



ANDRZEJ KRACH*

DETERMINING DIAGONAL BRANCHES IN MINE VENTILATION NETWORKS

WYZNACZANIE BOCZNIK PRZEKĄTNYCH W KOPALNIANEJ SIECI WENTYLACYJNEJ

The present paper discusses determining diagonal branches in a mine ventilation network by means of a method based on the relationship $\mathbf{A} \otimes \mathbf{P}^T(k, l) = \mathbf{M}$, which states that the nodal-branch incidence matrix \mathbf{A} , modulo-2 multiplied by the transposed path matrix $\mathbf{P}^T(k, l)$ from node no. k to node no. l , yields the matrix \mathbf{M} where all the elements in rows k and l – corresponding to the start and the end node – are 1, and where the elements in the remaining rows are 0, exclusively. If a row of the matrix \mathbf{M} is to contain only „0” elements, the following condition has to be fulfilled: after multiplying the elements of a row of the matrix \mathbf{A} by the elements of a column of the matrix $\mathbf{P}^T(k, l)$, i.e. by the elements of a proper row of the matrix $\mathbf{P}(k, l)$, the result row must display only „0” elements or an even number of „1” entries, as only such a number of „1” entries yields 0 when modulo-2 added – and since the rows of the matrix \mathbf{A} correspond to the graph nodes, and the path nodes level is 2 (apart from the nodes k and l , whose level is 1), then the number of „1” elements in a row has to be 0 or 2. If, in turn, the rows k and l of the matrix \mathbf{M} are to contain only „1” elements, the following condition has to be fulfilled: after multiplying the elements of the row k or l of the matrix \mathbf{A} by the elements of a column of the matrix $\mathbf{P}^T(k, l)$, the result row must display an uneven number of „1” entries, as only such a number of „1” entries yields 1 when modulo-2 added – and since the rows of the matrix \mathbf{A} correspond to the graph nodes, and the level of the i and j path nodes is 1, then the number of „1” elements in a row has to be 1. The process of determining diagonal branches by means of this method was demonstrated using the example of a simple ventilation network with two upcast shafts and one downcast shaft.

Keywords: ventilation of mines, ventilation network, diagonal branches

W artykule przedstawiono metodę wyznaczania bocznic przekątnych w sieci wentylacyjnej kopalni metodą bazującą na zależności $\mathbf{A} \otimes \mathbf{P}^T(k, l) = \mathbf{M}$, która podaje, że macierz incydencji węzłowo bocznicowej \mathbf{A} pomnożona modulo 2 przez transponowaną macierz ścieżek $\mathbf{P}^T(k, l)$ od węzła nr k do węzła nr l daje w wyniku macierz \mathbf{M} o takich własnościach że ma same jedynki w wierszach k i l , odpowiadającym węzłom początkowemu i końcowemu i same zera w pozostałych wierszach. Warunkiem na to, aby w wierszu macierzy \mathbf{M} były same zera jest aby po pomnożeniu elementów wiersza macierzy \mathbf{A} przez elementy kolumny macierzy $\mathbf{P}^T(k, l)$, czyli przez elementy odpowiedniego wiersza macierzy $\mathbf{P}(k, l)$, w wierszu

* STRATA MECHANICS RESEARCH INSTITUTE OF THE POLISH ACADEMY OF SCIENCES, REYMONTA 27, 30-059 KRAKOW, POLAND

wynikowym były same zera lub parzysta liczba jedynek, ponieważ tylko taka liczba jedynek zsumowana modulo 2 daje w wyniku 0, a ponieważ wiersze macierzy \mathbf{A} odpowiadają węzłom grafu, a węzły ścieżki są stopnia 2 (oprócz węzłów k i l , które są stopnia 1), to liczba jedynek w wierszu musi być równa 0 lub 2. Natomiast warunkiem na to, aby w wierszach k i l macierzy \mathbf{M} były same jedynki jest aby po pomnożeniu elementów wiersza k lub l macierzy \mathbf{A} przez elementy kolumny macierzy $\mathbf{P}^T(k, l)$ w wierszu wynikowym była nieparzysta liczba jedynek, ponieważ tylko taka liczba jedynek zsumowana modulo 2 daje w wyniku 1, a ponieważ wiersze macierzy \mathbf{A} odpowiadają węzłom grafu, a węzły k i l ścieżki są stopnia 1, to liczba jedynek w wierszu musi być równa 1. Wyznaczanie bocznic przekątnych tą metodą pokazano na przykładzie prostej sieci wentylacyjnej z dwoma szybami wydechowymi i jednym wdechowym.

Słowa kluczowe: wentylacja kopalń, sieć wentylacyjna, bocznicę przekątne

1. Introduction

Workings of an underground mine constitute a vast network through which fresh air is transported to mining sections, „spent” air is rushed out of the mine, and the output is hauled to the surface. Maintaining the stability of the directions of air currents ventilating work stations and evacuation routes is an issue of key importance. Thus, the differentiation between diagonal and standard branches was introduced. There are two basic definitions of a diagonal and standard branch. Czeczot (1925, 1957a, 1957b), Budryk (1961) and Bystron (1957, 1958) define diagonal and standard branches in relation to the sensitivity of the air flow in a given branch to changes in the resistance of other branches within a ventilation network: a diagonal branch is a branch where the direction of the air current alters as a result of the changes in the resistance of other branches; a standard branch is a branch where the direction of the air current does not alter with changes in the resistance of other branches. Another definition, based only on the network structure, is given by E. Simode and J. Sułkowski (Kolarczyk, 1994, 1998): a diagonal branch is such a network element whose orientation – with the established inlet and outlet of the network – can be described in two ways; a standard branch is such a network element whose orientation can be described in just one way.

Kolarczyk (1993, 1994, 1998) lists the following reasons for determining diagonal branches:

- unfavorable changes in the direction of the air flow, resulting from changes in the resistance of other branches,
- local ventilation reversions performed with the aim of reducing the area of direct smoke presence, or rescuing the crew during a fire,
- speeding up the process of fire suppression in sealed areas of diagonal branches,
- fire prevention in situations where a risk of endogenous fire occurs.

Apart from M. Kolarczyk, who investigated the issue of air flow stability on the basis of studies into the properties of the so-called drag changes sensitivity matrix (Kolarczyk et al., 2002; Kolarczyk, 2008), as well as determined diagonal branches by means of studying their orientation in various paths between selected nodes (Kolarczyk, 1994, 1998), the issue of air flow stability was also tackled by A., B. Strumiński (2006). Additionally, N. Szlązak and co-authors (1998) developed an algorithm and computer software for determining diagonal branches, based on the method of depth-first search (DFS) for a graph structure and adapted to direct reading of data recorded by means of the *Edtxt* program of the *Ventgraph* system, developed at the Strata Mechanics Research Institute of the Polish Academy of Sciences (Dziurzyński et al., 2014).

2. Determining diagonal branches

It has been assumed that a mine ventilation network shall be viewed as a connected directed graph, in which the edges or branches of the graph correspond to the branches of the ventilation network, and the vertices or nodes of the graph correspond to the nodes of the ventilation network.

The method for determining diagonal branches presented in this paper is based on the following theorem (Narsingh Deo, 1980):

$$\mathbf{A} \otimes \mathbf{P}^T(k, l) = \mathbf{M}$$

where

- \mathbf{A} — an incidence matrix of an (undirected) graph,
- $\mathbf{P}^T(k, l)$ — a transposed path matrix from the k -th start node to the l -th end node,
- \mathbf{M} — a matrix with the following properties:
 - only „1” entries in two rows: k and l ,
 - only „0” entries in the remaining rows,
- \otimes — multiplication modulo 2.

If a row of the matrix \mathbf{M} is to contain only „0” entries, the following condition has to be fulfilled:

- after multiplying the elements of a row of the matrix \mathbf{A} by the elements of a column of the matrix $\mathbf{P}^T(k, l)$, i.e. by the elements of a proper row of the matrix $\mathbf{P}(k, l)$, the result row must display only „0” elements or an even number of „1” entries, as only such a number of „1” entries yields 0 when modulo-2 added – and since the rows of the matrix \mathbf{A} correspond to the graph nodes, and the path nodes level is 2 (apart from the nodes k and l , whose level is 1), then the number of „1” elements in a row has to be 0 or 2.

If, in turn, the rows k and l of the matrix \mathbf{M} are to contain only „1” elements, the following condition has to be fulfilled:

- after multiplying the elements of the row k or l of the matrix \mathbf{A} by the elements of a column of the matrix $\mathbf{P}^T(k, l)$, the result row must display an uneven number of „1” entries, as only such a number of „1” entries yields 1 when modulo-2 added – and since the rows of the matrix \mathbf{A} correspond to the graph nodes, and the level of the i and j path nodes is 1, then the number of „1” elements in a row has to be 1.

The application of the above rules is best demonstrated with a practical example. Figure 1 shows an isometric diagram of a simple ventilation network with three ventilation shafts (a downcast one and two upcast ones), and a graph depicting this network, with numbered branches and nodes.

The start node is node no. 1, which corresponds to the inlet of the downcast shaft, and the end node is node no. 5, which corresponds to the outlets of the upcast shafts.

The graph structure can be presented in various manners – most often, as a list of branches, or as an incidence matrix. Below, the list of branches and the incidence matrix of an undirected graph, for the graph shown in Fig. 1, are given.

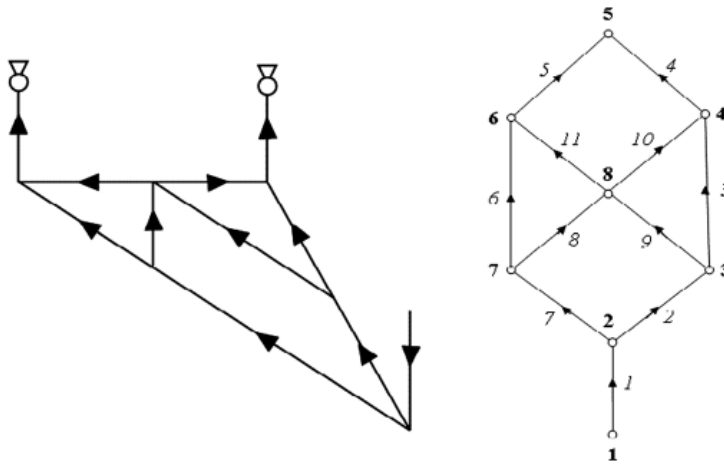


Fig. 1. An isometric diagram of a sample ventilation network, together with a graph depicting it

List of branches

No.	SN	EN
1	1	2
2	2	3
3	3	4
4	4	5
5	6	5
6	7	6
7	2	7
8	7	8
9	3	8
10	8	4
11	8	6

Nodal-branch incidence matrix of an undirected graph

	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>
1	1	0	0	0	0	0	0	0	0	0	0
2	1	1	0	0	0	0	1	0	0	0	0
3	0	1	1	0	0	0	0	0	1	0	0
4	0	0	1	1	0	0	0	0	0	1	0
5	0	0	0	1	1	0	0	0	0	0	0
6	0	0	0	0	1	1	0	0	0	0	1
7	0	0	0	0	0	1	1	1	0	0	0
8	0	0	0	0	0	0	1	1	1	1	1

The abbreviations in the headlines of the columns in the list of branches have the following meaning:

- No. – the branch number,
- SN – the start node (of a branch) number,
- EN – the end node (of a branch) number.

In the incidence matrix, the numeration of columns corresponds to the numeration of air splits, and the numeration of rows corresponds to the numeration of nodes.

The chart below shows what values the elements of the matrix rows must have so that, after multiplying the elements of a row of the incidence matrix \mathbf{A} (the underlined row) and a row of the path matrix, the end result would be: a row containing „0” entries and one „1” entry – for the start and end node (rows 1 and 5 of the incidence matrix); a row containing only „0” entries, or „0” entries and two „1” entries – for the remaining nodes (rows 2, 3, 4, and 6, 7, 8). The letter „x” signifies any value of an element in a row of the path matrix.

1) 1000000000 1xxxxxxx	2) 1100001000 00xxxx0xxx 01xxxx1xxx 10xxxx1xxx 11xxxx0xxx	3) 0110000100 x00xxxx0xx x01xxxx1xx x10xxxx1xx x11xxxx0xx	4) 0011000010 xx00xxxx0x xx01xxxx1x xx10xxxx1x xx11xxxx0x
5) 0001100000 xxx01xxxxx xxx10xxxxx	6) 0000110001 xxxx00xxxx0 xxxx01xxxx1 xxxx10xxxx1 xxxx11xxxx0	7) 0000011100 xxxxx000xxx xxxxx011xxx xxxxx101xxx xxxxx110xxx	8) 0000001111 xxxxxxx0000 xxxxxxx0011 xxxxxxx0101 xxxxxxx0110 xxxxxxx1001 xxxxxxx1010 xxxxxxx1100

The result is 8 path matrices \mathbf{B}_i ($i = 1 \dots 8$), encompassing altogether 30 rows, with a significant number of unknown element values. K -th element of j -th row in i -th matrix was denoted as $b_{i,j,k}$. A temporary path matrix \mathbf{C} is created, with the initial value $\mathbf{C} = \mathbf{B}_1$. Subsequently, the following operation on the rows of path matrices should be performed:

$$i = 2 \dots I : [j = 1 \dots J_i, m = 1 \dots M,$$

$$(k = 1 \dots K : q_k = 1, c_{m,k} \neq b_{i,j,k} \wedge b_{i,j,k} \neq x \wedge c_{m,k} \neq x \Rightarrow q_k = 0),$$

$$\prod_{k=1}^K q_k = 1 \Rightarrow k = 1 \dots K : (c_{m,k} = x \Rightarrow c_{m,k} = b_{i,j,k})]$$

where

- I — number of \mathbf{B}_i matrices (here, $I = 8$),
- J_i — number of rows in \mathbf{B}_i matrices (here, $J_1 = 1, J_2 = 4, J_3 = 4, J_4 = 4, J_5 = 2, J_6 = 4, J_7 = 4, J_8 = 7$),
- M — number of rows in the matrix \mathbf{C} ,
- K — number of elements in the rows in the matrices \mathbf{B}_i and \mathbf{C} ,
- q_k — auxiliary variable.

The above procedure compares the values of the elements included in the rows of \mathbf{B}_i matrices with the elements of the matrix \mathbf{C} . In the rows where compatibility of the element values occurs (0 or 1, and not x), element $c_{m,k}$ in a row of the matrix \mathbf{C} whose value is x becomes replaced with values of element $b_{i,j,k}$ in a row of the matrix \mathbf{B}_i .

Below, the consecutive values of the matrix \mathbf{C} for $i = 1 \dots 8$ are shown:

1+2 10xxxx1xxx 11xxxx0xxxx	1+2+3 100xxx1x0xx 101xxx1x1xx 110xxx0x1xx 111xxx0x0xx	1+2+3+4 1000xx1x00x 1100xx0x10x 1001xx1x01x 1101xx0x11x 1010xx1x11x 1110xx0x01x 1011xx1x10x 1111xx0x00x	1+2+3+4+5 10001x1x00x 11001x0x10x 10101x1x11x 11101x0x01x 10010x1x01x 11010x0x11x 10110x1x10x 11110x0x00x
----------------------------------	---	---	---

1+2+3+4+5+6	1+2+3+4+5+6+7	1+2+3+4+5+6+7+8	
1001001x010	11010000110	11110000000	
1101000x110	11110000000	10001110000	
1011001x100	11001000101	11101000011	
1111000x000	11101000011	10010110011	
1001011x011	10010011010	11001000101	
1101010x111	10110011100	10110110101	
1011011x101	10001011001	11010000110	
1111010x001	10101011111	10101110110	
1000101x001	11010101111	10001011001	
1100100x101	11110101001	11110101001	
1010101x111	11001101100	10010011010	
1110100x011	11101101010	11101101010	
1000111x000	10010110011	10110011100	
1100110x100	10110110101	11001101100	
1010111x110	10001110000		
1110110x010	10101110110		

Thus obtained path matrix **C** describes all the objects within a graph with nodes of the 1st and 2nd level, i.e. paths and perimeters. The length of a path equals the number of „1” entries in a row of the matrix **C**, and, as such, the number of branches in paths and perimeters described by means of a given row of the matrix **C**.

Path matrix **C**:

Path no.	Branch no.											Path length	
	<u>1</u>	<u>2</u>	<u>3</u>	<u>4</u>	<u>5</u>	<u>6</u>	<u>7</u>	<u>8</u>	<u>9</u>	<u>10</u>	<u>11</u>		
1	1	1	1	1	0	0	0	0	0	0	0	0	4
2	1	0	0	0	1	1	1	0	0	0	0	0	4
3	1	1	1	0	1	0	0	0	0	1	1	1	6
4	1	0	0	1	0	1	1	0	0	1	1	1	6
5	1	1	0	0	1	0	0	1	0	1	0	1	5
6	1	0	1	1	0	1	1	0	1	0	1	0	7
7	1	1	0	1	0	0	0	1	1	0	1	0	5
8	1	0	1	0	1	1	1	0	1	1	0	1	7
9	1	0	0	1	0	1	1	0	0	1	0	1	5
10	1	1	1	1	0	1	0	1	0	0	1	0	7
11	1	0	0	1	0	0	1	1	0	1	0	1	5
12	1	1	1	0	1	1	0	1	0	1	0	1	7
13	1	0	1	1	0	0	1	1	1	0	0	1	6
14	1	1	0	0	1	1	0	1	1	1	0	0	6

Figure 2 shows a path and perimeter described by means of row 8 in the matrix **C**:

In order to eliminate rows with perimeters from the matrix **C** and find the diagonal air slits, it is necessary to create lists of path branches. To this end, the list of branches from the graph in Fig. 1 is presented in the form of a row matrix **L**, whose elements are pairs of numbers (no. of the start node of the branch and no. of the end node of the branch). The elements of this

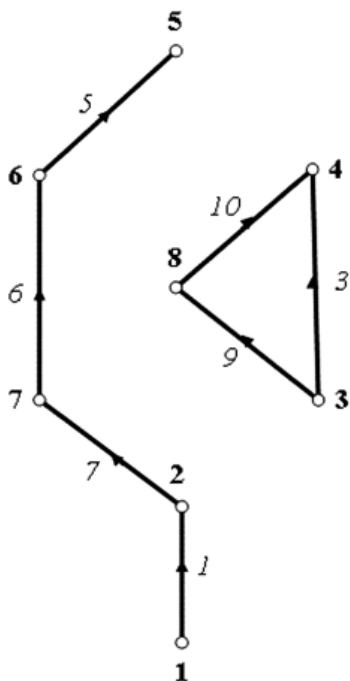


Fig. 2. Path and perimeter

matrix are then multiplied by the corresponding elements in the rows of the path matrix $C: I_n \times c_{k,n}$, where n – branch number, k – path number. The end result is lists of path branches shown in a chart below, where the numbers of columns are the numbers of branches, and the numbers of rows are the numbers of paths. Subsequently, moving along the path from the start node (1) to the end node (5), branches in a path are counted – and the branches where the movement occurs in the opposite direction, i.e. from the end node to the start node of a branch, are marked (in the chart below, such instances have been underlined). In the column X of the chart below, the number of branches in a row/the number of branches in a path is given. If the first number is bigger than the second number, it means that a given row describes a path and one or more perimeters. These rows should be removed from the path matrix, and the underlined branches should be assigned value -1 in the path matrix.

The lists of branches in paths

Path no.	1	2	3	4	5	6	7	8	9	10	11	X
1	1, 2	2, 3	3, 4	4, 5	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	4/4
2	1, 2	0, 0	0, 0	0, 0	6, 5	7, 6	2, 7	0, 0	0, 0	0, 0	0, 0	4/4
3	1, 2	2, 3	3, 4	0, 0	6, 5	0, 0	0, 0	0, 0	0, 0	<u>8, 4</u>	8, 6	6/6
4	1, 2	0, 0	0, 0	4, 5	0, 0	7, 6	2, 7	0, 0	0, 0	8, 4	<u>8, 6</u>	6/6
5	1, 2	2, 3	0, 0	0, 0	6, 5	0, 0	0, 0	0, 0	3, 8	0, 0	8, 6	5/5
6	1, 2	0, 0	3, 4	4, 5	0, 0	7, 6	2, 7	0, 0	<u>3, 8</u>	0, 0	<u>8, 6</u>	7/7
7	1, 2	2, 3	0, 0	4, 5	0, 0	0, 0	0, 0	0, 0	3, 8	8, 4	0, 0	5/5
8	1, 2	0, 0	3, 4	0, 0	6, 5	7, 6	2, 7	0, 0	3, 8	8, 4	0, 0	7/4
9	1, 2	0, 0	0, 0	0, 0	6, 5	0, 0	2, 7	7, 8	0, 0	0, 0	8, 6	5/5
10	1, 2	2, 3	3, 4	4, 5	0, 0	7, 6	0, 0	7, 8	0, 0	0, 0	8, 6	7/4
11	1, 2	0, 0	0, 0	4, 5	0, 0	0, 6	2, 7	7, 8	0, 0	8, 4	0, 0	5/5
12	1, 2	2, 3	3, 4	0, 0	6, 5	7, 6	0, 0	<u>7, 8</u>	0, 0	<u>8, 4</u>	0, 0	7/7
13	1, 2	0, 0	3, 4	4, 5	0, 0	0, 0	2, 7	7, 8	<u>3, 8</u>	0, 0	0, 0	6/6
14	1, 2	2, 3	0, 0	0, 0	6, 5	7, 6	0, 0	<u>7, 8</u>	3, 8	0, 0	0, 0	6/6

Below, the path matrix corrected accordingly is shown:

Path no.	Branch no.										
	1	2	3	4	5	6	7	8	9	10	11
1	1	1	1	1	0	0	0	0	0	0	0
2	1	0	0	0	1	1	1	0	0	0	0
3	1	1	1	0	1	0	0	0	0	-1	-1
4	1	0	0	1	0	1	1	0	0	1	-1
5	1	1	0	0	1	0	0	0	1	0	1
6	1	0	1	1	0	1	1	0	-1	0	-1
7	1	1	0	1	0	0	0	0	1	1	0
8	1	0	0	0	1	0	1	1	0	0	1
9	1	0	0	1	0	0	1	1	0	1	0
10	1	1	1	0	1	1	0	-1	0	-1	0
11	1	0	1	1	0	0	1	1	-1	0	0
12	1	1	0	0	1	1	0	-1	1	0	0

Now, it is enough to check which columns of the oriented matrix contain „1” entries of opposite signs. The numbers of these columns are the numbers of diagonal branches. Here, these are the branches 8, 9, 10, and 11.

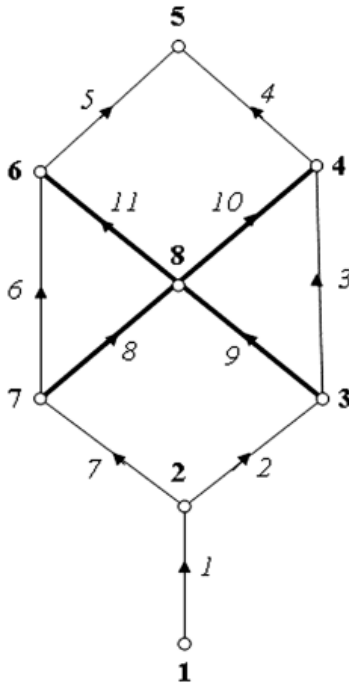


Fig. 3. Diagonal branches no. 8, 9, 10, 11

Developing an algorithm and a module of the *Ventgraph* computer program, based on the method for determining diagonal branches discussed above, will make it possible to determine such branches in large ventilation networks. It will also provide an alternative to the software using the traditional DFS method (Szlązak et al., 1998).

The study was accomplished as part of the statutory research conducted in 2013 at the Strata Mechanics Research Institute of the Polish Academy of Sciences, and financed by the Ministry of Science and Higher Education.

References

- Bystroń H., 1957. *Wpływ elementów sieci i pożaru na kierunki prądów powietrznych w kopalni*. Przegląd Górniczy, Nr 7-8.
- Bystroń H., 1958. *Sposób kreślenia kanonicznych schematów przewietrzania*. Przegląd górniczy, Nr 3.
- Budryk W., 1961. *Wentylacja kopalń cz. I. Przewietrzanie wyrobisk*. EGH, Katowice.
- Czczot H., 1925. *Teoria prądów przekątnych*. Prace AG w Krakowie.
- Czczot H., 1957. *Metoda badań i wyniki prac nad teorią wentylacji*. Wybór pism H. Czczotta, oprac. J. Litwiniszyn. PWN, Warszawa.
- Czczot H., 1957. *Przewietrzanie kopalni trzema szymbami*. Wybór pism H. Czczotta, oprac. S. Knothe, J. Bodziony, PWN, Warszawa.
- Dziurzyński W., Krach A., Pałka T., 2014. *Computer simulation of the propagation of heat in abandoned workings insulated with slurries and mineral substances*. Arch. Min. Sci., Vol. 59, No 1, p. 3-23.
- Kolarczyk M., 1993. *Wpływ struktury kopalnianej sieci wentylacyjnej na wrażliwości prądów powietrza przy zmianach oporów bocznic*. Zeszyty Naukowe Politechniki Śląskiej, Górnictwo, zeszyt 214, Gliwice.
- Kolarczyk M., 1994. *Bocznic normalne i przekątne w kopalnianej sieci wentylacyjnej, metoda wyznaczania*. XX Dni Techniki ROW'94.
- Kolarczyk M., 1998. *Bocznic normalne i przekątne w zredukowanych płaskich kopalnianych sieciach wentylacyjnych*. Szkoła Eksploatacji Podziemnej 98.
- Kolarczyk M., 2008. *The analogy between the conditions concerning airflow outputs sensitivity signs to changes in the resistance of side branches and H. Czczot's conditions concerning the flow direction in a simple diagonal network*. Arch. Min. Sci., Vol. 53, No 2, p. 270-292.
- Kolarczyk M., Gawenda A., Pach G., 2002. *Uwagi o możliwości oceny trwałości kierunku prądów przekątnych z wykorzystaniem wrażliwości wydatków powietrza na zmiany oporów bocznic*. 2 Szkoła Aerologii Górniczej, październik 2002.
- Narsingh Deo, 1980. *Teoria grafów i jej zastosowania w technice i informatyce*. PWN, Warszawa.
- Strumiński A., Madeja-Strumińska B., 2006. *Wrażliwość prądów powietrznych na zmianę kierunków przepływu w czasie pożaru podziemnego*. Rudy i metale nieżelazne, nr 10, s. 577.
- Szlązak N., Jian L., Borowski M., Obracaj D., 1998. *Numerical determination of diagonal branches in mining ventilation networks*. Arch. Min. Sci., Vol. 43 No 4, s. 270.

Received: 14 May 2014