Comparative analysis of optimisation methods applied to thermal cycle of a coal fired power plant

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Abstract   The paper presents a thermodynamic optimization of 900 MW power unit for ultra-supercritical parameters, modified according to AD700 concept. The aim of the study was to verify two optimisation methods, i.e., the finding the minimum of a constrained nonlinear multivariable function (*fmincon*) and the Nelder-Mead method with their own constrain functions. The analysis was carried out using IPSEpro software combined with MAT-LAB, where gross power generation efficiency was chosen as the objective function. In comparison with the Nelder-Mead method it was shown that using *fmincon* function gives reasonable results and a significant reduction of computational time. Unfortunately, with the increased number of decision parameters, the benefit measured by the increase in efficiency is becoming smaller. An important drawback of *fmincon* method is also a lack of repeatability by using different starting points. The obtained results led to the conclusion, that the Nelder-Mead method is a better tool for optimisation of thermal cycles with a high degree of complexity like the coal-fired power unit.

Keywords: Thermodynamic optimisation; Nelder-Mead method; Thermal cycle; Coal fired power plant; IPSEpro

1   Introduction

All over the world, including Poland, the power generation system is based mainly on hard and brown coal. However, the ageing of installations of

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Polish energy sector and new, more stringent environmental regulations, associated recently with CO_2 emission, are driving the power generation market toward more efficient cycles than the conventional subcritical steam plant. The current status of technology achieves efficiencies of 45% (lower heating value (LHV) basis) with live steam parameters limited to about 30 MPa and 600 ºC. This limit is mainly imposed by the materials of the boiler and high pressure turbine. However, the prospects of technology development will aim at 700 ºC and higher pressures in the near future. The transition to ultra-supercritical parameters is a clear qualitative change, as it could give rise to the net efficiency of electricity generation even by 7–8 pp [6]. Nevertheless such solution requires, not only the development of new high-temperature alloys, but also some modifications of the power unit structure. Choosing among technologies is difficult and requires a means of making comparisons across different solutions. One of the latest technology (AD700) is that proposed by Dong Energy [6,10], involving a tuning turbine (TT), which solves a problem of very high steam bleed temperatures and the exergy loss in high and intermediate pressure regenerative heaters. According to this configuration, different plant designs are developed and analyzed based on efficiency, economic and technical feasibility [6,8]. However, such a serious structural change of steam power plant triggers, the need for careful calculation of thermal cycle. The requirement of a proper selection of key elements of thermal cycle, as well as process parameters, makes a room for thermodynamic optimisation of such a plant. This is the main issue of the paper. With each modification of the structure the optimization of process parameters should be carried out, to provide the most effective solution. However, the optimization of such complex multi-element system is a tough problem because of a large number of variables. To solve the problem, some widely available commercial software could be used [3]. Unfortunately, they have often several drawbacks, like limitation of the number of variables, huge consuming time, a lack of reproducibility of solutions. Moreover, during the course of power plant optimization some constrain functions need to be defined. The difficulty associated with the use of commercial procedures is that these constraints can not be implemented as any nonlinear functions, they functions usually have to be given in the explicit form (functional). The specificity of the parametric optimization of thermal cycles excludes the clear formulation of limiting function [5]. Therefore it is necessary to verify whether a given set of parameters belongs to the allowable area. For this purpose, the calculated value of the objective

function is estimated as a result of the relatively costly simulation process using IPSEpro (ver. 5.1) [11], as it was shown in the paper by Elsner *et al.* [3]. The authors drew attention to the fundamental differences between standard IPSEpro optimization package and *fmincon* optimization function (1), included in MATLAB'Optimililzation Toolbox (ver. 7.10) [10]. They confirmed that the use of *fmincon* function gives the possibility to use several decision variables, instead of one (IPSEpro approach) and leads to the reduction of computation time.

The paper presents a comparative analysis of *fmincon* function with a newly developed algorithm, based on the Nelder-Mead method. This algorithm together with novel constrain functions, was implemented into the MATLAB package. The calculations were done with the use of heat- and mass-balance commercial software package IPSEpro combined with MATLAB, where efficiency was chosen as an objective function. As an object of study a 900 MW power unit for ultra-supercritical parameters, modified according to AD700 concept [6,10], was chosen. It consisted of a boiler with a single reheat, three main turbine sections, high pressure (HP), intermediate pressure (IP), and low pressure (LP) system, an additional tuning turbine (T-T), a condenser, five preheaters of low pressure regeneration system, a deaerator, and three preheaters of high pressure regeneration system. The diagram of thermal cycle of the power plant is presented in Fig. 1. For the complete calculation of this power plant design almost 150 variables need to be used. Optimization in such multi-dimensional space is a complex and time consuming process, that is why only the part of thermal cycle, consisting of high-pressure regenerative system (three preheaters with *dt_ in* and *dt_ out* as decision parameters), and high pressure turbine bleeds (with a bleed pressure and an exit pressure as decision parameters) was taken into account. The basic operation values of the thermal cycle are based on [3,8] and are listed in Tab. 1, while the operation parameters are given in Tab. 2. The mass flux of the live steam and the mass flux before tuning turbine are the values taken from calculations.

2   Methodology

The literature data and our own experience show that it is difficult to identify the best method for optimization of thermal cycle. That is why it is necessary to compare a number of advanced multidimensional optimization methods. In the study, described in this paper, *fmincon* function [9] as

Figure 1. Scheme of ultra-supercritical steam cycle: B – boiler; HP – high pressure tur-
bine; IP – intermediate pressure turbine; LP – low pressure turbine; T-T –
tuning turbine; G – generator, CON – condenser; DEA – deaerator; MIX –
mixing heat exchanger; W6,W7 – preheaters; M – external motor.

Table 1. Basic values adopted during calculation.

Parameters		Value	Unit
Live steam	Temperature	700	°C
	Pressure	35	MPa
Reheated steam	Temperature	720	°C
	Pressure	7.4	MPa
High pressure regeneration system	dt_out^*	2	°C
	dt_in^{**}	10	°C
Low pressure regeneration system	dt_out	3	°C
	dt_in	10	°C
Pressure in the condenser		5	kPa
Feed water temperature		330	°C
Boiler efficiency		94.5	%
Gross electric power		900	MW

dt_out^* – temperature difference at the 'out' end
dt_in^{**} – temperature defference at inlet,

well as the Nelder-Mead method were taken into account as it was previ-
ously mentioned.

The *fmincon* function attempts to find a constrained minimum of a scalar

Table 2. Operation parameters of the power unit.

Parameters	Representation	Value	Unit
Mass flux of live steam	m	584.17	kg/s
Gross electricity generation efficiency	η_{ge}	52.18	%
Mass flux before the T-T	mT-T	110.49	kg/s
Electric power of T-T	PT-T	41.67	MW

function of several variables, starting at an initial estimate. This function has a complex structure, however for thermal cycle optimization some of nonlinear terms were not used and the function took the simplified form

$$x = fmincon(f, x0, lb, ub, options) \,, \tag{1}$$

$$options = optimset(DiffMinChange, DiffMaxChange, TolX) \,, \quad \text{(2)}$$

where x is the name of chosen object function, f is an optimized function, $x0$ is a starting point and lb anf ub are lower and upper boundaries of the matrix. Function *optimset* is an internal function to set or to change values in the structure options. The *DiffMinChange* and *DiffMaxChange* are respectively the minimum and the maximum change of variables for finite difference gradients, and *TolX* is a termination tolerance of x [3,8].

The Nelder-Mead algorithm falls into the more general class of direct search algorithms [1,4,7]. In these method, the calculation procedure starts from the matrix of predefined points, and than a new test position are generated by extrapolating the behaviour of the objective function, obtained at each test point. Then, the algorithm replaces the predefined points with the new matrix, which is used for the next iteration. The Nelder-Mead algorithm maintains the simplex method, which is an approximation of the optimal point. The vertices are sorted according to the values of the objective function. The algorithm attempts to replace the worst vertex with a new point, which depends on the worst point and the centre of the best vertices. The basic idea of this method is an assumption, that each of n-dimensions simplex has $n + 1$ vertices. Assuming that the relevant functions are two-dimensional, the simplexes consists of three vertices: the best (B), good (G) and the worst (W) (Fig. 2a). The extreme is searched for through some geometric operations, called: midpoint (M), reflection point (R), expansion (E), contraction (C) and shrink points (R^1, R^2). The

graphical interpretations of the basic triangle transformations are shown in
Figs. 2 b-e (two-dimensional case) [2]. The advantage of this method is its
relatively low sensitivity to numerical noise and a low dependence on some
other properties of the objective function (e.g., convexity), since no specific
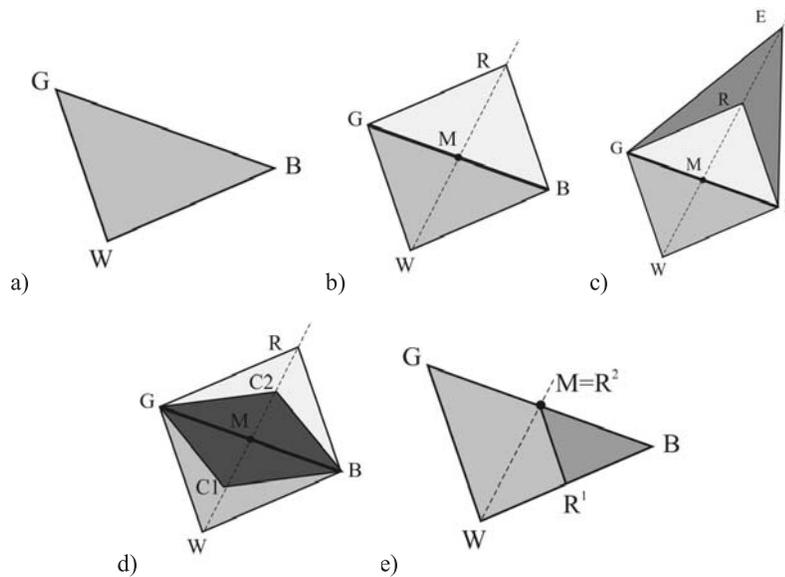continuity or other assumptions were incorporated in its design.

Figure 2. Graphical interpretation for a two-dimensional simplex method: a) basic idea,
b) midpoint and the reflection, c) expansion, d) contraction, e) shrink points
[2].

The method was extended by the procedure allowing to use constrain func-
tions, which for our case were defined as heat exchangers. The advantage
of this procedure was the fact, that during the search of the optimum it
is allowed to leave the acceptable area. Then without indication of an er-
ror, the procedure returns to the defined area and continues the optimum
searching.

3   Results of calculations

Optimization calculations were carried out in such a way, that the number
of decision parameters gradually increased. Initially, the calculations were
carried out considering only one preheater no. W7 (case 1) with two decision

parameters, i.e., temperature difference at inlet (dt_in) and temperature difference at the 'hot' end (dt_out) of the preheater. Then optimisation task was extended, and two W7 and W6 (case 2) and later W7, W6, W5 preheaters (case_3) were analysed. In the last step, the number of decision parameters was further extended by the bleed pressure (p1) and the exit pressure (p2) of high pressure steam turbine (case_4). In all cases a gross power generation efficiency defined as

$$\eta_{ge} = \frac{p_T}{Q_{in}} \; , \tag{3}$$

was used as an object function, where p_T is the total power, and Q_{in} is the fuel chemical energy.

The starting points were varied and depended on the considered method. Concerning *fmincon*, the starting points for each preheaters were 2 °C for dt_out and 10 °C for dt_in and for a pressure p1 13.4 MPa and for p2 8.0 MPa. The starting points for the Nelder-Mead method were dependent on the considered case, however it was always one of the points that was the same as the initial point used for *fmincon* method. In all cases the calculations were performed for the range of dt_in equal to 1-30 °C and dt_out equal to 0.5-5 °C. For calculations using *fmincon*, the minimum step of the variables was $1.0E^{-3}$ and the maximum step of the variables was 1.0. The termination tolerance of both optimization methods was $1.0E^{-12}$. For each case the *htc_area* (MATLAB Optimilization Toolbox [9]) parameter was used as a constrain factor, which maximum value was 15% of reference case. The *htc_area* is defined as a product of heat transfer coefficient k and heat transfer area A. Using the assumption that heat transfer coefficient does not vary with temperature, the *htc_area* is a good estimation of heat transfer area [3]. The results of calculations are given in Tab. 3, where optimal points obtained using both methods are presented. For completeness, in Fig. 3.a the *htc_area* as a function of dt_in and dt_out for case 1 were presented. It is seen (Fig 3.a) that with the drop of these parameters the efficiency grows almost linearly, while the heat exchange area increases strongly nonlinearly, especially for small values of dt_in (Fig. 3.b). This result indicates the need for the introduction of the constrain function that must be imposed on the *htc_area*.

It is easy to observe that the optimal points differ, which is not surprising, from the staring points, but they are also different depending on the model. It is also seen that the differences are strongly dependent on the case and the largest discrepancies are noted for case 4. The effectiveness
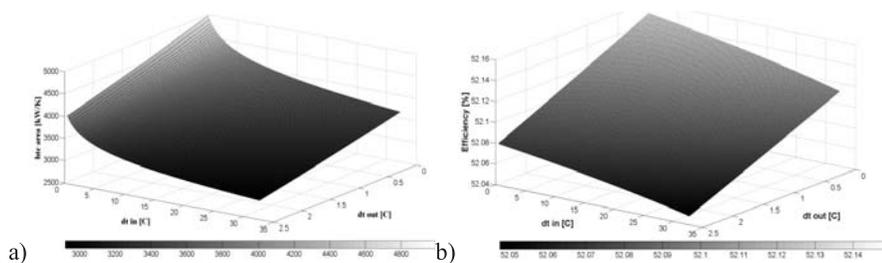
Figure 3. The *htc_area*: a) and efficiency b) as a function of *dt_in* and *dt_out*.

of the optimization algorithm can be assessed by the analysis of the objective function. The results are presented in Tab. 4. It is interesting that, despite the discrepancy discussed above, the efficiency is barely the same for both methods, at least for three first cases. Figure 4 shows that for the following cases *fmincon* function does not respond for the higher number of decision parameters. On the other hand, the Nelder-Mead method does, and for case 4 the gain of efficiency is 0.1% compared with *fmincon* and 0.15% compared with the reference case. It can lead to the conclusion that the Nelder-Mead method is a better tool for optimisation of a complex thermal cycle. Moreover, the analysis shows that optimization of regeneration system only, without turbine bleed/exit parameters is ineffective.
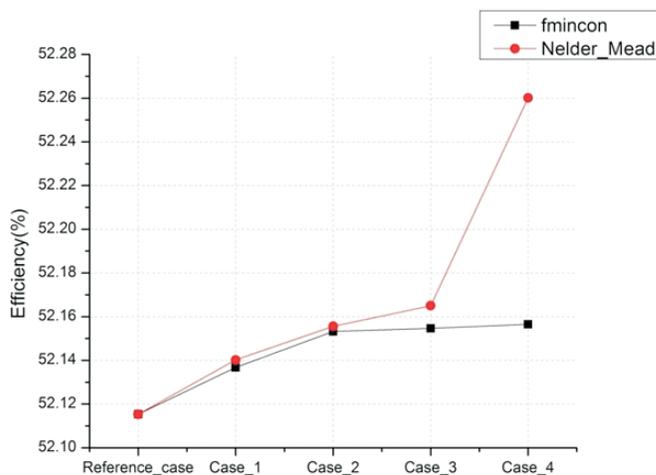

Figure 4. The comparison of efficiency obtained with the *fmincon* function and Nelder-Mead methods.

Table 3. The comparison of an optimal point for the *fmincon* function and the Nelder-Mead method.

		Optimal point for η_{ge} as the objective function				Unit
		fmincon		Nelder-Mead		
		Case 1				
		dt_in	dt_out	dt_in	dt_out	
	W7	10	0.5	5.06	0.5	°C
		Case 2				
	W7	6.51	0.5	3.9	0.5	°C
	W6	6.25	0.5	6.01	0.5	°C
Preheaters		Case 3				
	W7	9.78	0.5	3.62	0.67	°C
	W6	9.63	0.5	5.15	0.69	°C
	W5	9.54	0.68	4.79	1.38	°C
		Case 4				
	W7	9.98	1.62	10.17	0.7	°C
	W6	9.98	1.83	11.38	1.76	°C
	W5	7.96	0.8	7.38	2.13	°C
Parameters	p1	13.51		17.68		MPa
	p2	8.00		8.90		MPa

Table 4. The efficiency and the computation time for consecutive cases.

Parameters	η_{ge} [%]		Time [s]	
	fmincon	Nelder_Mead	*fmincon*	Nelder_Mead
Reference_case	52.115	52.115	–	–
Case 1	52.137	52.140	56	136
Case 2	52.153	52.156	263	739
Case 3	52.155	52.165	394	3120
Case 4	52.157	52.260	502	3428

On the other hand, the most important advantage of *fmincon* function is its significantly shorter computation time compared to the Nelder-Mead method. The relevant data is given in Tab. 4 and in Fig. 5. The difference clearly grows depending on the number of variables. However, it should be noted, that the time consumption is closely dependent on the initial point as well as on the analyzed parameter. For the last, considered case

two additional variables were introduced, i.e., pressures p1 and p2. This modification results with significant increase in efficiency (see Fig. 4), while the rise of computational time using the Nelder-Mead method was smaller in comparison with the time increment between case 2 and case 3.
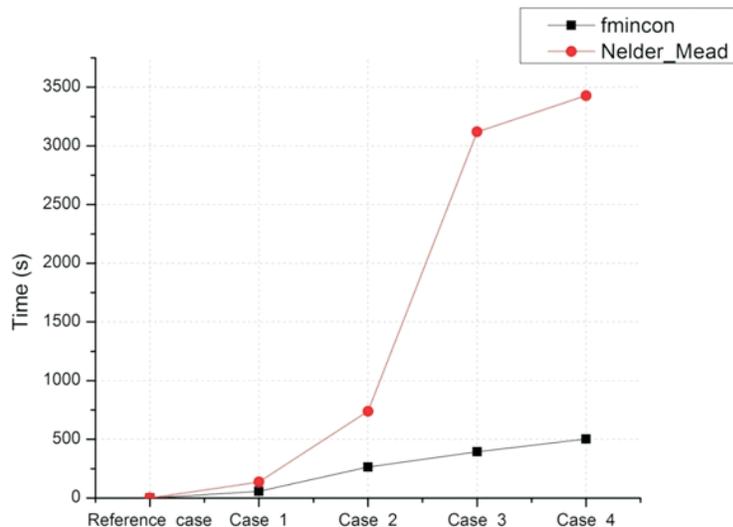

Figure 5. The time consumption distribution for *fmincon* function and Nelder-Mead method.

Comparing these two methods, it was decided in the last step, to check the repeatability of the solutions. The test was done for a simple case with only two decision parameters (case 1), where the starting point for each case was different. Figure 6 shows the result of the repeatability for both methods, where triangles refer to *fmincon* and circles to the Nelder-Mead method. Because in each case the optimum value of parameter dt_out was close to 0.5 °C, the graph presents only the efficiency as a function of dt_in. It is seen that the optimal point for *fmincon* is strongly dependent on the initial value. Farther more, in spite of the application of a constrain function, the method sometimes gives the solution outside the allowed domain (the red triangles). Therefore it is clear that another important drawback of *fmincon* method is a lack of repeatability, while the Nelder-Mead method gives almost the exact solution independent of initial values.
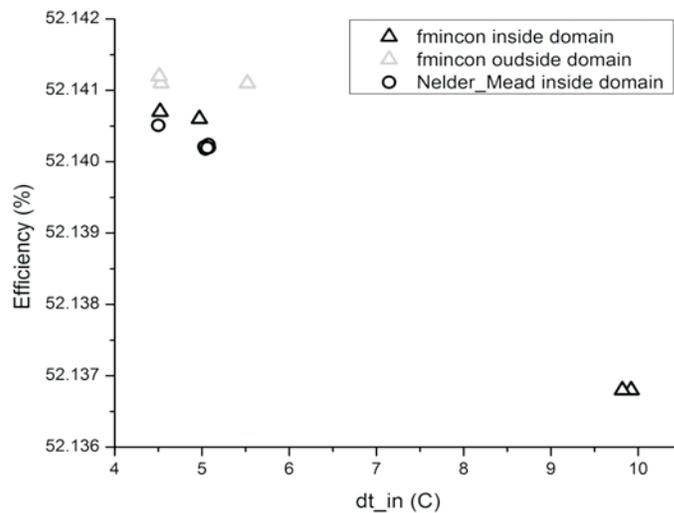
Figure 6. The repeatability of the solution for Nelder-Mead and *fmincon* methods.

4   Summary

The aim of the paper was to verify two optimisation methods, i.e., *fmincon* function and the Nelder-Mead method with their own constrain functions applied for 900 MW power unit for ultra-supercritical parameters, modified according to AD700 concept. In the research the gross power generation efficiency was chosen as an objective function, while 15% of *htc_ area* parameter for each preheaters was assumed as a constrain function. The analysis was performed using such decision parameters as *dt_ in*, *dt_ out* for preheaters W7, W6, W5 as well as bleed pressure and exit pressure of high pressure turbine.

It was shown that *fmincon* function gives reasonable results and a significant reduction of computational time in comparison with the Nelder-Mead method. Unfortunately, with the increased number of decision parameters, the benefit measured by the increase in efficiency is becoming smaller. Moreover, results indicate the increased discrepancies of the objective function with the number of decision parameters, and it is clear that the Nelder-Mead method responds much stronger. An important drawback of *fmincon* method is a lack of repeatability by using different starting points. Last but not least, the difficulty associated with the use of this standard procedure is that constraints could not be implemented as nonlinear functions, but had

to be given in the explicit form (functional).

The obtained results led to the conclusion that the Nelder-Mead method is a better tool for optimisation of a complex thermal cycle. Moreover, the analysis shows that optimization of regeneration system, without turbine bleed/exit parameters is ineffective.

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