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DYNAMIC NON – AXIS – SYMMETRICAL SUM ABOUT THE TORSION OF THE ELASTIC HALF-SPACE WITH THE PUNCH

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Summary. In the work the sum about the joint oscillation of the elastic isotropic half-space and rigid while stretching (compression) of the punch of an arbitrary shape in the plan to which the rotational moment changing according to the harmonic law in time is applied. The asymptotic formulas for defining contact shearing stresses under the punch, the angle of lagging and module of complex amplitude of the punch oscillation.

Keywords: elastic half-space, integral transformations, frequency of oscillations, contact shearing stresses, complex amplitude of oscillations.

INTRODUCTION

Nowadays the mechanics of contact interactions of solid deformable bodies represents a big and actively developing branch of mechanics of continuums. Static contact sums are quite well researched. A big problem is created by the solutions of dynamic contact sums which have a scientific and practical value.

The main publications on the given problem are given in the works [Galin 1980, Vorovich, Alexandrov, Babeshko 1974, Vorovich, Babeshko 1979, Seymov 1976, Novatskiy 1970, 1975, Kilchevskiy 1976, Cherepanov 1974, Alexandrov, Kovalenko 1986, Goryacheva, Dobychin 1988, Alexandrov, Pozharskiy 1998, Alexandrov, Chebakov 2005, Grinchenko, Meleshko 1981] which contain the review of main scientific results dedicated to the solution of contact static dynamic and thermoelastic sums for elastic and viscoelastic bodies. Mathematical methods of solution flat and spatial sums while different boundary conditions on the contact squares are set out. The main correlations of mechanics of continuums and theory of elasticity are given.

OBJECT AND PROBLEMS

The aim of the given work is the research of dynamic non-axis-symmetrical sum about the torsion of the elastic half-space (fig. 1) with the punch and determination of contact shearing stresses under the punch the angle of lagging and module of complex amplitude of the oscillation punch. Henceforth for shortness speaking about stresses transferences lagging their amplitude values are meant. True values are received with the multiplication by the multiplier e^{iwt} . As far as it is known to the authors the similar sum wasn't earlier considered.

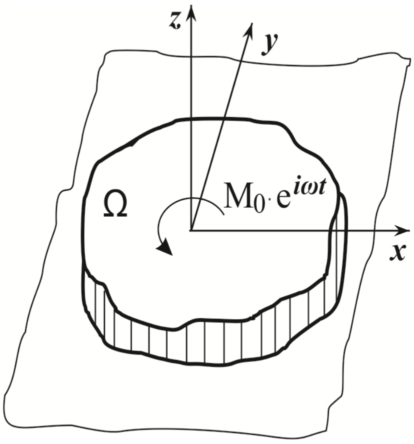

Fig. 1. The loading diagram

1. Постановка задачи. Putting the sum. From a mathematical point of view the sum comes to the solution of Lame's [Mushelishvili 1966] equation while the absence of body forces with the boundary conditions

$$\left.\begin{aligned} u &= f_1(x,y,o) \\ v &= f_2(x,y,o) \end{aligned}\right\} \quad (x,y)\in\Omega, \tag{1}$$

$$\sigma_z(x,y,o)=0,\ \ \tau_{xz}(x,y,o)=\tau_{yz}(x,y,o)=0,\quad (x,y)\notin\Omega.$$

Here u and v are elastic transferences on the axes x and y, and $\sigma_z,\tau_{xz},\tau_{yz}$ tension on the square with the normal z.

The use of the principle of saturable absorption [Vorovich, Babeshko 1979, Tihonov, Samarskiy 1972, Starchenko 2005, Starchenko, Buryak 2005] and twofold Fourier transform [Uflyand 1968], the given mixed sum will be led to the system of two twofold integral equations of the first type

$$\iint_\Omega \tau_1(\xi,\eta)K_{11}(x-\xi,y-\eta)d\xi d\eta+\iint_\Omega \tau_2(\xi,\eta)K_{12}(x-\xi,y-\eta)d\xi d\eta= \\ =4\pi^2\mu\, f_1(x,y),\quad (x,y)\in\Omega, \tag{2}$$

$$\iint_\Omega \tau_1(\xi,\eta)K_{12}(x-\xi,y-\eta)d\xi d\eta+\iint_\Omega \tau_2(\xi,\eta)K_{22}(x-\xi,y-\eta)d\xi d\eta= \\ =4\pi^2\mu\, f_2(x,y),\quad (x,y)\in\Omega,$$

Here: $\tau_1(x,y)=\tau_{xz}(x,y)=\tau_{11}(x,y)+i\tau_{12}(x,y),\ \tau_2(x,y)=\tau_{yz}(x,y)=$ $=\tau_{21}(x,y)+i\tau_{22}(x,y)$ – is shearing stresses in the area of the contact,

$$K_{11}(p,s)=\int\int_{-\infty}^{\infty}F_1(\beta,\gamma,k)[F(\gamma,k)]^{-1}e^{i(\alpha p+\beta s)}d\alpha d\beta,$$

$$K_{12}(p,s)=\int\int_{-\infty}^{\infty}F_2(\alpha,\beta,\gamma,k)[F(\gamma,k)]^{-1}e^{i(\alpha p+\beta s)}d\alpha d\beta,$$

$$K_{22}(p,s)=\int\int_{-\infty}^{\infty}F_1(\alpha,\gamma,k)[F(\gamma,k)]^{-1}e^{i(\alpha p+\beta s)}d\alpha d\beta,$$

$$F_1(\beta,\gamma,k) = -4\beta^2\gamma^2 + \left(3\beta^2+\gamma^2-k^2\right)k^2 + 4\beta^2\sqrt{\gamma^2-k^2}\sqrt{\gamma^2-b_0^2k^2},$$

$$F_2(\alpha,\beta,\gamma,k) = \alpha\beta\left(4\gamma^2-3k^2-4\sqrt{\gamma^2-k^2}\sqrt{\gamma^2-b_0^2k^2}\right),$$

$$F(\gamma,k) = \sqrt{\gamma^2-k^2}\left[4\gamma^2\sqrt{\gamma^2-k^2}\sqrt{\gamma^2-b_0^2k^2}-\left(2\gamma^2-k^2\right)^2\right],$$

$$F_1(\alpha,\gamma,k) = F_1(\beta,\gamma,k)\big|_{\beta=\alpha},\ \gamma^2=\alpha^2+\beta^2,$$

$$k^2 = p\omega^2\mu^{-1}(1-i\varepsilon),\ b_0^2 = (1-2v/2(1-v)).$$

ρ, μ – is the density and module of lagging of elastic half-space;
ε – is the coefficient of proportionality which characterizes the internal friction;
v – is Puassona coefficient.

For big values of the parameter $|k|$ and bandpass area of the contact the system of equations (2) to the members of the order of values $\frac{1}{|k|^2}$ is disintegrated into two independent equations:

$$\int_{-a}^{a} d\xi \int_{-\infty}^{\infty} \tau_j(\xi,\eta)d\eta \int\int_{-\infty}^{\infty} \frac{1}{\sqrt{\tau^2-k^2}} e^{i[\alpha(x-\xi)+\beta(y-\eta)]} d\alpha d\beta = 4\pi^2\mu f_j(x,y), \tag{3}$$

$$(j=1,\ 2).$$

Taking into consideration that $f_1(x,y) = -\theta y,\ f_1(x,y) = \theta x$ and searching for the solution of the equations (3) accordingly in the form of

$$\tau_1(x,y) = -y\tau_1^*(x), \quad \tau_2(x,y) = \tau_2^*(x), \tag{4}$$

with the regard of the equalities

$$\frac{1}{2\pi}\int\int_{-\infty}^{\infty} \eta e^{i\beta(y-\eta)} d\eta d\beta = y, \quad \frac{1}{2\pi}\int\int_{-\infty}^{\infty} e^{i\beta(y-\eta)} d\eta d\beta = 1.$$

understood in the sense of the theory of general functions [Vladimirov 1976] we will persuade that $\tau_1^*(x)$ and $\tau_2^*(x)$ must be found from one-dimensional integral equations of the first type which in dimensionless variables will have the look.

$$\int_{-1}^{1} \tau_j(\xi) k_\varepsilon[\chi(x-\xi)] d\xi = \pi\Delta f_j^*(x), \quad (|x| \le 1,\ j=1,2). \tag{5}$$

$$k_\varepsilon[\chi(x-\xi)] = \int_0^\infty \frac{\cos[\chi(x-\xi)]m|dm}{\sqrt{m^2-(1-i\varepsilon)^2}}. \tag{6}$$

Here: $\Delta = \mu a^{-1}$, $2a$ – is the width of the plus punch,
$f_1^*(x) = 0,\ f_2^*(x) = \theta x,\ \theta = \theta_i + i\theta_2$ – is the amplitude of the angle of the turn of the punch, $\chi = \omega a(\rho/\mu)^{1/2}$ – is the relative frequency of the oscillations.

Using the method of work [Nobl 1962], we'll get the main member of asymptotics of solution of equations (5) for big χ. With the regard to indications (4) while $\varepsilon \to 0$ we'll have

$$\tau_1(x,y) = -\Delta\theta\chi i\left[e^{-i\chi(1+x)} \Big/ \sqrt{i\pi\chi(1+x)} + erf\sqrt{i\chi(1+x)} + \right.$$
$$\left. + e^{-i\chi(1-x)} \Big/ \sqrt{i\pi\chi(1-x)} + erf\sqrt{i\chi(1-x)} - 1\right]y, \tag{7}$$

$$\tau_2(x,y) = \Delta\theta\chi i\left[\left(x - \frac{1}{2}i\chi\right)e^{-1\chi(1+x)} \Big/ \sqrt{i\pi\chi(1+x)} + \right.$$

$$+ x\,erf\sqrt{i\chi(1+x)} + \left(1 + \frac{1}{2}i\chi\right)e^{-i\chi(1-x)} \Big/ \sqrt{i\pi\chi(1-x)} +$$

$$\left. + x\,erf\sqrt{i\chi(1-x)} - x - \right], \qquad \left(erf\; x = \frac{2}{\sqrt{\pi}}\int_0^x e^{-s^2} ds\right).$$

Further we'll define the reactive moment which acts on the punch from the side of the half-space referred to the unit of length

$$M_z^{/} = \frac{1}{2b}\int_{-1}^{1} dx \int_{-b}^{b} \left[x\tau_2(x,y) - y\tau_1(x,y)\right]dy = M_1^{`} + iM_2^{`}. \tag{8}$$

Substituting (7) and (8) we'll get

$$M_z^{/} = \Delta\theta\left[(1+4xi)erf\sqrt{2xi} + \sqrt{2xi}e^{-2xi} \Big/ \sqrt{\pi} - 2xi\right]\Big/3, \tag{9}$$
$$\left(M_z = M_z^{`}\big/b^2, \quad M_z = M_1 + iM_2\right).$$

In the formulas (8) and (9) it's known that the punch isn't endlessly long but has the final but quite big length.

2. We'll get the formulas for counting the angle of lagging φ and module of complex amplitude of the oscillation of the punch θ_0. We'll write down the equation of the rotating movement of the punch relative to the ax z.

$$J_z \cdot \frac{d^2}{dt^2}\left(\theta e^{i\omega t}\right) = M_0 e^{i\omega t} - M_z e^{i\omega t}, \tag{10}$$

where:

$$JmM_0 = 0, \; J_z = J_z^{/`}\big/b^2, \; M_0 = M_0^{`}\big/b^2,$$

$J_z^{`}$ – is the moment of the inertia of the punch relative to the ax z.

Having done the differentiation in (10) and the division of real and imaginary parts we'll get taking into consideration (9)

$$M_0 = \theta_1\left(A_{11} - J_z\omega^2\right) + A_{12}\theta_2, \quad 0 = \theta_1 A_{21} + \theta_2\left(A_{22} - J_z\omega^2\right). \tag{11}$$

Solving the system (11) relative to θ_1 and θ_2, we'll find

$$tg(-\varphi) = \theta_2\big/\theta_1 = A_{21}^*\left(\chi^2 J_z^* - A_{22}^*\right)^{-1},$$
$$\theta_0^* = \left[\left(\chi^2 J_z^* - A_{11}^*\right)^2 + \left(A_{12}^*\right)2\right]^{-1/2}. \tag{12}$$

Here:

$$J_z^* = J_z(a\rho)^{-1},\ A_{nj}^* = A_{nj}/\Delta,\ (n,\ j = 1,2),$$
$$M_1 = (A_{11}\theta_1 + A_{12}\theta_2)\Delta,\ M_2 = (A_{21}\theta + A_{22}\theta_2)\Delta, \tag{13}$$
$$A_{22} = A_{11},\quad A_{21} = -A_{12},\quad \theta_0^* = \theta_0\Delta/M_0,\ \theta_0 = \left(\theta_1^2 + \theta_2^2\right)^{1/2}.$$

The results of the calculation of values done according to the formulas (12), (13) are given in table 1.

Table1. **«The results of the calculation of the module of complex amplitude of the oscillation of the punch and the angle of lagging»**

$\chi \setminus J_z^*$	θ_0^*			$\varphi(pa\partial)$		
	5	10	20	5	10	20
0,125	3,785	4,476	4,623	0,890	1,165	1,891
0,250	3,635	2,421	1,028	1,555	2,413	2,855
0,375	2,074	0,907	0,403	2,378	2,835	3,007
0,500	1,043	0,463	0,215	2,722	2,960	3,058
0,625	0,612	0,282	0,134	2,862	3,014	3,081
0,750	0,403	0,190	0,092	2,933	3,044	0,094
1,000	0,214	0,104	0,051	3,002	3,074	3,109
1,250	0,133	0,065	0,032	3,034	3,089	3,116
1,500	0,091	0,045	0,022	3,053	3,098	3,120
2,000	0,051	0,025	0,012	3,075	3,109	3,125

From table 1 the dependences of values φ and θ_0^* on the non-dimensional frequency χ while different values of the non-dimensional moment of the inertia J_z^*. can be seen.

It's seen that for values $\chi \geq 0{,}25$ the module of complex amplitude decreases with the increase χ and J_z^* but the angle of lagging φ increases with the increase χ and J_z^* that quiet corresponds to the physical meaning of the sum.

CONCLUSIONS

The strict conclusion of integral equations with taking the principle of the limited absorption into consideration is received. The asymptotic formulas for defining contact shearing stresses in dependence on the amplitude of the angle of the turn of the punch which can be used for specified calculations on the durability and rigidity in transport and general machine-building are given.

REFERENCES

1. Alexandrov V., Chebakov M, 2005.: The introduction into the mechanics of contact interactions. Rostov-on-Don: Publishing house OOO “TsVVR”, 108 pages.
2. Alexandrov V., Kovalenko Ye., 1986.: Sums of mechanics of continuums with mixed boundary conditions. M.: Nauka, 335 pages.
3. Alexandrov V., Pozharskiy D., 1998.: Non-classical spatial sums of mechanics of contact interactions of elastic bodies. M.: Faktorial, 288 pages.
4. Cherepanov G., 1974.: The mechanics of fragile destruction. M.: Nauka, 640 pages.
5. Galin L., 1980.: Contact sums of the theory of elasticity and viscoelasticity. M.: Nauka, 304 pages.
6. Goryacheva I., Dobychin M, 1988.: Contact sums in tribology. M.: Mashinostroyeniye, 254 pages.
7. Grinchenko V., Meleshko V., 1981.: Harmonic oscillations and waves in elastic bodies. Kiev: Naukova dumka, 284 pages.
8. Kilchevskiy N., 1976.: Dynamic contact compression of solid bodies. Hit. Kiev: Naukova dumka, 319 pages.
9. Mushelishvili N., 1966.: Some main sums of the mathematical theory of the elasticity. M.: Nauka, 708pages.
10. Nobl B., 1962.: Winer-Hopf’s method. M.: IL, 280 pages.
11. Novatskiy V.,1970.: Dynamic sums of the thermoelasticity. M.: Mir, 256 pages.
12. Novatskiy V., 1975.: The theory of elasticity. M.: Mir, 872 pages.
13. Seymov V., 1976.: Dynamic contact sums. Kiev: Naukova dumka, 284 pages.
14. Starchenko V., 2005.: Spatial dynamic contact sum for the elastic half-space. Dniprppetrovsk: National Miner’s University, p. 21-28.
15. Starchenko V., Buryak V., 2005.: Spatial dynamic mixed sum about the shear of elastic half-space. The bulletin of Eastern-Ukrainian National University named after V.Dal. № 6(88), p. 51-56.
16. Tihonov A., Samarskiy A., 1972.: The equations of mathematical physics. M.: Nauka, 567 pages.
17. Uflyand Ya., 1968.: Integral transformations in the sums of the theory of the elasticity. L.: Nauka, 403pages.
18. Vladimirov V., 1976.: Generalized functions in mathematical physics. M.: Nauka, 280 pages.
19. Vorovich I., Alexandrov A., Babeshko V., 1974.: Non-classical mixed sums of the theory of the elasticity. M.: Nauka, 456 pages.
20. Vorovich I., Babeshko V., 1979.: Dynamic mixed sums of the theory of elasticity for non-classical areas. M.: Nauka, 320 pages.

ДИНАМИЧЕСКАЯ НЕОСЕСИМЕТРИЧНАЯ ЗАДАЧА О КРУЧЕНИИ ШТАМПОМ УПРУГОГО ПОЛУПРОСТРАНСТВА

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Аннотация. В работе рассматривается задача о совместном колебании упругого изотропного полупространства и жесткой на растяжение (сжатие) пластинки (штампа) произвольной формы в плане, к которой приложен крутящий момент, изменяющийся по гармоническому закону во времени. Получены асимптотические формулы для определения контактных касательных напряжений под штампом, угла сдвига фаз и модуля комплексной амплитуды колебания штампа.

Ключевые слова: упругое полупространство, интегральные преобразования, частота колебаний, контактные касательные напряжения, комплексная амплитуда колебаний..