Optimum heating of cylindrical pressure components weakened by holes

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Abstract   A method for determining time-optimum medium temperature changes is presented. The heating of the pressure elements will be conducted so that the circumferential stress caused by pressure and fluid temperature variations at the edge of the opening at the point of stress concentration, do not exceed the allowable value. In contrast to present standards, two points at the edge of the opening are taken into consideration. The first point, P_1, is located at the cross section and the second, P_2, at the longitudinal section of the vessel. It will be shown that the optimum temperature courses should be determined with respect to the total circumferential stress at the point P_2, and not, as in the existing standards due to the stress at the point P_1. Optimum fluid temperature changes are assumed in the form of simple time functions. For practical reasons the optimum temperature in the ramp form is preferred. It is possible to increase the fluid temperature stepwise at the beginning of the heating process and then increase the fluid temperature with the constant rate. Allowing stepwise fluid temperature increase at the beginning of heating ensures that the heating time of a thick-walled component is shorter than heating time resulting from the calculations according to EN 12952-3 European Standard.

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1   Introduction

Optimization of heating and cooling of thick boiler components is the subject of many studies [1–4], since too rapid heating or cooling element causes high thermal stresses [5]. The paper presents a new method of determining the optimum fluid temperature changes during heating and cooling of thick walled pressure vessels weakened by holes. Optimum temperature curve is determined from the condition that the total circumferential stress, caused by the thermal load and pressure, at the edge of the hole at the point P_2 (Fig. 1) is equal to the permissible stress. Current standards limit the boiler
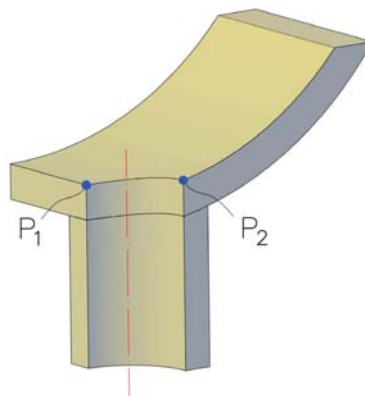

Figure 1. Pressure vessel – connector junction.

heating rate taking into account the stress at the point P_1 (Fig. 1), because at this point there is the greatest concentration of stresses from the pressure. However, during pressure vessel heating, the stresses due to pressure are tensile while the stresses from the thermal load are compressive and they compensate each other.

At the same heating rate of the pressure element during the boiler start-up, total circumferential or equivalent stress at the point P_1 is smaller than the corresponding stresses at the point P_2. This is due to much lower concentration of stress from the pressure at the point P_2. In determining the optimum heating rate or the optimum time changes of fluid temperature in the vessel when with temperature increases the pressure, one must take into account the point P_2.

2   Mathematical formulation of the problem

The previous optimization analysis shows [1] that the optimum fluid temperature changes, $T_f(t)$, obtained from the solution of the Volterra integral equation of the first kind, can be well approximated by

$$T_f = T_0 + a + b\,t + c/t\,,\tag{1}$$

where: T_0 – initial fluid temperature, a, b, c, – constants, t – time. At first, the optimum fluid temperature changes are approximated by the function $T_f(t)$ (Fig. 2)

$$T_f = T_0 + a + b\,t\,,\tag{2}$$

which can easily be carried out in practice. The symbols in Eq. (3) stand for: a – initial stepwise temperature increase, b – constant rate of fluid temperature changes (Fig. 1).

The optimum values of parameters a, b and c appearing in the function (1) or the parameters a and b in the function (3) will be determined from the condition

$$\sigma_\phi\left(\mathbf{r}_{P_2}, t_i\right) \cong \sigma_a\,, \quad i = 1, ..., n_t\,,\tag{3}$$

where: $\sigma_\phi\left(\mathbf{r}_{P_2}, t_i\right)$ – summary circumferential stress at the point P$_2$ due to pressure and thermal load, $\mathbf{r}_{P_2}$ – position vector of the point P$_2$ at the edge of the hole where the stress should be equal to the allowable stress (Fig. 1), t_i – i-th time step, n_t – number of time steps in the analyzed period of time, σ_a – allowable stress determined using current standards.
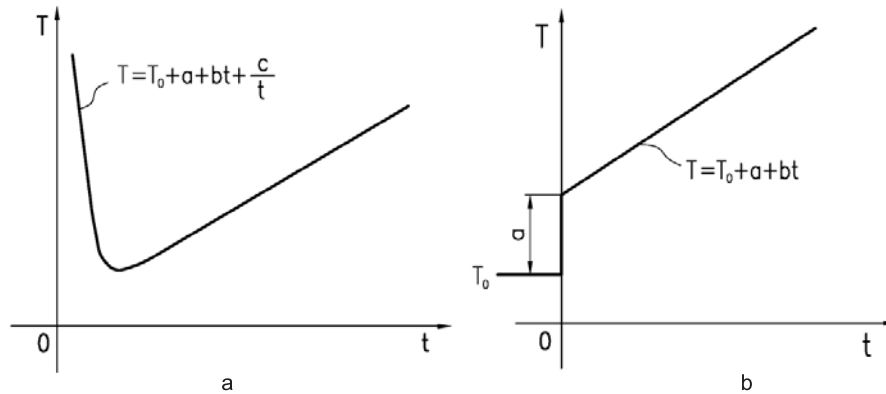

Figure 2. Functions used for approximation of optimum time changes of fluid temperature; a) function defined by Eq. (1), b) function defined by Eq. (3).

The parameters a and b will be determined by the method of least squares. The sum of squared differences of calculated stresses and allowable stresses at selected n_t times should be minimum

$$\sum_{i=1}^{n_t}\left[\int_0^{t_i} T_f\left(\theta\right)\frac{\partial\, u\left(\mathbf{r}_{P_2}, t-\theta\right)}{\partial t}d\theta + \alpha_m\left(p-p_o\right)\frac{d_{in}+s}{2s} - \sigma_a\right]^2 = \min, \quad (4)$$

where: u – thermal stress caused by stepwise temperature increase also called influence function, α_m – concentration factor for circumferential stress caused by the pressure at the point P_2 on the edge of the hole, p – absolute pressure in the drum, p_o – ambient pressure, d_{in} – inner diameter of the drum, s – drum wall thickness. Fluid temperature $T_f(\theta)$ was assumed as a function (1) or (3).

Problem of seeking a minimum of function (6) is a parametric least squares problem. Parameters $x_1 = a$, $x_2 = b, x_3 = $ c in the function (1), or parameters $x_1 = a$ and $x_2 = b$ in the function (3) are to be searched. Parameter values at which the sum of squares (6) is a minimum have been determined by Levenberg-Marquardt method. This method is a combination of two methods: the method of steepest descent and the Gauss-Newton method. At the beginning of the iterative process method of steepest descent is applied, which is slowly convergent, but allows to find a solution of the optimization problem even with not very accurate selection of initial values $x_1^{(0)}$ and $x_2^{(0)}$. To determine the values of parameters x_1 and x_2 near the minimum of the function (6) the Gauss-Newton method is applied in the next iteration steps. The advantage of this method is its high accuracy. This method can not be used from the very beginning of the iterative process, because the wrong choice of initial values of searched parameters causes the method is divergent. In the general case n-th dimensional vector of unknown parameters $\mathbf{x} = (x_1, \ldots, x_n)^T$ is determined using the following iterative procedure:

$$\mathbf{x}^{(k+1)} = \mathbf{x}^{(k)} + \delta^{(k)}, \quad (5)$$

where

$$\delta^{(k)} = \left[\left(\mathbf{J}_m^{(k)}\right)^T \mathbf{G}_f \mathbf{J}_m^{(k)} + \mu^{(k)}\mathbf{I}_n\right]^{-1}\left(\mathbf{J}^{(k)}\right)^T \mathbf{G}_f\left[\mathbf{f} - \mathbf{T}_m\left(\mathbf{x}\right)\right], \quad (6)$$

and

$$\mathbf{J}_m = \frac{\partial \mathbf{T}_m\left(\mathbf{x}\right)}{\partial \mathbf{x}^T} = \left[\left(\frac{\partial T_i\left(\mathbf{x}\right)}{\partial x_j}\right)\right]_{mxn}. \quad (7)$$

The following nomenclature has been adopted in Eqs. (6–7): $\mathbf{J}_m$ – Jacobian matrix, $\mathbf{G}_f$ – matrix of weights, μ – weighting factor, $\mathbf{I}_n$ – identity matrix. The upper index T denotes the matrix transpose, and (k) number of iteration steps. The advantage of the method of Levenberg-Marquardt is its fast convergence. The solution is usually found after a few iterations.

3 The results of calculations

Optimum fluid temperature changes during warm-up of the boiler drum with an inner diameter $d_{in} = 1700$ mm and wall thickness $s = 90$ mm were determined. The inner diameter of the downcomer is $d_{wo} = 90$ mm and wall thickness $s_o = 6$ mm. The following properties of steel were adopted for the calculation: $\lambda = 42$ W/(mK); $c = 538.5$ J/(kgK); $\rho = 7800$ kg/m^3; $E = 1.96 \times 10^{11}$ N/m^2; $\beta = 1.32 \times 10^{-5}$ 1/K, and $\nu = 0.3$. The heat transfer coefficient on the inner surface of the drum and downcomer is: $\alpha_{in} = 1000$ W/(m^{2}K). Allowable stress is: $\sigma_a = -138.7$ MPa. The outer surface of the drum and downcomer are thermally insulated. Stress concentration factor for the circumferential stress caused by the pressure at the point P_2 was determined by the finite element method (FEM) and is: $\alpha_m = 0.51$. The division analyzed drum-downcomer junction on the finite elements are shown in Fig. 3. The influence function, u, at the point P_2 is shown in Fig. 4 for different heat transfer coefficients, α_{in}, at the inner surface of drum and downcomer. This is the circumferential thermal stress at the point P_2, which has been determined using the FEM with a stepwise temperature increase by 1 K. The analysis of Fig. 4 shows that the maximum absolute value of the stress at the point P_2 is greater the larger is the value of the heat transfer coefficient, α_{in}. The influence function, u, at the point P_2 as a function of time, t, and can be approximated by a continuous function using the method of least squares. The following function approximates well the calculation results:

$$u\left(t\right) = \sigma_\phi(t) = \frac{a + ct^{0.5} + et + gt^{1.5} + it^2}{1 + bt^{0.5} + dt + ft^{1.5} + ht^2} \, , \tag{8}$$

where $u(t)$ is expressed in MPa/K, and time t in seconds.
The values of the coefficients appearing in function (8) for selected values of the heat transfer coefficient, α_{in}, are summarized in Tab. 1.

Optimum fluid temperature changes were estimated using the influence function for the heat transfer coefficient $\alpha_{in} = 1000$ W/(m^{2}K). The
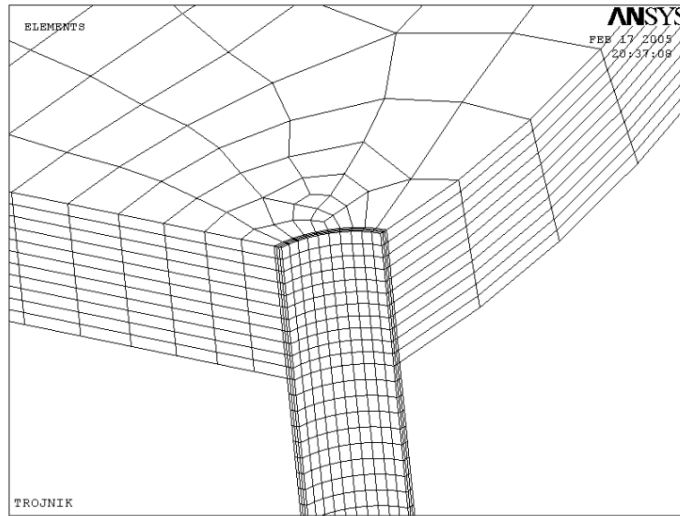
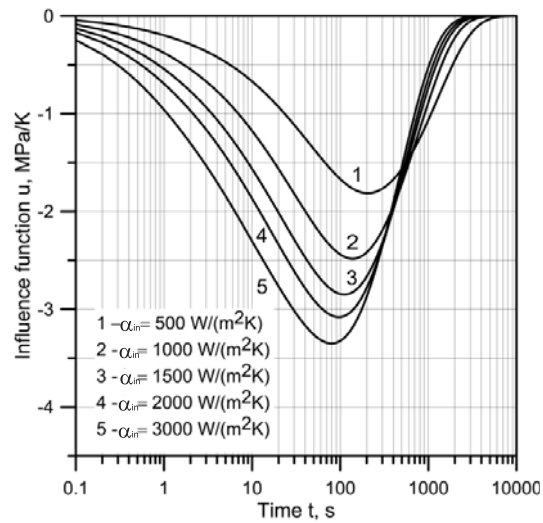Figure 3. Mesh of finite elements used for thermomechanical analysis of the drum – downcomer junction.

Figure 4. Time changes of influence function.

course of circumferential stress at the point P_2 as a function of time, which is required to apply the method of Levenberg-Marquardt, was determined using the Duhamel's integral. Duhamel's integral was evaluated by the

Table 1. Constants appearing in function (8) for different heat transfer coefficients.

α_{in}, W/(m²K)	a	b	c	d
500	0.0286681407184853	-0.0415608144447150	-0.2219682901729810	0.0048369728329664
1000	-0.0977213218969060	-0.0586092573488484	-0.3253429594268530	0.0077552803821987
1500	-0.1474428215903510	-0.0424849278266589	-0.4616645407079040	0.0074847664664757
2000	-0.0324517872429579	0.0243651192207815	-0.7325026091177960	0.0028930274058112
3000	-0.1799581418109710	0.0503498030917522	-0.9218388730309230	0.0029065544314647
α_{in}, W/(m²K)	e	f	g	h
500	0.0065091396270584	-0.0001226804747822	-0.0000644617856678	0.0000015233096532
1000	0.0103313135659350	-0.0002404002542458	-0.0001042456130844	0.0000041332088797
1500	0.0175368805692370	-0.0002409711328650	-0.0002180845439745	0.0000048699130545
2000	0.0308818887401169	-0.0000371731499695	-0.0004266156063316	0.0000025675425876
3000	0.0401831963316867	-0.0000568628281854	-0.0005699783438759	0.0000048767423716
α_{in}, W/(m²K)	i	Coefficient of determination r²		
500	0.0000002154380330	0.9999962577728130		
1000	0.0000003295808135	0.9999441165145940		
1500	0.0000008910457631	0.9999304103146090		
2000	0.0000019284582020	0.9999762462132270		
3000	0.0000026336042465	0.9999377933864930		

method of rectangles. The optimum fluid temperature changes have been determined for the pressureless state $p_n = 0$ MPa and for nominal operation pressure $p_n = 10.87$ MPa. The optimum fluid temperature changes described by function (1) are presented in Fig. 5. Figure 6 depicts the optimum fluid temperature changes approximated by the function (3). The initial jump of the temperature is 48.6 K for gauge pressure $p_n = 0$ MPa, and 51.2 K for $p_n = 10.87$ MPa. The analysis of the results illustrated in Figs. 5 and 6 indicates that the drum pressure has little effect on the optimum time changes of the fluid temperature. This is due to small value of stress concentration coefficient at the point P_2 for the stresses caused by the pressure, which is only $\alpha_m = 0.51$. Optimum fluid temperature changes approximated by the functions (1) and (3) were compared respectively for the pressure $p_n = 0$ MPa and $p_n = 10.87$ MPa in Figs. 7 and 8.

It is seen that the differences in the optimum fluid temperature changes are only in the beginning of the heating process. Plots of summary circumferential stress during the optimum heating process at the edge of the hole at points P_1 and P_2 as a function of time are presented in Figs. 9–12. During the start-up the total circumferential stress at the point P_1 caused by thermal load and the pressure is lower than at the point P_2. Small excesses over the allowable stresses at the point P_2 result from the assumed
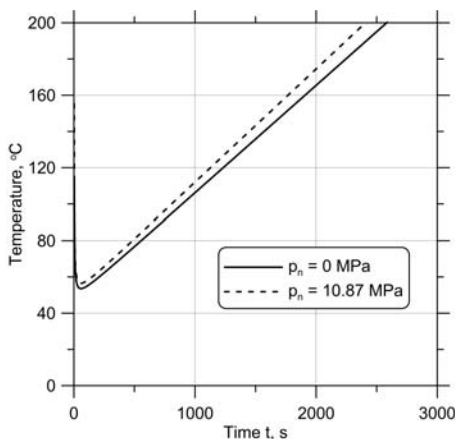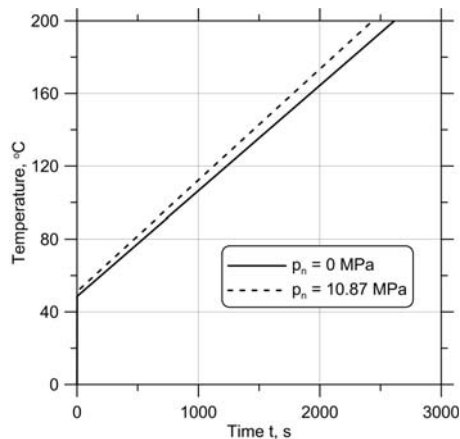
Figure 5. Optimum time changes of water temperature $T_f(t)$ in the drum approximated by function (1) $p_n = 0$ MPa ($a = 46.68\,°\mathrm{C}$, $b = 0.059\,°\mathrm{C/s}$, $c = 200.8\,°\mathrm{Cs}$) and $p_n = 10.87$ MPa ($a = 49.24\,°\mathrm{C}$, $b = 0.062\,°\mathrm{C/s}$, $c = 211.7\,°\mathrm{Cs}$).

Figure 6. Optimum time changes of water temperature $T_f(t)$ in the drum approximated by function (3) $p_n = 0$ MPa ($a = 48.56\,°\mathrm{C}$, $b = 0.057\,°\mathrm{C/s}$) and $p_n = 10.87$ MPa ($a = 41.22\,°\mathrm{C}$, $b = 0.061\,°\mathrm{C/s}$).
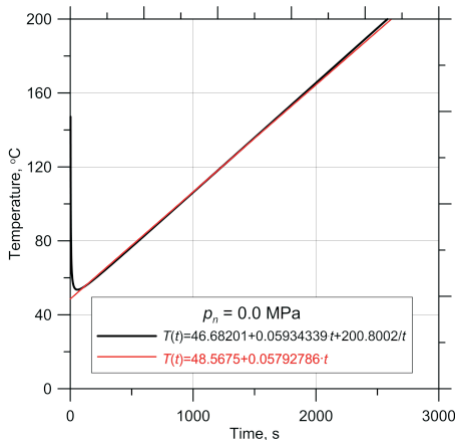
Figure 7. Optimum time changes of water temperature approximated by the function (1) or (3) for $p_n = 0$ MPa.
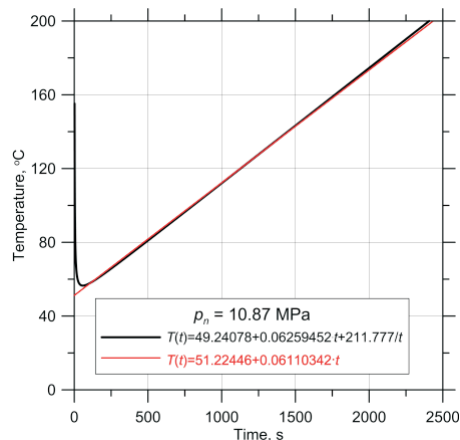
Figure 8. Optimum time changes of water temperature approximated by the function (1) or (3) for $p_n = 10.87$ MPa.
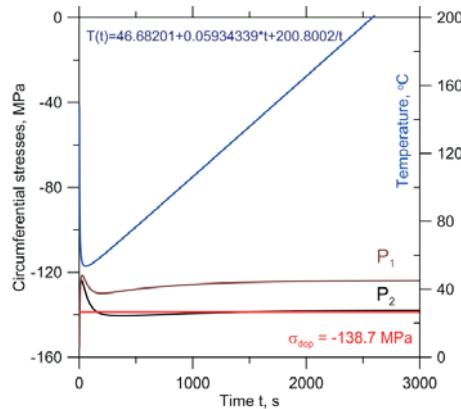
Figure 9. Total circumferential stresses due to pressure and thermal at points P_1 and P_2 during optimum drum heating for $p_n = 0$ MPa – optimum water temperature changes approximated by function (1).
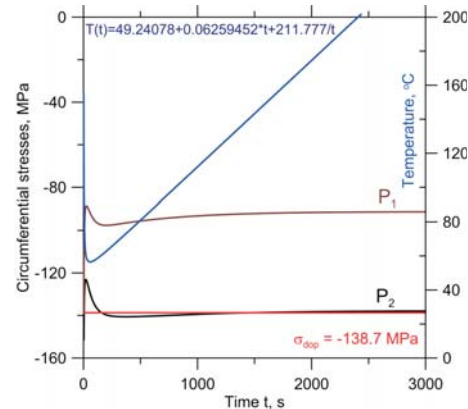
Figure 10. Total circumferential stresses due to pressure and thermal at points P_1 and P_2 during optimum drum heating for $p_n = 10.87$ MPa – optimum water temperature changes approximated by function (1).

forms of the functions given by Eq. (1) or (3). In the case of function (1) the total stress at the point P_2 are very close to the allowable stress. Only at the beginning of the heating total stresses are slightly smaller than the allowable stress. When the optimum fluid temperature is prescribed by the ramp function (3), then the allowable stress is exceeded a little more at the beginning of the heating process (Figs. 11 and 12).

This is due to too simple form of the function (3) approximating the optimum temperature changes. However, the process of optimum fluid temperature changes, which is characterized by an initial temperature jump above the initial temperature of the pressure element and further increasing the temperature witch a constant rate, is easy to implement in practice. The initial temperature jump is easy to conduct in practice by flooding the vessel with a hot water. Heating the drum with a constant rate can also be easily performed in practice. In the case of the drum boiler water temperature in the evaporator can be raised with a constant rate controlling the flow of the fuel mass supplied to the combustion chamber. From a mathematical point of view, it is possible to find a better form of the function approximating the optimum fluid temperature changes, it is however difficult to carry out in practice.
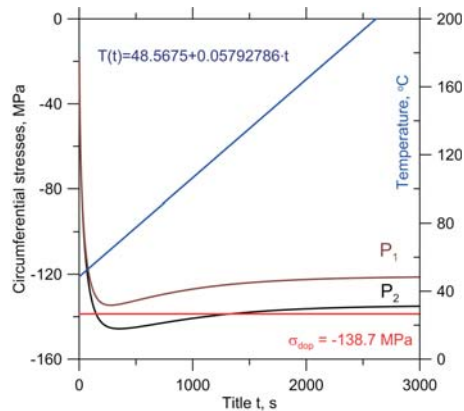
Figure 11. Total circumferential stresses due to pressure and thermal at points P_1 and P_2 during optimum drum heating for $p_n = 0$ MPa – optimum water temperature changes approximated by function (3).
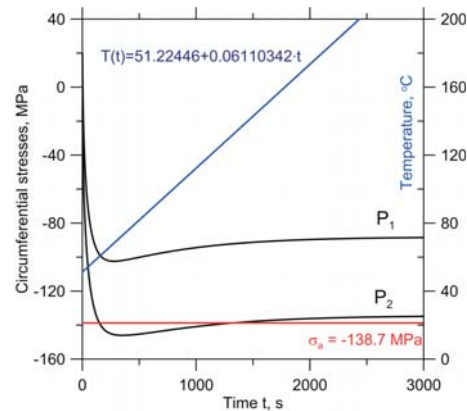
Figure 12. Total circumferential stresses due to pressure and thermal at points P_1 and P_2 during optimum drum heating for $p_n = 10.87$ MPa – optimum water temperature changes approximated by function (3).

4   Conclusions

The presented method for optimizing the start-up process can be used to determine the optimum fluid temperature during the heating steam boiler drums and pressure vessels of nuclear reactors. Because of the high thermal stresses occurring at the point P_2 at the edge of the hole, that is at the point lying on the edge of the hole in the drum cross section, this stresses affect the optimum process of heating. Compressive thermal stress at this point is compensated in a small way by the tensile stress from the pressure. The allowable fluid temperature changes during heating of thick-walled vessels should be determined due to the stress at the point P_2. Because of the possibility of practical implementation a more appropriate is the ramp function for approximating optimum fluid temperature changes.

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