=\frac{1+\varepsilon_{1}}{1+\varepsilon_{\Sigma R}}\left[\left(1+\varepsilon_{4}\right)-\frac{r_{A B}}{R_{40}}\right]$ $\begin{aligned} & w_{R 2}^{\prime}=-\frac{1+\varepsilon_{2}}{1+\varepsilon_{\Sigma R}}\left[\left(1+\varepsilon_{3}\right)+\frac{r_{A B}}{R_{30}}\right] \\ & w_{R 3}^{\prime}=-\frac{1+\varepsilon_{3}}{1+\varepsilon_{\Sigma R}}\left[\left(1+\varepsilon_{2}\right)+\frac{r_{A B}}{R_{20}}\right] \\ & w_{R 4}^{\prime}=\frac{1+\varepsilon_{4}}{1+\varepsilon_{\Sigma R}}\left[\left(1+\varepsilon_{1}\right)-\frac{r_{A B}}{R_{10}}\right] \end{aligned}$ |  |  | $\delta_{r C D} \equiv \frac{\Delta_{r C D}}{t_{0}{ }^{\prime \prime}}=\sum_{i=1}^{4} w_{R i}^{\prime \prime} \delta_{R i}=\sum_{i=1}^{4} w_{R i}^{\prime \prime}\left(\delta_{i 0}+\frac{\varepsilon_{i}}{1+\varepsilon_{i}} \delta_{\varepsilon i}\right)$ <br> (21b) $\text { where: } \begin{aligned} \dot{w_{R 1}} & =\frac{1+\varepsilon_{1}}{1+\varepsilon_{\Sigma R}}\left[\left(1+\varepsilon_{2}\right)-\frac{r_{C D}}{R_{40}}\right] \\ \dot{w_{R 2}}= & \frac{1+\varepsilon_{2}}{1+\varepsilon_{\Sigma R}}\left[\left(1+\varepsilon_{1}\right)-\frac{r_{C D}}{R_{30}}\right] \\ \dot{w_{R 3}} & =-\frac{1+\varepsilon_{3}}{1+\varepsilon_{\Sigma R}}\left[\left(1+\varepsilon_{4}\right)+\frac{r_{C D}}{R_{20}}\right] \\ \dot{w_{R 4}} & =-\frac{1+\varepsilon_{4}}{1+\varepsilon_{\Sigma R}}\left[\left(1+\varepsilon_{3}\right)+\frac{r_{C D}}{R_{10}}\right] \end{aligned}$ |  |  |
| $\begin{aligned} & R_{1}=R_{10}\left(1+\varepsilon_{1}\right) R_{2} \\ & =R_{20}\left(1+\varepsilon_{2}\right) \\ & R_{3}=R_{30}\left(1+\varepsilon_{3}\right) \end{aligned}$ | 可管 | $\left\|\delta_{r A B}\right\|=\sum_{i=1}^{4}\left\|w_{R i}^{\prime}\right\|\left\|\delta_{R i}\right\|=\sum_{i=1}^{4}\left\|w_{R i}^{\prime}\right\|\left(\left\|\delta_{i 0}\right\|+\frac{\left\|\varepsilon_{i}\right\|}{1+\varepsilon_{i}}\left\|\delta_{\varepsilon i}\right\|\right)$ <br> (22a) |  |  | $\left\|\delta_{r C D}\right\|=\sum_{i=1}^{4}\left\|w_{R i}^{\prime \prime}\right\|\left\|\delta_{R i}\right\|=\sum_{i=1}^{4} \left\lvert\, w_{R i}^{\prime \prime}\left(\left\|\delta_{i 0}\right\|+\frac{\left\|\varepsilon_{i}\right\|}{1+\varepsilon_{i}}\left\|\delta_{\varepsilon i}\right\|\right)\right.$ <br> (22b) |  |  |
|  |  | $\begin{gathered} \bar{\delta}_{r A B}=\sqrt{\sum_{i=1}^{4} w_{R i}^{\prime 2} \bar{\delta}_{R i}^{2}}=\sqrt{\sum_{i=1}^{4} w_{R i}^{\prime 2}\left(\bar{\delta}_{i 0}^{2}+\frac{\varepsilon_{i}^{2}}{\left(1+\varepsilon_{i}\right)^{2}} \bar{\delta}_{s i}^{2}\right)} \text { (23a) } \\ \text { correlation coefficient } k_{i j}=0 \end{gathered}$ |  |  | $\bar{\delta}_{r C D}=\sqrt{\sum_{i=1}^{4} w_{R i}^{\prime \prime 2} \bar{\delta}_{R i}^{2}}=\sqrt{\sum_{i=1}^{4} w_{R i}^{\prime 2}\left(\bar{\delta}_{i 0}^{2}+\frac{\varepsilon_{i}^{2}}{\left(1+\varepsilon_{i}\right)^{2}} \bar{\delta}_{s i}^{2}\right)}$ <br> (23b) correlation coefficient $k_{i j}=0$ |  |  |
| Measures for $r_{A B O}=0$ | Actual errors |  | $\delta_{r A B 0}=\delta_{10}-\delta_{20}-\delta_{30}+\delta_{40}$ | Limited errors | $\underset{\text { (24a) }}{\left\|\delta_{\text {rABO }}\right\|_{m}=\sum\left\|\delta_{i 0}\right\|}$ | Mean square measures $k_{i j}=0$ | $\begin{gathered} \bar{\delta}_{r A B 0}=\sqrt{\sum \bar{\delta}_{\mathrm{i}}^{2}} \\ (25 \mathrm{a}) \end{gathered}$ |
| Measures for $r_{C D 0}=0$ |  |  | $\delta_{r C D 0}=\delta_{10}+\delta_{20}-\delta_{30}-\delta_{40}$ |  |  |  | $\begin{gathered} \bar{\delta}_{\mathrm{rCDO}}=\sqrt{\sum \bar{\delta}_{\mathrm{io}}^{2}} \\ (25 \mathrm{~b}) \end{gathered}$ |

TABLE 3
Accuracy Measures of the Voltage to Current Parameters of 2J Bridge if All Arm Initial Resistances are Equal

| Parameters of circuit | $\begin{gathered} \hline \hline \text { Measures } \\ R_{i} \end{gathered}$ | Related accuracy measures of bridge of $r_{A B}, r_{C D}$ |  |
| :---: | :---: | :---: | :---: |
|  |  | related to initial sensitivities $t_{0}{ }^{\prime}$ or $t_{0}{ }^{\prime \prime}$ | of increments $r_{A B}-r_{A B 0}$ and $r_{C D}-r_{C D 0}$ |
| $\Delta R_{1}$ and $\Delta R_{2}$ variable $\varepsilon_{1}=\varepsilon_{A}+\varepsilon_{B},$ | arbitrary | $\delta_{r A B}=\frac{4-2 \varepsilon_{A}+6 \varepsilon_{B}}{4\left(1+\varepsilon_{B}\right)}\left(\delta_{10}-\delta_{20}-\delta_{30}\right)+\frac{4+2 \varepsilon_{A}+6 \varepsilon_{B}}{4\left(1+\varepsilon_{B}\right)} \delta_{40}+\frac{\varepsilon_{A}+\varepsilon_{B}}{1+\varepsilon_{B}} \delta_{11}+\frac{\varepsilon_{A}-\varepsilon_{B}}{1+\varepsilon_{B}} \delta_{r 2}$ <br> (26a) | $\left.\delta_{r A B E}=\frac{2+\varepsilon_{\mathrm{B}}}{4 \varepsilon_{A}}\left[\frac{3-2 \varepsilon_{A}+6 \varepsilon_{\mathrm{B}}}{4\left(1+\varepsilon_{B}\right)} \delta_{10}-\delta_{20}-\delta_{30}\right)+\frac{3+2 \varepsilon_{A}+6 \varepsilon_{\mathrm{B}}}{4\left(1+\varepsilon_{B}\right)} \delta_{\text {so }}+\frac{\varepsilon_{A}+\varepsilon_{\mathrm{B}}}{1+\varepsilon_{B}} \delta_{21}+\frac{\varepsilon_{A}-\varepsilon_{\mathrm{B}}}{1+\varepsilon_{\mathrm{B}}} \delta_{22}\right]$ <br> (26b) |
| $\begin{gathered} \varepsilon_{2}=-\varepsilon_{A}+\varepsilon_{B}, \\ \varepsilon_{3}=0, \varepsilon_{4}=0, \\ \varepsilon_{A}^{2} \approx 0, \varepsilon_{B}^{2} \approx 0, \end{gathered}$ | $\begin{gathered} \text { if } \\ \left\|\delta_{01}\right\|=\left\|\delta_{0}\right\| \\ \left\|\delta_{a x}\right\|\left\|\delta_{x 2}\right\|=\delta_{i} \mid \end{gathered}$ | $\left\|\delta_{\mathrm{rAB}}\right\|=\frac{4+2\left\|\varepsilon_{A}\right\|+6\left\|\varepsilon_{\mathrm{B}}\right\|}{1+\left\|\varepsilon_{B}\right\|}\left\|\delta_{0}\right\|+\frac{2\left(\left\|\varepsilon_{A}\right\|+\left\|\varepsilon_{B}\right\|\right)}{1+\left\|\varepsilon_{B}\right\|}\left\|\delta_{\varepsilon}\right\|$ <br> (27a) | $\left.\left.\left\|\delta_{\mathrm{rABr}}\right\|=\frac{2+\left\|\varepsilon_{\mathrm{B}}\right\|}{4\left\|\varepsilon_{A}\right\|}\left\|\frac{3+2\left\|\varepsilon_{A}\right\|+6\left\|\varepsilon_{B}\right\|}{1+\left\|\varepsilon_{B}\right\|}\right\| \delta_{0}\left\|+\frac{2\left(\left\|\varepsilon_{A}\right\|+\left\|\varepsilon_{B}\right\|\right)}{1+\left\|\varepsilon_{B}\right\|}\right\| \delta_{\varepsilon} \right\rvert\,\right]$ <br> (27b) |
|  | arbitrary | $\delta_{r c \mathrm{D}}=\frac{1+2 \varepsilon_{B}}{1+\varepsilon_{B}}\left(\delta_{10}+\delta_{20}\right)-\left(\delta_{30}+\delta_{40}\right)+\frac{\varepsilon_{A}+\varepsilon_{B}}{1+\varepsilon_{B}} \delta_{\epsilon 1}+\frac{\varepsilon_{B}-\varepsilon_{A}}{1+\varepsilon_{B}} \delta_{\varepsilon 2}$ | $\begin{equation*} \delta_{r ~ C D r}=\frac{\left(2+\varepsilon_{B}\right)\left(\delta_{10}+\delta_{20}\right)}{2\left(1+\varepsilon_{B}\right)}+\frac{2+\varepsilon_{B}}{4 \varepsilon_{B}}\left[\frac{\varepsilon_{A}+\varepsilon_{B}}{1+\varepsilon_{B}} \delta_{\varepsilon 1}+\frac{\varepsilon_{B}-\varepsilon_{A}}{1+\varepsilon_{B}} \delta_{\varepsilon 2}\right] \tag{28a} \end{equation*}$ <br> (28b) |
| $\begin{aligned} & r_{A B}=\frac{R_{10} \varepsilon_{A}}{2+\varepsilon_{B}} \\ & r_{C D}=\frac{R_{10}\left(2 \varepsilon_{B}-\varepsilon_{i}^{2}+\varepsilon_{B}^{2}\right)}{4+2 \varepsilon_{B}} \end{aligned}$ | $\begin{gathered} \text { if } \\ \left\lvert\, \begin{array}{c} \delta_{0}\left\|=\left\|\delta_{0}\right\|\right. \\ \left\|\delta_{s_{1}}\right\|\left\|\delta_{a x}\right\|=\left\|\delta_{d}\right\| \end{array}\right. \end{gathered}$ | $\left\|\delta_{\mathrm{rCD}}\right\|=\frac{4+6\left\|\varepsilon_{B}\right\|}{1+\left\|\varepsilon_{B}\right\|}\left\|\delta_{0}\right\|+\frac{2\left(\left\|\varepsilon_{A}\right\|+\left\|\varepsilon_{B}\right\|\right)}{1+\left\|\varepsilon_{B}\right\|}\left\|\delta_{\varepsilon}\right\|$ <br> (29a) | $\left\|\delta_{r C D r}\right\|=\frac{2+\left\|\varepsilon_{B}\right\|}{1+\left\|\varepsilon_{B}\right\|} \left\lvert\,\left[\left\|\delta_{0}\right\|+\frac{\left(\left\|\varepsilon_{A}\right\|+\left\|\varepsilon_{B}\right\|\right)}{2\left\|\varepsilon_{B}\right\|}\left\|\delta_{\varepsilon}\right\|\right]\right.$ <br> (29b) |



Fig. 2. Scheme of double-current bridge (2x1J) ( $J$-current source, $R_{1}, R_{2}$ - sensors, $R_{3}, R_{4}$ - resistors, K1, K2 - electronic switches)

Itwas conductedan experimental test on the 2x1J circuit. The real change in temperature of strain gauges in the point of their placement and the mechanical stress (deflection of cantilever beam) wereexamined. The circuit was tested with two metal (foil) strain gauges. Both output voltages are the means of voltages measured in two cycles [2]:

$$
\begin{align*}
U_{\text {DCavg }} & =0.5 t_{0}{ }^{\prime \prime}\left(\varepsilon_{1}+\varepsilon_{2}\right)  \tag{30}\\
U_{\text {ABavg }} & =0.5 t_{0}{ }^{\prime}\left(\varepsilon_{1}-\varepsilon_{2}\right) \tag{31}
\end{align*}
$$

In such a situation the resistance incrementsof strain gauges consists of two components: $\varepsilon_{1}=\varepsilon_{A}+\varepsilon_{B}, \varepsilon_{2}=-\varepsilon_{A}+\varepsilon_{B}$. The first one is resistance increment or decrement which is caused by bending force. The second one is the increment of temperature change $\Delta T$.In a case of using two identical strain gauges in the circuit it will cause the same value and sign of the relative temperature increments. When one gauge is stretched (Fig. 3) and the other one iscompressed at the same time, the increments of the mechanical stress will have the opposite signs:

$$
\begin{align*}
& \varepsilon_{1}^{\prime}(\Delta T)=\varepsilon_{2}^{\prime}(\Delta T)=\varepsilon_{B}  \tag{32}\\
& \varepsilon_{1}^{\prime \prime}\left(e_{B}\right)=-\varepsilon_{2}^{\prime \prime}\left(e_{B}\right)=\varepsilon_{A} \tag{33}
\end{align*}
$$



Fig. 3. Heater, strain gaugeand temperature sensorson cantilever beam.
For several (constant) temperatures of a cantilever beam (from $20^{\circ} \mathrm{C}$ until $60^{\circ} \mathrm{C}$ ) the measurements were conducted. The reference temperature was $20^{\circ} \mathrm{C}$ and the beam was bentwith the use of micrometer screw in the range from 0 until 10 mm . As shown in Fig. 4there is a significant influence of rising temperature $T$ on the $\varepsilon_{i}$ intercept. The slope is almostidentical.


Fig. 4. The relative resistance increments $\varepsilon_{1}, \varepsilon_{2}$ of strain gauges in the function of the beam deflection $X$,temperature Tisvariable.

## V. Conclusions

The accuracy measures of parameters $r_{A B}$ and $r_{C D}$ are obtained after transformation of error propagation formulas. The two-component method [10] of the bridge transfer functionsgives accuracy representation separately for its initial value and for increment. It is similar like unified one used for digital instruments and sensor transducers (Table 2). The parameters $r_{A B}, r_{C D}$ and their actual and limited errors as the functions of relative increments of resistance $\varepsilon_{i}$ are shown for the bridge circuit with two sensors $R_{1}, R_{2}$ (Table 3).The error formulas are presentedfor the measurement of two quantities $\left(\varepsilon_{1}=\varepsilon_{A}+\varepsilon_{B}, \varepsilon_{2}=-\varepsilon_{A}+\varepsilon_{B}\right)$.

The application of this circuit in simultaneous measurement of strain and temperature change wasfeatured. The experiment confirmed that there was linear relationship between and deflection and relative resistance increments of strain gauges (Fig. 4).

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