METHOD OF DETERMINING THE TRIAXIAL STRESS STATE IN THE ROCK MASS WITH DIRECTED HYDROFRACTURING

METODA OKREŚLANIA TRÓJOSIOWEGO STANU NAPRĘŻENIA W GÓRÓTWARZE Z UŻYCIEM UKIERUNKOWANEGO HYDROSZCZELINOWANIA

Stress state in the rock mass determines design of mine workings, their proper maintenance, and safety of the personnel working there. That is why numerous methods of determining stress state in the rock mass have been devised. One of the categories of such methods are hydraulic ones. The most commonly known methods of hydrofracturing and the HTPF have certain limitations resulting from their assumptions. The presented method based on directed hydrofracturing of rocks enables determining values and directions of principal stresses in the rock mass, independently on the complexity of local stress state. The essence of the method, conditions of conducting tests and the method of calculating components of stress state basing on underground measurements are presented.

Keywords: stress state in the rock mass, directed hydrofracturing of rocks, determining stresses with directed hydrofracturing


Słowa kluczowe: stan naprężenia w górotworze, ukierunkowane hydro szczelinowanie skala, określanie naprężeń z użyciem ukierunkowanego hydro szczelinowania
1. Introduction

Knowledge of stress state in the rock mass is one of more important factors influencing how mining works are designed and then conducted. It is particularly important if the designed mining activities go deep or take place in the area of disturbed geological structure. Asymmetric stress state has significant influence on the values of failure and post-peak failure parameters of coal and rocks (Bukowska et al., 2007; Bukowska, 2013) and thereby the state of rockburst hazard (Bukowka, 2012; Kabiesz, 2012; Kabiesz et al., 2013). It also could be used to properly design of mining workings support (Brodny, 2010, 2011).

That is why there have been developed numerous methods of determining stresses in the rock mass. Basing on Amadei and Stephansson’s systematics (2007) and the ISRM works (Ljunggren et al., 2003), the methods of determining stresses can be divided into the following groups:

1. Hydraulic methods:
   - hydraulic fracturing,
   - sleeve fracturing,
   - hydraulic tests of pre-existing fractures, HTPF,
   - directed rock hydro fracturing method N-UHS.

2. Relief methods:
   - surface relief method,
   - methods of relieving in boreholes:
     - Overcoring: CSIR and CSIRO cell,
     - USBM,
   - methods of relieving large rock volume.

3. Jacking methods:
   - flat jack method,
   - curved jack method.

4. Strain recovery methods:
   - anelastic strain recovery method, ASR,
   - differential strain curve analysis method, DSCA.

5. Method of analyses of fractures inside a borehole.

6. Other:
   - tectonics analyses,
   - analyses of tremor source mechanisms,
   - indirect methods,
   - inclusions in materials of time dependant characteristics,
   - measurement of residual stresses.

The underlined methods are the ones recommended by the International Society of Rock Mechanics (ISRM).

Among the commonly known and acknowledged methods there is one, employing directed hydrofracturing of rocks to determine stresses, which has been constantly developed at the Central Mining Institute since the early 2000s. The basics of the method were presented by Makówka (2006), its further development was signalled by Makówka (2010) and then by Makówka and Drzewiecki (2011). Its latest version was presented by Makówka (2014).
2. Classical method of hydrofracturing and the HTPF method

Determining stresses with directed hydrofracturing of rocks is a method similar to the classical method of hydrofracturing, yet as far as the measurement principles are concerned it is similar to the hydraulic test on pre-existing fracture (HTPF) method. In the classical hydrofracturing method the test is conducted in a vertical hole (i.e.: in a borehole drilled towards the most principal stress $\sigma_1$), where a selected section of the hole is sealed and fluid (most often water) is injected into such an area under appropriately high pressure. Increasing pressure of the fluid leads to a decrease in circumferential stress on the walls of the hole until it reaches opposite values, i.e. tensile stress on the part of borehole wall of the least principal stress $\sigma_3$. It determines the direction of the stress. Value of the stress is determined in the second (or any further) injection cycle based on fracture reopening pressure and closing pressure of the fracture induced in the first injection cycle.

In the HTPF method fluid is injected into fractures intersecting the borehole, pre-existing in the rock mass and intersecting it at different angles. Injecting the fluid into selected sections of the borehole enables opening them, which then enables determining the component of stress in the rock mass acting perpendicularly towards the plane of a given fracture.

Hydrofracturing method has significant limitations as it requires drilling a test borehole following the direction of the most principal stress. Hence the direction ought to be either known or assumed beforehand, while the HTPF method requires a borehole to find sufficient number of fractures of various orientations. That is why using the methods it is difficult to determine the full stress tensor – or alternatively: values and directions of principal stresses. Directed hydrofracturing (N-UHS) may be a solution to the problem.

3. Idea and conditions necessary to determine principal stresses with directed hydrofracturing

The prerequisite of determining triaxial stress state with directed hydrofracturing is conducting a series of measurements of closing pressure of fractures induced perpendicularly towards the axes of boreholes of differentiated spatial orientation. In each of the boreholes a starting notch is made, i.e. a kind of a cut in the plane perpendicular towards the axis of the borehole, as it is shown in Figure 1.

Then the section with a starter notch ought to be isolated with the proper packer and injected with the fluid until the rocks breaks. Injecting the fluid ought to be repeated in at least one cycle to determine fracture closing pressure. Value of the pressure is identified as a component of stress acting perpendicularly to the plane of the induced fracture. An ideal course of changes in pressure and flow of the injected fluid is presented in Figure 2.

The assumptions made and detailed conditions of taking measurements with the N-UHS method are presented by Makówka (2014). Boreholes for hydrofracturing ought to be located in the rock mass free of existing discontinuities, which can be verified with an endoscope camera. Depending on the type of a packer, fracturing can be conducted at the bottom of the hole, which enables using a single packer. Because of the risk of ejecting such a packer from the borehole once it loses grip of the borehole walls, it is recommended to conduct hydrofracturing at some distance from the bottom with a straddle packer, as it is safer. In such a situation it is possible
Fig. 1. Simplified schematics of N-UHS measuring system
1 – mobile pump, 2 – high-pressure hoses, 3 – valves, 4 – manometer, 5 – recorder, 6 – signal cable, 7 – drain hose, 8 – pressure and flow sensors, 9 – cable/high-pressure hose, 10 – packer (Makówka, 2014)

Fig. 2. Simplified graph of pressure and flow in hydrofracturing process with two injection cycles (Makówka, 2006)

$P_i =$ fracture initiation pressure
$P_f =$ fracture reopening pressure
$P_c =$ fracture closing pressure
$P_w =$ water pressure in pores

Cycle 1
Cycle 2
to make more measurements in different sections of boreholes. In one test site there ought to be
drilled at least five boreholes of different orientations. The boreholes ought to be long enough to
place a starter notch beyond the influence of a roadway where the boreholes were drilled from.
There are possible various layouts of boreholes in a test site. The simplest and most convenient
one to make is the divergent borehole grid. The best concentration of the volume of the tested
rock mass is obtained with the convergent grid, where bottoms of boreholes are located close
to one another.

During hydrofracturing it is necessary to record current changes in pressure and transient
flow of the injected liquid. Recording the flow is helpful in determining fracture opening pressure
and fracture closing pressure, yet having gained sufficient proficiency in interpreting the course
of changes in values of the pressure it is not necessary.

For each of the boreholes it is necessary to determine vertical and horizontal deflection
angle of the borehole from the locally assumed reference system. Together with fracture closing
pressure the data form the basis for calculating values and directions of principal stresses in the
volume of the rock mass, where the tests were conducted.

4. Determining principal stresses

Determining spatial orientation of principal stresses requires assuming the way of translat-
ing the arbitrarily chosen coordinate system into a system of any orientation. Jaeger, Cook and
Zimmerman (2007) present a translation method with a rotation of zenith angle $\theta$ and horizontal
angle $\lambda$.

May Oxyz coordinate system be a system of principal stresses (Fig. 3). May Px’y’z’ co-
ordinate system be a coordinate system converted in the way presented in Figure 3. In the new
coordinate system Pz’ axis jest rotated by angle $\theta$, Px’ axis is in the plane of OPz; and Py’ axis
substitutes the system to obtain the right-hand coordinate system and it follows the direction of
build up of angle $\lambda$. The component of versor coordinate system, expressed with angles $\theta$ and $\lambda$,
determining the direction of Pz’ axis is given as:

$$\mathbf{e}_{z'} = \begin{pmatrix} \sin \theta \cos \lambda & \sin \theta \sin \lambda & \cos \theta \end{pmatrix}$$

The direction of Px’ axis may be determined through substituting angle $\theta$ with $\theta + \pi/2$, to
obtain the component of versor

$$\mathbf{e}_{x'} = \begin{pmatrix} \cos \theta \cos \lambda & \cos \theta \sin \lambda & -\sin \theta \end{pmatrix}$$

Versor of Py’ axis is perpendicular to Oz axis and forms $\lambda + \pi/2$ with Ox as well as angle $\lambda$
with Oy, thus

$$\mathbf{e}_{y'} = \begin{pmatrix} -\sin \lambda & \cos \lambda & 0 \end{pmatrix}$$

Total versor of the new coordinate system looks as follows

$$\mathbf{e}_{x'} = \begin{pmatrix} \cos \theta \cos \lambda & \cos \theta \sin \lambda & -\sin \theta \\ -\sin \lambda & \cos \lambda & 0 \\ \sin \theta \cos \lambda & \sin \theta \sin \lambda & \cos \theta \end{pmatrix}$$
Applying the method of converting a coordinate system in the conditions of the coordinate system assumed at the very beginning requires reverse translation. May angles $\phi$ and $\gamma$ mean respectively angles of horizontal deflection and elevation, relating to angles $\lambda$ and $\theta$ – as it is shown in Figure 4:

$$\gamma = \pi/2 - \theta$$

and

$$\phi = -\lambda$$

it means that if:

$$\sin\gamma = \sin (\pi/2 - \theta) = \cos\theta$$
$$\cos\gamma = \cos (\pi/2 - \theta) = \sin\theta$$
$$\sin\theta = \sin (\pi/2 - \gamma) = \cos\gamma$$
$$\cos\theta = \sin (\pi/2 - \gamma) = \sin\gamma$$

and

$$\sin\phi = \sin (-\lambda) = -\sin\lambda$$
$$\cos\phi = \cos (-\lambda) = \cos\lambda$$
$$\sin\lambda = \sin (-\phi) = \cos\phi$$
$$\cos\lambda = \sin (-\phi) = \sin\phi$$

Fig. 3. Initial coordinate system and coordinate system after with angles $\theta$ and $\lambda$ (Jaeger et al., 2007)

Fig. 4. Coordinate system to convert Px’y’z’ into Oxyz
then solving (4) with (5) and (6) we obtain

\[
f = \begin{bmatrix}
    \cos\left(\frac{\pi}{2} - \gamma\right) \cos(-\phi) & \cos\left(\frac{\pi}{2} - \gamma\right) \sin(-\phi) & -\sin\left(\frac{\pi}{2} - \gamma\right) \\
    -\sin(-\phi) & \cos(-\phi) & 0 \\
    \sin\left(\frac{\pi}{2} - \gamma\right) \cos(-\phi) & \sin\left(\frac{\pi}{2} - \gamma\right) \sin(-\phi) & \cos\left(\frac{\pi}{2} - \gamma\right)
\end{bmatrix}
\]  

(9)

which, after simplifying considering dependences (7) and (8), enables obtaining versor of conversion from Px’y’z’ system into Oxyz one (as it is shown in Figure 5):

\[
f = \begin{bmatrix}
    \sin\gamma \cdot \cos\phi & -\sin\gamma \cdot \sin\phi & -\cos\gamma \\
    \sin\phi & \cos\phi & 0 \\
    \cos\gamma \cdot \cos\phi & -\cos\gamma \cdot \sin\phi & \sin\gamma
\end{bmatrix}
\]  

(10)

Converting tensor of stresses from the reference system Px’y’z’ into Oxyz one we obtain

\[
\sigma = f \cdot \sigma' \cdot f^T
\]

(11)

If we assume, that the tensor of stress s is tensor in Oxyz coordinate system, and in Px’y’z’ coordinate system tensor s’ as follows

\[
\sigma' = \begin{bmatrix}
    \sigma_1 & 0 & 0 \\
    0 & \sigma_2 & 0 \\
    0 & 0 & \sigma_3
\end{bmatrix}
\]

(12)

where \(\sigma_1, \sigma_2, \sigma_3\) are principal stresses of disordered values, then after conversions we obtain the following dependences of components of stress tensor:

\[
\begin{align*}
\sigma_{xx} &= \sin^2\gamma \left(\sigma_1 \cos^2\phi + \sigma_2 \sin^2\phi\right) + \sigma_3 \cos^2\gamma \\
\sigma_{yy} &= \sigma_1 \sin^2\phi + \sigma_2 \cos^2\phi \\
\sigma_{zz} &= \left(\sigma_1 \cos^2\phi + \sigma_2 \sin^2\phi\right) \cos^2\gamma + \sigma_3 \sin^2\gamma \\
\tau_{xy} &= \sin\gamma \sin\phi \cos\phi \left(\sigma_1 - \sigma_2\right) \\
\tau_{xz} &= \sin\gamma \cos\phi \left(\sigma_1 \cos^2\phi + \sigma_2 \sin^2\phi\right) - \sigma_3 \sin\gamma \cos\phi \\
\tau_{yz} &= \left(\sigma_1 - \sigma_2\right) \cos\gamma \sin\phi \cos\phi
\end{align*}
\]

(13)

Analogically to the coordinate system associated with a mine working, a coordinate system can be associated with a borehole oriented with respect to a mine working described with angles \(\gamma_1, \phi_1\) – Figure 6.
If in the rock mass with stress state defined in such a way a borehole is drilled and then, in the plane perpendicular to the borehole, a discontinuity is induced with the N-UHS method, then compressive strength will act perpendicularly to the plane, which may be identified as fracture closing pressure of the induced discontinuity

\[ P^{O1} = n_x^{O1} \cdot \sigma_x + n_y^{O1} \cdot \sigma_y + n_z^{O1} \cdot \sigma_z \]  \hspace{1cm} (14)

where:

\[ \sigma_x = \begin{bmatrix} \sigma_{xx} & \tau_{xy} & \tau_{xz} \end{bmatrix}^T \]
\[ \sigma_y = \begin{bmatrix} \tau_{yx} & \sigma_{yy} & \tau_{yz} \end{bmatrix}^T \]
\[ \sigma_z = \begin{bmatrix} \tau_{zx} & \tau_{zy} & \sigma_{zz} \end{bmatrix}^T \]  \hspace{1cm} (15)

\[ \tau_{xy} = \tau_{yx}, \tau_{xz} = \tau_{zx}, \tau_{yz} = \tau_{zy} \], whereas versor of the borehole direction, determined with angles \( \phi_1 \) and \( \gamma_1 \), is expressed as:

\[ n_x^{O1} = \cos \phi_1 \cos \gamma_1 \]
\[ n_y^{O1} = \cos \phi_1 \sin \gamma_1 \]
\[ n_z^{O1} = \sin \gamma_1 \]  \hspace{1cm} (16)

To determine stress state expressed with principal stresses \( \sigma_1, \sigma_2, \sigma_3 \) and angles describing their orientation with respect to local reference system \( \gamma \) and \( \phi \), it is necessary to provide five independently measured data \( P_s^{O1}, P_s^{O2}, \ldots, P_s^{O5} \) as well as the appropriate angles determining orientation of boreholes, where the values of pressure \( \gamma_n \) and \( \phi_n \) were measured:
\[ P_s^{O1} = n_x^{O1} \cdot \sigma_x + n_y^{O1} \cdot \sigma_y + n_z^{O1} \cdot \sigma_z \]
\[ P_s^{O2} = n_x^{O2} \cdot \sigma_x + n_y^{O2} \cdot \sigma_y + n_z^{O2} \cdot \sigma_z \]
\[ P_s^{O3} = n_x^{O3} \cdot \sigma_x + n_y^{O3} \cdot \sigma_y + n_z^{O3} \cdot \sigma_z \]
\[ P_s^{O4} = n_x^{O4} \cdot \sigma_x + n_y^{O4} \cdot \sigma_y + n_z^{O4} \cdot \sigma_z \]
\[ P_s^{O5} = n_x^{O5} \cdot \sigma_x + n_y^{O5} \cdot \sigma_y + n_z^{O5} \cdot \sigma_z \] (17)

where \( n_x^{Oi}, n_y^{Oi}, n_z^{Oi} \) mean components of versor determining directions of further boreholes of angles \( \gamma_i \) and \( \phi_i \). Conversion of stress tensor from the assumed coordinate system into a coordinate system associated with principal stresses is possible with one of methods known from classical mechanics. Two of them are presented by Makówka (2014).

The presented above method of calculating directions and values of principal stresses does not assume vertical direction of the biggest principal stress, thus it can be applied in the conditions of highly disturbed stress state, in the vicinity of faults or other dislocations.

If fewer measurements are available (e.g. failed measurements), it is possible to make assumptions concerning direction and/or value of one of components of stress state. It reduces the number of equations in a system (17) and enables applying a system of equations which is easier to calculate, known from an earlier version of the method presented by Makówka (2006).

5. Sample measurement

To determine stresses with UHS, although it is relatively easy from the technical point of view, it is necessary to meet certain geological, mining and technical requirements to make appropriate number of measurements of the right pressure. In given conditions of underground mine workings they are hard to meet.

A few measurements have been already made with the presented method. In Wujek coal mine a series of measurements involved five sites with, in total, 56 boreholes. Fracture closing pressure was properly determined in 23 of them. It enabled calculating 10 sets of values and directions of principal stresses in the version assuming vertical direction of the most principal stress s1. The results were thoroughly discussed in the article (Makówka, 2006).

Another measurement with upgraded equipment was made in Rydułtowy-Anna coal mine. The tests were conducted in area N-E2 of seam 713/1-2. The measurement site was situated in incline II 1200-E2, at the depth of 1180m. Approximately 700 m from the site in 2009 and 2010 there were two rockbursts. Their course and phenomena accompanying them made the specialists who analyse them look for unusual causes. One of them was a probable unusual and asymmetric state of stress, caused by sedimentation disturbances in the seam and rocks surrounding it and nearby NNE-SSW syncline axis. As a result the coal mine agreed on measurements of the state of stress.

Due to the above mentioned structural irregularities the tests turned out to be hard to conduct properly. It was particularly hard to find an appropriate section of the rock mass where the boreholes would go through homogeneous rock for at least 2 metres. Such conditions were obtained in three boreholes and appropriate measurements were made there.

The pattern of boreholes drilled at the measurement site is presented in Figure 1. The boreholes where interpretable data were collected are shaded in the figure.
Out of nine boreholes drilled there, values of fracture closing pressure $P_s'$ were obtained in measurements in three of them no.: 2, 3, 4. Table 1 presents values of the pressure and angles of elevation and deflection of boreholes where the measurements were made.

**TABLE 1**

<table>
<thead>
<tr>
<th>Borehole</th>
<th>Fracture closing pressure, $P_s'$</th>
<th>Angle of horizontal deflection of borehole from perpendicular to sidewall, $\phi$</th>
<th>Angle of elevation, $\gamma$</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>28.2 MPa</td>
<td>$7.7^\circ$</td>
<td>$58^\circ$</td>
</tr>
<tr>
<td>3</td>
<td>25.6 MPa</td>
<td>$37.7^\circ$</td>
<td>$58^\circ$</td>
</tr>
<tr>
<td>4</td>
<td>20.5 MPa</td>
<td>$-26^\circ$</td>
<td>$43^\circ$</td>
</tr>
</tbody>
</table>
Such a range of data does not enable full determination of values and directions of principal stresses. However, assuming that the most principal stress acts vertically, and its value results from the seam depth at the measurement site, i.e. 32.1 MPa, it is possible to use a dependence from the article (Makówka, 2006), simplified as follows:

\[
\begin{align*}
    P_s & = \left( S_H \cos^2 \phi + S_h \sin^2 \phi \right) \sin^2 \gamma + S_V \cos^2 \gamma \\
    P_{s1} & = \left( S_H \cos^2 (\phi + \Delta \phi_1) + S_h \sin^2 (\phi + \Delta \phi_1) \right) \sin^2 (\gamma + \Delta \gamma_1) + S_V \cos^2 (\gamma + \Delta \gamma_1) \\
    P_{s2} & = \left( S_H \cos^2 (\phi + \Delta \phi_2) + S_h \sin^2 (\phi + \Delta \phi_2) \right) \sin^2 (\gamma + \Delta \gamma_2) + S_V \cos^2 (\gamma + \Delta \gamma_2)
\end{align*}
\]

where $\phi$ and $\gamma$ are angles describing borehole 2, $P_s$ – fracture closing pressure in borehole 2, and values $\phi_1$ and $\gamma_1$ and $\phi_2$ and $\gamma_2$ refer respectively to boreholes 3 and 4.

The simplifications enabled obtaining the following results:
- value of the most principal horizontal stress $S_H = 26.88$ MPa,
- value of the least principal horizontal stress $S_H = 2.015$ MPa,
- angle of horizontal deflection from the direction perpendicular to the sidewall $\phi = 6.55^\circ$, i.e. $20.3^\circ$ from North. The results are presented in graphic form in Figure 8.

![Fig. 8. Results of determining values and directions of stresses in Rydultowy-Anna coal mine](image)

It must be remembered that the obtained and presented above results of determining values and directions of horizontal stresses are burdened with a potential error which is an effect of assuming the direction and value of the most principal stress, as a vertical one. It is worth noting that the direction of the most principal horizontal stress follows the direction of the nearby syncline axis.
6. Summary

The presented method of determining stresses in the rock mass originates from the known methods of hydrofracturing with a vertical measurement borehole and the HTPF method. Thanks to the fact that the fracture is made in arbitrarily selected direction (perpendicular towards a borehole of any orientation), it is possible to overcome limitations of the above mentioned known methods. Additionally, technical regime of the in situ tests enables conducting them by qualified personnel of a mine. Then the results should be sent to the research centre for further interpretation and analyses. Difficulties in solving systems of equations which enable calculating values and directions of principal stresses, which nowadays prompt use of numerical methods, make it necessary to simplify the calculation method. It is also necessary to confirm the method through further in situ tests.

Knowledge of the state of stresses in the rock mass where mining operations are conducted is an issue of crucial importance for stability of mine workings, especially the very deep ones. It refers both to the personnel’s safety aspect and the economic one as workings driven along the direction of the most principal stress are easier to maintain for a long time (Mark, 1991; Mark & Mucho, 1998). So far in Polish coal and copper ore mining industry mainly the vertical component of stress has been in the centre of attention. In copper ore mining industry the importance of knowing horizontal stresses has been recently noticed and in new sections of deposits there are attempts made to adjust deposit exploitation to the directions of horizontal stresses (Butra & Pytel, 1998; Butra et al., 2011; Fabjanczyk, 1997). Coal mining industry is yet to take such steps, as the articles suggest (Tajduś et al., 2004; Pawelus, 2008). In the presented case of Rydultowy-Anna coal mine, one of the possible causes of the rockbursts was a change of the direction of development workings and driving longwalls. The direction of the most principal stress is roughly perpendicular to the workings in area N-E2, and it is unfavourable from the maintenance point of view. The coal mine plans mining works in the part of the seam and driving new roadways and longwall panels following the probable direction of the stress, which may be more beneficial and safer for them.

References


Makówka J., 2010. *Interpretacja i wstępne wyniki badań nad zastosowaniem ukierunkowanego hydroszczelinowania skal do określania naprężeń w the rock mass. (Measures of interpretation and preliminary results of investigations over directed hydrofracturing use to determine stresses in rock mass).* Kwartalnik Akademii Górniczo-Hutniczej „Górnictwo i Geologia”, zeszyt 2, rok 34, Kraków, p. 463-470


Received: 13 February 2015