

## APPLICATION OF BIAS RANDOMIZATION IN EVALUATION OF MEASURING INSTRUMENT CAPABILITY

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### Abstract

The paper deals with the problem of bias randomization in evaluation of the measuring instrument capability. The bias plays a significant role in assessment of the measuring instrument quality. Because the measurement uncertainty is a comfortable parameter for evaluation in metrology, the bias may be treated as a component of the uncertainty associated with the measuring instrument. The basic method for calculation of the uncertainty in modern metrology is propagation of distributions. Any component of the uncertainty budget should be expressed as a distribution. Usually, in the case of a systematic effect being a bias, the rectangular distribution is assumed. In the paper an alternative randomization method using the Flatten-Gaussian distribution is proposed.

Keywords: measurement uncertainty, bias randomization, measuring instrument capability.

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### 1. Introduction

The paper [1] presents a method for randomizing the bias of a measuring instrument. The method is based on the Flatten-Gaussian distribution [2]. Usually, the bias of a measuring instrument is treated as a systematic effect, although is a part of coverage interval [3]. In metrology the bias is defined as estimate of a systematic measurement error [4]. In many cases the known systematic effect is not corrected, but instead is treated as an uncertainty contribution. This approach is presented in the international standard concerning the capability of measurement processes [5]. The measurement uncertainty, as a parameter, is used to assessment of the measuring instrument quality. This quality depends on a relation between the measurement uncertainty and a reference. Usually, a maximum permissible error associated with the measuring instrument is used as a reference.

### 2. Capability of measuring instrument

The capability ratio of a measuring instrument is defined as:

$$Q_{MS} = \frac{U_{MS}}{E_{max}} \cdot 100\%, \quad (1)$$

where:  $U_{MS}$  is the expanded uncertainty expressed usually for the coverage probability of 95%, associated with the measuring instrument, and  $E_{max}$  is the maximum permissible error acceptable for that instrument. The combined standard uncertainty associated with the measuring instrument consists of several components:

$$u_c^2 = u_{rep}^2 + u_{res}^2 + u_{bias}^2 + u_{cal}^2 + u_{temp}^2, \quad (2)$$

where:  $u_{\text{rep}}$  – the standard uncertainty associated with repeatability of measurement using a reference standard;  $u_{\text{res}}$  – the standard uncertainty associated with the resolution of measuring instrument;  $u_{\text{bias}}$  – the standard uncertainty arising from the bias;  $u_{\text{cal}}$  – the standard uncertainty associated with calibration of the measurement standard;  $u_{\text{temp}}$  – the standard uncertainty associated with temperature differences.

The first component is directly associated with the measuring instrument and refers to dispersion of indications on the reference standard carried out under repeatable conditions. The standard uncertainty of this component is an experimental standard deviation of a single indication  $q_i$  obtained from a series of  $n$  observations:

$$u_{\text{rep}} = s(q) = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (q_i - \bar{q})^2}. \tag{3}$$

It is recommended that the minimum number of indications is a series of  $n = 30$  observations [5]. This component is associated with the normal distribution.

The second component is the resolution of measuring instrument. The standard uncertainty associated with the resolution  $R$  is based on the rectangular distribution:

$$u_{\text{res}} = \frac{R}{2\sqrt{3}}. \tag{4}$$

The third component is the bias, regarded as the difference between the average of a series of observations and the reference value  $q_s$ :

$$B = |\bar{q} - q_s|. \tag{5}$$

In this case the reference value is represented by the measurement standard. The bias  $B$  is treated as a component of the uncertainty [5]. The bias is randomized by the rectangular distribution giving the standard uncertainty:

$$u_{\text{bias}} = \frac{B}{\sqrt{3}}. \tag{6}$$

The next component of uncertainty is associated with calibration of the measurement standard. In the calibration certificate there is given the expanded uncertainty  $U$  and the coverage factor  $k$  for the confidence level of approximately 95%. Therefore, the standard uncertainty is:

$$u_{\text{cal}} = \frac{U}{k}. \tag{7}$$

Usually, the factor  $k = 2$ , when we assume the normal distribution.

The last component is associated with an impact of environmental conditions on the value represented by the reference standard. In general, it is the effect of temperature. In this case changes of the standard value will be defined by the following relationship:

$$\Delta L = \Delta t \cdot \alpha \cdot L, \tag{8}$$

where:  $\Delta t$  is the variation of temperature during testing of the measuring instrument,  $\alpha$  is the coefficient of thermal expansion of the standard value  $L$ . The standard uncertainty is [5]:

$$u_{\text{temp}} = \frac{\Delta L}{\sqrt{3}}. \tag{9}$$

To this component the rectangular distribution is attributed.

Based on the above assumption, we can select five components of uncertainty associated with the measurand, and we can form a measurement equation defining the output quantity  $y$  as the sum of five input quantities  $x_i$ :

$$y = x_{\text{rep}} + x_{\text{res}} + x_{\text{bias}} + x_{\text{cal}} + x_{\text{temp}}, \tag{10}$$

where:  $x_{\text{rep}}$  – the input quantity associated with repeatability of measurement using the reference standard, having the normal distribution;  $x_{\text{res}}$  – the input quantity associated with the resolution of measuring instrument, having the rectangular distribution;  $x_{\text{bias}}$  – the input quantity associated with the bias, having the rectangular distribution;  $x_{\text{cal}}$  – the input quantity associated with calibration of the measurement standard, having the normal distribution;  $x_{\text{temp}}$  – the input quantity associated with temperature differences, having the rectangular distribution.

The input quantity represents the set of possible values having determined probability distributions. We can use propagation of distributions through the measurement model to calculate the output quantity, as in the method recommended in [6]. The measurement model is represented by (10). The set of possible values associated with the output quantity can be calculated using the Monte Carlo procedure. This procedure creates a numerical distribution function of the measurand, which covers the coverage interval for the coverage probability of 95%, with low  $y_{\text{low}}$  and high  $y_{\text{high}}$  endpoints. The expanded uncertainty is a half-width of this interval:

$$U_{\text{MS}} = \frac{y_{\text{high}} - y_{\text{low}}}{2}. \tag{11}$$

### 3. Randomization of measuring instrument bias

The bias of measuring instrument is a systematic effect, estimated by (5). Two quantities define the bias: the average of measuring instrument indications on the reference standard, and the value of reference standard [5]. Two components of uncertainty are associated with the bias. One component is associated with the average, and is expressed by the experimental standard deviation of the mean [3]:

$$s(\bar{q}) = \frac{s(q)}{\sqrt{n}} = \sqrt{\frac{\sum_{i=1}^n (q_i - \bar{q})^2}{n(n-1)}}. \tag{12}$$

The second component of the bias is associated with calibration of the measurement standard, and is expressed by (7). Then, the combined uncertainty attributed to the bias is:

$$u(B) = \sqrt{s^2(\bar{q}) + u_{\text{cal}}^2}. \tag{13}$$

We can create a new random variable containing the bias  $B$  and its uncertainty  $u(B)$ , as in Fig. 1. This random variable may be characterized by the Flatten-Gaussian distribution [2]. The Flatten-Gaussian distribution has a parameter  $r$  approximated by the following formula [1]:

$$r = \frac{2 \cdot B}{3 \cdot u(B)} + 1, \tag{14}$$

and may be generated, using the Monte Carlo procedure, by the following equation:

$$z_{\text{RN}} = \frac{r z_{\text{R}} + z_{\text{N}}}{\sqrt{r^2 + 1}}, \tag{15}$$

where  $z_R$  is a random variable having the rectangular distribution, and  $z_N$  is a random variable having the normal distribution, because the Flatten-Gaussian distribution is a convolution of rectangular and normal distributions with the parameter  $r$ , as the quotient of its standard deviations [2].

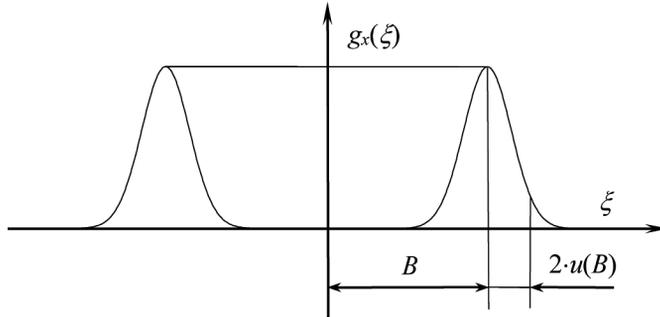


Fig. 1. Randomizing of the bias:  $g_x(\xi)$  is the probability density function with the variable  $\xi$  for the input quantity  $x_{rand}$ .

When we use the above randomization, the equation of measurand defined by (10) is reduced to four input quantities:

$$y = x_{rep} + x_{res} + x_{rand} + x_{temp}, \tag{16}$$

where the input quantity  $x_{rand}$  has the Flatten-Gaussian distribution.

#### 4. Practical example

The above considerations can be used to evaluate the capability of a typical measuring instrument which is, for example, a micrometer. This capability is assessed using a gauge block as the measurement standard. The measurement standard has a calibration certificate stating that the length of gauge block is 20,0002 mm, which was determined with the expanded uncertainty 0,1  $\mu\text{m}$ , for the coverage probability of approximately 95%. The 30 micrometer readings on the gauge block are indicated (Table 1).

Table 1. Indications of the micrometer on the gauge block.

20,001 mm	20,001 mm	20,001 mm
20,001 mm	20,000 mm	20,001 mm
20,001 mm	20,001 mm	20,000 mm
20,002 mm	20,001 mm	20,001 mm
20,001 mm	20,001 mm	20,001 mm
20,001 mm	20,001 mm	20,001 mm
20,001 mm	20,001 mm	20,002 mm
20,000 mm	20,001 mm	20,001 mm
20,001 mm	20,002 mm	20,001 mm
20,001 mm	20,001 mm	20,001 mm
$\bar{l} = 20,001 \text{ mm}$ $s(l) = 0,00045 \text{ mm}$		

The first component is the scattering of micrometer indications on the gauge block. The standard uncertainty associated with this component is:

$$u_{rep} = s(l) = 0,45 \mu\text{m}. \tag{17}$$

The second component under consideration is the resolution  $R = 1 \mu\text{m}$  of micrometer display. The standard uncertainty associated with this component is:

$$u_{\text{res}} = \frac{R}{2\sqrt{3}} = 0,29 \mu\text{m}. \quad (18)$$

The third component is the bias. The estimate of a gauge block length measured by the micrometer, as the mean indication, is  $20,001 \text{ mm}$ , but the length of gauge block, based on the calibration certificate, is  $l_s = 20,0002 \text{ mm}$ . Then, the bias is:

$$B = |\bar{l} - l_s| = 0,8 \mu\text{m}, \quad (19)$$

and the standard uncertainty associated with the bias is:

$$u_{\text{bias}} = \frac{B}{\sqrt{3}} = 0,46 \mu\text{m}. \quad (20)$$

The fourth component under consideration is the uncertainty associated with the gauge block length. The calibration certificate of measurement standard states that the expanded uncertainty  $U = 0,1 \mu\text{m}$ , as the standard uncertainty multiplied by the coverage factor  $k = 2$  is such that the coverage probability corresponds to approximately 95%. Then the standard uncertainty is:

$$u_{\text{cal}} = \frac{U}{k} = 0,05 \mu\text{m}. \quad (21)$$

The fifth component is related to the temperature effect on the measurement standard. The thermal coefficient of expansion of steel alloy, which the gauge block is made of, is  $\alpha = 12 \cdot 10^{-6} \text{ } 1^\circ\text{C}$ . The measurement was carried out at the temperature changing within the range  $\Delta t = \pm 1^\circ\text{C}$ . The limit of changing the gauge block dimension is:

$$\Delta L = \Delta t \cdot \alpha \cdot L = 0,24 \mu\text{m}. \quad (22)$$

Then, the standard uncertainty associated with this component is:

$$u_{\text{temp}} = \frac{\Delta L}{\sqrt{3}} = 0,14 \mu\text{m}. \quad (23)$$

We can formulate a measurement model defining the measurand  $l$  associated with the measurement of gauge block length by the micrometer:

$$l = \bar{l} + \delta l_{\text{rep}} + \delta l_{\text{res}} + \delta l_{\text{bias}} + \delta l_{\text{cal}} + \delta l_{\text{temp}}, \quad (24)$$

where:  $\bar{l}$  is the average of micrometer indications on the measurement standard, and  $\delta l_i$  are the influence quantities corresponding with above components. The uncertainty budget associated with the output quantity is presented in Table 2. The measurement model enables calculation using the Monte Carlo method. We can use the procedure recommended by [6]. Selecting the number of Monte Carlo trail  $M = 10^4$ , we calculate  $M$  times (24) by drawing every time the numerical value from each distribution attributed to the input quantities. The set of  $M$  numerical values attributed to the output quantity are sorted in increasing order. The sorted values have the probability from  $1/M$  to one. The set of output quantities (measurand) creates a discrete representation of the distribution function, as in Fig. 2. The output quantity values having the probabilities 2,5% and 97,5% define the endpoints of 95% coverage interval. Half of the coverage interval length is the expanded uncertainty.

The endpoints of coverage interval, calculated with the above procedure, are  $l_{\text{low}} = 19,9996 \text{ mm}$  and  $l_{\text{high}} = 20,0024 \text{ mm}$ , what gives the expanded uncertainty value

$U_{MS} = 1,4 \mu\text{m}$ . The standard uncertainty associated with measurand is  $u(l) = 0,72 \mu\text{m}$ . Assuming the maximum permissible error  $E_{\text{max}} = \pm 5 \mu\text{m}$ , we obtain the micrometer measurement capability ratio  $Q_{MS} = 28\%$ .

Table 2. The uncertainty budget of the measurand defined by (24).

Quantity	Estimate	Probability distribution	Standard uncertainty
$\delta l_{\text{rep}}$	0 mm	normal	0,00045 mm
$\delta l_{\text{res}}$	0 mm	rectangular	0,00029 mm
$\delta l_{\text{bias}}$	0 mm	rectangular	0,00046 mm
$\delta l_{\text{cal}}$	0 mm	normal	0,00005 mm
$\delta l_{\text{temp}}$	0 mm	rectangular	0,00014 mm
$l$	20,001 mm		0,00072 mm

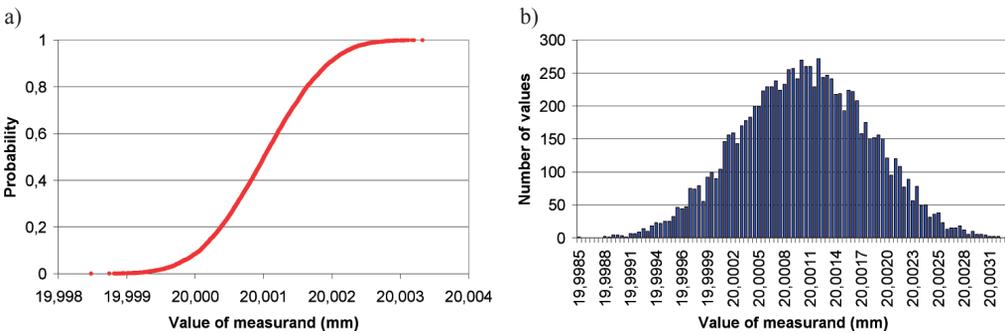


Fig. 2. The distribution function a), and the histogram b), for the measurand calculated with (24).

Using the method of randomization of the measuring instrument bias, we can form the following equation for the measurand:

$$l = \bar{l} + \delta l_{\text{rep}} + \delta l_{\text{res}} + \delta l_{\text{rand}} + \delta l_{\text{temp}} \tag{25}$$

The number of input quantities is reduced by one, and is summarized in Table 3. The quantity  $\delta l_{\text{rand}}$  randomized by the Flatten-Gaussian distribution has parameters:  $u(B) = 0,1 \mu\text{m}$ , and  $r = 6,5$ , calculated with (13) and (14), respectively. Then, the standard uncertainty associated with this quantity calculated by the Monte Carlo method with the formula (15) is  $u(\delta l_{\text{rand}}) = 0,6 \mu\text{m}$ .

Table 3. The uncertainty budget of the measurand defined by (25).

Quantity	Estimate	Probability distribution	Standard uncertainty
$\delta l_{\text{rep}}$	0 mm	normal	0,00045 mm
$\delta l_{\text{res}}$	0 mm	rectangular	0,00029 mm
$\delta l_{\text{rand}}$	0 mm	Flatten-Gaussian	0,00060 mm
$\delta l_{\text{temp}}$	0 mm	rectangular	0,00014 mm
$l$	20,001 mm		0,00082 mm

The endpoints of coverage interval, calculated for (25) with the Monte Carlo method are as follows:  $l_{\text{low}} = 19,9994 \text{ mm}$  and  $l_{\text{high}} = 20,0026 \text{ mm}$ , what gives the expanded uncertainty value

$U_{MS} = 1,6 \mu\text{m}$  (the standard uncertainty associated with the measurand  $u(l) = 0,82 \mu\text{m}$ ). This value is higher than that calculated with the equation (24), giving the value of the measuring instrument capability ratio  $Q_{MS} = 32\%$ . The distribution function and the histogram for the measurand defined by (25) are presented in Fig. 3.

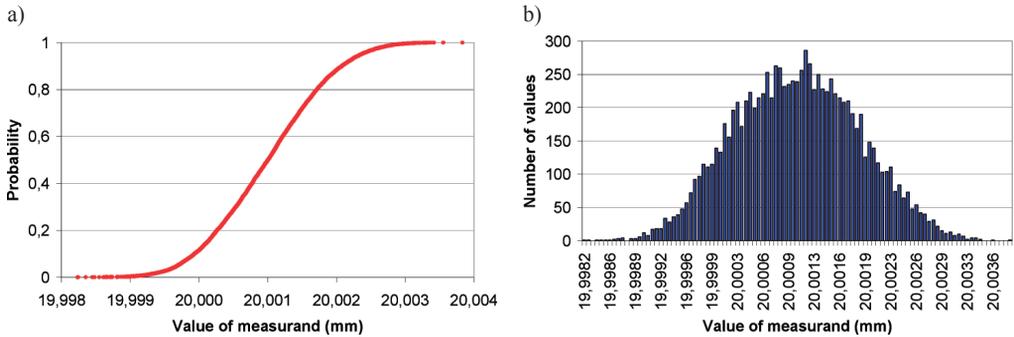


Fig. 3. The distribution function a), and the histogram b), for the measurand calculated with (25).

The above presented procedure may be used for testing the capability of any measuring instrument. For example, we can apply this method also for evaluation of the manometer measurement capability using a pressure gauge tester. The typical manometer has the measuring range of up to 25 MPa. The pressure gauge tester, as the reference standard, forces  $p = 20 \text{ MPa}$ . We can use the deadweight piston gauge with the accuracy class 0,05. The average of manometer readings is 19,9 MPa, with the resolution 0,1 MPa, as one tenth of the scale interval. Then, we can form the following measuring equation, as the measurand:

$$p = \bar{p} + \delta p_{\text{rep}} + \delta p_{\text{res}} + \delta p_{\text{rand}} \tag{26}$$

summarizes all components in Table 4, as the uncertainty budget.

Table 4. The uncertainty budget of the measurand defined by (26).

Quantity	Estimate	Probability distribution	Standard uncertainty
$\delta p_{\text{rep}}$	0 MPa	normal	0,045 MPa
$\delta p_{\text{res}}$	0 MPa	rectangular	0,029 MPa
$\delta p_{\text{rand}}$	0 MPa	Flatten-Gaussian	0,072 MPa
$p$	19,9 MPa		0,090 MPa

The (26) has only three components, because the temperature does not affect the pressure forced by the deadweight piston gauge. The bias of pressure is  $B = 0,1 \text{ MPa}$  with the uncertainty  $u(B) = 0,01 \text{ MPa}$ , calculated with (5) and (13). The standard uncertainty  $u_{\text{cal}}$ , in (13) is determined on the basis of the accuracy class of deadweight piston gauge, with:

$$u_{\text{cal}} = \frac{0,05 \cdot p}{100 \cdot \sqrt{3}} \tag{27}$$

Using the Monte Carlo procedure we can calculate the endpoints of coverage interval  $p_{\text{low}} = 19,73 \text{ MPa}$  and  $p_{\text{high}} = 20,07 \text{ MPa}$ , what gives the expanded uncertainty value  $U_{MS} = 0,17 \text{ MPa}$ . If we assume that the maximum permissible error  $E_{\text{max}} = \pm 0,5 \text{ MPa}$ , as half of the scale interval of manometer, then the measuring instrument capability ratio is  $Q_{MS} = 34\%$ .

## 5. Conclusions

The measurement uncertainty can be a convenient parameter for metrological evaluation of the measuring instrument quality. Analysis of the uncertainty should take into account all metrological influences associated with a measurement, such as a randomized bias. The bias may be randomized by the rectangular distribution, but the Flatten-Gaussian distribution contains the uncertainty components associated with the bias value. Then, the capability of measuring instrument is greater, but more reliable.

Calculation of the measurement uncertainty is used in assessment of conformity. The common rule of conformity assessment is to define the maximum permissible uncertainty MPU [7]. The MPU may be called also TUR (*test uncertainty ratio*), which usually has to be not greater than 1/3 of the MPE (*maximum permissible error*) [8]. This relation between MPU and MPE is commonly acceptable in trade [9] and industry [10], and usually is treated as the criterion of legal metrology [11]. The calculation of measuring instrument uncertainty performed above may be used to satisfy this criterion. Using the Monte Carlo method makes credible the acceptance criteria in the conformity assessment [12].

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