

REDUCTION IN THE NUMBER OF COMPARISONS REQUIRED TO CREATE MATRIX OF EXPERT JUDGMENT IN THE COMET METHOD

Wojciech Sałabun

Department of Artificial Intelligence Methods and Applied Mathematics, Faculty of Computer Science and Technology Information, West Pomeranian University of Technology, Szczecin, Poland

Corresponding author:

Wojciech Sałabun

West Pomeranian University of Technology

Faculty of Computer Science and Technology Information

żołnierska 49, 71-210 Szczecin, Poland

phone: (+48) 77 423 40 31

e-mail: wsalabun@wi.zut.edu.pl

Received: 7 June 2014

Accepted: 17 July 2014

ABSTRACT

Multi-criteria decision-making (MCDM) methods are associated with the ranking of alternatives based on expert judgments made using a number of criteria. In the MCDM field, the distance-based approach is one popular method for receiving a final ranking. One of the newest MCDM method, which uses the distance-based approach, is the Characteristic Objects Method (COMET). In this method, the preferences of each alternative are obtained on the basis of the distance from the nearest characteristic objects and their values. For this purpose, the domain and fuzzy numbers set for all the considered criteria are determined. The characteristic objects are obtained as the combination of the crisp values of all the fuzzy numbers. The preference values of all the characteristic object are determined based on the tournament method and the principle of indifference. Finally, the fuzzy model is constructed and is used to calculate preference values of the alternatives. In this way, a multi-criteria model is created and it is free of rank reversal phenomenon. In this approach, the matrix of expert judgment is necessary to create. For this purpose, an expert has to compare all the characteristic objects with each other. The number of necessary comparisons depends squarely to the number of objects. This study proposes the improvement of the COMET method by using the transitivity of pairwise comparisons. Three numerical examples are used to illustrate the efficiency of the proposed improvement with respect to results from the original approach. The proposed improvement reduces significantly the number of necessary comparisons to create the matrix of expert judgment.

KEYWORDS

MCDM, COMET, decision making, decision support systems, multiple-criterion optimization.

Introduction

Multi-criteria decision-making (MCDM) is a very popular branch of decision analysis (DA). The main purpose of MCDM is to support decision makers in facing multi-criteria problems. However, the list of commonly used MCDM methods is not long. Available methods include Weighted Sum Model (WSM) [1–6], Weighted Product Model (WPM) [7–9], Elimination and Choice Expressing Reality (ELECTRE), Analytic Hierarchy Process (AHP) [15–23], Analytic

Network Process (ANP) [18, 19, 21, 24–28], Technique for Order of Preference by Similarity to Ideal Solution (TOPSIS) [29–35] and Preference Ranking Organization Method for Enrichment of Evaluation (PROMETHE) [36–39], all of which have been developed with numerous improvements. Unfortunately, these methods and their improvements are susceptible to rank reversals phenomenon. The issue of rank reversals is usually caused by the addition or deletion of an alternative. Five major types of rank reversals can currently be identified.

This paper describes the new approach called the Characteristic Objects method (COMET), which is completely free of the rank reversal phenomenon, and takes into account correlations between all criteria. The final ranking is constructed based on characteristic objects and fuzzy rules.

A simple improvement of this approach is proposed using rules of transitivity in pairwise comparisons. Thanks to this improvement, the number of comparisons required to create a matrix of expert judgments is reduced.

The paper is organized as following. First, fundamental notions and concepts of the fuzzy sets that are necessary to understand the COMET method are presented. Then, the five steps of the COMET algorithm are shown, after which the proposed improvement is described, and three numerical examples are used to illustrate the efficiency of the modifications with respect to the results from the original approach. Finally, the conclusion summarizes the main findings of the paper.

Fuzzy Sets

The development of fuzzy sets theory was initiated by Lofti Zadeh, who presented the idea and first conception of fuzzy sets in his seminal paper "Fuzzy Sets" [40]. Today fuzzy set theory is a very important approach to control and modeling in a various scientific fields. Modeling using fuzzy sets has proven to be an effective way for formulating multi-criteria decision problems [41]. The basic definitions of notions and concepts of fuzzy sets are presented as following definitions:

Definition 1. Fuzzy set and membership function.

The characteristic function μ_A of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X inasmuch as crisp sets only allow full membership ($\mu_A(x) = 1$) or non-membership at all ($\mu_A(x) = 0$). This function can be generalized to a function $\mu_{\tilde{A}}$ such that the value assigned; to the element of the universal set X fall within a specified range, i.e., $\mu_{\tilde{A}} : X \rightarrow [0, 1]$. The assigned value indicates the membership grade of the element in the set A . The function $\mu_{\tilde{A}}$ is called the membership function and the set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x))\}$, where $x \in X$, defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set [42–44].

Definition 2. Triangular fuzzy number (TFN).

A fuzzy set \tilde{A} , defined on the universal set of real numbers \mathfrak{R} , is said to be a triangular fuzzy number

$\tilde{A}(a, m, b)$ if its membership function has the following form (1) [45]:

$$\mu_{\tilde{A}}(x, a, m, b) = \begin{cases} 0, & x \leq a \\ \frac{x-a}{m-a}, & a \leq x \leq m \\ 1, & x = m \\ \frac{b-x}{b-m}, & m \leq x \leq b \\ 0, & x \geq a \end{cases} \quad (1)$$

and the following characteristics (2, 3):

$$x_1, x_2 \in [a, b] \wedge x_2 > x_1 \Rightarrow \mu_{\tilde{A}}(x_2) > \mu_{\tilde{A}}(x_1), \quad (2)$$

$$x_1, x_2 \in [b, c] \wedge x_2 > x_1 \Rightarrow \mu_{\tilde{A}}(x_2) < \mu_{\tilde{A}}(x_1). \quad (3)$$

Definition 3. The support of a TFN \tilde{A} .

This is the crisp subset of the set \tilde{A} whose all elements have non-zero membership values in the set \tilde{A} (4):

$$S(\tilde{A}) = \{x : \mu_{\tilde{A}}(x) > 0\} = [a, b]. \quad (4)$$

Definition 4. The core of a TFN \tilde{A} .

This is the singleton (one-element fuzzy set) with the membership value equal to one (5):

$$C(\tilde{A}) = \{x : \mu_{\tilde{A}}(x) = 1\} = m. \quad (5)$$

Definition 5. The fuzzy rule.

The single fuzzy rule can be based on tautology Modus Ponens [43, 46]. The reasoning process uses logical connectives *IF-THEN*, *OR* and *AND*.

Definition 6. The rule base.

The rule base consists of logical rules determining causal relationships existing in the system between fuzzy sets of its inputs and output [47].

Definition 7. T-norm operator: product.

The t-norm operator is a function T modeling the intersection operation AND of two or more fuzzy numbers, e.g. \tilde{A} and \tilde{B} . In this paper, only product is used as t-norm operator [43, 46, 47]:

$$\mu_{\tilde{A}}(x) \text{ and } \mu_{\tilde{B}}(y) = \mu_{\tilde{A}}(x) \cdot \mu_{\tilde{B}}(y). \quad (6)$$

The characteristic objects method

The COMET is a very intuitionistic approach, but to be able to understand this, the basic knowledge on the Fuzzy Sets is necessary [48, 49].

Formal notation of this method can be presented using the following five steps:

Step 1: Define the space of the problem as follows:

- determine dimensionality of the problem by selecting number r of criteria, C_1, C_2, \dots, C_r ,
 - select the set of triangular fuzzy numbers for each criterion \tilde{C}_i , i.e., $\tilde{C}_{i1}, \tilde{C}_{i2}, \dots, \tilde{C}_{ic_i}$.
- In this way, the following result is obtained (7):

$$\begin{aligned} C_1 &= \{\tilde{C}_{11}, \tilde{C}_{12}, \dots, \tilde{C}_{1c_1}\} \\ C_2 &= \{\tilde{C}_{21}, \tilde{C}_{22}, \dots, \tilde{C}_{2c_2}\} \\ &\dots\dots\dots \\ C_r &= \{\tilde{C}_{r1}, \tilde{C}_{r2}, \dots, \tilde{C}_{rc_r}\} \end{aligned} \quad (7)$$

where c_1, c_2, \dots, c_r are numbers of the fuzzy numbers for all criteria.

Step 2: Generate the characteristic objects.

The characteristic objects (CO) are obtained by using the Cartesian Product of triangular fuzzy numbers cores for each criteria as follows (8):

$$CO = C(C_1) \times C(C_2) \times \dots \times C(C_r). \quad (8)$$

As the result of this, the ordered set of all CO is obtained (9):

$$\begin{aligned} CO_1 &= \{C(\tilde{C}_{11}), C(\tilde{C}_{21}), \dots, C(\tilde{C}_{r1})\} \\ CO_2 &= \{C(\tilde{C}_{11}), C(\tilde{C}_{21}), \dots, C(\tilde{C}_{r2})\} \\ &\dots\dots\dots \\ CO_t &= \{C(\tilde{C}_{1c_1}), C(\tilde{C}_{2c_2}), \dots, C(\tilde{C}_{rc_r})\} \end{aligned} \quad (9)$$

where t is a number of CO (10):

$$t = \prod_{i=1}^r c_i. \quad (10)$$

Step 3: Rank and evaluate the characteristic objects.

Determine the Matrix of Expert Judgment (MEJ). This is a result of the comparison of the characteristic objects by the knowledge of an expert. The MEJ structure is as follows (11):

$$MEJ = \begin{pmatrix} \alpha_{11} & \alpha_{12} & \dots & \alpha_{1t} \\ \alpha_{21} & \alpha_{22} & \dots & \alpha_{2t} \\ \dots & \dots & \dots & \dots \\ \alpha_{t1} & \alpha_{t2} & \dots & \alpha_{tt} \end{pmatrix} \begin{matrix} CO_1 \\ CO_2 \\ \dots \\ CO_t \end{matrix} \quad (11)$$

where α_{ij} is a result of comparing CO_i and CO_j by the expert. The more preferred characteristic object gets one point and the second object get null point. If the preferences are balanced, the both objects get half point. It depends solely on the knowledge and opinion of the expert and can be presented as (12):

$$\alpha_{ij} = f(CO_i, CO_j)$$

$$= \begin{cases} 0.0, & f_{\text{exp}}(CO_i) < f_{\text{exp}}(CO_j) \\ 0.5, & f_{\text{exp}}(CO_i) = f_{\text{exp}}(CO_j) \\ 1.0, & f_{\text{exp}}(CO_i) > f_{\text{exp}}(CO_j) \end{cases} \quad (12)$$

where f_{exp} is an expert judgment function. The most important properties are described by the formulas (12) and (13):

$$\alpha_{ii} = f(CO_i, CO_i) = 0.5, \quad (13)$$

$$\alpha_{ji} = 1 - \alpha_{ij}. \quad (14)$$

On the basis of formulas (12) and (13), the number of comparisons is reduced from t^2 cases to p cases (14):

$$p = \binom{t}{2} = \frac{t(t-1)}{2}. \quad (15)$$

Afterwards, we obtain a vertical vector of the summed Judgments (SJ) as follows (15):

$$SJ_i = \sum_{j=1}^t \alpha_{ij}. \quad (16)$$

The last step assigns to each characteristic object the approximate value of preference. In the result, we obtain a vertical vector P , where i -th row contains the approximate value of preference for CO_i . This algorithm is presented as a fragment of Matlab code:

```

1: k = length(unique(SJ));
2: P = zeros(t,1);
3: for i = 1:k
4:     ind = find(SJ == max(SJ));
5:     P(ind) = (k - i) / (k - 1);
6:     SJ(ind) = 0;
7: end
    
```

In line 1, we obtain number k as a number of unique value of the vector SJ. In line 2, we create vertical vector P of zeros (with identical size as vector SJ). In line 4, we obtain index with maximum value from vector SJ. This index is used to assign the value of preference to adequate position in vector P (based on the principle of indifference of Laplace'a). In line 6, the maximum value of the vector SJ is reset.

Step 4: The rule base.

Each one characteristic object and value of preference is converted to a fuzzy rule as follows, general form (17) and detailed form (18):

$$\text{if } CO_i \text{ then } P_i, \quad (17)$$

$$\text{if } C(\tilde{C}_{1i}) \text{ and } C(\tilde{C}_{2i}) \text{ and } \dots \text{ then } P_i. \quad (18)$$

In this way, the complete fuzzy rule base is obtained, which can be presented as (19):

$$\begin{aligned} &\text{if } CO_1 \text{ then } P_1 \\ &\text{if } CO_2 \text{ then } P_2 \\ &\dots\dots\dots \\ &\text{if } CO_t \text{ then } P_t \end{aligned} \quad (19)$$

Step 5: Inference in a fuzzy model and final ranking.

The each one alternative is a set of crisp number, which corresponding with criteria C_1, C_2, \dots, C_r . It can be presented as follows (20):

$$A_i = \{a_{1i}, a_{2i}, \dots, a_{ri}\}, \tag{20}$$

where condition (21) must be satisfied

$$\begin{aligned} a_{1i} &\in [C(\tilde{C}_{11}), C(\tilde{C}_{1c_1})] \\ a_{2i} &\in [C(\tilde{C}_{21}), C(\tilde{C}_{2c_2})] \\ &\dots\dots\dots \\ a_{ri} &\in [C(\tilde{C}_{r1}), C(\tilde{C}_{rc_r})] \end{aligned} \tag{21}$$

Each one alternative activates the specified number of fuzzy rules, where for each one is determined the fulfillment degree of the conjunctive complex premise. Fulfillment degrees of all activated rules sum to one. The preference of alternative is computed as sum of the product of all activated rules, as their fulfillment degrees, and their values of the preference. The final ranking of alternatives is obtained by sorting the preference of alternatives.

Proposed improvement

The COMET method is completely free of the rank reversal phenomenon. This is one of the most important attributes of the new method. It has been achieved through the use of characteristic objects, which divide the space of the problem on several smaller subspaces. In each subspace, preferences of alternatives are obtained by using independent characteristic objects and their values of preferences. For this purpose, the matrix of expert judgment (MEJ) must be determined. A required number of comparisons grows squarely with the number of characteristic objects.

In this paper, the author proposes using the rules of transitivity to reduce the number of comparisons required to create MEJ.

Between two objects, there are three basic preferences relations. If decision-maker prefers x over y, it will be written as: $x > y$ (for the inverse relationship: $x < y$), and for two equal preferences, it will be written as: $x = y$. The rules of transitivity are as follows (22) (for three objects A, B and C) [50]:

$$\begin{aligned} \text{if } A > B \text{ and } B > C \text{ then } A > C \\ \text{if } A > B \text{ and } B = C \text{ then } A > C \\ \text{if } A = B \text{ and } B > C \text{ then } A > C \\ \text{if } A = B \text{ and } B < C \text{ then } A < C \\ \text{if } A < B \text{ and } B < C \text{ then } A < C \\ \text{if } A < B \text{ and } B = C \text{ then } A < C \end{aligned} \tag{22}$$

The Eq. (22) presents only six of the nine possibilities. In the remaining three cases, an expert must explicitly specify the preference between objects in

the third pair. These rules can be written using the notation from Eq. (11) as follows:

$$\begin{aligned} \text{if } \alpha_{ij} > \alpha_{jk} \text{ and } \alpha_{jk} > \alpha_{ik} \text{ then } \alpha_{ij} > \alpha_{ik} \\ \text{if } \alpha_{ij} > \alpha_{jk} \text{ and } \alpha_{jk} = \alpha_{ik} \text{ then } \alpha_{ij} > \alpha_{ik} \\ \text{if } \alpha_{ij} = \alpha_{jk} \text{ and } \alpha_{jk} > \alpha_{ik} \text{ then } \alpha_{ij} > \alpha_{ik} \\ \text{if } \alpha_{ij} = \alpha_{jk} \text{ and } \alpha_{jk} < \alpha_{ik} \text{ then } \alpha_{ij} < \alpha_{ik} \\ \text{if } \alpha_{ij} < \alpha_{jk} \text{ and } \alpha_{jk} < \alpha_{ik} \text{ then } \alpha_{ij} < \alpha_{ik} \\ \text{if } \alpha_{ij} < \alpha_{jk} \text{ and } \alpha_{jk} = \alpha_{ik} \text{ then } \alpha_{ij} < \alpha_{ik} \end{aligned} \tag{23}$$

The proposed approach assumes filling the MEJ starting from the main diagonal. Then, the each column is filled in order from main diagonal to the first row, and if it is possible, the rules (23) should be used. In the next section, will be presented three numerical examples of this approach to check the efficiency of the proposed improvement.

Experiment and results

The experiment with three simple examples will be presented to check the efficiency of proposed reduction. The experiment assumes, that expert functions must be known, and each function consists only of two criteria. Additionally, the number of TFNs will be equal for both criteria, and TFNs will have uniform distribution.

The first example concerns the function (24) with two monotonic criteria, which are called x and y . The Fig. 1 presents the surface of this function

$$f_1(x) = 0.49x^2 + 0.51y^2. \tag{24}$$

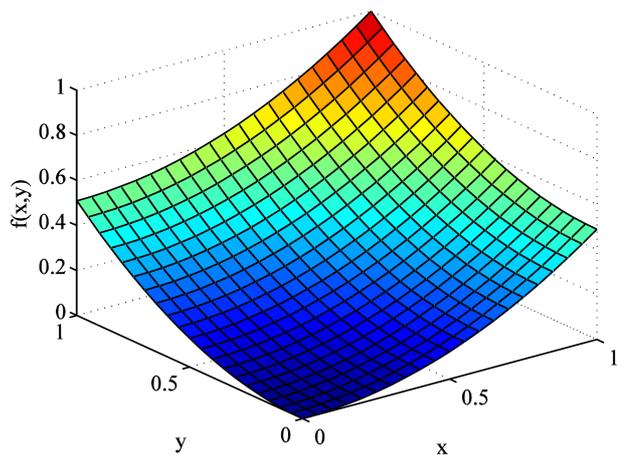


Fig. 1. The surface of the multi-criteria decision-making function (f_1) with two monotonic criteria (x and y).

The second example concerns the function (25) with two nonmonotonic criteria, which are called x and y . The Fig. 2 presents the shape of this function

$$f_2(x) = 1 - 0.5(x - 0.5)^2 - 0.5(y - 0.5)^2. \tag{25}$$

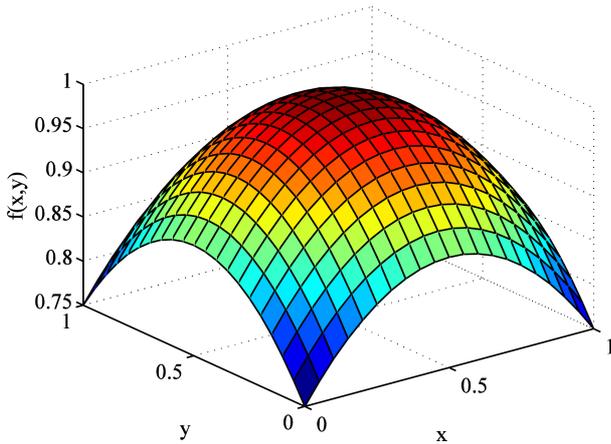


Fig. 2. The surface of the multi-criteria decision-making function (f_2) with two nonmonotonic criteria (x and y).

The third example concerns the function (26) with one nonmonotonic criterion (x) and one monotonic criterion (y). The Fig. 3 presents the shape of this function

$$f_3(x) = 1 - 0.45(\sin(x) - 0.5)^2 - 0.55(\cos(y) - 0.5)^2 \tag{26}$$

These functions (24–26) have been chosen in such a way to analyze the most common types of MCDM problems with two criteria. Each MEJ matrix is obtained with the reduction of comparisons and without.

All calculations were performed in MATLAB. The special script was written to count the number of comparisons in each particular case. Investigation involves 9, 16, 25, 36, 49, 64, 81, and 100 characteris-

tic objects for comparison the both approaches. The complete comparison of results of this investigation is presented in Table 1.

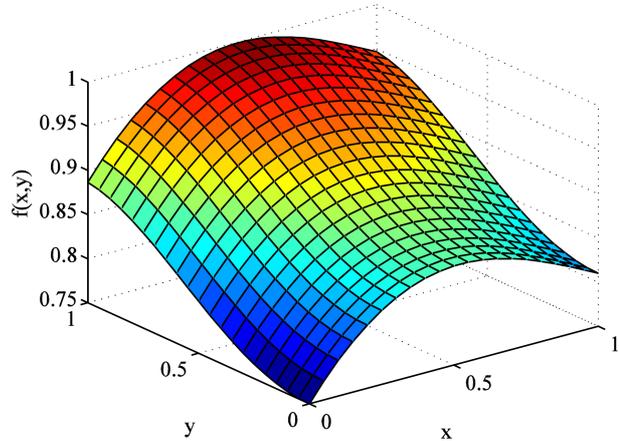


Fig. 3. The surface of the multi-criteria decision-making function (f_3) with one non-monotonic criterion (x) and one monotonic criterion (y).

The most significant is fact, that the all obtained MEJ matrices have the same structure in each considered example. This means that the proposed approach returns the same result, but with a reduced number of comparisons required to create MEJ matrix. The minimal percent of the reduction has been achieved for nine characteristic objects, and it was 36.11%. In the rest cases, the number of comparisons is reduced by at least 41.67%. If the number of characteristic objects increases, then the efficiency of reduction also increases. In the best case, the number of comparisons is reduced by 66.67%.

Table 1
The comparison of results on the reduction efficiency for three investigated examples.

Number of characteristic objects		9	16	25	36	49	64	81	100
Number of comparisons		36	120	300	630	1176	2016	3240	4950
Ex. #1	Number of comparisons after reduction	21	57	125	242	423	692	1074	1595
	Reduced number of comparisons	15	63	175	388	753	1324	2166	3355
	Percent of reduction [%]	41.67	52.50	58.33	61.59	64.03	65.67	66.85	67.78
Ex. #2	Number of comparisons after reduction	23	58	160	280	605	929	1650	2266
	Reduced number of comparisons	13	62	140	350	571	1087	1590	2684
	Percent of reduction [%]	36.11	51.67	46.67	55.56	48.55	53.92	49.07	54.22
Ex. #3	Number of comparisons after reduction	23	70	163	331	593	994	1566	2364
	Reduced number of comparisons	13	50	137	299	583	1022	1674	2586
	Percent of reduction [%]	36.11	41.67	45.67	47.46	49.57	50.69	51.67	52.24

Conclusions

A new approach to reduce in the number of comparisons required to create MEJ matrix in the COMET method is proposed. It has been developed on the basis of transitivity rules in the pairwise comparison. In this paper, the simple procedure to reduce the number of comparisons is presented and tested on three numerical examples. This improvement reduces the number of comparisons at least 36.11%. This result is achieved by using 9 characteristic objects, but if the number of characteristic objects increases, then the number of required comparisons decreases. In that way, the efficiency of reduction can rise up to 66.67%. This reduction does not change the structure of MEJ matrix. It means that the results are the same with reduced workload. However, this is true, but only for reliable sources of data, e.g., mathematical functions or experts.

This research work was co-financed by the European Commission in the frame of the European Social Fund, by the European Social Fund and the State Budget according to the Operational Programme Human Capital, Priority VIII, Action 8.2, Under-action 8.2.2: 'Regional Innovative Strategy' within the system project of the Regional Labour Office in Szczecin 'Investment in knowledge as a driving force for development of innovation in the region – third edition'.

References

- [1] Afshari A., Mojahed M., Yusuff R., *Simple Additive Weighting approach to Personnel Selection problem*, International Journal of Innovation, Management and Technology, 1, 5, 511–515, 2010.
- [2] Salih Y., See O., Ibrahim R., Yussof S., Iqbal A., *A Novel Noncooperative Game Competing Model Using Generalized Simple Additive Weighting Method to Perform Network Selection in Heterogeneous Wireless Networks*, International Journal of Communication Systems, first published online, 3 FEB 2014.
- [3] Huang Y.S., Chang W.C., Li W.H., Lin Z.L., *Aggregation of utility-based individual preferences for group decision-making*, European Journal of Operational Research, 229, 2, 462–469, 2013.
- [4] French S., *Decision behavior, analysis and support*, Cambridge, New York, 2013.
- [5] Kontos T.D., Komilis D.P., Halvadakis C.P., *Siting MSW landfills with a spatial multiple criteria analysis methodology*, Waste Management, 25, 8, 818–832, 2005.
- [6] Simanaviciene R., Ustinovichius L., *Sensitivity analysis for multiple criteria decision making methods: TOPSIS and SAW*, Procedia – Social and Behavioral Sciences, 2, 6, 7743–7744, 2010.
- [7] Zavadskas E.K., Turskis Z., Antucheviciene J., Zarkarevicius A., *Optimization of Weighted Aggregated Sum Product Assessment*, Electronics & Electrical Engineering, 122, 3–6, 2012.
- [8] Triantaphyllou E., *Multi-Criteria Decision Making Methods*, Springer, New York, 2000.
- [9] Savitha K., Chandrasekar C., *Vertical Handover decision schemes using SAW and WPM for Network selection in Heterogeneous Wireless Networks*, Global Journal of Computer Science and Technology, 11, 9, 19–24, 2011.
- [10] Brito A.J., de Almeida A.T., Mota C.M., *A multicriteria model for risk sorting of natural gas pipelines based on ELECTRE TRI integrating utility theory*, European Journal of Operational Research, 200, 3, 812–821, 2010.
- [11] Figueira J., Greco S., Ehrgott M., *Multiple Criteria Decision Analysis: State of the Art Surveys*, Springer, New York, 2004.
- [12] Hatami-Marbini A., Tavana M., *An extension of the ELECTRE I method for group decision-making under a fuzzy environment*, Omega, 39, 4, 373–386, 2011.
- [13] Montazer G.A., Saremi H.Q., Ramezani M., *Design a new mixed expert decision aiding system using fuzzy ELECTRE III method for vendor selection*, Expert Systems with Applications, 36, 8, 10, 837–10, 847, 2009.
- [14] Mousseau V., Dias L., *Valued outranking relations in ELECTRE providing manageable disaggregation procedures*, European Journal of Operational Research, 156, 2, 467–482, 2004.
- [15] Blair A.R., Mandelker G.N., Saaty T.L., Whitaker R., *Forecasting the resurgence of the u.s. economy in 2010: An expert judgment approach*, Socio-Economic Planning Sciences, 44, 3, . 114–121, 2010.
- [16] Dong Y., Zhang G., Hong W.C., Xu Y., *Consensus models for AHP group decision making under row geometric mean prioritization method*, Decision Support Systems, 49, 3, 281–289, 2010.
- [17] Karami E., *Appropriateness of farmers adoption of irrigation methods: The application of the AHP model*, Agricultural Systems, 87, 1, 101–119, 2006.
- [18] Saaty T.L., *Decision making the analytic hierarchy and network processes (AHP/ANP)*, Journal of Systems Science and Systems Engineering, 13, 1, 1–35, 2004.

- [19] Saaty T.L., *Time dependent decision-making; dynamic priorities in the AHP/ANP: Generalizing from points to functions and from real to complex variables*, Mathematical and Computer Modelling, 46, 78, 860–891, 2007.
- [20] Saaty T.L., *Decision making the analytic hierarchy and network processes (AHP/ANP)*, International Journal Services Sciences, 1, 1, 83–98, 2008.
- [21] Saaty T.L., Brandy C., *The encyclicon, volume 2: a dictionary of complex decisions using the analytic network process*, RWS Publications, Pittsburgh, 2009.
- [22] Saaty T.L., Shang J.S., *An innovative orders-of-magnitude approach to AHP-based multi-criteria decision making: Prioritizing divergent intangible humane acts*, European Journal of Operational Research, 214, 3, 703–715, 2011.
- [23] Saaty T.L., Tran L.T., *On the invalidity of fuzzifying numerical judgments in the analytic hierarchy process*, Mathematical and Computer Modelling, 46, 78, 962–975, 2007.
- [24] Hsu T.H., Hung L.C., Tang J.W., *A hybrid ANP evaluation model for electronic service quality*, Applied Soft Computing, 12, 1, 72–81, 2012.
- [25] Kirytopoulos K., Voulgaridou D., Platis A., Leopoulos V., *An effective markov based approach for calculating the limit matrix in the analytic network process*, European Journal of Operational Research, 214, 1, 85–90, 2011.
- [26] Niemira M.P., Saaty T.L., *An analytic network process model for financial-crisis forecasting*, International Journal of Forecasting, 20, 4, 573–587, 2004.
- [27] Wang Y.L., Tzeng G.H., *Brand marketing for creating brand value based on a MCDM model combining DEMATEL with ANP and VIKOR methods*, Expert Systems with Applications, 39, 5, 5600–5615, 2012.
- [28] Yang J.L., Tzeng G.H., *An integrated MCDM technique combined with DEMATEL for a novel cluster-weighted with ANP method*, Expert Systems with Applications, 38, 3, 1410–1424, 2011.
- [29] Kwanyoung I., Hyunbo C., *A systematic approach for developing a new business model using morphological analysis and integrated fuzzy approach*, Expert Systems with Applications, 40, 11, 4463–4477, 2013.
- [30] Kuo R.J., Wu Y.H., Hsu T.S., *Integration of fuzzy set theory and TOPSIS into HFMEA to improve outpatient service for elderly patients in Taiwan*, Journal of the Chinese Medical Association, 75, 7, 341–348, 2012.
- [31] La Scalia G., Aiello G., Rastellini C., Micale R., Cicalese L., *Multi-criteria decision making support system for pancreatic islet transplantation*, Expert Systems with Applications, 38, 4, 3091–3097, 2011.
- [32] Taleizadeh A.A., Akhavan Niaki S.T., Aryanezhad M.B., *A hybrid method of Pareto, TOPSIS and genetic algorithm to optimize multi-product multiconstraint inventory control systems with random fuzzy replenishments*, Mathematical and Computer Modeling, 49, 5–6, 1044–1057, 2009.
- [33] Kim Y., Chung E.S., Jun S.M., Kim S.U., *Prioritizing the best sites for treated wastewater instream use in an urban watershed using fuzzy TOPSIS*, Resources Conservation and Recycling, 73, 23–32, 2013.
- [34] Sun Y.F., Liang Z.S., Shan C.J., Viernstein H., Unger F., *Comprehensive evaluation of natural antioxidants and antioxidant potentials in Ziziphus jujuba Mill. var. spinosa (Bunge) Huex H. F. Chou fruits based on geographical origin by TOPSIS method*, Food Chemistry, 124, 4, 1612–1619, 2011.
- [35] Salabun W., *The mean error estimation of TOPSIS method using a fuzzy reference models*, Journal of Theoretical and Applied Computer Science, 7, 3, 40–50, 2013.
- [36] Eppe S., De Smet Y., *Approximating Promethee II's net flow scores by piecewise linear value functions*, European Journal of Operational Research, 233, 3, 651–659, 2014.
- [37] Amaral T.M., Costa A.P.C., *Improving decision-making and management of hospital resources: An application of the PROMETHEE II method in an Emergency Department*, Operations Research for Health Care, 3, 1, 1–6, 2014.
- [38] Makan A., Mountadar M., *Sustainable management of municipal solid waste in Morocco: Application of PROMETHEE method for choosing the optimal management scheme*, African Journal of Environmental and Waste Management, 1, 1, 1–13, 2013.
- [39] Ziolkowska J.R., *Evaluating sustainability of biofuels feedstocks: A multi-objective framework for supporting decision making*, Biomass and Bioenergy, 55, 425–440, 2013.
- [40] Zadeh L.A., *Fuzzy sets*, Information and Control, 8, 3, 338–353, 1965.
- [41] Zimmermann H.J., *Fuzzy Set Theory – and Its Applications*, Kluwer, Boston, 2001.
- [42] Kaufmann A., Gupta M.M., *Fuzzy mathematical models in engineering and management science*, Elsevier Science Publishers, Amsterdam, Netherlands, 1988.

- [43] Piegat A., *Fuzzy Modeling and Control*, Springer-Verlag, New York, 2001.
- [44] Kumar A., Singh P., Kaur A., Kaur P., *RM approach for ranking of generalized trapezoidal Fuzzy numbers*, *Fuzzy Information and Engineering*, 2, 1, 37–47, 2010.
- [45] Pedrycz W., Ekel P., Parreiras R., *Fuzzy Multicriteria Decision-making: Models, Methods and Applications*, John Wiley & Sons, Chichester, 2011.
- [46] Wang G., Wang H., *Non-Fuzzy versions of Fuzzy reasoning in classical logics*, *Information Sciences*, 138, 1–4, 211–236, 2001.
- [47] Ross T.J., *Fuzzy Logic with Engineering Applications*, John Wiley & Sons, Chichester, 2010.
- [48] Piegat A., Sałabun W., *Nonlinearity of human multicriteria in decision-making*, *Journal of Theoretical and Applied Computer Science*, 6, 3, 3–49, 2012.
- [49] Sałabun W., *The use of Fuzzy logic to evaluate the nonlinearity of human multi-criteria used in decision making*, *Przegląd Elektrotechniczny*, 88, 10b, 235–238, 2012.
- [50] Xu Y., Gupta J.N.D., Wang H., *The ordinal consistency of an incomplete reciprocal preference relation*, *Fuzzy Sets and Systems*, 246, 62–77, 2014.