Application of evaluationary approach to thermo-mechanical optimization of gas turbine airfoil cooling configuration

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Abstract   Cooling of the hot gas path components plays a key role in modern gas turbines. It allows, due to efficiency reasons, to operate the machines with temperature exceeding components' melting point. The cooling system however brings about some disadvantages as well. If so, we need to enforce the positive effects of cooling and diminish the drawbacks, which influence the reliability of components and the whole machine. To solve such a task we have to perform an optimization which makes it possible to reach the desired goal. The task is approached in the 3D configuration. The search process is performed by means of the evolutionary approach with floating-point representation of design variables. Each cooling structure candidate is evaluated on the basis of thermo-mechanical FEM computations done with Ansys via automatically generated script file. These computations are parallelized. The results are compared with the reference case which is the C3X airfoil and they show a potential stored in the cooling system. Appropriate passage distribution makes it possible to improve the operation condition for highly loaded components. Application of evolutionary approach, although most suitable for such problems, is time consuming, so more advanced approach (Conjugate Heat Transfer) requires huge computational power. The analysis is based on original procedure which involves optimization of size and location of internal cooling passages of cylindrical shape within the airfoil. All the channels can freely move within the airfoil cross section and also their number can change. Such a procedure is original.

Keywords: Turbine airfoil; Optimization; Evolutionary algorithm; Airfoil cooling

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Nomenclature

A	–	passage cross-sectional area, m^2
I	–	individual
C	–	constant value parameter
c_p	–	specific heat, J/kgK
f	–	objective function
F	–	fitness function
h	–	convective heat transfer coefficient, W/m^{2}K
$\dot{m}$	–	mass flow rate, kg/s
Nu	–	Nusselt number
Pr	–	Prandtl number
r	–	passage diameter, mm
Re	–	Reynolds number
T	–	temperature, K
x, y, z	–	space coordinates, m
σ	–	stress, MPa

Subscripts

c	–	coolant
max	–	maximum
i	–	nodal value
in	–	inlet

Superscripts

k	–	iteration (generation) number
1	–	principal (stress)
eqv	–	von Mises stress

1   Introduction

The increase of effectiveness of a gas turbine cycle can be obtained by a higher temperature at the inlet of the expander, a selection of adequate compression and an optimization of the cycle structure (intercasing cooling, sequential burning, regeneration, etc.). The increase of temperature is however one of the main goals of engineering activity. The development is possible due to the progress in material engineering as well as the application of cooling systems for hot gas path components. The practice of raising the inlet temperature is still common with aircraft engines and stationary turbines [14]. The cooling of the hot components (blades and vanes) is usually provided by the compressor bleed air flowing through internal passages whose shape is sometimes fairly complex. The heat from the component is mostly transferred into the coolant via convection.

The problem of cooling system optimization seems to be quite new and has been the point of interest only in the last few years. This is due to the high computational costs of such problems. Although several authors dealt with the problem in question, very few considered solving the task in 3D. Some background of cooling system optimization has been given by Dulikravich *et al.* [4]. This work discuses different approaches and algorithms to the problem and presents a solution of a multidisciplinary optimization of internally cooled blade with shaped passages and an outflow of the coolant at the trailing edge. Later Martin and Dulikravich [10] presented the aero-thermo-elastic optimization problem of a convective cooled blade. The computational procedures are described as well as the use of various objective functions and constraints. Dennis *et al.* [2] described the problem of cooling passages optimization with the use of a genetic algorithm. The authors optimized the cooling structure with respect to the heat flux transferred into the blade, which indirectly minimized the coolant consumption. Since the genetic approach is time consuming, the authors introduced the parallel computations. A very interesting seems to be another work of Dennis *et al.* [3], where the optimization of serpentine-like cooling passage was presented. This task was realized in 3D with the thermo-mechanical criteria involved. The aim of the optimization was to find some shape parameters of the internal cooling passages as well as the heat transfer conditions within those channels. The results showed large potential hidden in a cooling system; appropriate size and location of channels had a great influence on the coolant usage, thermal stress level and in consequence lifetime of the component. Bucchieri *et al.* [1] presented a paper dealing with optimization of the tip cooling holes of a gas turbine rotor blade. The design process aimed to find such a configuration of cooling passages which provided, on one hand the minimum coolant usage and wall heat flux and, on the other, maximum blade efficiency. Haasenritter and Weigand [8] studied the optimization problem of the rib structure within internal cooling channels as a tool for heat transfer intensification. The problem was to select the optimal dimensions and distance between neighbouring ribs. Another work [5] dealt with the optimization design parameters for the impingement cooling. The task was to find the four geometrical design variables defining the cooling structure in question. Talya *et al.* [15] took film cooling into consideration in the process of the profile shape optimization, but its structure was fixed and so were the inner channels, which were only scaled according to the size and shape of the profile. Nowak and Wroblewski [13] analysed

problem of thermal optimization where the temperature and thermal gradients were minimized. This paper extends the problem with mechanical response of the material to the presence of cooling. Verstraete *et al.* [16] also took on the problem and they focused on introducing the CHT calculations into the optimization. The authors worked out an Artificial Neural Network and Radial Basis Function modules whose aim was a rough but quick estimation of concurrent solutions. Then full CHT analysis was made for selected candidates.

Optimization of internal cooling prodided with a network of passages was the subject of the paper presented by Gonzalez *et al.* [7]. The authors carried out the calculations for four levels of fractal branching cooling channels, where the number of branches within a specific level was optimized. The design process was based on the hybrid multi-objective optimization which took into consideration the thermo-fluid objectives, which were determined in a quasi 1D analysis.

The solution to the full CHT problems together with optimization tasks seem to be the next step in the process of cooling system development. Nevertheless, at present simultaneous solution to both problems is unlikely due to high computational costs of such a job.

In this paper one of the optimization problems is discussed. It deals with the search of the optimal distribution and size of circular cooling passages within a turbine vane from the point of view of the thermo-mechanical criteria. It assumed that the search process was to be performed by means of an evolutionary algorithm and FEM. Such a selection of optimization method is brought about due to its essential advantage, namely its capability of searching for the absolute extreme [6]. The optimization problems dealing with turbine blade cooling systems usually involved quite high number of design variables as well as complex, implicit and susceptible to slight parameter changes, response (function) of the system in question. This response in a form of temperature, stress or other parameters was not monotonic but usually included the large number of local extremes which was a significant problem for many gradient-based optimization methods [12]. In such cases, the stochastic methods were preferred, which was observed in almost all works mentioned before. The most common was the evolutionary approach which apart from some disadvantages, was suitable to solve complex technical problems.

Since the cooling system is described by a significant number of design variables (location and radius of each passage), the objective function can

have many local extremes located far from the optimal solution. A disadvantage (for the user) of the evolutionary approach is its time consumption, especially in the case of large problems where other computational tools [FEM (Finite Element Method), CFD (Computational Fluid Dynamics), BEM (Boundary Element Method) etc.] for fitness determination are involved.

It is planned that the devised and tested optimization procedure would be a module in a general design tool of blade cooling. Similar problem was stated by Zecchi *et al.* [17], but without the optimization module.

2   Optimization technique

The problem discussed in this paper involves three types of criteria: thermal, stress and economical. On one hand the temperature of the airfoil is to be reduced. Unfortunately such an action leads towards higher thermal gradients and in consequence thermal stress. On the other hand, from the point of view of material durability, it is necessary to keep the stress as low as possible. The second criterion is in a charge of it. Additionally the economical objective tends to reduce the coolant usage. There are two most common approaches to solve the optimization problems with many criteria involved. One is to deal with the task as a multi-objective one, which leads to a set of Pareto optimal solutions, whereas the other is based on combining the decision vector into a single-objective scalar function. The latter approach has been chosen and the Single Objective Evolutionary Algorithm is used for the optimization search. The reason is that the single-objective algorithm usually requires less geometries to be evaluated to reach the optimum. This process is automatically coupled with the FEM software Ansys utilized for the computation of each cooling configuration. These two parts are connected with a module for an automatic FEM model generation and result data processing. Evolutionary search originates from the genetic algorithm with the difference of design variable treatment. Genetic algorithm operates on the binary coded variables, whereas the evolutionary approach uses the floating-point representation. According both to the literature [11] and the experience gained so far, the latter method is more relevant while continuous technical problems are considered. The reason is that the binary representation, although rooted in biology, is discrete. A definition of such coding depends on the original variable domain and a number of bits used for coding (in the problem in question at least 8, and preferably 16 bits

per variable are required). So, in the case of robust optimization with several design variables the binary representation consists of a very long string of bits.

The optimization based on the evolutionary algorithm combine the evolutionary principle of survival of an individual which is best adapted with the systematic, though randomised exchange of information [6]. The essence of the genetic algorithm is to get, in subsequent generations, individuals with higher and higher values of the fitness function, which is the measure of their quality. To introduce a change in an individual some biological processes take place in time. These are the selection, crossover and mutation. The evolution process in a population comprised of abstract individuals, prepared for the optimization problem needs, takes place to produce better individuals in the successive populations till the optimal solution (best fitted individual) is reached. So the first step in the evolutionary search is the adequate preparation of the individual, which should include all the design variables. The design variables are a set of floating-point coded numbers which compose the chromosome of a single individual. From the mathematical point of view the chromosome is a vector built of the design variables

$$I = [a_1, a_2, a_3, \ldots a_n] \quad (a_i \to \Re) . \tag{1}$$

Evolutionary algorithm tends to find the best adopted individual by maximizing the fitness function defined.

The population composed of a number of individuals, where the size of the population depends on the problem in question, is then subject to the genetic operations. Such operations change the genotype (in our case equal to the chromosome) of the members, which in consequence changes their fitness. The main genetic operations have a very simple mathematical representation and their aim is to mimic the actual biological behaviour of organisms:

- *Crossover* — two individuals (parents) are sampled (with some selection criteria) and their genotype is combined to produce offspring. This operation is usually based on weighted averaging of the variables,

$[a_1, a_2, ...]$ $\qquad\qquad$ $[m_1 a_1 + (1 - m_1) b_1, m_2 a_2 + (1 - m_2) b_2, ...]$

$\xrightarrow{\text{crossover}}$

$[b_1, b_2, ...]$ $\qquad\qquad$ $[(1 - m_1) a_1 + m_1 b_1, (1 - m_2) a_2 + m_2 b_2, ...]$.

$(m_i \to \{0, 1\})$

- *Mutation* — each variable in the chromosome can be, with some probability, randomly disturbed with ε, which is some fraction (user defined) of its value,

$$[a_1, a_2, ...] \quad \xrightarrow{\text{mutuation}} \quad [a_1 + \varepsilon_1, a_2 + \varepsilon_2, ...] \ .$$

- *Selection* — process which selects individuals for reproduction (crossover) and creates the succeeding population. In current work the roulette wheel method is applied, where the probability of survival to the next generation is proportional to individual's fitness. At this stage a care is taken to keep the best fitted individuals within the population. The processes of crossover and mutation are controlled by the user defined probabilities of each process to take place.

3   Test case

The aim of the present optimization process is to find the optimal location and size (i.e. radius of cooling hole) of cooling passages within a gas turbine airfoil. The problem is solved for the well known C3X airfoil taken from literature [9], which was extensively investigated by NASA. The vane profile is assumed to be aerodynamically optimal and fixed during the computation process. The report of the NASA research has been the source of information for the purpose of this work. The vane in question is originally a convective cooled one with ten internal passages as presented in Fig. 1. The airfoil was investigated within a linear 3-vane cascade supplied with exhaust gases from a burner. Cooling of the 76.2 mm height vane was provided with air. The vane was fabricated of ASTM 310 stainless steel.

For the profile model presented in Fig. 1 the optimization calculation was started for the original number of cooling passages, whose location and size varied. During the search process the number of cooling channels could change. It was decided to keep the cylindrical shape of each passage as it was in the reference case. The problem was solved for 3-D configuration which made it possible to take into account thermal gradients and in consequence the thermal stress in the radial direction. This stress component would depend strictly on the cooling intensity.

Each individual of the population represented the cooling system configuration. Assumed in this work cylindrical passage shape this configuration

Figure 1. Airfoil (C3X) geometry.

contains two space coordinates (Cartesian) and the radius of each passage

$$I = [x_1, y_1, r_1, x_2, y_2, r_2, ..., x_n, y_n, r_n] \ , \qquad (2)$$

so an individual was composed of 30 floating-point design variables. Although the number of cooling passages could vary the total length of the individual's chromosome was the algorithm parameters and remained fixed in the search process. Passages which did not meet the minimum radius constraint were excluded from the FEM model, but they could reappear anytime in the successive generations. The objective function was defined in the domain of the airfoil cross-section, so the constraints needed to be properly determined. The cooling passages may freely move within the blade domain, but for each individual all of them must be located inside the vane. Even a single passage which violated the boundaries made the individual discredited for further calculations. Also sufficient distance from the boundary has to be provided as well as the mutual distance between the channels (Tab. 1).

Since the optimization domain is rather complex it was necessary to introduce and operate in a local curvilinear coordinate system (ξ, η) connected with the airfoil (Fig. 2). The local system was normalised to the interval (0,1), where ξ defined the location along the pressure wall and η described

Table 1. Domain constraints.

Location	airfoil cross section
Min radius	0.8 mm
Max radius	6 mm
Min distance between passages	$r_i + r_{i+1} + 2$ mm
Min distance of passage from wall	$r_i + 1$ mm

the position as a fraction of actual airfoil thickness. The whole algorithm operated with the local system and only for the needs of FEM computations the design variables were transformed into the global coordinates.
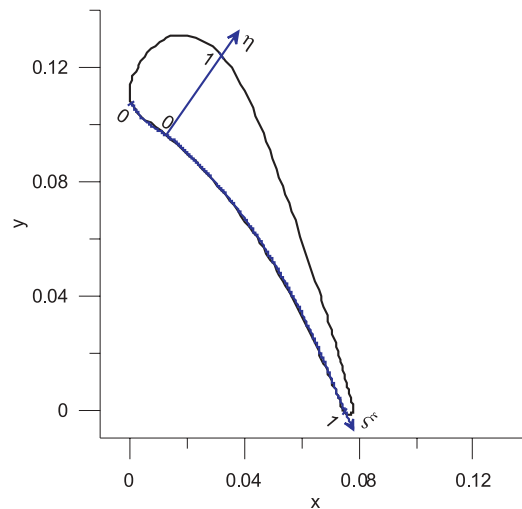

Figure 2. Local coordinate system.

At the beginning of the calculation process the base (initial) population was sampled. The procedure leading to the production of a single individual was performed in such a manner that the passages were successively sampled with the constraint criteria being satisfied. So the whole base population was in the range of fitness function. To determine the fitness function value for each vane candidate the chromosome was decoded in order to obtain the location and size of each channel. Then a command file for FEM software was prepared and the analysis was performed giving the temperature distribution and resulting thermal stress within the vane. Those two quantities were then used for the fitness value calculation. This process was repeated

for each population member. After that, the population was subjected to
genetic operations and in consequence a new offspring population was ob-
tained. Then the constraints for each individual of the children population
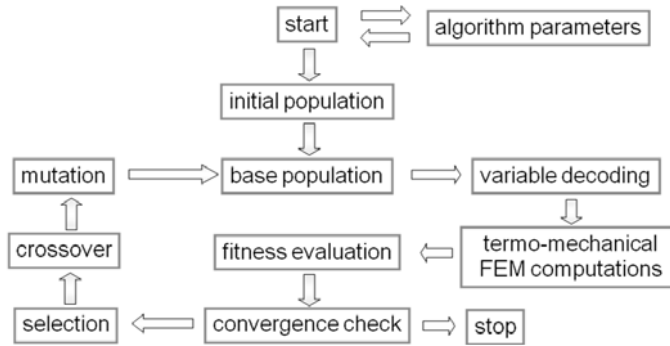were checked and the process was repeated (Fig. 3).

Figure 3. Algorithm flow chart.

The thermal boundary conditions at the profile were taken from the ex-
periment report [9]. They were determined at 68 locations along the vane
profile cross section and were linearly approximated between these points.
Since the experiment was performed in 2D at the midspan of the vane, we
assumed constant parameters along the vane height. The justification for
such an assumption could be the small height of the vane in question.

The determination of the boundary conditions in the cooling passages
was very important for the thermal and strength analysis and should take
into account three-dimensional character of the airfoil. They were estab-
lished on the basis of inviscid, adiabatic pipe-flow. It assumed that all the
passages are fed from a single plenum at total pressure $p = 0.3$ MPa and
temperature $T_{in} = 375$ K and the static pressure at the outlet was assumed
$p = 0.27$ MPa for each passage The coolant flow is assumed to be laminar.
Results of the flow calculations made it possible to determine the convec-
tive heat transfer coefficient. Its distribution with pipe diameter, which was
worked out on the basis of relationships between Re, Pr and Nu numbers [9]

$$\text{Nu} = 0.023\text{Re}^{0.8}\text{Pr}^{0.4} \tag{3}$$

is shown in Fig. 4. Coolant temperature change along the passage was
evaluated on the basis of the relationship defining fluid heating up in a pipe

of constant temperature:

$$T(z) = (T_{in} - T_{wall}) \exp \left(-\frac{h\pi d}{\dot{m}c_p} z \right) + T_{wall} , \qquad (4)$$

where z is the coordinate along the pipe. Coolant temperature at the outlet of cooling passage as a function of diameter and wall temperature is given in Fig. 5.
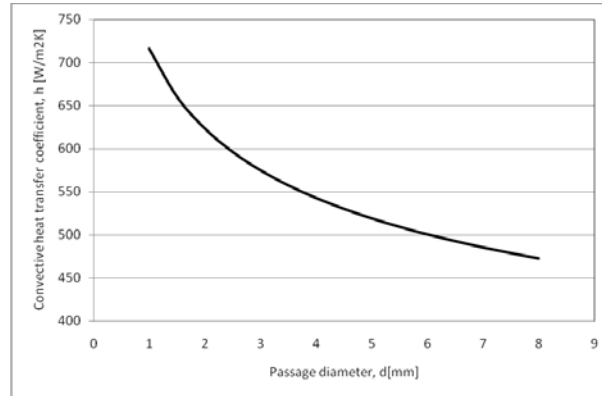

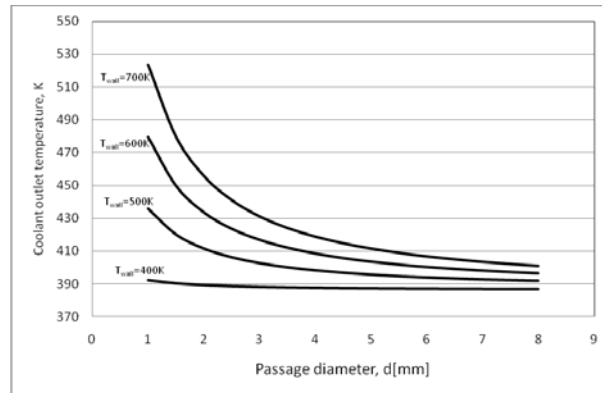Figure 4. Convective heat transfer coefficient in function of passage diameter.

Figure 5. Coolant output temperature with respect to passage diameter and wall temperature.

Coolant temperature variations along the passage depend on current passage wall temperature, which came from FEM evaluation for the best individual in the last generation.

4   Numerical tests

Some test numerical calculations dealing with the optimization of cooling structure within the C3X airfoil has been performed. The aim of the analysis was to find best possible configuration of cooling passages on the basis of the criteria posed. The calculation should shed some light on the optimization method performance and its convenience to such problems.

4.1   Optimization objective

Turbine component cooling is provided due to keep its temperature below the allowable limit. This is because of the safety and reliability reasons. On the other hand, the presence of cooling introduces a negative impact on turbine performance and thermal load of cooled component. So, in order to meet the requirements mentioned the objective function composed of airfoil temperature and resulting thermal stresses, as well as size of passages is proposed:

$$
f = \begin{cases} \sum_i C_i \left(\frac{Y_{imax} - Y_{aim}}{Y_{aim}} \right) + 1 & \text{if} \quad Y_{max} > Y_{aim} \\ 1 & \text{if} \quad Y_{max} \leq Y_{aim} \end{cases} , \tag{5}
$$

where

$$
Y_i = \{T_i, \sigma_i^1, \sigma_i^{eqv}, A\} ,
$$

and T_i stands for temperature at node i, whereas σ_i^1, σ_i^{eqv} represents the first principle and von Mises stress, respectively and A is the total cross-sectional area of the passages. Constant C_i is a weight factor. Index aim means the target value of variable Y. The C multiplier makes it possible to adjust the terms of the function to each other. The values of the specific parameters are:

$$
T_t = 650 \text{ K} , \ \sigma_t^1 = 200 \text{ MPa} , \ \sigma_t^{eqv} = 200 \text{ MPa} ,
$$
$$
A = 200 \text{ mm}^2 , \ C_i = \left\{ \frac{1}{2}, \frac{1}{6}, \frac{1}{6}, \frac{1}{6} \right\} . \tag{6}
$$

The objective function of the above defined form keeps both the airfoil maximum temperature and the thermal stress both principal and von Mises at the desired level with simultaneous reduction of the heat exchange area. The last parameter indirectly moderates coolant usage, which can have a positive influence on turbine performance.

For the needs of mechanical calculations the vane in question was supported on its lower end in the radial direction, whereas constraints in the horizontal plane were applied in two, carefully selected points, which did not lead to a "numerical" stress concentration. Their only aim was to prevent a free body movement. The tip of the vane was free to deform.

Since the genetic algorithm searches for the best fitted (maximum fitness value) solution, the fitness function (F) was defined as $1/f$ in order to achieve the desired goal. Individuals who did not satisfy the constraints (the solution was not feasible) were "punished" by exclusion from the population.

The domain for the optimization process was defined in a geometrical manner (Tab. 1), which meant that the individuals (cooling passage size and location) had to be feasible. All the passages could have been located anywhere within the airfoil cross-section unless they were too close to the vane wall or to each other. In such cases the automatic FE model generation failed and the so severe "punishment" was necessary.

To maintain the best solution (modification of individuals due to genetic operations) obtained from the previous generation the 10% of the best fitted individuals were copied into the successive population. The iteration process was terminated when the convergence criteria were met, usually if fitness function variations adhered to a certain limit for an assumed number of generations. Due to the domain complexity (airfoil shape) a particular care had to be taken to run the search procedure effectively, which was mentioned before.

The numerical model for each cooling configuration was generated automatically by a user defined module. The model was discretized automatically with assumed element size, which had been approved on the basis of mesh testing. The airfoil contour was divided into 202 elements, which produced the total number of elements within the blade at the range 20–30k DOF for thermal calculations and triple that much for stress evaluation, depending on the passage configuration. Finer mesh did not improve the results. The time to perform a single FEM computation was about 50–60 seconds. Taking into account the number of cases required by the algorithm it took 5–6 days for the whole optimization. Application of parallel computing (4 threads) realized on a single PC with Pentium 4 Quad 1.8 GHz and 8GB RAM made it possible to almost linear reduction of time to about 30–36 hours.

4.2   Optimization results

Several runs of the same algorithm were performed for the assumed objective function. Each run gave a little different result in terms of thermal stress and temperature field, and in consequence the fitness value. It of course resulted in slightly different passage distributions. This was because of the non-deterministic character of the genetic search where the final solution was within a certain range, which was the method accuracy. The precision of the method was in turns dependant on the objective function form (its complexity). It has to be stressed that different configurations of the objective function components can result in the same final value of the whole function. Nevertheless, the optimization outcome of each run was very similar.
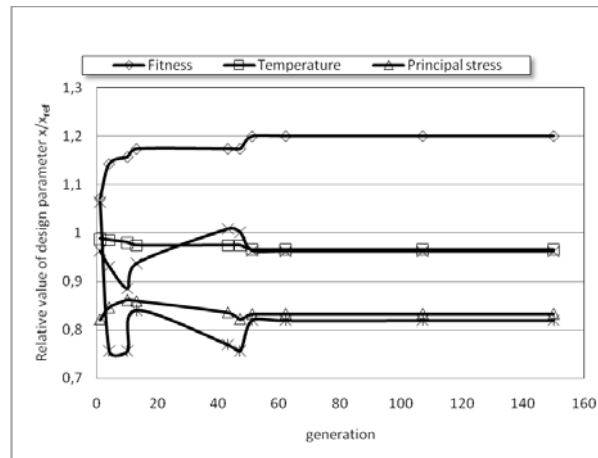

Figure 6. Relative fitness value and design parameters in the course of optimization process.

The results obtained were juxtaposed to the original vane treated as a reference. Figure 6 presents the relative value of the design parameter changes in the course of optimization whereas Figs. 7, 8 and 9 show the contour plots of these values for the optimized and reference case. The optimization lead to a redistribution of the cooling passages and changes in their size. The new configuration has two channels of bigger diameter that are responsible for cooling of hottest parts of the vane (Fig. 7). They are close to the vane wall, where the maximum temperature occur. Other passages are generally smaller than in the reference airfoil. There is also

less cooling in the trailing edge, which makes the temperature distribution more uniform and in consequence reduces the thermal gradients. The optimization showed that the vane in question required more cooling in its front part and less cooling at the trailing edge area.
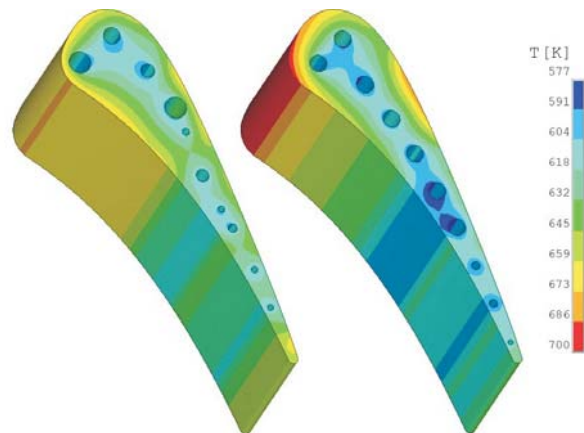

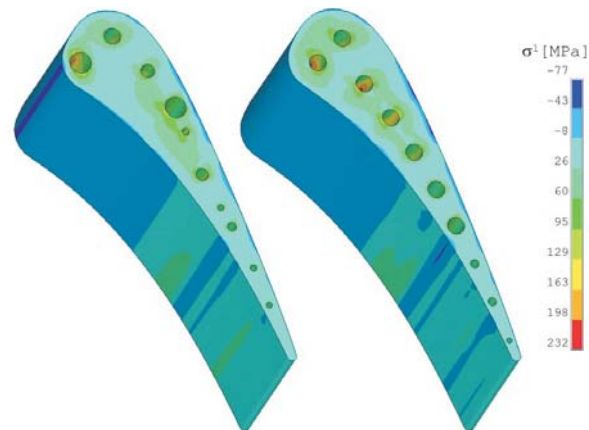Figure 7. Temperature distribution for the optimized and reference case.

Figure 8. Principal stress (σ^1) distribution for the optimized and reference case.

The results showed that relocation of the cooling passages within the vane could reduce temperature level by over 20 K and principal stress by 50 MPa. Only a slight moderation of von Mises stress was observed. It should be also pointed out that the total cross-sectional area of all pas-
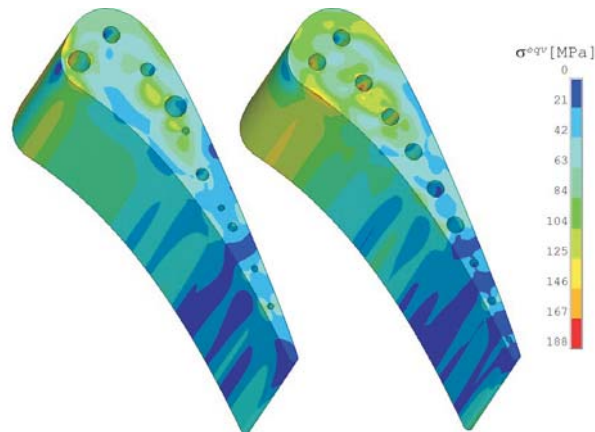
Figure 9. Von Mises stress (σ^{eqv}) distribution for the optimized and reference case.

sages was smaller as well (193.8 mm^2 comparing to 236.4 mm^2 for the reference case) which resulted in lower coolant usage by roughly 20%. Lower temperature of the airfoil was a consequence of the proximity of passages to the wall especially at the leading edge, where maximum of temperature occurred. Maximum stress was always located at a passage wall but in each case a different one. The original cooling system produced both principal and von Mises maximum stress within the third (counting from the leading edge) passage, whereas after optimization they occurred in the forth (principal) and the first (von Mises) one. The total improvement in the airfoil performance measured in terms of the fitness value (Fig. 6) could be estimated as 20%. Since there is lower coolant usage, and the average temperature in the solid is higher, a heat flux from the hot gas decreased.

5   Conclusions

The paper presents some aspects of an optimization procedure utilizing the evolutionary algorithm for searching the best possible cooling system configuration. The thermo-mechanical field predictions were determined by means of FEM software. For the purpose of this research a convective cooled vane was assumed and the optimization consisted in the relocation and diameter changes of the cooling passages. The search procedure was run as a Single Objective Algorithm but the fitness function entailed three different criteria types: one was to minimize the blade maximum temper-

ature, next tended to reduce the thermal stress and the last one was in a charge of coolant usage moderation. The calculations took into account the 3D nature of cooling which was involved in the thermal boundary conditions. Applied optimization strategy is said to be one of the best tools for the global optimum search especially in the case of problems with complex geometries and a large number of design variables. Apart from the general advantages of this optimization method, the present research disclosed significant difficulties, which have to be overcome to use this search process successfully. Namely, a complex shape of the blade domain in connection with numerous design variables.

Since the cooling system significantly influences both the efficiency and lifetime of modern gas turbines it seems to be desirable to conduct further analyses on this topic by development of optimization methods and giving consideration to the important factors of cooling performance.

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