Arch. Min. Sci., Vol. 58 (2013), No 2, p. 395–410

Electronic version (in color) of this paper is available: http://mining.archives.pl

DOI 10.2478/amsc-2013-0027

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CONCEPT OF DETERMINATION AND ANALYSIS OF THE BREAK-EVEN POINT FOR A MINING ENTERPRISE

KONCEPCJA OBLICZANIA WIELOASORTYMENTOWEGO PROGU RENTOWNOŚCI DLA PRZEDSIĘBIORSTWA GÓRNICZEGO

The paper presents a new concept of determination of the break-even point (BEP) for multi-assortment production enterprises. Against the background of the discussed methods for the analysis and assessment used so far in relation to the mining sector, the complexity of the problem of determination of the BEP in the event of multi-assortment production is brought to a closer focus. The paper also presents two author's own methods for the calculation and for the practical application for the needs of management of a mine and of a group of mines (company, enterprise, holding) in present market conditions.

Keywords: The multi-assortment break-even point, the sensitivity analysis, the safety margins, the border sizes

Praca przedstawia nową koncepcję wyznaczania progu rentowności dla przedsiębiorstw o produkcji wieloasortymentowej. Na tle przedyskutowanych dotychczasowych sposobów jego analizy i oceny w odniesieniu do branży górniczej, przybliżono złożoność problemu określenia progu rentowności przy produkcji wieloasortymentowej oraz zaproponowano dwie autorskie metody jego obliczania i praktycznego wykorzystania dla potrzeb zarządzania kopalnią oraz grupą kopalń (spółka, kompania, holding) w obecnych uwarunkowaniach rynkowych.

Słowa kluczowe: wieloasortymentowy próg rentowności, analiza wrażliwości, wielkości graniczne, marginesy bezpieczeństwa

1. Introduction

In the event of enterprises producing and selling various assortments of products, which also include hard coal mines, reliable and transparent methods of the BEP analysis should be applied.

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The BEP analysis provides information on minimal (threshold) quantities of specific coal assortments the sale of which, under given conditions, guarantees that a mine does not generate losses. A comparison of threshold quantities of individual coal assortments with an optimal coal production and sale plan for the mine allows to directly estimate the differences in the sale quantities of said coal assortments and to assess which of them disadvantageously impact the financial result achieved by the mine. Being familiar with the threshold structure of sales of specific coal assortments constitutes helpful information for the mine management in planning the production and sale of coal while it is especially useful in planning the assortment production structure in adaptation to the quantitative and qualitative needs of consumers (Jaśkowski, 1998; Magda et al., 2009; Snopkowski, 2002). However, such an approach to the calculation of the BEP for multi-assortment production mines (enterprises) is rather complicated. On the other hand, in many publications and scientific studies, one can come across the approach to the determination of the BEP based on the single-assortment threshold method. In their calculations, authors treat sale of several assortments of coal as one coal type and average the sale price and variable unit costs of various assortments of coal. In the author's opinion, this leads to unreliable results and it cannot constitute the grounds for a dependable analysis, assessment, and decisions based thereon. Hence, the author has resolved to bring the complexity of the issue of determination of the BEP for multi-assortment production into closer focus and presented his own concept for the calculation thereof. He also introduces a, so far non-applied, take of the percentage threshold.

2. The BEP in single-assortment production

The BEP analysis covers the examination of the so-called "break even" point where the revenues from sales exactly cover the incurred costs. The financial result of the enterprise is then equal zero and the enterprise generates neither profit nor loss. In keeping with this definition, the BEP is in the point in which the value of sales (S) equals the level of total costs (Kc) which can be recorded as:

$$S = Kc \tag{1}$$

whereby:

$$S = P \cdot c \tag{2}$$

and

$$Kc = Ks + P \cdot kjz \tag{3}$$

where:

- P — quantity of production (sales), ton,
- c — price of coal, PLN/ton,
- kjz — variable cost, PLN/ton,
- Ks — fixed cost, PLN.

Substituting equations (1) and (2) to equation (3), the following relationship is obtained:

$$P \cdot c = Ks + P \cdot kjz \tag{4}$$

On the bases of which the BEP can be calculated in:
– a quantitative approach:

$$BEP = \frac{Ks}{c - kjz} \text{ [ton]} \tag{5}$$

– value approach:

$$BEP' = \frac{Ks}{c - kjz} \cdot c = BEP \cdot c \text{ [PLN]} \tag{6}$$

– as a degree of use of production capacity:

$$BEP'' = \frac{Ks}{P_m \cdot (c - kjz)} \cdot 100 = \frac{BEP}{P_m} \cdot 100 \text{ [\%]} \tag{7}$$

where:

P_m — maximum production (sale), ton.

To determine the BEP in the event of production (sale) of even two assortments, it is necessary to make use of the methods presented in the chapters to follow.

3. The BEP in multi-assortment production

The coal extracted by mines is sold in the form of a variety of assortments. Each assortment features its own sale price and, many a time, its own production costs (additional costs: grinding, enrichment, drying, etc.). The complexity of the problem of the BEP analysis in the event of multi-assortment production mines (enterprises) has had this result that the literature of the subject proposes a variety of approaches to the calculation thereof. In principle, three different methods of its analysis are distinguished. The choice of a specific method is conditioned by the possibility to estimate the fixed costs which, in turn, are impacted by the cost account kept in the enterprise and the accuracy of the methods for the isolation of fixed and variable costs. Hence, in individual methods, the fixed costs are (Nowak, 2001; Nowak, 2003):

a) settled in total between the individual assortments,
b) referred in total towards the enterprise,
c) settled in part between individual assortments and referred in part towards the enterprise – the segment analysis.

In the first method (point a), the fixed costs are settled into individual assortments according to the key constituting the coverage margin realised on individual products:

$$WNKs = \frac{Ks}{M} \tag{8}$$

where:

$WNKs$ — fixed costs overhead ratio, [-],

M — global coverage margin obtained on the sales of all products, [PLN]:

$$M = \sum_{i=1}^{n} M_i = \sum_{i=1}^{n} m_i \cdot P_i = \sum_{i=1}^{n} (c_i - kjz_i) \cdot P_i \ \text{[PLN]} \tag{9}$$

where:

m — unit gross margin, [PLN/ton],

i — assortment type index, $i = 1, 2, \ldots, n$.

Overhead fixed costs are calculated according to the following formula:

$$Ks_i = WNKs \cdot M_i \ \text{[PLN]} \tag{10}$$

Hence, the quantitative BEP of specific assortments is set from the relationship:

$$BEP_i = \frac{Ks_i}{c_i - kjz_i} \ \text{[ton]} \tag{11}$$

Whereas the value thresholds of individual assortments are set from the formula:

$$BEP_i' = c_i \cdot BEP_i \ \text{[PLN]} \tag{12}$$

The value BEP for the mine is calculated from the formula:

$$BEP' = \sum_{i=1}^{n} BEP_i' \ \text{[PLN]} \tag{13}$$

If the fixed costs are in total assigned to the enterprise (point b), three approaches to the multi-assortment BEP analysis are distinguished.

The first approach is based on a simplifying assumption according to which the share of total variable costs in the total production value is fixed and set *a priori*. Hence, the value BEP possible to calculate is computed as follows (Nowak, 2001; Wermut, 2000):

$$BEP_w' = \frac{Ks}{1 - \dfrac{\sum_{i=1}^{n} kjz_i \cdot P_i}{\sum_{i=1}^{n} c_i \cdot P_i}} \ \text{[PLN]} \tag{14}$$

The sales value ("guaranteeing" the reaching of the BEP) determined based on the formula above refers only to the assumed structure of production since each change in this structure may substantially change the value of the BEP. For that reason such a method of determination of the BEP may be used only for the purposes of an *ex post* evaluation. The denominator of the formula informs which part of the revenue from sales will be left after all the variable costs have been covered in relation to all types of products with the specific share of these products in this revenue. The revenue (formula 14) within the BEP can be converted into the number of tonnes of specific assortments making use of the information on the share of these assortments in the revenue and on their prices (Wermut, 2000).

The second approach consists in the graphic determination of the BEP. With that in view, a curve of the accumulated gross margin for all assortments is marked out. This method is more accurate than mathematical calculations which assume average values for the entire mine (Sobańska, 2003). From the diagram, it is possible to read the minimal revenue value which guarantees the reaching of the BEP.

The method based on the weighted average margin for covering is yet another method for determination of the value BEP (for fixed costs as a total). The calculations should commence with the calculation of quantitative BEPs of individual assortments (Nowak, 2001):

$$BEP_i = \frac{Ks \cdot \dfrac{S_i}{\sum_{i=1}^{n} S_i}}{\sum_{i=1}^{n}\left(m_i \cdot \dfrac{S_i}{\sum_{i=1}^{n} S_i}\right)} \text{ [ton]} \tag{15}$$

Multiplying the threshold quantities of specific assortments (formula 15) by their prices, we obtain the set of value BEPs. The mine's value BEP consists of the sum of individual value BEPs of individual assortments.

In the event the mine applies a multistage cost accounting, which enables the division of the fixed costs into two parts: specific assortments (Ks_i) and of the entire enterprise (Ks_o), then it is possible to apply the segment analysis of the BEP (point c). The fixed costs are divided into specific assortments proportionally to the global coverage margin of these assortments (Nowak, 2001). The quantitative thresholds of individual assortments are determined from the following relationship:

$$BEP_i = \frac{Ks_i \cdot \dfrac{M_i}{\sum_{i=1}^{n} M_i} \cdot Ks_o}{m_i} \text{ [ton]} \tag{16}$$

The value thresholds are determined as the product of threshold quantities of individual assortments and their prices.

4. Author's concept of the multi-assortment BEP analysis

In the event of a single-assortment production, the BEP is a point whereas in the event of production of multiple different products it is a set of finitely many points. Evening of total costs with sales revenues may be achieved in many different combinations of the quantitative coal assortment structure. The revenues from the sale of individual assortments and the total costs (for an individual mine) are calculated as follows:

$$S = \sum_{i=1}^{n} P_i \cdot c_i \tag{17}$$

$$Kc = \sum_{i=1}^{n} P_i \cdot kjz_i + Ks \tag{18}$$

Hence, the BEP can be recorded as:

$$\sum_{i=1}^{n} P_i \cdot c_i = \sum_{i=1}^{n} P_i \cdot kjz_i + Ks \tag{19}$$

In the event of production (sale) of two assortments, the BEP can be reached at finitely many combinations of the production structure. It will be a set of combinations of the quantities of individual assortments (a set of points) located on the section the end of which is determined based on the equation (19). To illustrate this case, the author used data originating from a real hard coal mine. The data shall be subject to appropriate modifications to make the illustration of the examples provided below possible.

Example 1

Table 1 presents the output structure of monthly production of mine "X" as well as the information on prices, variable unit costs of specific assortments of coal, and fixed costs.

TABLE 1

Assumed value of production and economic indicators of mine "X" in a monthly take

Specification	**Unit**	**Assortment**	
		Nut coal	**Fine coal**
Production	ton	25,200	94,800
Price of coal	PLN/ton	610.0	400.0
Unit variable cost	PLN/ton	38.5	41.5
Fixed cost	PLN	34,368,193	

We hypothetically assume that we shall produce only the *Nut coal* assortment, hence, on the basis of equation (19) its quantity (P_N) may be determined, the sale of which at a given price and costs shall result in mine's reaching the BEP:

$$610 \cdot P_N + 400 \cdot 0 = 34{,}368{,}193 + 38.5 \cdot P_N + 41.5 \cdot 0$$

$$P_N = \frac{34{,}368{,}193}{571.5} = 60{,}137 \ \text{[ton]}$$

Then threshold quantity of sales of the *Nut coal* assortment shall amount to 60,137 tonnes. In turn, while producing and selling exclusively the *Fine coal* assortment, the BEP will be reached at the quantity of 95,867 tonnes, in keeping with the calculations:

$$610 \cdot 0 + 400 \cdot P_F = 34{,}368{,}193 + 38.5 \cdot 0 + 41.5 \cdot P_F$$

$$P_F = \frac{34{,}968{,}193}{358.5} = 95{,}867 \ \text{[ton]}$$

The graphic solution for the example under analysis is presented in Fig. 1. The set out limit values of the coal assortments in the picture are designated with letters α (for the *Nut coal* assortment) and β (for the *Fine coal* assortment). They are intersections with the axes representing the quantities of the assortments under analysis. Connecting points α and β, we receive a section $\alpha\beta$ constituting a finite set of points (a combination of quantities of *Nut coal* and *Fine coal* assortments) fulfilling the equation (19). Each point of the said section guarantees that the BEP is reached.

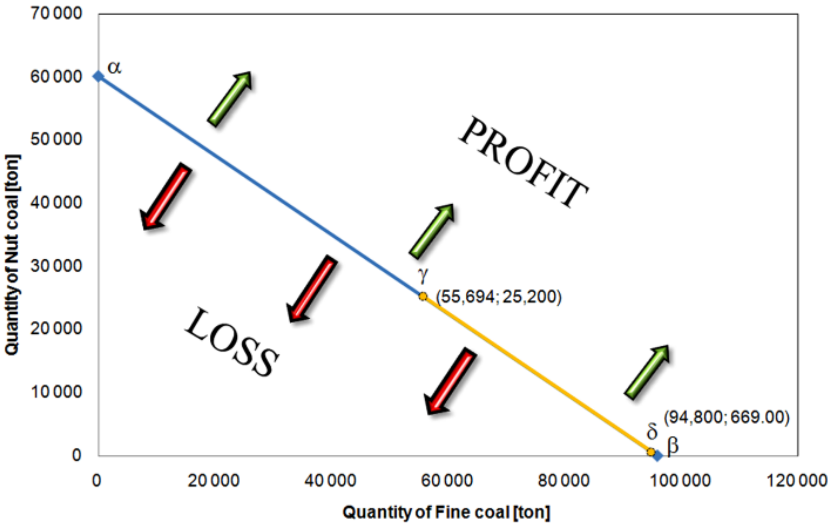

Fig. 1. Graphic presentation of the BEP for example no. 1

However, it is not an admissible solution due to the fixed production structure: 25,200 tonnes of the *Nut coal* assortment and 94,800 tonnes of the *Fine coal* assortment. Faced with which, the admissible solution of the threshold quantities of the assortments under analysis for the case subject to the analysis, will be the fragment of the section $\alpha\beta$, and namely section $\gamma\delta$, which results from the adopted structure (Fig. 1).

In the event of production of three assortments, the limit and threshold quantities of individual assortments shall constitute a set of points located on a plane. To illustrate this case, the author makes use of the data from example no. 1, expanding the production structure to three assortments of coal, without changing the mine's extraction capacity.

Example 2

Table 2 includes the partly expanded data from example no. 1.

TABLE 2

Assumed value of production and economic indicators of mine "X" in a monthly take

Specification	Unit	Assortment		
		Nut coal	Fine coal II	Fine coal I
Production	ton	28,800	44,400	46,800
Price of coal	PLN/ton	610.0	450.0	510.0
Unit variable cost	PLN/ton	38.5	40.8	41.5
Fixed cost	PLN	34,368,193		

For this example, the equation (19) can be, in general, recorded in the following form:

$$P_1 \cdot (c_1 - kjz_1) + P_2 \cdot (c_2 - kjz_2) + P_3 \cdot (c_3 - kjz_3) = Ks \tag{20}$$

The author proposes to determine the maximum limit quantity of production (sales) of individual assortments pursuant to the following formulae:

$$P_{G1} = \frac{K_s}{c_1 - kjz_1} \qquad \text{for} \quad P_2 = 0 \text{ i } P_3 = 0 \tag{21}$$

$$P_{G2} = \frac{K_s}{c_2 - kjz_2} \qquad \text{for} \quad P_1 = 0 \text{ i } P_3 = 0 \tag{22}$$

$$P_{G3} = \frac{K_s}{c_3 - kjz_3} \qquad \text{for} \quad P_1 = 0,\ P_2 = 0 \tag{23}$$

The set of all possible combinations of limit quantities (quantitative BEPs) of the *Nut coal, Fine coal I* and *Fine coal II* assortments forms the surface of a triangle ABC (Fig. 2). Whereas the set of admissible threshold quantities is limited by the maximum production quantity of analysed assortments and is contained within the area of the hexagon DEFGHI.

The coordinates of hexagon DEFGHI points are presented in table 3.

TABLE 3

Coordinates of hexagon DEFGHI points, [ton]

Points	Axis		
	X	Y	Z
D	28,800.00	43,765.87	0.00
E	28,345.95	44,400.00	0.00
F	0.00	44,400.00	34,577.83
G	0.00	30,406.63	46,800.00
H	21,771.47	0.00	46,800.00
I	28,800.00	0.00	38,226.24

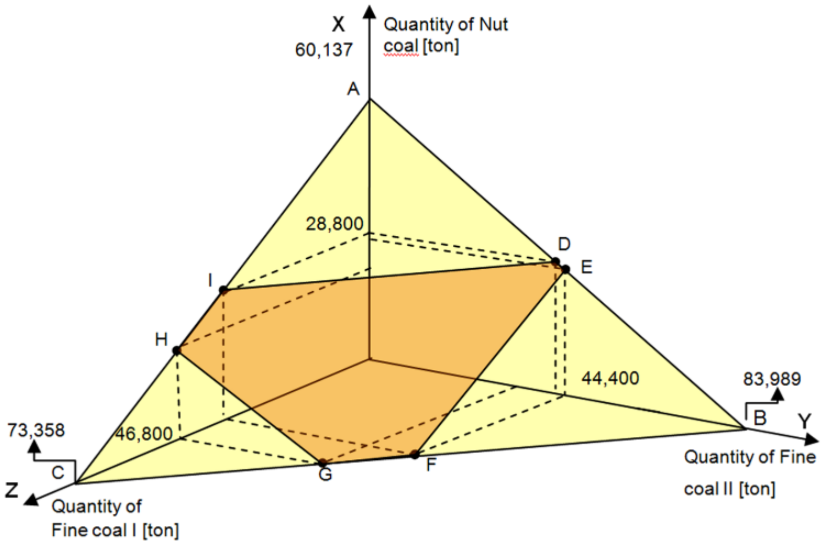

Fig. 2. Graphic presentation of the BEP for example no. 2

The area of solutions of admissible threshold quantities shall assume various shapes depending on the percentage share of the quantities of individual assortments in the production structure. In the event of the change of production for the data from example no. 2, without the change of the remaining values, the admissible solutions area will be obtained in the form of a pentagon – example no. 3.

Example 3

For the data from example 2, only a change of the production structure was performed without interfering with the maximum production capacity (tab. 4). Figure 3 illustrates the area of solutions admissible for this case. It is a pentagon DEFGH. The coordinates of the points of this pentagon are presented in table 5.

TABLE 4

Assumed value of production and economic indicators of mine "X" at the change of the production structure

Specification	Unit	Assortment		
		Nut coal	Fine coal II	Fine coal I
Production	ton	26,400	34,800	58,800
Price of coal	PLN/ton	610.0	450.0	510.0
Unit variable cost	PLN/ton	38.5	40.8	41.5
Fixed cost	PLN	34,368,193		

TABLE 5

Coordinates of pentagon DEFGH points, [ton]

Points	Axis		
	X	Y	Z
D	26,400.00	34,800.00	10,758.66
E	0.00	34,800.00	42,962.72
F	0.00	16,667.63	58,800.00
G	11,934.20	0.00	58,800.00
H	26,400.00	0.00	41,153.88

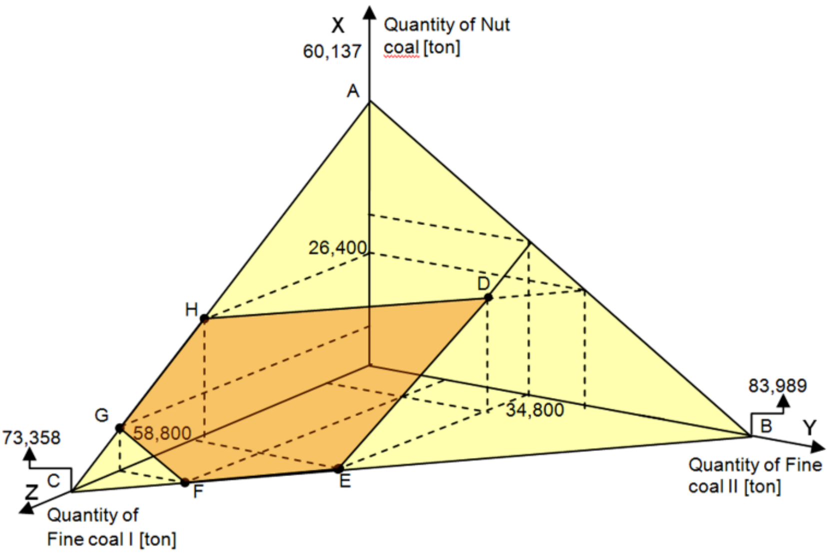

Fig. 3. Graphic presentation of the BEP for example no. 3

In the event of production of more than three assortments, the graphic representation of the admissible solutions area is, unfortunately, technically impossible – a hyper dimensional space.

As it can be seen, determination of threshold quantities of production (sale) of individual assortments is a very complex problem due to the occurrence of infinitely many combinations of the sale quantities of individual assortments “guaranteeing” that the mine will achieve zero profit. For that reason, setting of one of the solution variants, in the form of specific threshold quantities cannot constitute the basis for the adoption of relevant production decisions, which lies at the basis of the methods for the determination of the multi-assortment BEP proposed in the literature (Nowak, 2001; Sobańska, 2003; Wermut, 2000). Moreover, each of these methods provides different solutions. Determining whether the mine is close to the BEP is quite difficult and unreliable. For example, a situation can occur where the quantity of sale of only one of the assortments is lower than the threshold quantity – the enterprise may be either above or below the BEP. In both cases, it is influenced by the price of a given assortment, the unit gross margin to be more precise. In the event of a higher price, a higher revenue from the sales of a given assortment will be achieved (and the bigger percentage share in the revenue covering the costs incurred). Then, a slight decrease in the given assortment sale quantity may cause a significant drop in revenues and a drop below the BEP. In the event of a lower price, we will deal with the lower revenue sensitivity to the change in the quantity of the assortment being sold which does not necessarily need to lead the enterprise to the loss zone.

The methods for the calculation of the multi-assortment BEP provided by the literature and based on the determination of threshold quantities of specific assortments are of little use at the same time being rather complicated, in particular as regards the interpretation of results.

In relation to the above, the author should like to propose his own concept for the determination of the multi-assortment BEP for hard coal mines with two different approaches to the issue within this proposal (Fuksa, 2011).

Method I

In relation to a single mine, the equation (20) – for n coal assortments can be taken down in the following form:

$$P_1 \cdot (c_1 - kjz_1) + P_2 \cdot (c_2 - kjz_2) + \ldots + P_n \cdot (c_n - kjz_n) = Ks \tag{24}$$

Analogically to the formulas (21)-(23) and at the assumption of the zero production of the remaining assortments, determination of the maximum border production (sales) quantity of a specific assortment is proposed to be set according to the following formula:

$$P_{Gn} = \frac{K_s}{c_n - kjz_n} \qquad \text{for} \quad P_1 = 0, P_2 = 0, \ldots, P_{n-1} = 0 \tag{25}$$

Whereas the threshold quantity (P_{pn}) for a specific assortments is determined as the product of its threshold quantity and weight (w_n), – the said weight is calculated as the quotient of the maximum production of such assortment (P_n) to the maximum production (Q_s):

$$P_{pn} = P_{Gn} \cdot w_n \tag{26}$$

Upon substitution, we receive:

$$P_{pn} = \frac{Ks}{c_n - kjz_n} \cdot \frac{P_n}{Q_s} \ \text{[ton]} \tag{27}$$

The total of threshold quantities of specific assortments shall constitute a quantitative BEP (*PRI*), as a weighted average expressed with the use of weights the total of which equals one (convex combination):

$$PRI = \frac{\frac{Ks}{c_1 - kjz_1} \cdot \frac{P_1}{Q_s} + \frac{Ks}{c_2 - kjz_2} \cdot \frac{P_2}{Q_s} + \ldots + \frac{Ks}{c_n - kjz_n} \cdot \frac{P_n}{Q_s}}{\frac{P_1}{Q_s} + \frac{P_2}{Q_s} + \ldots + \frac{P_n}{Q_s}} \ \text{[ton]} \tag{28}$$

Knowing that:

$$\sum_{i=1}^{n} \frac{P_i}{Q_s} = 1 \tag{29}$$

The formula (28) will adopt the form:

$$PRI = \frac{Ks}{m_1} \cdot \frac{P_1}{Q_s} + \frac{Ks}{m_2} \cdot \frac{P_2}{Q_s} + \ldots + \frac{Ks}{m_n} \cdot \frac{P_n}{Q_s} \quad [\text{ton}] \tag{30}$$

The formula (10) can be recorded in a more simple form:

$$PRI = \frac{Ks}{Q_s} \cdot \sum_{i=1}^{n} \frac{P_i}{(c_i - kjz_i)} \quad [\text{ton}] \tag{31}$$

The calculation of the threshold quantities of individual assortments (formula 27) as well as the one calculated quantity of the threshold production (formula 31), being the weighted average of the production quantities of various assortments cannot, however, constitute the basis for the accurate BEP analysis. As already mentioned, the infinitely many combinations of the quantities of the analysed assortments reflect the point where the revenues equal the costs. Table 6 yet again presents the coordinates of the points of pentagon DEFGH, constituting the solution for example no. 3. The threshold quantities of assortments subject to analysis are totalled in the last column. Based on the formula (31), the quantity BEP was calculated for this example and it amounted to 73,532 tonnes.

TABLE 6

Quantitative BEP for the points of pentagon DEFGH, [ton]

Points	**Axis**			**Threshold quantities total**
	X	**Y**	**Z**	
D	26,400.00	34,800.00	10,758.66	71,958.66
E	0.00	34,800.00	42,962.72	77,762.72
F	0.00	16,667.63	58,800.00	75,467.63
G	11,934.20	0.00	58,800.00	70,734.20
H	26,400.00	0.00	41,153.88	67,553.88

The threshold quantities total (tab. 6) assumes values higher or lower than the threshold quantity calculated according to the formula (31) and it is precisely such combinations that in each case cause the enterprise to reach the BEP. Hence, determination of the threshold quantity of one of the assortments for the assumed or guaranteed sales of the remaining assortments according to the formula:

$$P_{pn} = \frac{Ks - \sum_{i=1}^{n-1} P_i \cdot (c_i - kjz_i)}{c_n - kjz_n} \quad [\text{ton}] \tag{32}$$

is more useful.

Example 4

Maximum production quantities of individual assortments for mine "X" as well as the assumed quantities of sale thereof (so that the mine would be unprofitable) are presented in table 7. Based on the formula (32), the minimum quantity for the unsold *Fine coal II* assortment "guaranteeing" that the mine reaches the BEP (table 7, column 6) was set. For the mine to reach the BEP and not to generate the loss, it is enough to find a consumers for at least 4,202 tonnes of *Fine coal II* (tab. 7).

TABLE 7

Maximum quantities, according to the plan and the threshold quantities of individual coal assortments for mine "X"

Consumers	**Assortments**	**Maximum production quantities**	**Threshold quantities Method I**	**Sales quantities according to the plan**	**Minimum threshold quantity of sales of *Fine coal II* assortment**
		[ton]	**[ton]**	**[ton]**	**[ton]**
Consumer 1	Nut coal	26,400	13,230	14,500	14,500
Consumer 2	Fine coal I	58,800	35,945	52,000	52,000
Dumping ground	Fine coal II	34,800	24,357	0.0	4,202

Whereby it is proposed that the BEP in the value take, constituting the revenue value covering the incurred costs, be calculated according to the following formula:

$$PRW = \frac{Ks}{m_1} \cdot \frac{P_1}{Q_s} \cdot c_1 + \frac{Ks}{m_2} \cdot \frac{P_2}{Q_s} \cdot c_2 + \ldots + \frac{Ks}{m_n} \cdot \frac{P_n}{Q_s} \cdot c_n \quad \text{[PLN]} \tag{33}$$

Upon reduction, we receive:

$$PRW = \frac{Ks}{Q_s} \cdot \sum_{i=1}^{n} \frac{P_i \cdot c_i}{(c_i - kjz_i)} = \frac{Ks}{Q_s} \cdot \sum_{i=1}^{n} \frac{P_i}{(1 - \frac{kjz_i}{c_i})} \quad \text{[PLN]} \tag{34}$$

The presented methods for the determination of the multi-assortment BEP to be found in the literature (point 3) facilitate calculation thereof in both the quantitative as well as the value approach. However, they do not take into consideration the method for determination of the threshold in the percentage approach as a degree of utilisation of the production capacity which, in the author's opinion, is more transparent and easier as regards the interpretation of results. The author proposes that its value be determined in the following way:

$$PRP = \frac{\frac{Ks}{Q_s} \cdot \sum_{i=1}^{n} \frac{P_i \cdot c_i}{(c_i - kjz_i)}}{\sum_{i=1}^{n} P_i \cdot c_i} = \frac{\frac{Ks}{Q_s} \cdot \sum_{i=1}^{n} \frac{P_i}{(1 - \frac{kjz_i}{c_i})}}{\sum_{i=1}^{n} P_i \cdot c_i} \quad [\%] \tag{35}$$

Method II

The second method for the calculation of the multi-assortment BEP proposed by the author is simpler in use and, in author's opinion, leads to more reliable results. It is proposed to calculate the percentage BEP (*PRP*), as the quotient of fixed costs and gross global margin, according to the following formula:

$$PRP = \frac{Ks}{\sum_{i=1}^{n} P_i \cdot (c_i - kjz_i)} \cdot 100 \quad [\%] \tag{36}$$

The knowledge of the value of the "percentage" BEP (*PRP*), allows for the calculation of its take:

– quantitative:

$$PRI = PRP \cdot P_m \quad [\text{ton}] \tag{37}$$

or the threshold quantity of any assortment according to the formula:

$$P_{pn} = PRP \cdot P_n \quad [\text{ton}] \tag{38}$$

– value:

$$PRW = PRP \cdot \sum_{i=1}^{n} P_i \cdot c_i \quad [\text{PLN}] \tag{39}$$

One of the significant elements of the BEP analysis is the sensitivity analysis covering the examination of: the border level of the unit sale price, border level of variable unit costs and safety margins for the price and variable unit cost. The author proposes to set:

– **the border level of the unit sale price**, with invariability of the remaining factors impacting the BEP (e.g.: guaranteed or contracted sale quantities). On the basis of the equation (24), it is proposed that the minimum price of the specific assortment be determined according to the formula:

$$c_{n\,\min} = \frac{Ks + P_n \cdot kjz_n - \sum_{i=1}^{n-1} P_i \cdot (c_i - kjz_i)}{P_n} \quad [\text{PLN/ton}] \tag{40}$$

Sale of the assortment at the minimal price, with the invariability of the remaining factors (constituting the basis for the calculation of this price), shall have the result that the mine will reach the BEP.

– **the border level of variable unit costs,** with the assumptions identical to those above, for a specific assortment, is calculated from the formula:

$$kjz_{n\,\max} = \frac{Ks - P_n \cdot c_n - \sum_{i=1}^{n-1} P_i \cdot (c_i - kjz_i)}{P_n} \quad [\text{PLN/ton}] \tag{41}$$

– **safety margins** for the minimal price and maximum variable unit cost of a specific assortment shall be calculated according to the following formulas:

$$Mc = \frac{c_n - c_{n\,\min}}{c_n} \cdot 100 \quad [\%] \tag{42}$$

$$Mk = \frac{kjz_{n\,\max} - kjz_n}{kjz_n} \cdot 100 \quad [\%] \tag{43}$$

The second proposed method for the BEP calculation has been positively verified by the author in the course of the conducted research, both as regards its quantitative and the percentage approach. In relation to the percentage BEP, both methods yield results that are very close to each other. The maximum difference between the results obtained in the course of the research conducted by the author on the example of five hard coal mines equalled only 0.54 percentage point. Table 8 presents the BEP values calculated with the use of two methods proposed by the author for the analysed mine "X".

TABLE 8

Maximum quantities, according to the plan and the threshold quantities of individual coal assortments for mine "X"

Consumers	**Assortments**	**Maximum production quantities**	**Threshold quantities Method I**	**Threshold quantities Method II**
		[ton]	**[ton]**	**[ton]**
Consumer 1	Nut coal	26,400	13,230	15,953
Consumer 2	Fine coal I	58,800	35,945	35,531
Dumping ground	Fine coal II	34,800	24,357	21,029
PRI [ton]			73,532	72,512
PRW [PLN]			37,363,040	37,314,879
PRP [%]			60.50	60.43

The analysis of break-even point of the analyzed mining plant, is carried on the per cent break-even point (PRP). The nominal value of the mine's break-even point is calculated in accordance with formula (36). Next, the percentage share of the value of these sale revenues in the mine's maximum revenue is determined according to the formula (Fuksa, 2011):

$$\frac{S^l}{S_{\max}} \cdot 100 \quad [\%] \tag{44}$$

where:

S^l — sales revenue of a mine, at l iteration step, [PLN],

l — subsequent iteration steps, $l = 1, 2, \ldots, y$,

$S_{\max}$ — maximum revenue of the mining plant when whole coal is sold, [PLN].

Each time, the obtained result must be compared against the nominal value of the mine's break-even point PRP. The result smaller than the nominal value mean represent that the mining plant is below the break-even point.

5. Conclusion

The concept for the calculation and analysis of the BEP at multi-assortment production proposed by the author constitutes a simple and effective tool assisting the process of making rational production-related decisions. The results obtained through these methods have been positively verified by the author in the course of the research conducted at the angle of their practical application in the mining sector.

The objectives set by the owner (the Minister of Economy) and resulting from the government programmes for the restructuring of hard coal mining constitute the basis for the development of the strategic plan for a coal company and the mines it is composed of. The conditions, both internal and external, force to verify the set objectives and, to a substantial degree, impact the level of sale of individual coal assortments, their share in the production structure, and the sale price of coal. For example, the need to decrease the sale price of coal, the changes in the production assortment structure (demand for higher quantities of culm resulting in the necessity to grind, for example, medium assortments), the necessity to increase the extraction, and many other [factors] have this result that the knowledge of the BEP, border values, and safety margins plays a substantial role.

This study is supported through the statutory research registered in AGH University of Mining and Metallurgy in Krakow at no. 11.11.100.481 in the year 2013

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Received: 18 April 2012