Ice slurry flow and heat transfer during flow through tubes of rectangular and slit cross-sections

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Abstract The paper presents the results of experimental research of pressure drop and heat transfer coefficients of ice slurry during its flow through tubes of rectangular and slit cross-sections. Moreover, the work discusses the influence of solid particles, type of motion and cross-section on the changes in the pressure drop and heat transfer coefficient. The analysis presented in the paper allows for identification of the criterial relations used to calculate the Fanning factor and the Nusselt number for laminar and turbulent flow, taking into account elements such as phase change, which accompanies the heat transfer process. Ice slurry flow is treated as a generalized flow of a non-Newtonian fluid.

Keywords: Ice slurry; Pressure drop; Heat transfer; Generalized non-Newtonian fluid flow

Nomenclature

\[ a, b \] lengths of minor and major sides of the rectangular duct, m
\[ A_w \] internal heating surface area of the tube, m\(^2\)
\[ c, d \] Kozicki constants
\[ c_f \] Fanning friction factor
\[ c_p \] mean value of specific heat, J/(kg K)
\[ d_h \] hydraulic diameter, m
\[ d_i \] inner diameter of pipe, m

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\( d_s \) — diameter of solid particles, m

\( h \) — enthalpy

\( K^* \) — consistency index

\( L \) — total length of measurement section, m

\( L_h \) — length of heat measurement section, m

\( \dot{m} \) — mass flux of ice slurry, kg/s

\( n^* \) — characteristic flow-behavior index

\( \dot{q}_m \) — heat flux density, W/m²

\( r \) — ice melting capacity, J/kg

\( T_f \) — mass mean temperature, K

\( T_w \) — wall temperature, K

\( \ oppress \ m \) — mean flow velocity, m/s

\( x_a \) — carrying fluid concentration, %

\( x_s \) — mass fraction of ice, %

\( Gz_K \) — Graetz number, \( Gz_K = \frac{Pr_B Re_K d_h}{L_h} \)

\( He \) — Hedström number, \( He = d_s^2 \tau_p \rho / \mu_p^2 \)

\( K_F \) — phase change number, \( K_F = \frac{r}{(c_p B (T_w - T_f))} \)

\( Nu \) — Nusselt number, \( Nu = \alpha d_h / \lambda_{B,w=0} \)

\( Pe_K \) — Peclet number for ice slurry, \( Pe_K = Re_K Pr_B \)

\( Pr_B \) — Prandtl number for ice slurry, \( Pr_B = \mu_p c_p B / \lambda_{B,wm=0} \)

\( Re_B \) — Reynolds number for ice slurry, \( Re_B = \omega d_h \rho_B / \mu_p \)

\( Re_K \) — Reynolds number according to Kozicki, \( Re_K = (\rho_B \omega_m^2 n^* d_h^3) / (8 n^* K^*) \)

**Greek symbols**

\( \alpha \) — heat transfer coefficient, W/(mK)

\( \Gamma \) — tube shear rate (s⁻¹)

\( \delta \) — absolute value of uncertainty

\( \lambda \) — heat conductivity, W/(m K)

\( \mu_p \) — plastic viscosity, kg/(ms)

\( \varepsilon_B \) — quotient \( \tau_p / \tau_w \)

\( \varepsilon_{BC} \) — quotient \( \tau_p / \tau_w C \)

\( \rho \) — density, kg/m³

\( \rho_B \) — density of ice slurry, kg/m³

\( \tau_p \) — yield shear stress, N/m²

\( \tau_w \) — shear stress on the tube wall, N/m²

\( \tau_w C \) — critical shear stress on the tube wall, N/m²

**Subscripts**

\( a \) — carrying fluid

\( B \) — Bingham number

\( C \) — critical value

\( f \) — fluid

\( h \) — hydraulic

\( i \) — internal

\( in \) — inlet
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\[ m \quad – \quad \text{mixture; mean value} \]
\[ \text{out} \quad – \quad \text{outlet} \]
\[ s \quad – \quad \text{ice; solid particle} \]
\[ w \quad – \quad \text{wall} \]

1 Introduction

Presently there is a tendency to minimize the quantity of refrigeration fluid in the installation by implementing the intermediate systems, with a cooling agent acting as an intermediary medium in heat transfer between an object being cooled and a refrigerant. Ice slurry belongs to the group of heat carriers (secondary refrigerants). Ice slurry is a mixture of ice particles and water (or water with addition of a substance, such as salt, glycol, alcohol, which lowers the freezing point). Ice slurries show several advantages as compared to the traditionally used refrigerants. They have extremely high heat capacity (resulting from the high value of the ice heat of fusion), high heat transfer coefficient, and do not show environmental adverse effects.

Ice slurry is formed continuously in ice slurry generators and collected in storage tanks fitted with stirrers, from where it is fed into the cooling system. The size of ice crystals in the slurry depends on its generation method. The dimensions of solid particles in ice slurries generally do not exceed \( 10^{-4} \leq d_s \leq 0.5 \) mm, although studies dedicated to larger ice crystals (3–12.5 mm) also exist. The size of ice crystals and the type of the carrier liquid determine the rheological properties of ice slurries. Using a particular type of ice slurry generator (e.g., a scraped surface generator) and stirrers in the storage tank ensures a specific ice crystal size.

The basic advantage of ice slurry over refrigerants consists in the fact that the former can be effectively stored. Ice slurry is usually stored in periods of lower cooling demand, e.g., at night, when electricity rates are lower. Ice slurry storage makes it possible to reduce ice slurry generator capacity (by 50–80%) and therefore to limit investment costs. As compared to refrigerants, the advantages of ice slurry consist in its significant thermal capacity and high values of heat transfer coefficients. In consequence, ice slurry-based cooling systems make it possible to reduce the size of the storage tank (by 2–10 times) as well as the heat exchangers (e.g., by 30–50% for ice slurry with a mass fraction of \( x_s = 20\% \)), and pumping power (even up to 25%), with respect to glycol-based systems. The near-constant ice slurry temperature makes it possible to reduce primary air temperature and therefore also the volumetric flow rate of ventilation air (even by 40%).
technical and economic aspects of ice slurry use in various cooling systems were discussed at length in papers [1–3].

The rheological models most frequently assigned to ice slurries include Oswald-de Waele’s power law model [4], Bingham model [5–9], and Casson model [10]. Almost all experiments of ice slurry flow concern the flow in tube with circular cross sections [4–12]. In literature, there are no studies on ice slurry flow in ducts of rectangular cross-sections or between parallel plates (slit cross section). Only the work of Stamatiou and Kawai [13-14] deals with thermal and flow properties of ice slurry (6.2% sodium chloride solution) in vertical channels of rectangular cross-section (a = 0.025 m, b = 0.305 m, L = 0.61 m). Their work describes thermal studies and velocity profiles, without giving precise results concerning pressure drops. In the literature dealing with ice slurry rheology, there are no attempts to treat a flow of this medium as a generalized flow of a non-Newtonian liquid.

The objective of this paper is to present the phenomena accompanying the flow and heat transfer process in ice slurry during its flow through horizontal straight tubes of rectangular and slit cross-sections. The final result presented in the study consists in determining the criterial relations for the Fanning factor and Nusselt number both for laminar and turbulent flow of ice slurry obtained from a 10.6% ethanol water solution, treated as a Bingham fluid. The dependence of the Fanning factor and the Nusselt number on the generalized Reynolds number according to Kozicki allows to apply this relations also to non-Bingham fluids.

2 Experimental studies

Experimental research on thermal and flow processes of slurry ice was conducted using a test stand shown in Fig. 1a. The test stand is made of two basic systems: the system connected with production and storage of ice slurry (elements 11–14), and the meter circuit (elements 1–10) with exchangeable measuring lengths which contain the overall lengths including flume stabilization sectors, pressure drops measuring sectors (which equals 0.5 m) and sectors of reduction of the outlet effect.

The test program included measurements of flow and thermal parameters for ice slurry flow through:

- a rectangular tube with the following dimensions: a = 0.0078 m, b = 0.0265 m, L = 3.0 m (Fig. 1b),
Ice slurry flow and heat transfer during flow through tubes.

- a tube of a rectangular (slit) cross-section, with the following dimensions: \(a = 0.003\) m, \(b = 0.0358\) m, \(L = 2.0\) m.

The values of \(L_h/d_h (118 < L_h/d_h < 127)\) included in the present study are significantly higher than the ones commonly found in thermal studies of ice slurry flow in pipes \((L_h/d_h < 80)\) [3,13–15].

![Figure 1: a) Schematic diagram of the test stand: 1 – heated measuring segment, 2 – elbow, 3 – bend, 4 – calorimetric measurement, 5 – measurement of: density, volume change, ice and air content, 6 – mass flow-meter, 7 – heater, 8 – wattmeter and autotransformer, 9 – flow visualization, 10 – air-escape, 11 – ice generator, 12 – accumulation container, 13 – pump, 14 – volume flow-meter; b) Cross-section of a rectangular channel: 1 – thermal insulation, 2 – heating cable, 3 – seal, 4 – connector, 5 – copper wall, 6 – plexiglass wall, 7 – temperature sensor sleeve.](image)

The most important equipment used for the experiments is listed in Tab. 1. The measurement of ice slurry temperature is based on precise resistance sensors (Hart Scientific) with large dimensions Pt100(7013) (the diameter of the sensor mantle – 5 mm, active length of at least 0.02 m, accuracy 0.018K). In order to measure the difference in temperature of the wall and the flowing medium, Pt100(5622-05) sensors are used (the diameter of the sensor mantle – 0.5 mm, active length of at least 0.01 m) [11]. The measurement accuracy of the indicated temperature difference was ensured by a calibration of the sensors for the whole expected temperature range. The calibration was performed using precise sensors Pt100(7013), directly at the measurement stand, after the prior installation of all sensors. The calibration of the sensors was carried out under adiabatic flow conditions. The accuracy of thus created system of precise measurement of temperature...
Table 1: List of most important measuring instruments.

<table>
<thead>
<tr>
<th>Instrumentation</th>
<th>Type</th>
<th>Range</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Differential</td>
<td>Fuji FKCV 33V4LKAYYAA</td>
<td>0–32 kPa</td>
<td>±0.07% of measuring range</td>
</tr>
<tr>
<td>pressure</td>
<td>Fuji FKCV 11V4LKAYYAA</td>
<td>0–1 kPa</td>
<td></td>
</tr>
<tr>
<td>transducers</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Temperature</td>
<td>HART Scientific</td>
<td></td>
<td>±0.018 °C</td>
</tr>
<tr>
<td>sensors</td>
<td>Pt100(7013) Ø5 mm</td>
<td>-100–100 °C</td>
<td>±0.15 °C nominally</td>
</tr>
<tr>
<td>mass (density)</td>
<td>Pt100(5622-05) Ø0.5 mm</td>
<td>-200–350 °C</td>
<td>±0.1% of flow</td>
</tr>
<tr>
<td>flowmeter</td>
<td>Danfoss 2100</td>
<td>0–5600 kg/h</td>
<td>±0.0005 g/cm³</td>
</tr>
<tr>
<td>power consumption</td>
<td>Wattmeter</td>
<td>0–100</td>
<td>± 0.5</td>
</tr>
</tbody>
</table>

The measurement of ice slurry temperature was facilitated by thermometric sleeves installed at the inlet and outlet of the measurement length, as well as at some other installation points. Within the measurement length, ice slurry core temperature was measured by means of a Pt100(5622-05) resistance thermometer, inserted directly into the liquid through a special stuffing-box. Wall temperature was taken with the help of a thermometer of the same type, inserted into a thermometer sleeve placed in the hole drilled in the interior of the wall (7 in Fig. 1b).

The heat transfer coefficient was determined for the end section of the thermal measurement element, according to the following formula:

$$\alpha = \frac{\dot{q}_m}{T_w - T_f},$$

where heat flux density, $\dot{q}_m$, corresponds to the actual heat flux transferred to the ice slurry.

The change in the mass fraction of ice caused by its melting, $\Delta x_s = x_{s,in} - x_{s,out}$, was calculated using the equation of ice slurry heat balance

$$\dot{q}_m A_w = \dot{m} [h_{out}(T_{f,out}, x_{s,out}) - h_{in}(T_{f,in}, x_{s,in})].$$

The mass flux of ice slurry, $\dot{m}$, is a constant value. The change in the mass flux of ice in ice slurry, due to its melting, equals: $\Delta \dot{m}_s = \Delta x_s \dot{m}$. Enthalpy,
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$h$, of ice slurry, treated as a mixture of ethanol and water ice, is determined on the basis of the relations included in [11]. Ice slurry temperatures $T_{f,in}$ and $T_{f,ou}$ measured at the measurement lengths’ inlet and outlet, respectively, correspond to the equilibrium temperature values of the medium after it has been mixed. In order to take into consideration the influence of the adopted rheological model of the slurry on the value of the heat transfer coefficient, the actual value of flow resistances within the length were measured as well, by means of the differential pressure transducer. Experimental studies presented in [11] indicate, that for $2000 \leq \dot{q}_m \leq 8000$ W/m$^2$ the value of heat transfer coefficient depends on the heat flux density only to a very small degree.

The test program included the measurements for:

- mean flow velocity $0.1 \leq w_m < 3.5$ m/s;
- heat flux of $\dot{q}_m = 5000$ W/m$^2$;
- mass fraction of ice $0 \leq x_s \leq 30\%$;
- average size of ice crystals (width/length) $d_s = 0.1/0.15$ mm [11].

The ice crystal measurements were taken for a slurry with a temperature of $t = -5.95$°C; this was done by means of a I.ÖR. Bucharest-Romania MC6 microscope (used magnification 42.3:1) located in a cold room vis-à-vis the work stand. The temperature in the cold room was maintained at $t = -8$ °C. The measurement errors of important experimental values are listed in Tab. 2.

3 Results of experimental studies

3.1 Pressure drop

Measurements in channels having a circular cross-section also enabled the rheological identification of the studied medium. Ice slurry of ethanol solution is a Bingham fluid [8]. With pipe flow curves equations in the coordinate system $(\Gamma, \tau_w)$, it is possible to determine the values of yield shear stress $\tau_p$ and dynamic coefficient of plastic viscosity $\mu_p$ for ice slurries with various ice mass fractions. The results of measurements in circular tube and relations for $\tau_p$ and $\mu_p$, were given in [10]. Measurements of pressure drops in channels were performed for two channels of the following side-to-side ratios: $a/b = 0.294$ and $a/b = 0.084$ it is for rectangular and slit channel respectively [16].
Table 2: The list of measurement errors of important experimental values.

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Formula</th>
<th>Uncertainty</th>
<th>Relative uncertainty, %</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average velocity ( w_m ), m/s</td>
<td>( w_m = \frac{m}{\rho_B \cdot v} )</td>
<td>( d_h, \ m )</td>
<td>( \delta w_m \times 10^2, \ m/s )</td>
</tr>
<tr>
<td>Friction factor ( c_f )</td>
<td>( c_f = \frac{d_h \cdot \Delta \tau}{2L \cdot \rho_B \cdot w_m^2} )</td>
<td>( d_h, \ m )</td>
<td>( \delta c_f \times 10^3 )</td>
</tr>
<tr>
<td>Reynolds number ( Re_K )</td>
<td>( Re_K = \frac{\rho_B \cdot w_m^2 \cdot d_h}{\mu_B \cdot \Delta \rho \cdot \Delta \tau} )</td>
<td>( d_h, \ m )</td>
<td>( \delta Re_i )</td>
</tr>
<tr>
<td>Hedström number ( He )</td>
<td>( He = \frac{d_h^2 \cdot \tau}{\rho_B \cdot \mu_B^2} )</td>
<td>( d_h, \ m )</td>
<td>( \delta He )</td>
</tr>
<tr>
<td>Prandtl number ( Pr_B )</td>
<td>( Pr_B = \frac{\mu_B \cdot \rho_B \cdot \Delta \rho}{\lambda_B \cdot w_m = 0} )</td>
<td>( d_h, \ m )</td>
<td>( \delta Pr_B )</td>
</tr>
<tr>
<td>Greatz number ( Gz_K )</td>
<td>( Gz_K = Pr_B \cdot Re_K \cdot d_h / L_h )</td>
<td>( d_h, \ m )</td>
<td>( \delta Gz_K )</td>
</tr>
<tr>
<td>Nusselt number ( Nu )</td>
<td>( Nu = \frac{\alpha d_h}{\lambda_B \cdot w_m = 0} )</td>
<td>( d_h, \ m )</td>
<td>( \delta Nu )</td>
</tr>
<tr>
<td>Heat transfer coefficient ( \alpha )</td>
<td>( \alpha = \frac{\delta u}{\tau - \tau_f} )</td>
<td>( d_h, \ m )</td>
<td>( \delta u )</td>
</tr>
</tbody>
</table>

In ice slurry rheology, it is preferred to determine the Fanning friction factor \( c_f = 2 \tau_w / (\rho_B w_m^2) \) by matching specific rheological models of fluids (Ostwald-de Waele’s, Bingham’s, Casson’s) to their own specific methods of determining pressure drop. In this case, Reynolds number is a simple function of parameters describing the rheological properties of fluid, which are not dependent on the average velocity and flow geometry. However, the Fanning friction factor \( c_f \) is a function of criterial numbers (i.e. Reynolds, Hedström’s, Casson’s) which are defined independently for a given rheological model. In the range of a turbulent flow, for a given rheological model, there are various semi-empirical and empirical relations which enable to define the Fanning friction factor of the ice slurry [10]. Criteria for a change of the flow character are also applied to a given rheological model [10,17,18]. Figure 2a shows the Fanning friction factor as a function of Reynolds num-
Figure 2: Ice slurry flow through channels having rectangular cross-sections: a) Experimental values of the Fanning friction factor $c_f$ as a function of $Re_B$ and $He$ numbers; b) Fanning friction factors as a function of generalized Reynolds number according to Kozicki and Metzner-Reed.
ber $Re_B = Re_B/(c + d)$ [16] for the two investigated channels.

Another method of determining pressure drop for non-Newtonian liquids is introducing a generalized Reynolds number, which enables to treat any non-Newtonian liquid flow as a general Newtonian liquid flow. At present, the Metzner-Reed method (the laminar flow) [19] and the Dodge-Metzner method [19] (the turbulent flow) are the most commonly used methods of determining pressure losses in a rheologically stable non-Newtonian liquid flow. Those methods, given for the pipe flows, were generalized by Kozicki [20] for other cross-section flows.

Similarly to Metzner and Reed, Kozicki assumed that relations between Fanning friction factors and the Reynolds number are the same as in non-Newtonian liquids. Due to this assumption, Kozicki defined the Reynolds number as

$$Re_K = \frac{\rho_B w_m^{2-n^*} d^h}{8^{n^*-1} K^*}. \quad (3)$$

In case of a pipe flow, generalized indices $n^*$ and $K^*$ correspond to characteristic flow-behavior index and consistency index. Generalized indices $n^*$ and $K^*$ are dependent on: rheological properties of fluid, average tangential stress on the wall, average flow velocity, and nondimensional geometric constants $c, d$, which are defined respectively for various geometries of cross-section flows [20]. The values of the Kozicki’s constants for the studied cross-sections are: $c = 0.3027; d = 0.798$ for rectangular channel and $c = 0.4269; d = 0.9278$ for slit channel. In Eq. (3), characteristic flow-behavior index $n^*$ and consistency index $K^*$ for the Bingham fluid and for the section of arbitrary geometry, can be determined on the basis of the following relations [20]:

$$n^* = \frac{c \left[ 1 - \frac{1+d/c}{c+d} \right]}{1 - \varepsilon_B - d \left[ 1 - \frac{1+d/c}{c+d} \right] - \frac{\varepsilon_B \left( 1 - \frac{d/c}{d} \right)}{d}}, \quad (4)$$

$$K^* = \left( \frac{\varepsilon_B}{c} \right)^{1-dn^*/c} \left[ \frac{c}{c+d} \tau_w^{1+dn^*/c} - \frac{c}{d} \tau_p^{1+dn^*/c} + \frac{c^2}{d(c+d)} \tau_p^{1+dn^*/c} \right]^{-n^*}. \quad (5)$$

Generalized Reynolds number enables comparisons of non-Newtonian liquids of different rheological models, and facilitates the use of the same relations in defining pressure drop and flow character change criteria for the non-Newtonian liquids. Figure 2b shows Fanning friction factors of ice slurry
as a function of the generalized Reynolds number according to Kozicki and Metzner-Reed. Introducing a generalized Reynolds number according to Kozicki Eq. (3) in the case of a rectangular cross-section makes it possible to determine the Fanning friction factor using the same relations as for pipes of a circular cross-section and to treat a flow of a non-Newtonian liquid as a liquid for which the Fanning friction factor is given by the Fanning relation (in the laminar region)

\[ c_f = \frac{16}{Re_K}, \]  

and of the Dodge-Metzner Eq. (7) or Blasius relation for \( c_f \), generalized by Kozicki, in the turbulent region [19]:

\[ \frac{1}{\sqrt{c_f}} = \frac{4}{(n^*)^{0.75}} \log(Re_K c_f^{1-0.5n^*}) - \frac{0.4}{(n^*)^{1.2}} + 4(n^*)^{0.25} \log \left[ \frac{4(c + d n^*)}{1 + 3n^*} \right]. \]  

The Fanning friction factors obtained experimentally and calculated from Eqs. (6)–(7), for a slit cross-section and for a rectangular one, are shown in Fig. 2b. In this cases, the mean relative difference between the calculated and measured \( c_f \) values is less than 10%.

The analogy between a Newtonian and non-Newtonian flow is not complete: the critical value of Reynolds number \( Re_{BC} \) – unlike for a Newtonian liquid – is not constant. Therefore, in the generalized flow of non-Newtonian liquids, relations (6)–(7) must be supplemented with a criterion connected with the change of flow character. Hanks criterion [21], Shah-Sutton criterion [22], and Maglione [23] criterion for the change of the flow character can be directly applied to the rheologically stable Bingham fluids. The Hanks and Shah-Sutton criteria have been developed for Bingham fluids, while the Maglione criterion concerns fluids which obey the Herschel-Bulkley model. Among the above criteria, the Hanks criterion makes (8)–(10) it possible to obtain the lowest values of the critical Reynolds number. According to Hanks [21], the critical Reynolds number is dependent on the Hedström number (8):

\[ Re_{BC}^* = \frac{1}{8} \sqrt{\frac{2}{3}} \frac{1 - 3/2 \varepsilon_{BC} + 1/2 \varepsilon_{BC}^3}{\varepsilon_{BC}} He^*, \]  

critical quotient \( \varepsilon_{BC} = \tau_p/\tau_{wC} \) is calculated using the Eq.(9)

\[ \frac{\varepsilon_{BC}}{(1 - \varepsilon_{BC})^3} = \frac{He^*}{22400}, \]  

Reynolds ($Re_{BC}^*$ and Hedström ($He^*$) numbers are calculated using hydraulic diameter which is defined as

$$d_h = 2a\sqrt{\frac{2}{3}}.$$  \hspace{1cm} (10)

Figure 3 shows the comparison of the critical Reynolds numbers obtained experimentally and calculated from the Hanks criterion for an ice slurry flow through channels of rectangular cross-sections.

![Figure 3: Comparison of experimentally obtained critical $Re_{BC}^*$ numbers for ice slurry with the values determined based on the Hanks criterion.](image)

3.2 Heat transfer coefficients

Figures 4a and 4b show the dependence of the Nusselt number on the generalized Reynolds number according to Kozicki for the flow through a slit and a rectangular channel, respectively. In the case of the flow through rectangular tubes, similarly to the flow in pipes [11], it is possible to observe then increase in heat transfer coefficient of the ice slurry in comparison to the heat transfer coefficient of the carrying liquid.

The influence of solid particles on the increase of heat transfer coefficient of the ice slurry in comparison to the commercially available ethanol solution depends on the hydraulic diameter of the channel. In laminar flow regimes, this ratio equals 3.2–4.8 for a rectangular channel ($d_h = 0.012$ m), as well as 1.9–3.4 for the slit channel ($d_h = 0.0055$ m). A similar trend could be observed in the case of a pipe flow. In that respect, the flow through a slit channel corresponded to the flow through a pipe with the diameter of $d_i = 0.01$ m (2.5–4). Meanwhile, the flow through a rectangular tube with the hydraulic diameter of $d_h = 0.012$ m approximated the flow through
Figure 4: The Nusselt number as a function of generalized Reynolds number: a) slit channel, b) rectangular channel.

a pipe with the diameter of $d_i = 0.016$ m (3.5–4.7) [11,16]. The observable increase in the heat transfer coefficient of the slurry in comparison to ethanol amounted to 20–30%. This phenomenon might be explained by the significant influence of heat conduction in the near-wall region on the heat transfer process for the laminar flow. On the other hand, the presence of ice crystals causes the heat conductivity coefficient to increase, by bringing on an additional microconvection effect. In the case of a turbulent flow, the microconvection process caused by solid particles is outweighed by the turbulence of the carrying liquid. The observations included in [11,16] indicate that the Nusselt number for the analyzed case of heat transfer can be expressed as a function of the Graetz number, Gz, the product of $\Delta x_s$ and the phase change number, $K_F$. Figure 5 show that the relations $\text{Nu}(\Delta x_s K_F/100)$ for different flow cross-sections are similar.
The influence of solid particles on the Nusselt number (which occurs due to the microconvection effect and the reaction with the near-wall layer) is shown in Fig. 6. Heat transfer between the fluid and the tube wall depend on the thickness of the near-wall layer and on heat conductivity of the liquid. In moving slurry, it is possible to observe the increase in heat conductivity, $\lambda_{BW}$, in comparison to the heat conductivity of a static slurry ($\lambda_{BW,\infty}$). Quantitatively, the result of the increase in the heat conductivity of a moving slurry is described by the Charunyakorn formula [24]. According to this equation, larger solid particles lead to greater heat conductivity. This is the cause of the increase in heat transfer coefficients of the slurry in comparison to the heat transfer coefficient of carrying liquids.

The above presented remarks concern types of slurry in which neither the melting process nor any related changes in the solid particles fraction take place. In laminar regimes, small mass fluxes make the amount of received heat sufficient to completely melt the ice particles within the near-wall layer. In consequence, a structure with properties corresponding to the features of the carrying liquid was created near the wall. In this case the heat transfer from the wall to the core of the fluid, where solid particles were present, was hindered. It needs to be kept in mind that in the analyzed case, the intensity of heat transfer process depends also on the heat transfer process which takes place between the carrying liquid and the solid particles. The greater the contact surface between the solid particles and the carrying liquid (more particles with a greater diameter $d_s$), the greater the intensity of heat transfer between the wall, the carrying liquid and the solid particles. As far as turbulent flow is concerned, greater mass fluxes and
an intensive mixing process impede complete melting of ice near the walls, and the influence of solid particles on the heat transfer process of ice slurry resembles a corresponding process in the case of the types of slurry which do not undergo phase change. Thus, graphs included in Fig. 6 demonstrate a slightly different influence of the $d_s/d_h$ parameter on the heat transfer process in laminar and turbulent regimes.
Greater temperature gradients ($\Delta T = T_w - T_f$) in transverse cross-section and a more intensive melting process of solid particles near the tube wall, which accompany laminar flow, cause a visible change in the rheological properties of the liquid in a transverse cross-section. The influence of the heterogeneity of rheological properties of slurry in a transverse flow cross-section on the heat transfer process could be taken into account by factor $(K^*(T_f)/K^*(T_w))^y$. In turbulent flow regimes, this effect is not so well visible and the accuracy of calculating the $(K^*(T_f)/K^*(T_w))^y$ quotient is outweighed by the measurement accuracy of the temperature difference $\Delta T$.

Finally for calculating the Nusselt number in laminar and turbulent regimes following equation have been adopted

$$
\text{Nu} = A(GzK)^m \left( \frac{\Delta x_s K_F}{100} \right)^n \left( \frac{d_s}{d_h} \right)^p \left( \frac{K^*(T_f)}{K^*(T_w)} \right)^y,
$$

where

$$
\text{Nu} = A(\text{Pe}_K)^m.
$$

(11)

(12)

Table 3: Coefficients in Eqs. (11) and (12) for the flow through rectangular tubes.

<table>
<thead>
<tr>
<th>Transversal cross-section</th>
<th>Type of movement</th>
<th>A</th>
<th>m</th>
<th>N</th>
<th>p</th>
<th>y</th>
<th>Applicability range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rectangular and slit cross-section</td>
<td>laminar</td>
<td>3.66</td>
<td>0.16</td>
<td>-0.28</td>
<td>-0.12</td>
<td>0.16</td>
<td>$5.6% &lt; x_s &lt; 30%$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$w_m &gt; 0.5 \text{ m/s}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$30 &lt; \text{Re}_K &lt; 2300$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$3% &lt; x_s &lt; 30%$</td>
</tr>
<tr>
<td></td>
<td>turbulent</td>
<td>0.0032</td>
<td>0.86</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>$w_m &lt; 3.1 \text{ m/s}$</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>$1900 &lt; \text{Re}_K &lt; 6000$</td>
</tr>
</tbody>
</table>

Equation (12) takes into account the fact that in turbulent flow regimes the temperature profile of the fluid was thermally developed and the influence of the thermal entry length on the heat transfer process is overlooked. The coefficients (A, m, n, p, y) (Tab. 3) in formulas (11) and (12) have been determined so that for every measurement point the difference between the measured and the calculated Nusselt numbers tends to zero [25].

Figure 7 presents a comparison of the measured and calculated Nusselt numbers by means of Eqs. (11) or (12), as well as the parameters included in Tab. 3, for analyzed cross-sections.
4 Conclusions

This paper discusses ice slurry flow and heat transfer through channels of rectangular and slit cross-sections.

- Irrespectively of flow cross-sections, by introducing a generalized Reynolds number according to Kozicki, it becomes possible to evaluate ice slurry pressure drops using the Fanning relation (in the laminar range) and the Blasius or Metzner-Dodge-Kozicki relation (in the turbulent range). A correct evaluation of pressure drops based on the above equations depends on a correct determination of the rheological parameters of fluid, which in the case of ice slurry is not always easy.

- Critical Reynolds numbers depend not only on rheological properties of fluid, but also on flow structures. The experimentally determined values of critical Reynolds numbers $Re_{BC}$ fall within the range $1800 < Re_{BC} < 3000$, and at low ice contents ($x_s < 10\%$) are lower than the critical Reynolds numbers for carrying fluid (10.6\% ethanol solution at -4.5 °C). The above observations indicate that it is necessary to formulate a more adequate criterion of flow character change (in comparison with the Hanks criterion). Such a criterion should
take into account the influence of rheological properties, size of solid particles, hydraulic diameter, as well as phase flow velocities.

- In the studied measurement range, the values of the measured heat transfer coefficients change within the range from 1200 to 7200 W/(m²K) for rectangular channel flow. For the same mass fractions of ice, mean velocities and types of movement, heat transfer coefficients obtained for slit and rectangular channels ($d_h = 0.0055$ and 0.012 m respectively) were greater than in the case of flow through pipes ($d_h = 0.010–0.020$ m).

- Irrespective of the transverse cross-section of the flow, it was possible to observe a stronger influence of the mass fraction of solid particles on heat transfer coefficients in the laminar rather than in the turbulent flow range.

- In laminar flow regimes for over 80%, and in turbulent regime for over 88% of measurement points, the divergence between the heat transfer coefficient values calculated on the basis of their own criterial relations and the ones obtained through measurement is smaller than 15%.

- In heat transfer processes, the parameters $d_h, w_m, x_s$ and $d_s$ should be selected so that ice slurry flow is a homogeneous laminar flow with heat transfer coefficient greater than the heat transfer coefficient of the carrying liquid.

The results presented in the paper apply to medium-sized solid ice particles formed in a scraped surface ice slurry generator. The use of a specific type of ice slurry generator and a storage tank with stirrers makes it possible to avoid solid particle agglomeration in the tank. A constant unchanging mean size of the solid particles is therefore assumed (no melting) during adiabatic flow (transport through pipelines). In heat transfer processes in heat exchangers, the ice melting process results in a change of the ice content and of the ice crystal size. The effect of the change in the ice content was reflected in the study by making the physical properties of the ice slurry dependent on the mean ice fraction. The presented results refer to a certain mean size of ice crystals at the inlet to the thermal measurement section. It is to be expected that a reduction of the size of the crystals will limit the microconvection effect, causing a drop in the substitute thermal conductivity coefficient for the moving slurry and the heat transfer coefficient. At the same time, as the ice melts, the plastic viscosity coefficient is reduced and the value of the Reynolds number for the slurry increases. The increasing
Nu(Re) function in turn causes the increase of heat transfer coefficient. The effects of the decrease of thermal conductivity and the increase of the Re number in the aspect of the obtained heat transfer coefficient values are of an opposing nature. Therefore, it is to be expected that changes in heat transfer coefficients in heat exchangers may be not very significant. In addition, in air coolers, heat transfer coefficients are determined by the values of the gas-side heat transfer coefficient. Therefore, the ultimate influence of the size of ice crystals on the thermal capacity of heat exchangers is not as significant as a change in the ice fraction of the slurry. Changes in the latter may easily be taken into account in the calculation algorithm.

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References


