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Effect of variable viscosity on free flow of non-Newtonian power-law fluids along a vertical surface with thermal stratification

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Abstract The aim of this paper is to investigate the effect of thermal stratification together with variable viscosity on free convection flow of non-Newtonian fluids along a nonisothermal semi infinite vertical plate embedded in a saturated porous medium. The governing equations of continuity, momentum and energy are transformed into nonlinear ordinary differential equations using similarity transformations and then solved by using the Runge-Kutta-Gill method along with shooting technique. Governing parameters for the problem under study are the variable viscosity, thermal stratification parameter, non-Newtonian parameter and the power-law index parameter.The velocity and temperature distributions are presented and discussed. The Nusselt number is also derived and discussed numerically.

Keywords: Free convection; Thermal stratification; Non-Newtonian; Variable viscosity; Porous medium

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Nomenclature

f	–	dimensionless stream function
g	–	gravitational acceleration, ms^{-2}
k	–	permeability, m^2
M	–	thermal stratification parameter
n	–	fluid power-law index
p	–	pressure, Pa
Ra_x	–	Rayleigh number
Nu	–	Nusselt number
T	–	temperature, K
T_0	–	reference temperature, K
T_w	–	wall temperature, K
T_∞	–	temperature of the uniform flow, K
u, v	–	velocity components, ms^{-1}
x, y	–	coordinate system, m

Greek symbols

α	–	thermal diffusivity, m^{2}s^{-1}
β	–	thermal expansion, K^{-1}
γ	–	constant defined in Eq. (7), K^{-1}
λ	–	power law index
η	–	dimensionless similarity variable
θ	–	dimensionless temperature
θ_c	–	variable viscosity defined in Eq. (19)
μ	–	viscosity, m^2/s
ν	–	kinematic viscosity, m^2/s
ρ	–	density, kgm^{-2}
ψ	–	dimensionless stream function

Subscripts

w	–	condition at the wall
∞	–	property related to the reference state

1 Introduction

In recent years heat transfer in the porous medium has received considerable attention because of its importance to geophysical, thermal engineering, geothermal system, crude oil extraction and energy related engineering problems such as thermal insulation of building, recovery of petroleum products, packed bed reactors and sensible heat storage beds etc. Kassoy [7] studied the effect of variable viscosity on the onset of convection in porous medium. Cheng and Minkowycz [4] have studied the effect of free convection about a vertical plate embedded in a porous medium. Chen and Chen [2,3] have analyzed the free convection of non-Newtonian fluids on different geometries. Nakayama and Koyama [13] investigated the effect of thermal stratification in a porous medium. Lai *et.al* [9,10] analyzed the effect of variable viscosity on convection heat transfer along a vertical surface in a saturated porous medium. Elbashbeshy [5] have studied the effect of variable viscosity and thermal diffusivity along a vertical plate. Moorthy and Govindarajulu [12] have investigated the effect of free convection flow of non-Newtonian fluids along a horizontal plate in a porous medium. Hossain *et al.* [6] have studied the conjugate free convection of non-Newtonian fluids about a vertical cylinder fin. Beithou *et al.* [1] have investigated the effect of porosity on free convection flow of non-Newtonian fluids along a vertical plate. Gorla [18] has studied the effect of mixed convection in non-Newtonian fluids along a vertical plate with variable surface heat flux in a porous medium. Kumari [8] has investigated the effect of variable viscosity effect on free and mixed convection boundary layer flow from a horizontal surface in a saturated porous medium with variable heat flux. Postelnicu *et al.* [17] investigated the effect of variable viscosity on forced convection over a horizontal flat plate in a porous medium with internal heat generation. Ali [11] studied the effect of variable viscosity on mixed convection heat transfer along a vertical moving surface. Pantokratoras [16] investigated the effect of the Falkner-Skan flow with constant wall temperature and variable viscosity. The purpose of the present work is to study the effect of thermal stratification on free convection flow of non-Newtonian fluids along a vertical plate in a porous medium with variable viscosity.

2 Analysis

The problem illustrated in Fig. 1, represents a non-Newtonian power-law fluids along a nonisothermal semi infinite vertical plate embedded in a saturated porous medium. The fluid and medium properties are assumed to be constant, except for the viscosity of the fluid. Using Boussinesq and boundary layer approximations, the governing equations of continuity, momentum and energy are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \,, \tag{1}$$

$$u^n = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} + \rho g\right) \,, \tag{2}$$

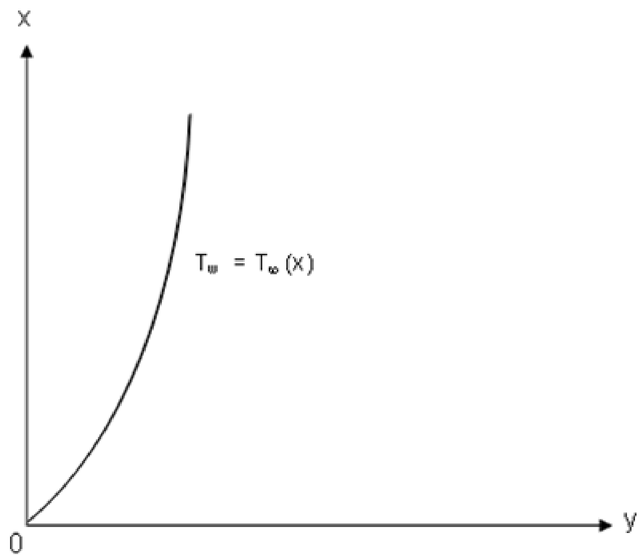

Figure 1. Physical model and its coordinate system.

$$v^n = -\frac{K}{\mu}\left(\frac{\partial p}{\partial y}\right) , \tag{3}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2} , \tag{4}$$

where

$$T_\infty (x) = (1 - M)\, T_0 + MT_w (x) \tag{5}$$

and

$$\rho = \rho_\infty \{1 - \beta (T - T_\infty)\} . \tag{6}$$

The viscosity of the fluid is assumed to be an inverse linear function of temperature and can be expressed as

$$\frac{1}{\mu} = \frac{1}{\mu_\infty}\{1 + \gamma (T - T_\infty)\} , \tag{7}$$

which is reasonable for liquids such as water and oil. Here γ is a constant.

The boundary conditions are

$$y = 0, \quad v = 0, \quad T = T_w , \tag{8}$$

$$y \to \infty, \quad u = 0, \quad T = T_\infty . \tag{9}$$

3 Method of solution

Introducing the stream function $\Psi(x, y)$ such that

$$u = \psi_y, \quad v = -\psi_x \ , \tag{10}$$

where

$$\psi = \alpha f \left(\frac{n}{\lambda + n}\right)^{\frac{1}{2}} (\mathrm{Ra}_x)^{1/2n} \ , \tag{11}$$

$$\eta = \left(\frac{\lambda + n}{n}\right)^{\frac{1}{2}} (\mathrm{Ra}_x)^{\frac{1}{2n}} \frac{y}{x} \ , \tag{12}$$

$$\mathrm{Ra}_x = \left(\frac{kg\beta\Delta T x^n}{\alpha^n \nu}\right) \ .$$

Define

$$\theta = \frac{T - T_\infty}{T_w - T_\infty} \ , \tag{13}$$

$$T_w - T_\infty = A x^\lambda \ . \tag{14}$$

Substitution of transformations (10)–(14) into equations (2)–(4) along with the equations (5)–(7), leads to nonlinear ordinary differential equations are

$$\frac{d}{d\eta}\left(f'\right)^n = \left(f'\right)^n \frac{\theta'}{\theta - \theta_c} - \left(\frac{\theta - \theta_c}{\theta_c}\right)\theta' \ , \tag{15}$$

$$\theta'' = \left(\frac{n}{\lambda + n}\right)\left(\theta + \frac{M}{1 - M}\right) f'\lambda - \frac{1}{2} f\theta' \ , \tag{16}$$

where prime symbols denote the first and second derivative with respect to time, together with the boundary conditions

$$\eta = 0, \quad f = 0, \theta = 1 \ , \tag{17}$$

$$\eta \to \infty, \quad f' = 0, \theta = 0 \ , \tag{18}$$

where

$$\theta_c = -\frac{1}{\gamma\left(T_w - T_\infty\right)} \tag{19}$$

is the parameter characterizing the influence of temperature on viscosity. For a given temperature difference, large values of θ_c implies a negligible effect of viscosity variation and vice versa. Since the viscosity of liquids decreases with increasing temperature while it increases for gases, θ_c is negative for liquids and positive for gases.

Equations (15) and (16) are integrated numerically by using the Runge-Kutta-Gill method along with the shooting technique.

The heat transfer coefficient in terms of the Nusselt and Rayleigh numbers is given by

$$\theta'(0) = \frac{\mathrm{Nu}}{\left(\frac{\lambda+n}{n}\right)^{\frac{1}{2}} (\mathrm{Ra}_x)^{\frac{1}{2n}}} \,. \tag{20}$$

4 Results and discussion

The velocity and temperature profiles are presented in Figs. (2) to (9). It is seen in Figs. (2) and Fig. (3) that as λ increases, the velocity and temperature decrease for given values of M, n and θ_c.

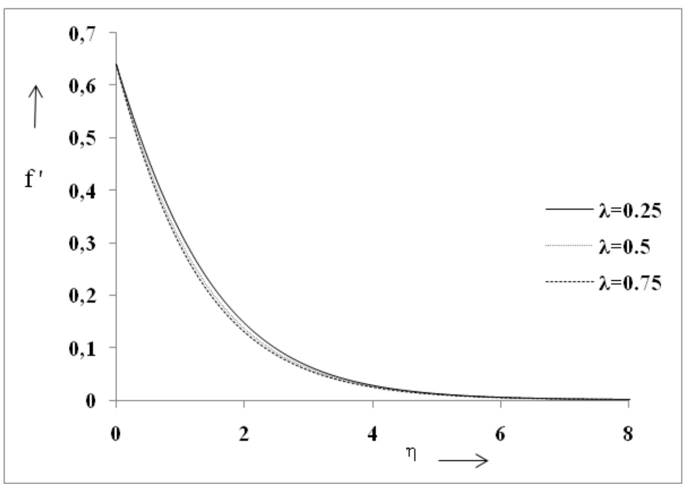

Figure 2. Velocity profiles for different values of λ for $n = 0.5$, $M = 0.25$ and $\theta_c = 5$.

From Fig. (4) it is observed that the velocity decreases near the plate and increases away from the plate as $\theta_c \to 0$ in the cases of gases ($\theta_c > 0$) and increases near the plate and decreases away from the plate as $\theta_c \to 0$ in the case of liquids ($\theta_c < 0$) for given values of M, n and λ. It is evident from Fig. (5) that temperature increases as $\theta_c \to 0$ for $\theta_c > 0$ (ie for gases) and decreases as $\theta_c \to 0$ for $\theta_c < 0$ (i.e. for liquids) for given values of M, n and λ.

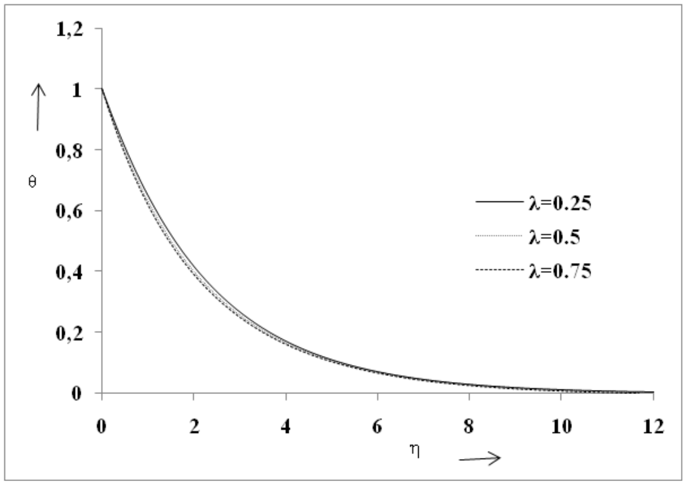

Figure 3. Temperature profiles for different values of λ for $n = 0.5$, $M = 0.25$ and $\theta_c = 5$.

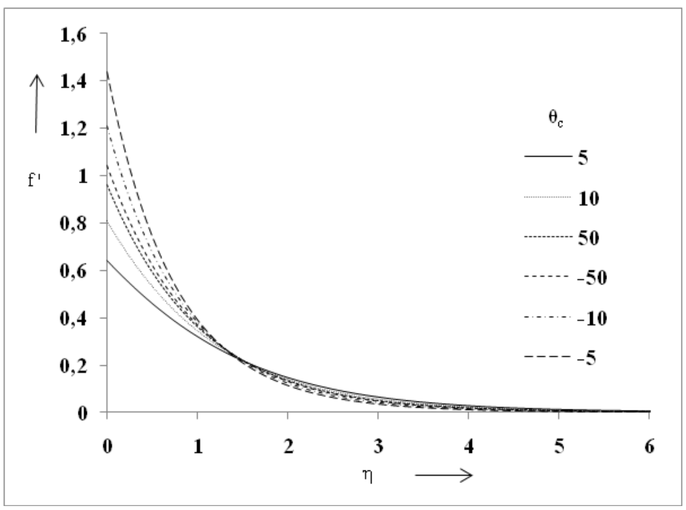

Figure 4. Velocity profiles for different values of θ_c for $n = 0.5$, $M = 0.25$ and $\lambda = 0.25$.

Figures (6) and (7) show that as M increases, the velocity and temperature decrease for given values of n, λ and θ_c. It is evident from Figs. (8) and Fig. (9) that for given values of M, λ and θ_c as n increases, the velocity also increases however the temperature decreases. Figure (10) illustrates the effect of θ_c on the slip velocity $f'(0)$ for different values of n and for fixed values of M and λ. It is observed that the slip velocity decreases as $\theta_c \to 0$

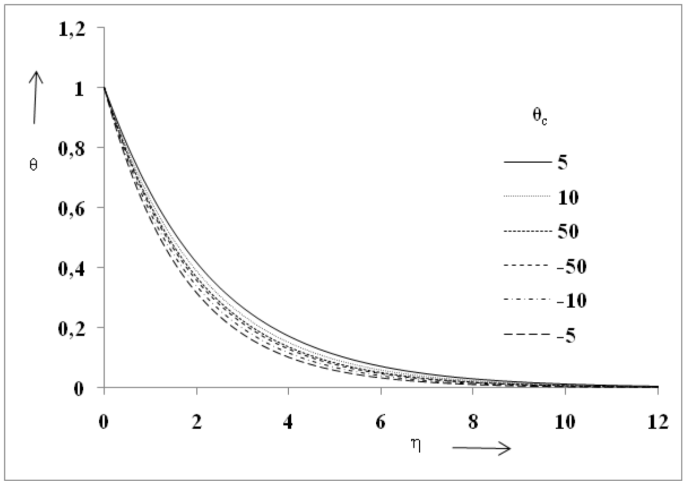

Figure 5. Temperature profiles for different values of θ_c for $n = 0.5$, $M = 0.25$ and $\lambda = 0.25$.

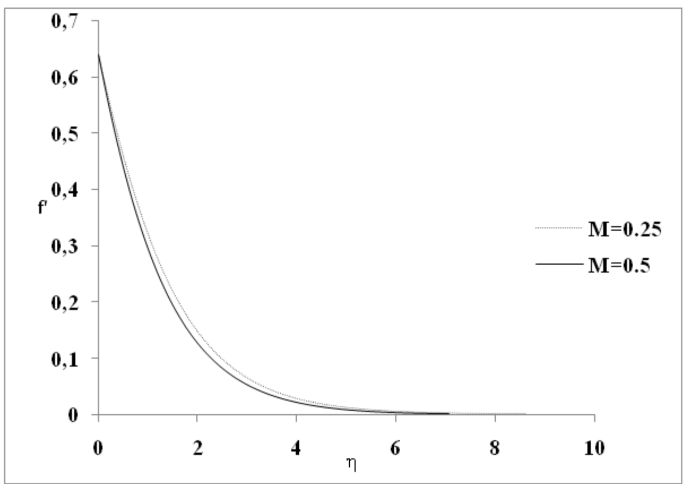

Figure 6. Velocity profiles for different values of M for $n = 0.5$, $\lambda = 0.25$ and $\theta_c = 5$.

for $\theta_c > 0$ (i.e. for gases) while increases as $\theta_c \to 0$ for $\theta_c < 0$ (i.e. for liquids). It is realized also that as n increases, the slip velocity increases for $\theta_c > 0$ and decreases for $\theta_c < 0$. The effect of θ_c on the rate of heat transfer, $\theta'(0)$, for different values of n and for fixed values of λ and M is shown in Fig. (11). It is observed that the heat transfer rate decreases as $\theta_c \to 0$ for $\theta_c > 0$ (i.e. for gases) and increases as $\theta_c \to 0$ for $\theta_c < 0$ (i.e. for

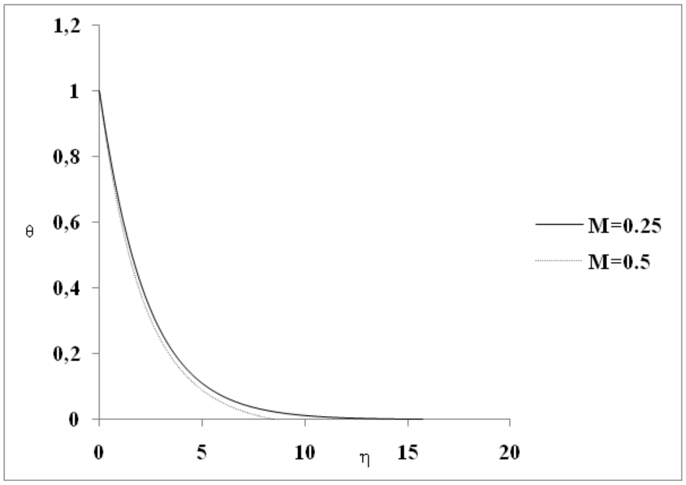

Figure 7. Temperature profiles for different values of M for $n = 0.5$, $\lambda = 0.25$ and $\theta_c = 5$.

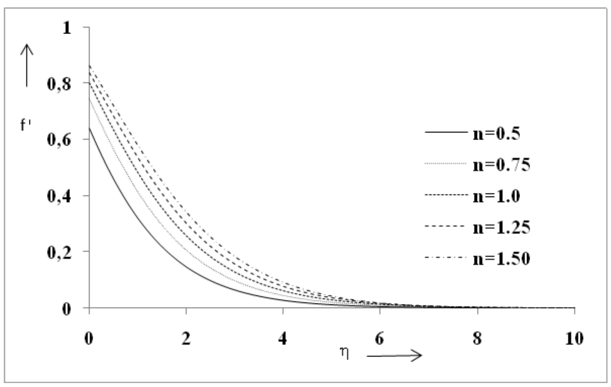

Figure 8. Velocity profiles for different values of n for $\lambda = 0.25$, $M = 0.25$, and $\theta_c = 5$.

liquids). However as n increases, the heat transfer rate increases for both $\theta_c > 0$ and $\theta_c < 0$.

Numerical results of -$\theta'(0)$ for constant viscosity case ($\theta_c \rightarrow \infty$) are presented in Tab. 1. In order to assess the accuracy of the numerical results, we compare our results for different values of n with those of Chen and Chen [2]. The obtained values are in satisfactionary agreement with the others.

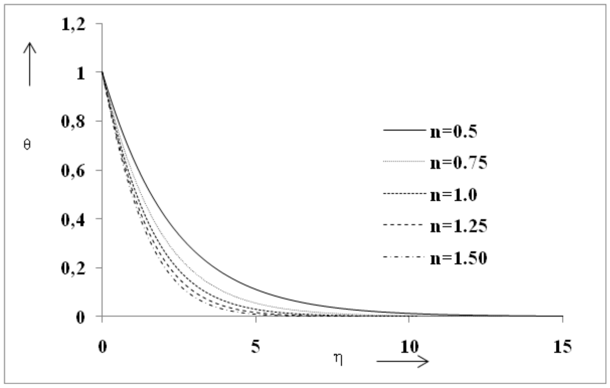

Figure 9. Temperature profiles for different values of n for $\lambda = 0.25$, $M = 0.25$, and $\theta_c = 5$.

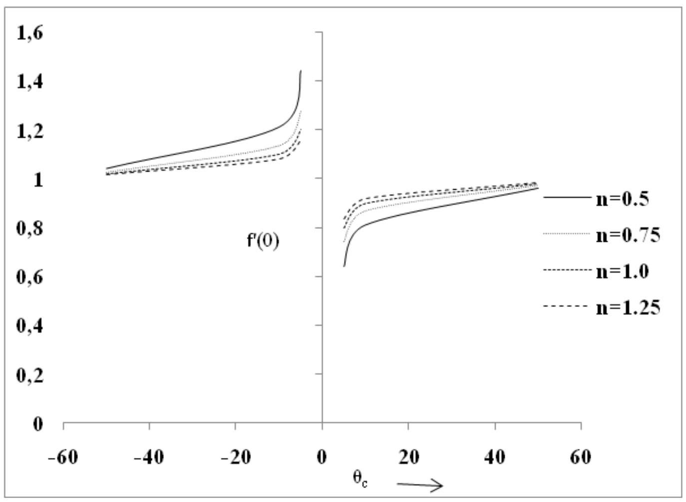

Figure 10. Effect of θ_c on the slip velocity for $M = 0.25$ and $\lambda = 0.25$.

5 Conclusion

The present investigation focuses on the effect of thermal stratification on free convection flow of non-Newtonian fluids on a vertical surface in a porous medium with variable viscosity. Boussinesq and boundary layer approximations are used in this study. Obtained results show that as the thermal stratification parameter increases the heat transfer rate also increases for $\theta_c > 0$ and $\theta_c < 0$.

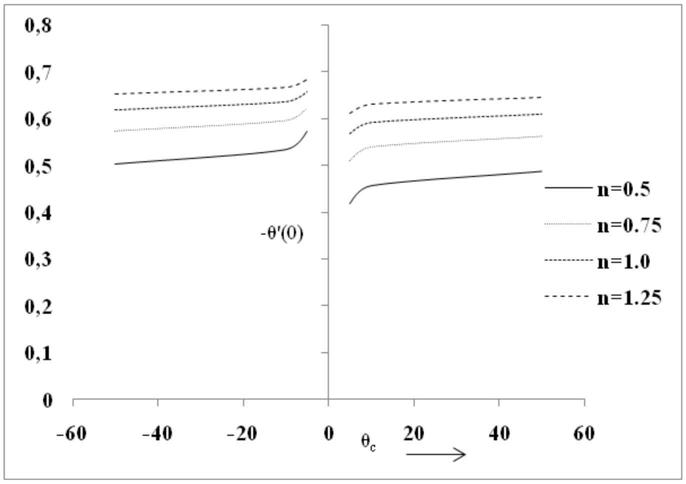

Figure 11. Effect of θ_c on the rate of heat transfer for $M = 0.25$ and $\lambda = 0.25$.

Table 1. Comparison of values of $\theta'(0)$ for $\lambda = 0$, $M = 0$, $\theta_c \to \infty$.

n	Present results	Results of Chen H.T& Chen C.K [2]
0.5	0.3765	0.3768
0.8	0.4237	0.4238
1.0	0.4437	0.4437
1.5	0.4753	0.4752
2.0	0.4936	0.4938

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