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A NEW ICAPM APPROACH TO MULTIFACTOR STOCK PRICING USING BOOTSTRAP

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ABSTRACT

The aim of this work is the use of bootstrap methods for assessing of returns and risk of stock described by a small-to-moderate time series data. The paper presents the possibility of using bootstrap for testing the selected ICAPM application. We estimate systematic risk and risk premium components, depending on the fundamental risk factors. We compare bootstrap and classical asymptotic GLS results.

The authors analyze quarterly returns of stocks listed on Warsaw Stock Exchange in 1995–2010. The full-sample observations are divided into two separate sub-periods: 1995–2004, the years preceding Poland's accession to the EU, and 2005–2010, the years of Poland's membership in the UE.

The components of risk premium change in the second sub-period. Also, we test the multifactor-efficiency (ME) of the generated portfolios. GRS and asymptotic Wald tests reject ME. However, the bootstrapped Wald test does not reject ME for the tested cases. Using the tested ICAPM application to forming ME portfolios makes it possible to offer a number of useful guidelines for portfolio managers.

STRESZCZENIE

S. Urbański, J. Leśkow. *Nowa aplikacja ICAPM do wieloczynnikowej wyceny akcji z zastosowaniem metod bootstrap*. Folia Oeconomica Cracoviensia 2014, 55: 15–33.

Celem niniejszej pracy jest zastosowanie metod bootstrap do oszacowania stóp zwrotu i ryzyka akcji opisanych krótkimi szeregami czasowymi. Artykuł prezentuje możliwość zastosowania metod bootstrap do testowania wybranej aplikacji ICAPM. My szacujemy składowe ryzyka systematycznego i premii za ryzyko, w zależności od fundamentalnych czynników ryzyka. Porównujemy wyniki otrzymane metodami bootstrap i klasyczną uogólnioną metodą najmniejszych kwadratów.

Analizie poddajemy kwartalne stopy zwrotu akcji notowanych na Giełdzie Papierów Wartościowych w Warszawie w latach 1995–2010. Wszystkie obserwacji dzielimy na dwa podokresy: 1995–2004 (okres poprzedzający wejście Polski do Unii Europejskiej) oraz 2005–2010 (okres członkostwa Polski w Unii Europejskiej). Składowe premii za ryzyko ulegają zmianie w drugim podokresie. My testujemy również wieloczynnikową efektywność (ME) generowanych portfeli. Test GRS oraz asymptotyczny test Walda odrzuca ME. Natomiast bootstrapowy test Walda, w żadnym badanym przypadku nie odrzuca ME.

Zastosowanie testowanej aplikacji ICAPM do budowy portfeli wieloczynnikowo efektywnych pozwala na wyciągnięcie wielu użytecznych wskazówek dla zarządzających portfelami inwestycyjnymi.

KEY WORDS — SŁOWA KLUCZOWE

asset pricing, bootstrap method, return changes, systematic risk, multifactor efficiency

wycena aktywów, metoda bootstrap, zmiany stop zwrotu, ryzyko systematyczne, wieloczynnikowa efektywność

INTRODUCTION

Testing the stock pricing that could be observed in the conditions of ICAPM validity can be referred to an analysis of multifactor-efficiency (ME) of a given portfolio. For this purpose, you can use the Wald statistics of the asymptotic distribution χ^2.

Wald test tends to over-reject the ME portfolio hypothesis (see Chou and Zhou (2006), p. 221) for finite samples. However, a small-sample case can be analyzed with the use of the GRS test — see Gibbons et al (1989) — on condition of the normality of the sample. Therefore, for non-normal small samples one should consider alternative scenarios like the bootstrap method. One of the purposes of this article is to show the validity of such an approach.

The Wald test can be applied for large samples and under the assumption of independence. However, the ICAPM applications in emerging markets can be tested with the help of samples of a small-to-moderate size for which only iid conditions can be assumed, but normality is usually rejected. One never knows what is the true distribution of the returns, therefore there is a need to consider good approximations.

Contemporary statistical inference provides resampling and bootstrap methods to create confidence intervals for cases of small non-normal samples. Recent research provides also resampling tools for time-series data. For more information, the reader is referred to Leśkow et al (2008, 2014). Chou and Zhou (2006) present the possibility of using the bootstrap method to test the ME of Fama–French (FF) portfolios and the portfolio representing the CRSP index for the U.S. market. Research works on testing the classic Capital Asset Pricing Model and other studies on the Polish market are presented, among others, by Osińska and Romański (1994), Jajuga (2000), Bołt and Miłobędzki (2002), Osiewalski and Pipień (2004), Gurgul and Majdosz (2007) and Zarzecki et al. (2004–2005).

In this work we test the application of the ICAPM for the Warsaw Stock Exchange (WSE) data in 1995–2010. The above approach was proposed by Urbański (2011). We use bootstrap procedures in stock pricing simulated by the aggregated three-factor model.

The aim of our research is to consider an approach for pricing of stocks, determined by the assessment of the systematic risk and risk premium components. As a result, the multifactor efficiency of the tested portfolio can be evaluated.

Section 1 discusses theoretical methods for testing the multifactor-efficiency of a given portfolio. Section 2 proposes the possible use of the bootstrap method in finance. Section 3 presents several procedures for data preparation in order to use the studied algorithms. Section 4 shows the results of calculations. Section 5 includes a summary and conclusions.

1. MULTIFACTOR-EFFICIENCY RESTRICTIONS

Multifactor application of stock pricing in light of the ICAPM can be described by the following equation:

$$E(R_t) = \beta E(f_t), \tag{1}$$

where $R_t = (r_{1t},...,r_{it},...,r_{Nt})'$ is N-vector of the excess returns over the risk-free rate on stock i in period t, $\beta = (\beta_1,...,\beta_i,...,\beta_N)'$ and f_t is the k-vector of factors.

Portfolios satisfying the equation (1) are ME. A statistical model testing a general form of the ICAPM can be described by the regressions (2) and (3) of the following two-step procedure:

$$r_{it} = \alpha_i + \beta_i f_t + e_{it},\ \forall i = 1, ..., N;\ t = 1,\dots, T, \tag{2}$$

$$r_{it} = \gamma_0 + \gamma_1 \hat{\beta}_i + \varepsilon_{it},\ i = 1, ..., N;\ t = 1,\dots, T, \tag{3}$$

where γ_1 is the k-vector of the second pass regression parameters and e_{it} and ε_{it} are error components. Here, N is the number of assets, and T is the number of observations.

Pricing in light of the ICAPM aims to estimate the parameters of regressions (2) and (3), as well as to prove that generated portfolios are ME.

The pricing restriction of ME portfolios can be formulated as the hypothesis testing problem:

$$H_0\!:\ \alpha = 0, \text{ where } \alpha = (\alpha_1,...,\alpha_N)'.$$

Such a null hypothesis can be tested using the asymptotic χ^2 distribution corresponding to the following Wald statistic:

$$W = \hat{\alpha}' \mathrm{var}[\hat{\alpha}]^{-1} \hat{\alpha} , \sim \chi^2_N . \quad (4)$$

If the errors e_{it} defined in (2) are iid, then (4) is of the form (Cochrane (2001), pp. 217–219):

$$W = \frac{T}{1 + E(f_t)' \mathrm{var}[f_t]^{-1} E(f_t)} \hat{\alpha}' \hat{\Sigma}_e^{-1} \hat{\alpha} , \quad (5)$$

where $\hat{\Sigma}_e = \hat{e}'\hat{e}/(T-k-1)$, and $\hat{e}$ is the $T \times N$ matrix of residuals.

In practice, applying the Wald test or GRS method requires estimating matrix Σ_e. This, in turn, induces imposing the normality assumption on the random error terms in (2) and (3) to ensure that the statistic $\hat{t} = \hat{\theta}_i / \mathrm{se}(\hat{\theta}_i)$ has a t-Student distribution.[1] In reality, however, the exact distribution of $\hat{t}$ is not known. The bootstrap method can overcome this problem.

2. RESAMPLING METHOD APPROACH

Contemporary statistical inference provides tools to deal with small and non-normal samples. We are now able to approximate the finite sample distribution of the estimators without invoking the normality assumption or large sample distributions. Extensive surveys of bootstrap methods can be found for example, in monograph texts by Politis (1999) and Lahiri (2003). Time series applications of bootstrap and other resampling methods can be found e.g. in Leśkow (2008, 2014).

For small samples, most of the resampling methods provide more reliable results than the normal approximation. For regression-type models, we study small-sample distributions of the estimates via bootstrapping the residuals. In such a case, the model errors are iid and the factors are treated as fixed constants. In this case, the fitted residuals are resampled.[2] In such a scenario, the bootstrap procedure can be designed in the following way:

1) Estimate the parameters of regressions (2) and (3) by a chosen asymptotic method. These regressions in bootstrap procedure are referred to in this paper as "null" regressions. Under such "null" regression it is necessary to:
   a) Determine the model residuals $\hat{e}_{it}$.
   b) Calculate the Wald statistic:

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[1] se $(\hat{\theta}_i)$ is a standard error of $\hat{\theta}_i$.

[2] The errors are not observable, thus fitted residuals are used.

$$W = \hat{\alpha}' \text{var}[\hat{\alpha}]^{-1} \hat{\alpha} . \quad (6)$$

2) Repeat the following procedure a large number of times.
   a) Draw the residuals e_{it}^*, t=1,…,T from $\hat{e}_{it}$ with replacement.
   b) Generate the bootstrap returns as follows:

$$r_{it}^* = \alpha_i + \beta_i f_t + e_{it}^* . \quad (7)$$

   c) Estimate the bootstrap parameters, of the first path of the model, α_i^* and β_i^* of the following regression:

$$r_{it}^* = \alpha_i^* + \beta_i^* f_t + e_{it} . \quad (8)$$

   d) Estimate the bootstrap parameters, of the second path of the model, γ_0^* and γ_1^* of the following regression:

$$r_{it}^* = \gamma_0^* + \gamma_1^* \hat{\beta}_i^* + \varepsilon_{it} . \quad (9)$$

   e) Calculate the bootstrapped Wald statistic:

$$W^* = \frac{T}{1 + E(f_t)' \text{var}[f_t]^{-1} E(f_t)} (\hat{\alpha}^*)' \hat{\Sigma}_e^{-1} \hat{\alpha}^* . \quad (10)$$

   f) Calculate the percentage of α_i^*'s and β_i^*'s and γ_0^*'s and γ_1^*'s, and W^*'s that are greater than α_i and β_i and γ_0 and γ_1, and W. The percentages are the p-values of the bootstrap test.

One of the main concerns while using the bootstrap method is consistency, i.e. concordance between the quantiles derived from the bootstrap distribution and the asymptotic one. The bootstrap quantiles can be derived using a computer algorithm described above. In this case, the consistency of bootstrap is presented in the monograph of Davison and Hinkley (1999).

3. DATA

In this section we analyze the quarterly returns of stocks listed on WSE in 1995–2010. The full-sample observations are divided into two separate sub--periods: 1995–2004, the years preceding Poland's accession to the EU, and 2005–2010, the years of Poland's membership in the UE. Data referring to the fundamental results of the inspected companies are taken from the database drawn up by Notoria Serwis Sp. z o.o. Data for defining returns on securities are provided by the Warsaw Stock Exchange.

The data presented by Urbański (2012b) indicate that the WSE is among the average-sized European stock exchanges. It justifies the choice of the WSE as an area for researching the returns in Central Europe's emerging markets.

The entire sample comprises 56 quarterly investment periods from May 10, 1996 to May 12, 2010. The first sub-period covers 36 quarters from May10, 1996 to May 19, 2005. The second sub-period covers 20 quarters from May 19, 2005 to May 15, 2010.

A rapid increase in the number of WSE companies is recorded after 2004, following Poland's accession to the EU. However, it has been accompanied by an increase in the number of speculative stocks whose returns are not linked to their financial results; see Urbański (2012a). Consequently, the tests are performed for two modes. Mode 1 considers all WSE stocks except of companies characterized by a negative book value. In mode 2, we eliminate speculative stocks meeting one of the following boundary conditions: a) $MV/BV > 100$, b) $ROE < 0$ and $BV > 0$ and $MV/BV > 30$ and $r_{it} > 0$, where *MV* is the stock market value, *ROE* is the return on book value (*BV*). The speculative stocks appear from Q1 of 2005. The number of analyzed companies decreased from 10% in 2005 to 30% in 2010, after exclusion of speculative stocks. All stock returns are calculated in excess of 91-day Polish Treasury bill return (*RF*).

The bootstrap quantile is based on 10,000 resamples of the data. The inspected securities are divided into quintile portfolios built on the basis of fundamental functional *FUN*, presented in equation (11), and *NUM* and *DEN* functions constituting the numerator and denominator of *FUN*, respectively.[3]

$$FUN = \frac{nor(ROE)*nor(AP)*nor(AZO)*nor(AZN)}{nor(MV/E)*nor(MV/BV)}, \tag{11}$$

where

$$ROE = F_1; AP = F_2 = \frac{\sum_{t=1}^{i} S(Q_t)}{\sum_{t=1}^{i} \overline{S(nQ_t)}}; AZO = F_3 = \frac{\sum_{t=1}^{i} ZO(Q_t)}{\sum_{t=1}^{i} \overline{ZO(nQ_t)}};$$

[3] The tested securities are divided into quintile portfolios in one direction; 5 portfolios are formed on *FUN*, 5 on *NUM* and 5 on *DEN*.

$$AZN = F_4 = \frac{\sum_{t=1}^{i} ZN(Q_t)}{\sum_{t=1}^{i} \overline{ZN(nQ_t)}}, MV/E = F_5; MV/BV = F_6.$$

Variables F_j ($j = 1,\ldots,6$) are transformed to standardized areas ranging $<a_j; b_j>$, in keeping with Equation (12):

$$nor(F_j) = \left[\ a_j + (b_j - a_j) * \frac{F_j - c_j * F_j^{\min}}{d_j * F_j^{\max} - c_j * F_j^{\min} + e_j}\ \right]. \tag{12}$$

In Equations (11) and (12), the corresponding indications are as follows. *ROE* is return on book equity; $\sum_{t=1}^{i} S(Q_t), \sum_{t=1}^{i} ZO(Q_t), \sum_{t=1}^{i} ZN(Q_t)$ are values that are accumulated from the beginning of the year as net sales revenue (*S*), operating profit (*ZO*) and net profit (*ZN*) at the end of "*i*" quarter (Q_i); $\sum_{t=1}^{i} \overline{S(nQ_t)}, \sum_{t=1}^{i} \overline{ZO(nQ_t)}, \sum_{t=1}^{i} \overline{ZN(nQ_t)}$ are average values, accumulated from the beginning of the year as *S*, *ZO* and *ZN* at the end of Q_i over the last *n* years;[4] *MV/E* is the market-to-earning value ratio; *MV/BV* is the market-to-book value ratio; a_j, b_j, c_j, d_j, e_j are variation parameters. Calculations prove, that in modeling equilibrium on the stock market, it is possible to assume identical values for all parameters; see Urbański (2011). The functions F_j ($j = 1,\ldots,6$) are transform into equal normalized area $<1;2>$.[5]

In comparison with the work conducted by FF (1995) and Cochrane (2001), it is assumed that *FUN* may constitute positive characteristics as a basis for the general description of returns. The function *NUM* represents an investor forming a portfolio which consists of the best fundamental companies. Whereas *DEN* represents an investor portfolio which consists of the undervalued stocks. Similarly, *FUN* represents an investor forming a portfolio which consists of the best fundamental and simultaneously undervalued stocks. *FUN*, *NUM* and *DEN* are calculated for all analyzed securities at the beginning of each investment period in which the return is to be calculated. *FUN*, *NUM* and *DEN* for portfolios consti-

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[4] The present research assumes that $n = 3$ years.

[5] If $\sum_{t=1}^{i} ZN(Q_t), \sum_{t=1}^{i} ZO(Q_t), \sum_{t=1}^{i} \overline{ZN(nQ_t)}$ or $\sum_{t=1}^{i} \overline{ZO(nQ_t)}$ in equation (11) is negative, the functions F_j ($j = 1, 3, 4$) are transformed into area (0,1); see Urbański (2011).

tute average arithmetical values of these functions of various portfolio securities. Returns on given portfolios are average stock returns weighted by market capitalizations. The factors f_t are assigned to company portfolios.

4. RESULTS

We test the aggregated three-factor model presented by Urbański (2011). This model analyses the influence of excess market returns (*RM*)[6] and factors f^{HMLN} and f^{LMHD} on returns in the analyzed portfolios. f^{HMLN} (high minus low) is the difference between the returns from the portfolio with the highest and lowest NUM_t values in the period t; f^{LMHD} (low minus high) is the difference between the returns from the portfolio with the lowest and highest DEN_t values in the period t.

Absolute values of correlation coefficient between the response variable and explanatory variables range from 0.05 to 0.92. Absolute values of the correlation coefficient between factors are reaching the levels of 0.23 for full-sample observations, 0.37 for sub-period 1995–2005, and 0.23 for sub-period 2005–2010, respectively. For the first and second sub-period the correlation between $RM_t–RF_t$ and f_t^{HMLN} is equal to 0.24 and 0.18, respectively, and between $RM_t–RF_t$ and f_t^{LMHD} is -0.37 and -0.16. It is possible, therefore, to duplicate information. The orthogonalized market factors are defined using the following regression:

$$RM_t - RF_t = \alpha + \beta_{HMLN} f_t^{HMLN} + \beta_{LMHD} f_t^{LMHD} + e_t; t = 1,...,T, \quad (13)$$

where:

Mode 1; full sample

α = -0.01;	β_{HMLN} =0.29;	β_{LMHD} = -0.27;	R^2 = 6.32%
(76.20%)	(15.87%)	(13.86%)	

Mode 2; full sample

α = -0.02;	β_{HMLN} =0.33;	β_{LMHD} = -0.010;	R^2 = 4.97%
(50.66%)	(11.38%)	(58.35%)	

Mode 1; first sub-period

α = -0.01;	β_{HMLN} =0.40;	β_{LMHD} = -0.59;	R^2 = 25.18%
(82.42%)	(5.43%)	(0.90%)	

[6] The market return (*RM*) is evaluated by the return on the WIG/ WSE index.

Mode 1; second sub-period

α = 0.03;	β_{HMLN} =-1.05;	β_{LMHD} = 0.48;	R^2 = 20.82%
(63.57%)	(8.01%)	(7.24%)	

Mode 2; second sub-period

α = -0.01;	-β_{HMLN} =0.32;	β_{LMHD} = 0.57;	R^2 = 29.07%
(82.36%)	(51.28%)	(1.72%)	

Under the regression model (13) the values of variable loadings are included for all tested periods. The corresponding *p*-values appear in brackets. Regression (13), especially for sub-periods, contains higher explanatory power. The value of the orthogonalized market factor is defined as follows:[7]

$$f_t^{MO} = \alpha + e_t . \qquad (14)$$

The response variable and the explanatory variables are subject to stationarity tests whose hypothesis is based on the Dickey–Fuller test. Dickey–Fuller tests and augmented Dickey–Fuller tests confirm lack of unit root for each test case at 1% significance level.[8] This leads to conclusions regarding the stationarity of the analyzed variables.

We test the aggregated three-factor model in two passes:

$$r_{it} - RF_t = \alpha_i + \beta_{i,HMLN} f_t^{HMLN} + \beta_{i,LMHD} f_t^{LMHD} + \beta_{i,MO} f_t^{MO} + e_{it}; t = 1,...,T \; ; \forall i=1,..., 15, \qquad (15)$$

$$r_{it} - RF_t = \gamma_0 + \gamma_{HMLN}\hat{\beta}_{i,HMLN} + \gamma_{LMHD}\hat{\beta}_{i,LMHD} + \gamma_{MO}\hat{\beta}_{i,MO} + \varepsilon_{it}; i = 1,...,15; \; t = 1,...,T. \qquad (16)$$

Beta values are estimators of the systematic risk. The second pass estimates the beta loadings which define risk premiums. Regression parameters in (15) and (16) are estimated via GLS — following Prais–Winsten procedure, and by three bootstrap methods: quantile bootstrap, BCα bootstrap, and t-bootstrap; see Efron and Tibshirani (1993). Homoskedasticity of the residuals is confirmed using

[7] A similar procedure concerning the orthogonalization of the market factor is applied by Fama and French (1993), p. 27–31, for the five-factor model. The loadings of all of the tested *HML*, *SMB*, *TERM* and *DEF* variables differ significantly from zero. The determination coefficient of the analyzed regression (by FF) is $R^2 = 38\%$.

[8] Dickey–Fuller tests are carried out for the three tested periods. 18 tested cases include the response variable for 5 portfolios formed on *FUN*, *NUM* and *DEN* and 3 explanatory variables: f_t^{MO}, f_t^{HMLN} and f_t^{LMHD}. The augmented Dickey–Fuller tests are carried out for lag, defined on the basis of minimizing the modified Akaike criterion, assuming that maximum lag equals 4. Test findings are available from the authors on request.

White and Breusch–Pagan methods. Therefore, the heteroscedascity correction is not required.[9]

The parameters of the second pass can be estimated by three variants:

1) the pooled time-series and cross-section estimate,
2) the "pure cross-sectional" estimate, on the basis of time series averages,
3) the Fama–MacBeth procedure that means running a cross-sectional regression at each point in time; estimated parameters $\hat{\gamma}_0$ and $\hat{\gamma}_1$ are average cross-sectional estimates of $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$.The time-series standard deviations of $\hat{\gamma}_{0t}$ and $\hat{\gamma}_{1t}$ are used to estimate the standard error of $\hat{\gamma}_0$ and $\hat{\gamma}_1$.[10]

If the explanatory variables of regression (16) do not vary over time, and if the errors are cross-sectionally correlated but not correlated over time, then the pooled time-series and cross-sectional OLS estimate, the "pure cross-sectional" OLS estimate, and the the Fama–MacBeth procedure are identical; see Cochrane (2001), pp. 247–250. WE estimate the risk premium vector using the pooled time-series and cross-section data. Independent variables (betas) remain permanent for all periods, while dependent variables constitute the returns which should by nature be random; see Cochrane (2001), p. 247. Therefore, we assume the lack of autocorrelation of the residual component. The impact of heteroskedasticity is taken into account by means of the change of variables method.[11]

Table 1

The Parameter Values of Time-Series Regression of Excess Stock Returns on the Orthogonalized Stock-Market Factor, f^{MO} and the Mimicking Returns for the *NUM* Value (f^{HMLN}) and *DEN* Value (f^{LMHD}) Factors

$$r_{it} - RF_t = \alpha_i + \beta_{i,HMLN} f_t^{HMLN} + \beta_{i,LMHD} f_t^{LMHD} + \beta_{i,MO} f_t^{MO} + e_{it}; t = 1,\ldots,T \,;\, \forall i = 1,\ \ldots,\ 15$$

Mode 1. The sample period is from 1995 to 2010, T=56 Quarters								
Portfolio „*i*"	Quantile bootstrap, θ^*		BCα bootstrap, θ^*		t-bootstrap	„null" regression		
	$\hat{\theta}^*_{0.025}$	$\hat{\theta}^*_{0.975}$	$\hat{\theta}^*_{0.025}$	$\hat{\theta}^*_{0.975}$	p-value[a]	θ^*	p-value[a]	R^2 %
$\hat{\theta}^* = \hat{\beta}_{i,HMLN}$						$\hat{\theta} = \hat{\beta}_{i,HMLN}$		
1	**-0.531**	**-0.180**	**-0.531**	**-0.174**	**0.002**	-0.357	**0.000**	88.60
5	0.331	0.715	0.331	0.715	**0.000**	0.520	**0.000**	85.01
6	**-0.716**	**-0.266**	**-0.702**	**-0.248**	**0.002**	-0.472	**0.000**	83.48

[9] The co-variance matrix of regression coefficients is also estimated by means of the Newey–West estimator where standard errors are corrected for autocorrelation and heteroskedasticity. The results are qualitatively similar. They are readily available upon request.

[10] $\hat{\gamma}_1$ is the vector $\hat{\gamma}_1[\hat{\gamma}_{HMLN}, \hat{\gamma}_{LMHD}, \hat{\gamma}_{MO}]$.

[11] See footnote 9.

10	**0.398**	**0.767**	**0.381**	**0.751**	**0.000**	0.591	**0.000**	85.49
11	-0.056	0.297	-0.054	0.298	0.230	0.120	0.210	83.91
12	**0.088**	**0.410**	**0.047**	**0.380**	**0.000**	0.250	**0.005**	87.98
13	**0.143**	**0.510**	**0.122**	**0.499**	**0.004**	0.330	**0.002**	78.67
14	**0.089**	**0.433**	**0.087**	**0.431**	**0.000**	0.273	**0.004**	86.02
15	-0.065	0.366	-0.039	0.421	0.248	0.138	0.220	86.12
$\hat{\theta}^{*}=\hat{\beta}_{i,MO}$						$\hat{\theta}=\hat{\beta}_{i,MO}$		
1	**1.011**	**1.251**	**1.019**	**1.260**	**0.000**	1.125	**0.000**	88.60
5	**0.878**	**1.118**	**0.895**	**1.139**	**0.000**	1.002	**0.000**	85.01
6	**0.996**	**1.309**	**1.006**	**1.313**	**0.000**	1.149	**0.000**	83.48
10	0.876	**1.100**	**0.878**	**1.102**	**0.000**	0.989	**0.000**	85.49
11	**0.851**	**1.075**	**0.847**	**1.074**	**0.000**	0.964	**0.000**	83.91
15	0.979	1.242	0.984	**1.246**	**0.000**	1.111	**0.000**	86.12
$\hat{\theta}^{*}=\hat{\beta}_{i,LMHD}$						$\hat{\theta}=\hat{\beta}_{i,LMHD}$		
1	**-0.722**	**-0.426**	**-0.742**	**-0.444**	**0.002**	-0.572	**0.000**	88.60
5	-0.477	**-0.172**	**-0.502**	**-0.191**	**0.002**	-0.331	**0.000**	85.01
6	**-0.667**	**-0.288**	**-0.665**	**-0.278**	**0.002**	-0.469	**0.000**	83.48
10	-0.549	-0.254	-0.574	**-0.269**	**0.002**	-0.404	**0.000**	85.49
11	**0.024**	**0.327**	**0.017**	**0.325**	**0.030**	0.180	**0.031**	83.91
12	0.069	0.345	0.069	0.345	0.006	0.203	**0.008**	87.98
13	**-0.324**	**-0.010**	-0.294	0.007	**0.058**	-0.163	**0.062**	78.67
14	**-0.897**	**-0.602**	**-0.903**	**-0.604**	**0.002**	-0.752	**0.000**	86.02
15	**-1.038**	**-0.672**	**-1.065**	**-0.691**	**0.002**	-0.872	**0.000**	86.12

Regression parameters for all bootstrap iterations and “null” regression are estimated by GLS. Portfolios for $i=1$ and $i=5$ are formed on minimal and maximal values of *FUN*. Portfolios for $i=6$ and $i=10$ are formed on minimal and maximal values of *NUM*. Portfolios for $i=11$ and $i=15$ are formed on minimal and maximal values of *DEN*. $\hat{\theta}^{*}_{0.025}$ is the bootstrapped value of the estimator for the 2,5% level and, similarly, $\hat{\theta}^{*}_{0.975}$ is the bootstrapped value of the estimator for the 97,5% level. The bootstrap quantile is based on 10,000 data resamples. Negative-*BV* stocks are excluded from the portfolios. The errors-in-variables are adjusted and follow Shanken (1992). [a] Corresponds to the significance test for model parameters in the null hypotheses. **Bold type** — the parameter is significantly different from zero at the level of 5%.

The impact of estimation errors of the true beta values in the first pass is considered by correcting the standard errors of beta loadings estimated in the second pass. With this purpose in mind, Shanken’s estimator is applied; see Shanken (1992).

Table 1 presents the values of parameters of regression (15) for the full-sample and for the portfolios of mode 1 type.[12] The regression parameters estimated in "null" regressions for the first and second sub-periods are subject to Chow's stability tests. In most cases, the results confirm the stability of the parameters at the level of 5%. The regression parameters for test cases, estimated in "null" regression and three bootstrap methods are similar. Also, the cross-section changes of systematic risk component, for the portfolios formed on mode 1 and mode 2 are similar.

For portfolios formed on *FUN* and *NUM*, the systematic risk component $\beta_{i,HMLN}$ increases monotonically from negative values for the smallest *FUN* and *NUM* quintiles to positive values for the largest quintiles. However, the risk component $\beta_{i,LMHD}$ assumes negative values for all quintiles.

For portfolios formed on *DEN*, the systematic risk component $\beta_{i,LMHD}$ decreases monotonically from positive values for the smallest *DEN* quintiles to negative values for the largest quintiles. The risk component $\beta_{i,HMLN}$ assumes positive values for all quintiles.

The schemes of return changes on portfolios formed on *FUN* and *DEN* (for the full-sample and for the portfolios of mode 1 type) are presented in Figure 1 and Figure 2.

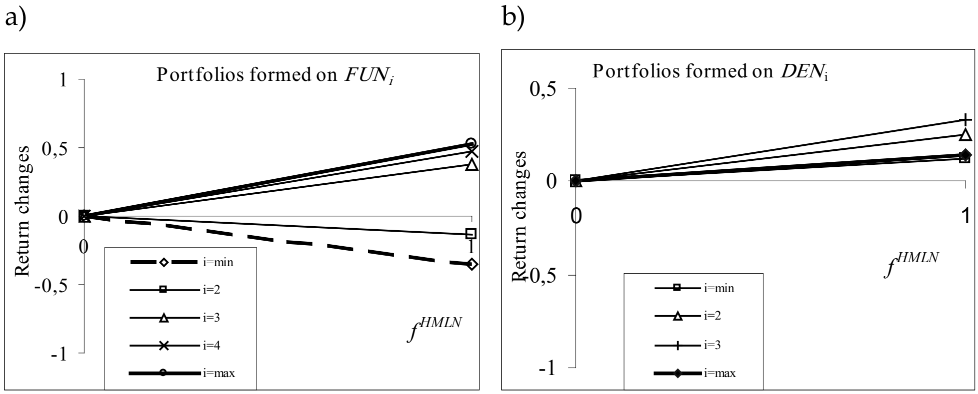

Figure 1. Influence of f^{HMLN} factor on returns of portfolios formed on *FUN* and *DEN*[a]

Figure 1 shows the influence of f^{HMLN} on returns of portfolios formed on *FUN* (Figure 1a) and *DEN* (Figure 1b). Portfolio for $i = 1$ is formed on minimal value of *FUN* or *DEN*. Portfolio for $i = 5$ is formed on maximal value of *FUN* or *DEN*. Negative–*BV* stocks are excluded from the portfolios. The sample period is from 1995 to 2010, 56 Quarters.

[12] Parameter values for the sub-periods, and for mode 2 are available on request.

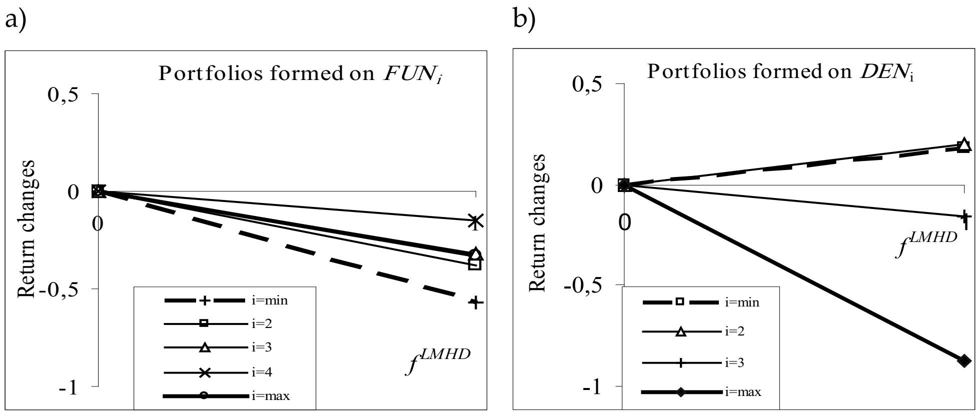

Figure 2. Influence of f^{LMHD} factor on returns of portfolios formed on *FUN* and *DEN* [a]

This figure shows the influence of f^{LMHD} on returns of portfolios formed on *FUN* (Figure 2a) and *DEN* (Figure 2b). Portfolio for $i=1$ is formed on minimal value of *FUN* or *DEN*. Portfolio for $i=5$ is formed on maximal value of *FUN* or *DEN*. Negative-*BV* stocks are excluded from the portfolios. The sample period is from 1995 to 2010, 56 Quarters.

The conducted research indicates that long investments in companies with large *FUN* or *NUM* values lead to higher returns for growing f^{HMLN} and decreasing f^{LMHD} values.

Long investments in companies with large *DEN* (low *BV/MV* and *E/MV*) demonstrate higher returns for growing f^{HMLN} and decreasing f^{LMHD} values. However, long investments in companies with small *DEN* values (high *BV/MV* and *E/MV*) demonstrate higher returns for growing f^{HMLN} and f^{LMHD} values. The values of the R^2 coefficient reach high values at 90%.

Cross-section changes of risk components $\beta_{i,HMLN}$ and $\beta_{i,LMHD}$ are similar for the whole sample and the first sub-period. Beta distributions in the second sub-period, for portfolios formed on *DEN*, are similar, while these changes for portfolios formed on *FUN* and *NUM* are more difficult to interpret.

The values of parameters of regression (16) are presenting in Table 2. Coefficients $\gamma_1 \equiv \gamma_{MO}$, $\gamma_2 \equiv \gamma_{HMLN}$ and $\gamma_3 \equiv \gamma_{LMHD}$ constitute systematic risk premium in terms of the factor connected with a market portfolio and the f^{HMLN} and f^{LMHD} factors.

The results for the whole sample are as follows: the risk premiums γ_{HMLN} and γ_{LMHD} estimated by three bootstrap methods are significantly higher than zero; these results for the portfolios formed on mode 1 and mode 2 are similar; however, if speculative stocks are not excluded from consideration, γ_{LMHD} estimated in "null" regression (also, in the second sub-period) is equal to zero.

The components γ_{HMLN} and γ_{LMHD} estimated by bootstrap are significantly higher than zero in the both sub-periods. The γ_{LMHD} component ranges from 4% in 1996–2005 to 8% in 2005–2010. However, γ_{HMLN} ranges from 6% in 1996–2005 to 2% in 2005–2010.

The component γ_{MO} estimated in “null” regression is insignificantly different from zero for all the tested periods. The corresponding *p*-values are higher than 0.19. However, the bootstrap estimations for the full sample and first sub-period indicate the significant positive results.

Table 2

The risk premium vector (γ) values estimated from the second-pass regression for the aggregated three-factor model

$$r_{it} - RF_t = \gamma_0 + \gamma_{HMLN}\hat{\beta}_{i,HMLN} + \gamma_{LMHD}\hat{\beta}_{i,LMHD} + \gamma_{MO}\hat{\beta}_{i,MO} + \varepsilon_{it}; i = 1,\dots,15; t = 1,\dots,T$$

Mode	Quantile bootstrap, θ^*			BCα bootstrap, θ^*		t-boot-strap	“null” regression	
	Para-meter	$\hat{\theta}^*_{2.5\%}$	$\hat{\theta}^*_{97.5\%}$	$\hat{\theta}^*_{2.5\%}$	$\hat{\theta}^*_{97.5\%}$	*p*-value [a]	$\hat{\theta}$	*p*-value [a]
The sample period is from 1995 to 2010, $T = 56$ Quarters								
1	$\hat{\gamma}_0$	**-0.12**	**-0.02**	**-0.09**	**-0.09**	**0.00**	-0.09	0.14
	$\hat{\gamma}_{HMLN}$	**0.03**	**0.06**	**0.03**	**0.08**	**0.00**	0.05	**0.02**
	$\hat{\gamma}_{MO}$	**0.01**	**0.11**	**0.05**	**0.18**	**0.00**	0.07	0.21
	$\hat{\gamma}_{LMHD}$	**0.01**	**0.06**	**0.02**	**0.08**	**0.00**	0.04	0.11
2	$\hat{\gamma}_0$	-0.15	0.01	-0.10	-0.12	**0.01**	-0.08	0.32
	$\hat{\gamma}_{HMLN}$	**0.03**	**0.08**	**0.04**	**0.10**	**0.00**	0.05	**0.01**
	$\hat{\gamma}_{MO}$	-0.04	0.14	**0.01**	**0.33**	*0.07*	0.06	0.48
	$\hat{\gamma}_{LMHD}$	0.03	**0.07**	**0.04**	**0.09**	**0.00**	0.05	**0.02**
The sample period is from 1995 to 2005, $T = 36$ Quarters								
1	$\hat{\gamma}_0$	**-0.14**	**-0.02**	**-0.11**	**-0.12**	**0.00**	-0.09	0.12
	$\hat{\gamma}_{HMLN}$	**0.03**	**0.08**	**0.04**	**0.11**	**0.00**	0.06	**0.03**
	$\hat{\gamma}_{MO}$	**0.01**	**0.12**	**0.04**	**0.20**	**0.00**	0.08	0.19
	$\hat{\gamma}_{LMHD}$	**0.01**	**0.06**	**0.02**	**0.09**	**0.00**	0.04	0.13

The sample period is from 2005 to 2010, $T = 20$ Quarters								
1	$\hat{\gamma}_0$	-0.06	0.04	-0.02	-0.03	0.50	-0.01	0.85
	$\hat{\gamma}_{HMLN}$	**0.00**	**0.03**	**0.01**	**0.00**	0.14	0.01	0.64
	$\hat{\gamma}_{MO}$	-0.05	0.09	0.00	-0.04	0.46	0.02	0.82
	$\hat{\gamma}_{LMHD}$	*-0.01*	*0.04*	**0.01**	**0.00**	0.15	0.02	0.60
2	$\hat{\gamma}_0$	-0.17	0.09	**-0.10**	**-0.19**	0.34	-0.06	0.67
	$\hat{\gamma}_{HMLN}$	*-0.01*	*0.05*	*0.01*	*-0.02*	**0.03**	0.02	0.42
	$\hat{\gamma}_{MO}$	-0.12	0.14	-0.05	0.18	0.63	0.02	0.87
	$\hat{\gamma}_{LMHD}$	**0.04**	**0.11**	**0.07**	**0.15**	**0.00**	0.08	**0.00**

Regression parameters for all bootstrap iterations and "null" regression are estimated by GLS. Portfolios for $i = 1$-5 are formed on *FUN*. Portfolios for $i = 6$-10 are formed on *NUM*. Portfolios for $i = 11$-15 are formed on *DEN*. $\hat{\theta}^*_{2.5\%}$ is the bootstrapped value of the estimator for the 2,5% level and, similarly, $\hat{\theta}^*_{97.5\%}$ is the bootstrapped value of the estimator for the 97,5% level. The bootstrap quantile is based on 10,000 data resamples. In mode 1 negative-*BV* stocks are excluded from the portfolios. In mode 2 speculative stocks are excluded from the portfolios. It is assumed that speculative stocks meet one of the following two conditions: 1) $MV/BV > 100$ and $r_{it} > 0$, 2) $ROE < 0$ and $MV/BV > 30$ and $r_{it} > 0$, where *MV* is the stock market value, *ROE* is the return on book value (*BV*), r_{it} is the return of portfolio *i* in period *t*. [a] Corresponds to the significance test for model parameters in the null hypotheses. **Bold type** — the parameter is significantly different from zero at the level of 5%. *Italic type* — the parameter is significantly different from zero at the level of 10%.

The value of γ_{MO} for the second sub-period is equal to zero also for bootstrap and "null" estimations pointing toward the decisive impact of risk in terms of the f^{HMLN} and f^{LMHD} factors on cross-section returns. This indicates that f^{MO} does not appear to be important factor in ICAPM confirming the previous studies (see, for example, Fama and French (1992), Jagannathan and Wang (1996), Lettau and Ludvigson (2001) and Pekova (2006)).

Table 3

The results of multifactor efficiency tests

$$r_{it} - RF_t = \alpha_i + \beta_{i,HMLN} f_t^{HMLN} + \beta_{i,LMHD} f_t^{LMHD} + \beta_{i,MO} f_t^{MO} + e_{it}; t = 1,\dots,T \text{ ; } \forall i = 1, \dots, 15$$

Mode	Quantile bootstrap, W^*			W		GRS	
	$\hat{W}^*_{5\%}$	$\hat{W}^*_{10\%}$	p-value (χ^2)	Statistic value	p-value (χ^2)	Statistic value	p-value (F)
Panel A: The sample period is from 1995 to 2010, $T = 56$ Quarters							
1	141.16	124.47	0.99	36.24	0.00	1.77	0.08
2	150.65	134.24	0.97	43.18	0.00	2.10	0.03
The sample period is from 1995 to 2010, $T = 36$ Quarters							
1	197.16	167.03	0.97	42.97	0.00	1.61	0.17
The sample period is from 2005 to 2010, $T = 20$ Quarters							
1	1140.63	787.38	0.65	166.53	0.00	1.39	0.50
2	2754.80	1825.96	0.99	57.43	0.00	0.48	0.84
Panel B: Chou and Zhou (2006), Fama-French's Factors							
Period: 1964–1993			0.03		<0.01		0.01
Panel C: Chou and Zhou (2006), CRSP index							
Period: 1926–1995			0.07		0.01		0.03
Period: 1986–1995			0.38		0.21		0.28

Panel A; $H_0 : \alpha_i = 0, \forall i = 1,\dots,n$. W is the statistic of Wald. GRS is the F-statistic of Gibbons et al (1989). In mode 1 negative-*BV* stocks are excluded from the portfolios. In mode 2 speculative stocks are excluded from the portfolios. It is assumed that speculative stocks meet one of the following two conditions: 1) $MV/BV > 100$ and $r_{it} > 0$, 2) $ROE < 0$ and $MV/BV > 30$ and $r_{it} > 0$, where *MV* is the stock market value, *ROE* is the return on book value (*BV*), r_{it} is the return of portfolio i in period t.

In Panel B the authors examine the joint efficiency of the Fama-French's factors in: $r_{it} - RF_t = \alpha_i + \beta_{i,HML} f_t^{HML} + \beta_{i,SMB} f_t^{SMB} + \beta_{i,MO}(RM_t - RF_t) + e_{it}$, where r_{it}'s are monthly returns on 25 Fama-French's portfolios and RM_t–RF_t is the excess return on a market index. In Panel C the authors examine the efficiency of the CRSP value-weighted index in the standard market model: $R_t = \alpha + \beta r_{pt} + e_t$, where R_t is a vector of returns on 10 CRSP size decile portfolios in excess of the 30-day T-bill rate. The bootstrap quantile is based on 10,000 data resamples.

ME is tested under the assumption that errors of the regression (15) are iid. Also, we test the normality of residuals.[13] We employ three efficiency tests, the GRS test, the asymptotic Wald test and bootstrap tests. The empirical results are reported in Table 3. Under iid assumption, the asymptotic Wald test reject ME of the tested portfolios for all the investigated periods at the significance level below 1%. The GLS test rejects ME for the whole sample for portfolios formed under assumption mode 1 at the 8% significance level, and under assumption mode 2 at the 4% significance level.

However, the bootstrapped Wald test, W^*, does not reject efficiency for investigated periods. We may conclude that the aggregated tree-factor model generates ME portfolios on the WSE when stock returns are assumed to come from iid models.

Moreover, we also compare our ME results to other procedures implemented on American market; see Chou and Zhou (2006). The results are presented in Panel B and C of Table 3. The p-values obtained suggest a strong rejection. It is, nevertheless, quite interesting to observe that the bootstrap derived p-values are greater than the non-boostrap ones.

5. CONCLUSIONS

The usage of bootstrap to test the ICPM application proposed by Urbański (2011) is presented for WSE stocks. The conducted research leads to the following conclusions:

1. The use of bootstrap procedures allows for an accurate assessment of return changes as compared with classical asymptotic methods.
2. Long investments in companies with large *FUN* or *NUM* demonstrate higher returns for growing f^{HMLN} and decreasing f^{LMHD} values.
3. Long investments in companies with large *DEN* (low *BV*/*MV* and *E*/*MV*) demonstrate higher returns for growing f^{HMLN} and decreasing f^{LMHD} values.
4. Long investments in companies with small *DEN* values (high *BV*/*MV* and *E*/*MV*) record higher returns for growing f^{HMLN} and f^{LMHD} values.
5. Estimates of systematic risk components for test cases using classical procedures and bootstrap methods are similar.
6. The cross-section changes of systematic risk component, for the portfolios formed on the basis of all analyzed stocks (Mode 1) and stocks with the exception the speculative stocks (Mode 2), are similar.
7. The risk premium components estimated by bootstrap are significantly different from zero in all tested cases.

[13] The Shapiro–Wilk tests confirm the residuals normality for the whole sample in 9 out of 15 tested portfolios.

8. If speculative stocks are not excluded from consideration, risk premium component, γ_{LMHD} estimated in ″null″ regression is insignificantly different from zero in all tested periods.
9. The risk premium γ_{HMLN} (determining the investor sensitivity to financial results) equals approx. 6% per quarter in the first sub-period and decreases to 1% in the second sub-period.
10. The risk premium γ_{LMHD} (determining the investor sensitivity to the value) equals approx. 4% per quarter in the first sub-period and grows in the second sub-period to 8%, after the elimination of speculative stocks.
11. GRS and asymptotic Wald tests reject ME of the most portfolios simulated by the tested ICAPM application. However, the bootstrapped Wald test does not reject efficiency for the tested cases.

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