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Compliance of the Results of Hypothesis Testing with Exponential Distribution for Selected Statistical Tests

Andrzej Rogowski*

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Abstract

The article discusses the problems related to the identification of the exponential probability distribution with use of the basic statistical consistency tests for continuous distributions, modified for testing the exponentiality of the distribution, especially Kolmogorov λ test. With use of simulation methods, the ability to differentiate the exponential distribution from other distributions out of gamma family and power distribution, depending on the parameters of the distributions and the quantity of the random sample. For small samples the composite hypothesis was tested. Out of transportation area few samples were presented, where the verification of the hypothesis, that the sample originates from the exponential distribution, was significant.

1. Introduction

The verification of hypothesis that the probability distribution of the tested property is the exponential distribution (shortly: testing the exponentiality of the tested property) is, besides the verification of the hypothesis that the probability distribution of the tested property is the normal distribution (testing the normality), is one of the most often verified statistical hypotheses about the consistency of the empirical distribution with the theoretical distribution of probability – so called the consistency testing. That is the result of special theoretical meaning of the exponential distribution in different areas of science and technology and its specific

* PhD. Eng., University of Technology and Humanities, Faculty of Transport and Electrical Engineering; 26600 Radom; Malczewskiego str. 29, e-mail: androgow@wp.pl

properties, for example the „lack of memory"[1]. The exponential distribution is the basic distribution used for the theory of reliability, so called the exponential law of reliability – the intensity of damage (at least in the normal exploitation period, that is omitting the periods adaptation and wear-out, in the second part of so called the bathtub curve [5, p. 72]) is constant, and that implies that the time of proper operation is a random variable with the exponential distribution. The same distribution is used in diagnostics, maintenance theory and safety theory, theory of traffic flow, telecommunications etc. The developed methods of mass maintenance theory (at least for the Poisson streams) and homogeneous stationary Markov processes in wide range use the exponential distribution, which additionally allows exact analytic solutions for different models. All that fields of science find their applications in the transportation which is not only the technical system. Throughout the years different theories in the field of transportation developed, different theoretical and practical problems were solved with use of the exponential distribution as a model for random events. As examples one could list (showing examples from the dim and distant past as well as contemporary):

- The model of pederastian crossing, it was assumed, that the vehicles passing the crossing constitute Poisson renewal stream (so that the periods between the passing vehicles are of exponential distribution – the moment of pederastian crossing the street constitute the stream of renewal, the distribution of periods between crossing was determined [5, example 5, p. 168];
- Generation of the arrivals in given point of the road (road vehicle traffic) [14, p. 128] and the periods in the railway traffic [13, pp. 57-59];
- While determining the delays on the crossroads with controlled traffic lights, Webster assumed the Poisson stream of arrival of the vehicles on the crossroads [14, p. 256];
- While determining the boundary accepted interval, Ashword assumed the Poisson stream of arrival in the main stream [14, p. 351];
- The stream of the main road traffic (at least in the one-lane road) is sometimes defined by the exponential distribution (although many authors questions that definition by assuming the shifted exponential distribution or other) [13, p. 18];
- The optimisation of intensity of arrival of vehicles to the service station of FIAT, it was assumed and verified with the chi-square test the exponentiality of the arrival stream and the service times (the sample n = 287, significance level α = 0,05) [6, p. 208];
- The optimisation of the technological process – the cyclic supply of the materials for the assembly stands (the number of transportation means greater than the number of the assembly stands, significance level α = 0,05) [6, p. 204];
- K. Kuśmińska-Fijałkowska, who dealt with the development of the method for intermodal reloading processes of transportation units by introduction of a sys-

[1] The definition of the term "the lack of memory" and a proof that the exponential distribution is the only continuous distribution with that property see [5, pp. 22-24].

tem for exchange, flow and processing of the RFID information, DECT, which together with the control system for the stream of the transportation units allows increase in the efficiency of the land terminal operation in the intermodal transportation chains, assumed exponential distribution of the service time and periods between entries and verified it with the Kolmogorov λ test, significance level α = 0,05 [8];
- T. M. Perzyński dealt with the problems of safety of computer networks used in control of the railroad traffic; he assumed that the distribution of the time of the proper operation of the computer systems is defined by exponential distribution, and the assumption was verified bu the Pearson χ^2 test and Kolmogorov λ test, significance level α = 0,05 [10];
- L. Bester dealt with the safety of the railroad crossings of category D and C, representing the object of the research (the railroad crossing, railroad traffic and road traffic) she assumed, that it could be defined with the apparatus of uniform stationary Markov processes and in later simulation research she generated the streams of entries of trains and vehicles as the Poisson streams [1].

But as often as the exponential distribution is used, the reservations are made, that the exponential distribution does not define the reality good enough, and its use is too much of a simplification and other distributions should be used, for example the Weibull or Erlang distributions as well as shifted distributions. A good example here is the research result by T. Szczuraszek and E. Macioszek [12]. They analysed the distribution of the time periods on the traffic lanes on the small roundabout. In the method for calculation of the roundabout throughput, that is used in Poland, the exponential distribution is used, but the research shows that the real distribution of the time periods on the roundabout differs from the exponential distribution (see [12]). The authors stated, at significance level α= 0,05, that for small intensity of traffic the time period distribution might be defined by all of the tested distributions, that is: exponential distribution, gamma distribution, Erlang distribution, logarithmic-normal distribution and Covan distribution. For medium intensity one could use the shifted exponential distribution and Covan distribution. They also state the Hyperlang distribution (convex combination of the shifted exponential distribution with Erlang distribution) to be used in their analyses. In their analyses they only used the chi-square Pearson test without using the specialised consistency tests for continuous distributions (except for the Covan distribution, which is mixed distribution, all other distributions are continuous distributions) [12]. Similar problems were handled by, among others, E. Duda and G. Sierpiński [15].

But using other distributions brings huge computational difficulties and restricts the researcher to use only simulation methods, which also tend to be controversial (or at least some methods of simulation). To omit the computational complications and the simulation methods and "find the explanation" for the possibility of use of the exponential distribution, the verifications of hypothesis of the exponentiality of the probability distribution are performed. Most often the Kolmogorov λ test is performed. The lack of bases for rejection of the hypothesis (that is incorrectly interpreted as the proof for the validity of the hypothesis[2]) constitutes the justification for admitance od the made assumptions. The classic Kolmogorov λ test, although presented in all the handbooks for statistics, is not the best test for verification of such hypothesis, especially with small sample and composite hypothesis (up to 100, which occurs often in empirical research; the use of Pearson χ^2 test is a fatal mistake since the required quantity of a sample should be at least 100). One should use the compatibility tests dedicated for the continuous distributions modified for the hypotheses of exponentiality of the distribution. But using several tests might be a significant difficulty in giving an answer to the question if the hypothesis of exponentiality of the distribution should be rejected or the are no bases for its rejection, which practically means the acceptance of the hypothesis. The situation gets complicated when different tests give contradictory results. The statisticians suggest using the use of stronger tests for given alternative hypothesis (that is those, that give smaller error of II type[3]) or assuming the most often repeated result. But testing the strenght of the tests is very complicated (and often impossible to express by numbers). Also not very useful for a practitioner. The work presents the attempt to estimate (in percent) the consistency of the results of verification of hypothesis of exponentiality of the distribution (rejection or the lack of bases for rejection) different tests and ability to "distinguish" the exponential distribution from the "real" (generated) distribution. Lets note that not distinguishing, by the statistical tests, the exponential distribution from other distributions, like Weibull, gamma, for given group of parameters is a significant practical argument for using

[2] In the theory of verification of hypotheses, for the sake that the tests (in general) do not verify error of type II (accepting the hypothesis H_0 when it is not valid) – so called the significance tests, the procedure of verification ends with rejection of the tested hypothesis (the significance test verifies the error of type I – rejection of hypothesis H_0 when it is valid) or statement of lack of bases for rejection of the hypothesis (which is not equal to its acceptance). Another problem is the definition of the hypotheses H_0 and H_1 – often, especially for the consistency tests, H_1 is the opposite to H_0. Often H_0 is defined in such a way that it is the negation of the hypothesis really tested – such situation often occurs in case of parametric hypotheses. It should be remembered that on the choice of H_0 and H_1 hypotheses depends the choice of tests and critical areas. In case of earlier selection of the test the form of the hypotheses should be consistent with the conditions of the test applicability.

Author assumed that in all following tests the H_0 hypothesis is in the form: „the sample originates from the population of the exponential distribution", H_1 is the negation of the H_0. The hypothesis does not specify the λ parameter for the exponential distribution (complex hypothesis) and is estimated based on the sample (as the inversion of the average value from the sample).

In case of the λ Kolmogorov test there exists the possibility of testing the hypothesis H0 in individual ranges (more, see [4], p. 73).

It should be emphasised that in some publication the term „the significance test" is used alternatively to the term „the parametric test" so it is to some extent the test that verifies the level of significance. Of course the parametric tests are in most cases also the test that only verify the significance level.

[3] From the fact, that we don't know the error of type II it does not emerge that we can not state for which test it is smaller, although it is not easy and the result depend on many factors, especially on the quantity of sample and the form of the alternative hypothesis.

the exponential distribution in different models, despite the theoretical reservations for its application.

2. The Statistical Apparatus

The analysis uses the Kolmogorov-Stephens test, Cramer-von Mises test, Watson test and Anderson-Darling test modified for testing the hypotheses of exponentiality of distribution for composite hypotheses (parameter of the exponential distribution estimated based on the sample). For comparison the classic Kolmogorov λ test was analysed. The samples were generated with quantity of 25, 50, 75 and 100. By estimation of the critical values the exact distributions were taken into consideration (depending of the quantity of sample, when such values were at hand) and/or the boundary distributions. The test statistics and the critical values of tests are gathered in the Table 1.

Samples were "taken" from the distributions: exponential, Erlang, Weibull, gamma, power and chi-square. The cumulative distribution function and the designation of parameters are gathered in the Table 2. The generation of samples and all the computations were performed in the spreadsheet Excel, using the built-in distribution functions and specially developed procedures (computations of the test statistics). The built-in generator for uniform distribution from the range (0,1) was used and the fact that X is the random continuous variable and the random variable $Y = F(X)$ has an uniform distribution on the section [0,1]. By building the inverse functions to the distribution functions (or using the built-in functions) and by generating the parameters of the distributions the simple samples were obtained out of the population of respective distributions. For such samples the hypothesis was tested that the samples originates from the population of exponential distribution estimating, based on the sample, the parameter of the exponential distribution (the composite hypothesis). Next it was counted in how many cases the result of the test showed the lack of bases for rejection of the hypothesis and in how many cases all the tests (and respectively 6, 5, 4, 3) showed the same result (the lack of bases for rejection of the hypothesis). Also the consistency of respective pairs of the test were assessed. One should note, that only in situation, when the sample was generated out of the population of exponential distribution, the lack of bases for rejection of the hypothesis is the correct result (we assume that the random generators are valid). In every other case (for samples generated out of other distributions) the lack of bases for rejection of the hypothesis means that the statistical test did not distinguish between non-exponential and exponential distribution (from our point of view that means false conclusion). But for testing only the consistency of the results of the test it does not matter – it is important if the results are identical.

One should note the important fact related to the generation of the random samples. For the sake of use of inverse functions to the relevant distribution functions (and properties of the gamma distribution) and testing the composite hypothesis –

Table 1

Used statistical tests and critical values of tests

Name and designation of the test	Test statistics	Critical value of the test for the significance level α = 0,05
Kolmogorov-Stephens test (**K**)	$D = \max\limits_{1 \le i \le n}\left(\left\lvert\frac{i}{n} - F(x_i)\right\rvert, \left\lvert F(x_i)\ \frac{i-1}{n}\right\rvert\right)$	n = 100 *0,13403* n = 75 *0,15442* n = 50 *0,21723* n = 25 *0,26404*
Kolmogorov-Stephens test (**K-S**)	$D_1 = \left(D + \frac{0,2}{n}\right)\left(\sqrt{n} + 0,26 + \frac{0,5}{\sqrt{n}}\right)$	1,094
Kolmogorov-Stephens test (**K-S1**)	$\sqrt{n}D$	n = 100 *0,9900* n = 75 *0,9888* n = 50 *0,9869* n = 25 *0,9824*
Cramer – von Mises test (**C-M1**)	$W_3^2 = W^2\left(1 + \frac{0,16}{n}\right)$ where the $W^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left(F(X_i) - \frac{2i-1}{2n}\right)^2$	n = 100 *0,220* n = 75 *0,220* n = 50 *0,219* n = 25 *0,217*
Cramer – von Mises test (**C-M**)	as above	*0,224*
Watson test **W**	$U_3^2 = U^2\left(1 + \frac{0,16}{n}\right)$ where the $U^2 = \frac{1}{12n} + \sum_{i=1}^{n}\left(F(X_i) - \frac{2i-1}{2n}\right)^2$ $-n\left(\left(\frac{1}{n}\sum_{i=1}^{n} F(X_i)\right) - 0,5\right)^2$	*0,159*
Anderson – Darling test (**A-D**)	$A_7^2 = A^2\left(1 + \frac{3}{10n}\right)$ where the $A^2 = -n - \frac{1}{n}\sum_{i=1}^{n}[(2i-1)\ln F(x_i)$ $+(2n-2i+1)ln(1 - F(x_i))]$	*1,321*

In tests: K-S, C-M, W and A-D for assessment of the critical value the boundary distribution was used.

Source: K test and critical values – [3], C-M, C-M1 and critical values – [8], other – [2].

calculation of the parameter λ of the exponential distribution based on the sample, the results of the statistical tests are independent on some of the parameters of the generated distributions, for example: if we generate the exponential distribution, by fixed sample originating from the uniform distribution in the section (0,1), the values of the test statistics do not depend on the value of parameter λ used in the inverse function to the distribution function of the exponential distribution (for each value λ are the same). They only depend on the value of elements in the generated sample originating from the uniform distribution in the section (0,1). The situation for other distributions is similar – if the values of significant parameters are fixed, then the results of the tests depend only on the sample generated from the uniform distribution, the values of the parameters and are identical for all values of other

Table 2

Distribution functions of the generated distributions

Name of distribution	Distribution functions
exponential distribution	$\mathbf{F(x)} = \begin{cases} \mathbf{0}, & x \le 0 \\ \mathbf{1 - e^{-\lambda x}}, & x > 0 \end{cases}$
Weibull distribution	$\mathbf{F(x)} = \begin{cases} \mathbf{0}, & x \le 0 \\ \mathbf{1 - e^{-(bx)^{\nu}}}, & x, b, \nu > 0 \end{cases}$
gamma distribution	$\mathbf{F(x)} = \begin{cases} \mathbf{0}, & x \le 0 \\ \dfrac{\boldsymbol{\Gamma(p, bx)}}{\boldsymbol{\Gamma(p)}}, & x > 0,\ b > 0,\ p > 0 \end{cases}$
power distribution	$\mathbf{F(x)} = \begin{cases} \mathbf{0}, & x \le 0 \\ \left(\dfrac{\mathbf{x}}{\mathbf{b}}\right)^{\frac{1}{\delta}}, & x \in (0, b] \quad, \mathbf{b}\boldsymbol{\delta} > 0 \\ \mathbf{1}, & x > b \end{cases}$
Erlang distribution	$\mathbf{F(x)} = \begin{cases} \mathbf{0}, & x \le 0 \\ \mathbf{1} - \sum_{\mathbf{j=0}}^{\mathbf{n-1}} \dfrac{\mathbf{(\lambda x)^j e^{-\lambda x}}}{\mathbf{j!}}, & x > 0,\ \lambda > 0,\ n = 1, 2, \ldots \end{cases}$
chi-square distribution	$\mathbf{F(x)} = \begin{cases} \mathbf{0}, & x \le 0 \\ \dfrac{\boldsymbol{\Gamma}(\frac{\mathbf{k}}{\mathbf{2}}, \frac{\mathbf{x}}{\mathbf{2}})}{\boldsymbol{\Gamma}(\frac{\mathbf{k}}{\mathbf{2}})}, & x > 0,\ k = 1, 2, 3, \ldots \end{cases}$
$\boldsymbol{\Gamma}\mathbf{(p)} = \int_{\mathbf{0}}^{+\infty} \left(\mathbf{x^{p-1} e^{-x} dx}\right)$ **funkcja gamma** $\boldsymbol{\Gamma}\mathbf{(p, bx)} = \int_{\mathbf{0}}^{\mathbf{bx}} \left(\mathbf{y^{p-1} e^{-y} dy}\right)$ **niepełna funkcja gamma**	

parameters. The significant meaning have the following parameters: ν in the Weibull distribution, p in the gamma distribution, δ in the power distribution and k in the Erlang distributions (other parameters can not be “random” by fixing their values) and chi-square. By generating the Erlang distribution the $k = 1$ parameter was ommited, and by chi-square the parameter $k = 2$, since that distributions, with given parameters, are exponential.

3. The Results of Analysis

The basic results of simulation are gathered in the Tables 3-8. The results are located with similar order: the type of distribution the simple sample originates (is generated) from, the quantity of sample, the value of the significant distribution parameter, the number of samples (for each quantity and value of the parameter), the ratio (in percent) of samples for which the result did not give bases for rejection of the hypothesis of the exponentiality of the distribution – for each of 7 tests, the ratio (in percent) for which the results of the tests were identical – separately for all 7 tests and for 6 test, ommiting the Kolmogorov(K) test.

The exponential distribution (Tab. 3). Since the significance level α = 0,05 was assumed, it was expected that the ratio of the results "the lack of bases for rejection of the hypothesis" would be around 95%. The results of simulation are different, at least for some tests. For the tests C-M, C-M1 and W are consistent with expectations, while there is almost no difference in the results between the use of exact distribution and boundary distribution for the Cramer-von Mises test, also the results of the Watson test do not differ much. The classic Kolmogorov (K) test is in this case not sensitive to the quantity of the sample giving over 99% of the "correct" answers – and, as it shows later, it is not an accident, the K test is "reluctant" to reject the null hypothesis. That result is clearly different from results of other tests, especially modified Kolmogorov-Stephens tests. The K-S1 test disqualified significantly more samples as coming from the population of exponential distribution as related to other tests. What is interesting, the biggest difference was observed for the samples of quantity 100 and 25, while in the latter case there is no clear difference between the K-S and K-S1 tests – it seems that the greater quantity of the sample should blur the difference in the results, but the observed dependency is opposite.

Table 3

The percentage of samples, for individual test, for which the result of the test did not give bases for rejection of the hypothesis of the exponentiality of the distribution, since samples were generated from the exponential distribution generator

The exponential distribution, the number of samples 800 for each of the quantity of sample *n*									
n	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
100	99,25%	92,38%	88,88%	94,13%	94,13%	94,13%	94,63%	88,00%	90,25%
75	99,25%	93,88%	92,13%	95,25%	95,00%	95,88%	95,00%	91,00%	93,00%
50	99,13%	94,25%	92,63%	95,63%	95,13%	96,00%	94,75%	92,13%	94,13%
25	99,38%	90,63%	90,75%	94,50%	94,13%	94,75%	92,00%	87,38%	89,63%

The Erlang distribution (Tab. 4). The differentiation ability of the Erlang distribution from the exponential distribution is strongly dependent on the quantity of the sample and the parameter k – it increases with the increase of the quantity of the sample and the increase of the parameter k. Except the K test there generally is no problem for the quantity 100 and 75, for the quantity 50 there are problems for $k = 2$, and for the quantity 25 for $k = 2, 3$, while for $k = 2$ the attempt to differentiate between the Erlang distribution and the exponential distribution is out of sense. If we take into account the stated limitations, then generally it is not important which test we use, though for the C-M1 test the ratio of "proper" answers is greatest (for the quantity 100 and 75 the tests C-M and C-M1 gave identical results) and for the W test it is the lowest. That conclusion is confirmed by high consistency of the results of the tests (without K test), except for the $k = 2$ the consistency is in excess of 94%. Similar results of consistency were obtained for K-S1 test with C-M and C-M1 tests for samples of quantity 50, 75 and 100. The K test is significantly worse – one could say it is by class worse.

Table 4

The percentage of samples, for individual test, for which the result of the test did not give bases for rejection of the hypothesis of the exponentiality of the distribution, since samples were generated from the Erlang distribution generator

The Erlang distribution, quantity of the sample 100, number of samples 800 for each value of *k*									
k	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
2	8,13%	0,63%	0,38%	0,25%	0,25%	0,63%	0,00%	91,9%	99,1%
3-6	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	0,0%	100,0%	100,0%
The Erlang distribution, quantity of the sample 75, number of samples 800 for each value of *k*									
k	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
2	25,00%	3,63%	2,25%	2,25%	2,25%	4,75%	1,38%	76,1%	95,9%
3	0,75%	0,75%	0,75%	0,75%	0,75%	0,75%	0,75%	100,00%	100,00%
4-6	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	100,00%
The Erlang distribution, quantity of the sample 50, number of samples 800 for each value of *k*									
k	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
2	52,50%	11,88%	9,75%	9,50%	8,75%	15,25%	8,00%	53,13%	88,13%
3	4,75%	0,13%	0,00%	0,13%	0,13%	0,88%	0,00%	95,25%	99,13%
4-6	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	100,00%
The Erlang distribution, quantity of the sample 25, number of samples 800 for each value of *k*									
k	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
2	84,88%	38,00%	38,25%	42,38%	40,75%	51,00%	47,63%	48,00%	79,25%
3	48,500%	6,875%	7,000%	7,125%	6,375%	10,875%	7,625%	57,00%	94,25%
4	19,125%	0,625%	0,625%	0,250%	0,250%	1,250%	0,500%	81,00%	98,63%
5	4,50%	0,13%	0,13%	0,00%	0,00%	0,13%	0,00%	95,50%	99,88%

The Weibull distribution (Tab. 5). In case of the Weibull distribution the differentiation from the exponential distribution is possible (effective) only in case of parameter ν in the range (0.25;0.65) and above 1.35 for sample quantity at least

50 for all tests except the K test, for the A-D test also for parameter value (0.65;0.85) but for samples of 100 element. For tests of quantity 25 the error made is at the level of 5-10% – one should exclude the Watson test, that clear is not suitable for such a situation. At the same moment there is strong consistency of the results of the tests for the value of parameter ν in the range (0.85;1.15) lowered by relatively low consistency of A-D and W tests' results for the value of parameter ν in the range (0.65;0.85) and (1.15;1.35). What is interesting, there is a clear supremacy of the pattern: negative results for five tests and the lack of bases for rejection of the hypothesis for one test over the opposite pattern: negative result for one test and the lack of bases for rejection of the hypothesis for five tests. The problem is significant since the Weibull distribution is preferred as the model more suitable to the reality than the exponential distribution. In situation when it is not possible to differentiate the distributions in the empirical manner, using the Weibull distribution starts being doubtful. So the question arises if there are more subtle (statistical) methods for differentiation of the two types of distributions. What should be the minimal quantity of samples to effectively differentiate (with existing statistical apparatus).

Table 5

The percentage of samples, for individual test, for which the result of the test did not give bases for rejection of the hypothesis of the exponentiality of the distribution, since samples were generated from the Weibull distribution generator

The Weibull distribution, the quantity of sample 100, the number of sample 500 for each range of parameter ν									
ν	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0,25;0,65)	0,60%	0,20%	0,20%	0,20%	0,20%	0,20%	0,20%	99,60%	100,00%
(0,65;0,85)	40,20%	18,20%	15,80%	15,00%	14,80%	26,40%	11,00%	69,00%	81,40%
(0,85;0,95)	91,00%	76,00%	71,60%	76,20%	75,80%	80,60%	72,40%	76,80%	85,80%
(0,95;1)	97,60%	90,20%	88,20%	92,40%	92,00%	92,00%	90,60%	88,00%	91,20%
(1;1,05)	98,40%	86,40%	84,60%	90,00%	88,80%	89,80%	90,80%	84,20%	88,20%
(1,05;1,15)	95,60%	79,00%	74,80%	83,60%	83,20%	85,80%	84,40%	78,40%	84,80%
(1,15;1,35)	59,00%	31,60%	26,00%	25,80%	24,80%	36,20%	25,00%	61,80%	82,80%
>1,35	1,80%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	98,20%	100,00%
The Weibull distribution, the quantity of sample 75, the number of sample 500 for each range of parameter ν									
ν	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0,25;0,65)	1,80%	0,60%	0,60%	0,40%	0,40%	1,20%	0,20%	98,20%	99,00%
(0,65;0,85)	53,80%	25,20%	22,80%	24,60%	24,20%	37,00%	17,60%	61,40%	77,40%
(0,85;0,95)	98,60%	84,60%	80,20%	84,60%	84,00%	89,80%	82,20%	76,80%	82,80%
(0,95;1)	98,40%	90,60%	87,60%	94,40%	94,20%	94,80%	92,80%	88,60%	91,20%
(1;1,05)	97,60%	90,00%	86,40%	89,40%	88,60%	91,60%	89,40%	86,80%	91,40%
(1,05;1,15)	99,00%	85,60%	82,20%	88,80%	88,00%	91,20%	88,60%	81,80%	87,60%
(1,15;1,35)	71,40%	32,60%	27,00%	28,20%	27,40%	38,00%	27,60%	49,00%	78,00%
>1,35	5,40%	1,80%	1,60%	1,20%	1,00%	1,60%	1,20%	95,60%	99,20%

Continued Tab. 5

The Weibull distribution, the quantity of sample 50, the number of sample 500 for each range of parameter v									
v	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0,25;0,65)	5,00%	1,20%	1,20%	1,00%	1,00%	3,00%	1,00%	95,60%	98,00%
(0,65;0,85)	73,20%	40,60%	38,20%	44,80%	43,60%	55,80%	37,60%	59,20%	75,40%
(0,85;0,95)	98,00%	88,40%	85,80%	90,40%	89,80%	93,40%	86,60%	85,00%	88,80%
(0,95;1)	99,00%	97,20%	96,60%	97,60%	97,60%	96,60%	95,40%	95,60%	97,00%
(1;1,05)	98,20%	93,00%	91,60%	94,60%	94,20%	93,80%	94,60%	92,00%	94,00%
(1,05;1,15)	100,00%	91,60%	90,40%	92,20%	91,40%	93,20%	92,60%	87,60%	92,00%
(1,15;1,35)	86,40%	60,20%	55,80%	59,40%	58,00%	65,20%	59,20%	63,80%	83,20%
>1,35	9,00%	2,00%	1,40%	1,60%	1,60%	2,40%	1,60%	92,20%	98,40%
The Weibull distribution, the quantity of sample 25, the number of sample 500 for each range of parameter v									
v	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0,25;0,65)	25,20%	8,80%	8,80%	8,60%	8,00%	17,60%	5,00%	78,80%	86,40%
(0,65;0,85)	88,60%	64,60%	64,60%	69,60%	67,80%	68,40%	41,60%	50,80%	63,80%
(0,85;0,95)	98,80%	85,80%	86,20%	92,40%	91,80%	91,40%	80,00%	77,00%	81,80%
(0,95;1)	98,40%	93,00%	93,00%	93,60%	93,40%	94,20%	87,20%	86,80%	89,00%
(1;1,05)	98,60%	90,40%	90,40%	94,00%	92,60%	93,20%	90,20%	86,40%	89,20%
(1,05;1,15)	97,80%	82,40%	82,40%	90,80%	89,40%	89,40%	91,40%	83,20%	86,20%
(1,15;1,35)	95,80%	67,00%	67,20%	71,60%	69,40%	74,80%	73,00%	67,20%	85,00%
>1,35	26,40%	8,80%	8,80%	9,60%	8,60%	11,40%	10,60%	81,80%	95,60%

Does inversion of the hypotheses (the Weibull distribution is null hypothesis and the exponential distribution is the alternative hypothesis) significantly change the situation? It should be emphasised that in case of the Weibull distribution (and especially gamma distribution) there emerge computational problems in the Excel spreadsheet for parameterg$v$ (p for gamma distribution) less than 0.25 (0.2).

The gamma distribution (Tab. 6). For the gamma distribution the situation is similar as for the Weibull distribution. For the value of parameter p in the range (0.2;0.4) there is effective possibility of differentiation of the exponential distribution from the gamma distribution – it does not relate to the quantity of sample 25 for the tests K and W. It points out that for those values of the p parameter the K test results for quantity 100, 75 and 50 are identical as for other tests. For those quantities, also for the parameter p >1.6 (excluding K test) the differentiation between the distributions is effective. Such situation occurs also for the parameter in the range (0.4;0.7) and quantity 100 and for A-D test by the quantity 75. In other cases the attempts to differentiate are out of sense. The closer is the p parameter to 1 the more consistent are the results of the tests. Lets note that for p in the range (0.9;1.1) (similarly for the Weibull distribution for v in the range (0.95;1.05)) the percentage of the results "the lack of bases for rejection of the hypothesis" is on the same level as in case of testing the hypothesis of exponentiality of the distribution, when the sample originated (was generated) from the population with exponential distribution (Tab. 3). That emerges from the fact that the gamma distribution for $p = 1$ and the Weibull distribution for $\nu = 1$ are exponential distributions, since the difficulty in differentiation between the Weibull and gamma distributions for respective parameters close to 1 is fully understandable.

Table 6

The percentage of samples, for individual test, for which the result of the test did not give bases for rejection of the hypothesis of the exponentiality of the distribution, since samples were generated from the gamma distribution generator

The gamma distribution, the quantity of sample 100, the number of sample 500 for each range of parameter p									
p	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0,2;0,4)	0,20%	0,20%	0,20%	0,20%	0,20%	0,20%	0,20%	100,00%	100,00%
(0,4;0,7)	26,00%	7,20%	6,20%	7,00%	6,80%	13,20%	3,40%	77,00%	89,40%
(0,7;0,9)	88,40%	66,80%	60,80%	66,20%	65,80%	75,20%	57,60%	65,20%	76,20%
(0,9;1)	97,20%	90,60%	87,40%	91,40%	90,60%	91,40%	89,20%	87,00%	90,60%
(1;1,1)	98,00%	86,20%	83,40%	89,40%	88,20%	87,80%	88,80%	82,60%	87,20%
(1,1;1,3)	93,60%	77,00%	72,20%	80,40%	79,80%	81,60%	79,60%	76,80%	84,60%
(1,3;1,6)	70,60%	36,40%	30,40%	33,80%	33,00%	41,40%	30,20%	54,20%	80,20%
>1,6	4,00%	0,40%	0,00%	0,20%	0,20%	1,20%	0,00%	96,00%	98,80%
The gamma distribution, the quantity of sample 75, the number of sample 500 for each range of parameter p									
p	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0,2;0,4)	0,40%	0,20%	0,20%	0,20%	0,20%	0,40%	0,20%	99,80%	99,80%
(0,4;0,7)	31,60%	13,40%	10,20%	12,20%	12,20%	19,00%	6,60%	73,80%	86,60%
(0,7;0,9)	95,80%	78,00%	72,40%	76,60%	75,80%	84,40%	70,80%	68,60%	77,60%
(0,9;1)	97,80%	90,20%	87,60%	93,40%	93,40%	94,20%	92,20%	89,00%	91,40%
(1;1,1)	98,40%	88,60%	84,40%	88,20%	87,60%	89,80%	89,40%	84,00%	89,60%
(1,1;1,3)	97,00%	84,20%	78,20%	83,20%	82,60%	86,40%	81,80%	78,40%	85,60%
(1,3;1,6)	83,20%	40,60%	33,20%	37,80%	36,40%	48,20%	34,80%	42,40%	71,00%
>1,6	14,40%	3,60%	2,60%	1,80%	1,80%	4,40%	1,40%	87,00%	96,40%
The gamma distribution, the quantity of sample 50, the number of sample 500 for each range of parameter p									
p	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0,2;0,4)	3,80%	1,40%	1,00%	1,20%	1,00%	1,40%	0,40%	96,40%	98,20%
(0,4;0,7)	53,40%	24,00%	21,20%	25,40%	24,00%	32,60%	16,60%	59,40%	78,20%
(0,7;0,9)	90,60%	74,60%	72,00%	79,80%	78,80%	83,20%	71,40%	71,40%	78,20%
(0,9;1)	99,20%	96,00%	95,40%	96,40%	95,80%	95,80%	93,80%	93,20%	95,20%
(1;1,1)	98,20%	93,20%	91,20%	94,60%	94,40%	93,80%	95,00%	91,20%	92,60%
(1,1;1,3)	88,80%	77,40%	73,80%	83,80%	83,00%	85,60%	83,80%	77,80%	83,00%
(1,3;1,6)	78,80%	57,60%	54,80%	63,00%	61,00%	64,80%	60,40%	67,60%	80,00%
>1,6	15,00%	2,40%	1,80%	3,20%	3,20%	4,60%	2,80%	86,20%	96,40%

Continued Tab. 6

The gamma distribution, the quantity of sample 25, the number of sample 500 for each range of parameter p									
p	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0,2;0,4)	22,80%	6,20%	6,20%	7,40%	7,20%	12,80%	2,00%	78,60%	89,00%
(0,4;0,7)	78,20%	45,80%	46,20%	51,20%	48,80%	50,60%	19,60%	40,20%	61,80%
(0,7;0,9)	97,20%	82,40%	82,40%	88,00%	87,00%	87,20%	70,40%	70,40%	76,40%
(0,9;1)	98,20%	92,40%	92,40%	93,20%	92,80%	92,80%	86,40%	86,20%	88,80%
(1;1,1)	98,60%	90,80%	90,80%	94,20%	92,40%	93,00%	91,80%	87,40%	90,40%
(1,1;1,3)	97,80%	84,60%	85,00%	92,00%	91,00%	91,00%	94,60%	85,20%	87,60%
(1,3;1,6)	97,00%	74,80%	74,80%	79,60%	77,60%	82,80%	79,00%	73,20%	85,00%
>1,6	42,80%	15,60%	15,80%	17,80%	17,40%	21,40%	18,20%	71,20%	91,60%

The power distribution (Tab. 7). The power distribution is significantly used in the theory of reliability and maintenance. It is quite easily (with enough quantity of sample) differentiatable from the exponential distribution (even for the K test). The percentage of the "correct answers" increases fast together with the quantity of sample and the decrease of the value of parameter δ, while keeping high consistency of the results (except for the K test). For the quantity of the sample 100 practically do not exist any problems with differentiation, for the quantity 75, by δ >1.15 the error shows in the range 2-9%, for the quantity 50 the differentiation becomes ineffective for $\delta > 1.15$, for δ in the range (1.05;1.15) the error appears in the range 6% (C-M1) do 11% (W, K-S). For the quantity of sample 25 only for the value of parameter $\delta < 0.85$ exist effective possibility of differentiation between power and exponential distribution with selected tests.

Table 7

The percentage of samples, for individual test, for which the result of the test did not give bases for rejection of the hypothesis of the exponentiality of the distribution, since samples were generated from the power distribution generator

The power distribution, the quantity of sample 100, the number of sample 500 for each range of parameter δ									
δ	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0;0,65)	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	100,00%
(0,65;0,85)	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	100,00%
(0,85;0,95)	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	100,00%
(0,95;1)	1,80%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	98,20%	100,00%
(1;1,05)	4,80%	1,00%	1,00%	0,00%	0,00%	0,00%	0,00%	95,20%	99,00%
(1,05;1,15)	11,40%	0,20%	0,00%	0,00%	0,00%	0,20%	0,00%	88,60%	99,60%
(1,15;1,35)	33,60%	2,40%	1,60%	1,40%	1,20%	2,00%	1,40%	67,20%	98,40%
>1,35	17,80%	3,60%	2,40%	2,20%	1,80%	1,00%	0,40%	82,60%	96,40%

Continued Tab. 7

The power distribution, the quantity of sample 75, the number of sample 500 for each range of parameter δ									
δ	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0;0,65)	0,20%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	99,80%	100,00%
(0,65;0,85)	0,20%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	99,80%	100,00%
(0,85;0,95)	1,20%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	98,80%	100,00%
(0,95;1)	9,20%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	90,80%	100,00%
(1;1,05)	13,80%	1,40%	1,00%	0,00%	0,00%	0,20%	0,00%	86,20%	98,40%
(1,05;1,15)	28,40%	2,20%	1,00%	1,20%	1,20%	1,00%	0,60%	72,00%	97,20%
(1,15;1,35)	46,20%	6,00%	3,40%	3,40%	2,80%	4,80%	1,40%	54,80%	94,00%
>1,35	23,60%	8,40%	6,60%	7,20%	7,00%	3,80%	2,20%	78,20%	92,40%
The power distribution, the quantity of sample 50, the number of sample 500 for each range of parameter δ									
δ	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0;0,65)	0,20%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	99,80%	100,00%
(0,65;0,85)	2,40%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	97,60%	100,00%
(0,85;0,95)	16,40%	0,20%	0,20%	0,00%	0,00%	0,00%	0,00%	83,60%	99,80%
(0,95;1)	30,20%	2,40%	2,00%	0,20%	0,00%	2,00%	0,40%	69,80%	96,80%
(1;1,05)	43,40%	4,00%	3,00%	2,20%	2,20%	3,60%	2,80%	58,60%	96,40%
(1,05;1,15)	55,60%	11,00%	7,80%	6,80%	6,20%	11,20%	7,20%	49,20%	91,60%
(1,15;1,35)	80,00%	29,20%	25,20%	25,20%	24,20%	29,00%	26,00%	38,60%	83,00%
>1,35	32,00%	13,60%	12,60%	12,80%	12,40%	10,60%	8,20%	74,20%	90,20%
The power distribution, the quantity of sample 25, the number of sample 500 for each range of parameter δ									
δ	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
(0;0,65)	2,20%	0,20%	0,20%	0,60%	0,40%	0,60%	0,40%	98,00%	99,60%
(0,65;0,85)	53,60%	9,40%	10,00%	6,80%	6,20%	8,40%	7,80%	50,60%	91,40%
(0,85;0,95)	65,00%	23,80%	23,80%	21,20%	19,40%	22,20%	22,40%	50,20%	83,20%
(0,95;1)	80,40%	33,80%	34,20%	31,00%	29,40%	31,00%	25,60%	40,40%	81,20%
(1,05;1,15)	84,60%	46,00%	46,20%	43,60%	41,00%	45,20%	39,00%	50,40%	85,60%
(1,15;1,35)	79,60%	34,40%	34,60%	34,40%	31,80%	39,20%	34,80%	45,40%	80,80%
>1,35	54,80%	26,80%	26,80%	29,80%	28,60%	25,00%	15,40%	59,20%	82,00%

The chi-square distribution (Tab. 8). The possibility to differentiate the chi-square distribution from the exponential distribution depends on the quantity of the sample and on the number of degrees of freedom k. For samples with quantity 25, only for distributions with at least 6 degrees of freedom chi-square distribution can be easily differentiated from the exponential distribution (up to 10% of faulty results). For the quantity of the sample 50 it is possible for $k = 1$ and $k > 3$, excluding the Watson test and the K-S test, which are effective for $k > 4$ (we remember that the K test has its own rules). But for$k = 1$ and $k = 4$ we should expect about 4-10% of faulty results. For samples with quantity 75 and 100 only for $k = 3$

the are significant problems with differentiation of the chi-square and exponential distributions[4]. In case of the chi-square distribution the best differentiating test is the Anderson-Darling test. For quantity of the sample 50, 75 and 100 and $k \neq 3$ the consistency of all 6 tests (excluding K test) significantly exceeds 90%.

Table 8

The percentage of samples, for individual test, for which the result of the test did not give bases for rejection of the hypothesis of the exponentiality of the distribution, since samples were generated from the chi-square distribution generator

The χ^2 distribution, the quantity of sample 100, the number of sample 800 for each range of parameter *k*									
k	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
1	6,75%	0,88%	0,75%	1,00%	1,00%	2,13%	0,25%	93,13%	97,75%
3	67,50%	26,00%	21,00%	23,88%	23,25%	31,75%	19,50%	47,75%	80,25%
4	8,25%	0,75%	0,50%	0,25%	0,25%	0,63%	0,00%	91,75%	99,00%
5	0,25%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	99,75%	100,00%
6	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	100,00%	100,00%
The χ^2 distribution, the quantity of sample 75, the number of sample 800 for each range of parameter *k*									
k	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
1	15,13%	2,63%	2,25%	1,75%	1,63%	5,50%	0,25%	84,75%	94,00%
3	79,75%	39,50%	35,13%	37,63%	36,50%	47,50%	34,88%	48,75%	77,88%
4	24,50%	3,38%	2,00%	2,13%	2,00%	4,63%	1,50%	76,63%	96,0%
5	2,88%	0,00%	0,00%	0,00%	0,00%	0,25%	0,00%	97,13%	99,750%
6	0,13%	0,00%	0,00%	0,00%	0,00%	0,00%	0,00%	99,88%	100,00%
The χ^2 distribution, the quantity of sample 50, the number of sample 800 for each range of parameter *k*									
k	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
1	36,88%	11,63%	9,50%	10,00%	9,38%	17,25%	3,63%	66,00%	84,13%
3	87,63%	53,38%	49,75%	54,00%	52,63%	61,25%	51,88%	55,25%	78,00%
4	52,38%	12,50%	9,63%	9,38%	8,63%	15,38%	7,50%	52,63%	87,13%
5	17,63%	0,63%	0,50%	0,63%	0,63%	1,38%	0,38%	82,63%	98,75%
6	4,00%	0,13%	0,00%	0,13%	0,13%	0,13%	0,00%	96,00%	99,88%

[4] For $k = 2$ simulations were not conducted since the chi-square distribution for $k = 2$ is the exponential distribution, and probably for the same reason there exists the problem with differenatiation of the chi-square distribution with 3 degrees of freedom from the exponential distribution (for 1 degree of freedom the difference between the density functions are explicit – the density function is not limited).

Continued Tab. 8

The χ^2 distribution, the quantity of sample 25, the number of sample 800 for each range of parameter *k*									
k	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**
1	72,875%	36,375%	36,500%	40,125%	38,000%	46,000%	16,875%	43,00%	65,13%
3	94,75%	69,63%	70,13%	77,88%	76,25%	79,13%	78,75%	71,00%	81,88%
4	84,75%	38,00%	38,25%	42,50%	40,88%	51,00%	47,75%	48,13%	79,25%
5	66,13%	15,38%	15,88%	18,00%	15,88%	25,50%	20,38%	45,38%	83,63%
6	47,13%	6,13%	6,25%	6,13%	5,38%	10,38%	6,50%	57,38%	93,88%

4. Examples

Example 1

In the Institute of Transportation Systems and Electrical Engineering of the University of Technology and Humanities the research is taken considering the safety of pederastians in the urban traffic. During the research Mrs. Dębowska-Mróz, M.Sc. Eng. Measured and analysed so called the "acceleration noise" of the vehicle as the potential measure of hazard. She verified the hypothesis that the distribution of the acceleration noise is an exponential distribution using the samples of quantity 25, 50, 75 and 100[5]. For samples of quantity 25 and 50 all 7 tests gave negative result – the hypothesis of exponentiality of the distribution should be rejected. But for samples of quantity 75 and 100 the classic Kolmogorov test "did not find" bases for rejection of the hypothesis, other tests unanimously gave negative result. The histograms (Fig. 1.), especially for a 100-element sample, do not give stron bases for confirmation of the supposition about the exponential distribution of the acceleration noise – at least bimodality and probably shifting are visible.

s – "acceleration noise" [m/s^2]

Also the hypothesis was verified that the distributions of the acceleration noise are liable to the shifted exponential distribution. In case of samples with quantity 75 and 100 all 7 tests gave negative result, but for samples of quantities 25 and 50 for all tests there were no bases to reject the hypothesis, and the values of the test statistics were significantly smaller than the critical values[6]. We have a clear

[5] More precisely: the data was recorded from one test passage and for building samples respectively 25, 50, 75 and 100 calculated (momentary) values were collected from the "acceleration noise" (the sample of smaller quantity was a part of sample of greater quantity.

[6] More precisely: for n = 50 – 0,106 for K test, 0,816 for K-S test, 0,751 for K-S1 test, 0,087 for C-M,– C-M1 and W tests, 0,635 for AD test; for n = 25 – 0,106 for K test, 0,612 for K-S test, 0,531 for K-S1 test, 0,041 for C-M and C-M1 tests, 0,38 for W test, 0,338 for A-D test.

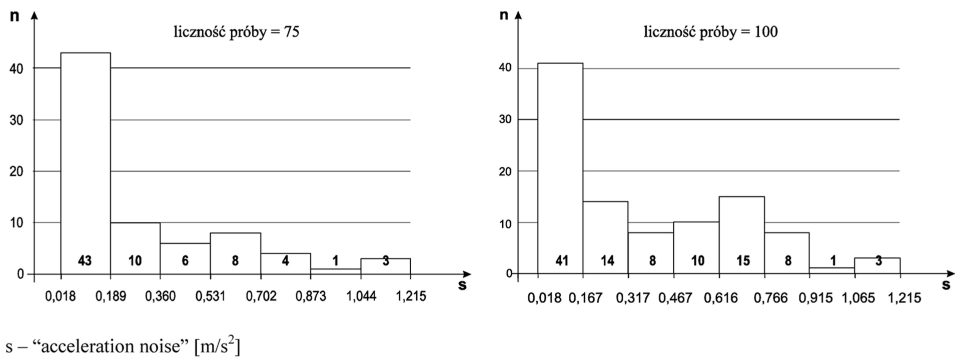

Fig. 1. The histogram of the "acceleration noise" for samples of quantity 75 and 100 measurements
Source: author's own work based on the measurements performed by M. Dębowska-Mróz

example, that the duration time of the experiment may significantly influence the distribution of the results of the measurements.

Example 2

Table 9

The results of testing the hypothesis that the results of the measurements of the traffic intensity originate from the shifted exponential distribution

	A				B				C				D				E			
	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV	I	II	III	IV
Average intensity	135	110	245	1006	158	162	320	1269	149	160	309	1241	170	169	339	1378	89	90	179	734
K	–	+	–	+	+	–	+	+	+	–	+	+	–	–	–	+	–	+	+	+
K-S	–	+	–	–	–	–	+	–	–	–	–	+	–	–	–	+	–	–	–	–
K-S1	–	+	–	–	–	–	+	+	–	–	–	+	–	–	–	+	–	+	–	–
C-M	–	+	–	–	–	–	+	+	–	–	–	+	–	–	–	+	–	+	–	+
C-M1	–	+	–	–	–	–	+	+	–	–	–	+	–	–	–	+	–	+	–	+
W	–	+	–	–	–	–	+	+	–	–	–	+	–	–	–	+	–	+	–	+
A-D	–	+	–	–	–	–	+	+	–	–	–	+	–	–	–	+	–	+	–	+

„–" means the rejection of the null hypothesis – attempt does not come from the shifted exponential distribution
„+" means lack of reason to reject the null hypothesis
Designation of the tests identical as in Table 1.
Source: author's own work based on the measurements performed under the guidance of M. Dębowska-Mróz

Under the supervision of M. Dębowska-Mróz, M. Sc., students measured the traffic intensity on the selected inter-node sections of the selected cross-sections of the traffic arteries of the city of Radom. The measurement was made between the 11a.m. and 5p.m. in 15 minute intervals regarding the travel direction structure. Based on the data collected for 5 sections the hypothesis was verified that the traffic intensity might be described with the shifted exponential distribution. Individual directions of travel were tested separately (samples I and II), jointly for both directions (sample III) and the hourly intensity with 15 minute step (sample IV). The results of the tests are showed in the Table 9.

Example 3

G. Krawczyk dealt with the estimation of the necessary energetic capacity of the container, that ensures maximal smoothing out the power peaks, that are generated in the Warsaw Metro [7]. He assumed that the sub-station should draw from the power grid the amount of power that equals to the average level of power in the traction power network. The momentary values of power that exceed the average should be handled by the sub-station and the container that supplies the traction power network. To all the values of power in the traction power network that are below average should be assigned a power draw by the container, that equals to the difference between the average power and the momentary power of the sub-station. Based on the measurements of current and voltage performed on the DC-side on one of the sub-stations of metro he determined the momentary power and momentary energy as well as other post-processed values for the developed method. Based on the collected data the hypotheses were verified, that they originate from the populations of exponential distributions – built by G. Krawczyk histograms did not exclude the rationality of such hypotheses (G. Krawczyk did not identify the distributions). For all the regarded values the 100% consistency of results was achieved – the hypotheses should be rejected. The tested values are not subjected to the exponential distribution.

5. Conclusions

The conclusions that follow the analysis are not (and rather they may not be) unambiguous. The choice of tests depends on the real aims of testing and possible alternative hypotheses (the "competitive" distributions). Unquestionable seems to be the fact that the classic Kolmogorov test should not be used for testing the composite hypotheses about the exponentiality of the distribution for small samples (n <100), unless it is used only for "justification" of acceptance for use of the exponential distribution. In situation when we intend to strongly confirm the exponentiality of the distribution we should rather use the K-S1 test. Between the results of the K-S and K-S1 tests there exists the significant difference – the K-S1 test more often rejects the null hypothesis. On the other hand between C-M and C-M1 tests the difference is not so significant, so it is possible to use the boundary distribution, the more that the tables for exact distribution are difficult to obtain. There exists also high consistency of the C-M and C-M1 tests with the Watson test (with significant difference in case of samples originating from the chi-square distribution), but there exist clear differences to the A-D test. In the literature it is often stated that the Anderson-Darling test is

Table 10

The results of testing the hypothesis of exponentiality of the distribution for samples from other than exponential distributions

Quantity of sample ***n***	**K**	**K-S**	**K-S1**	**C-M**	**C-M1**	**W**	**A-D**	**Consistency of 7 tests**	**Consistency of 6 tests**	**Consistency of 7 tests [%]**	**Consistency of 6 tests [%]**	**Number of samples**
100	5885	3994	3715	3980	3934	4312	3784	17527	18882	87,64%	94,41%	20000
75	7022	4538	4185	4440	4389	4916	4215	16774	18641	83,87%	93,21%	20000
50	8830	5528	5248	5551	5459	6000	5258	15835	18413	79,18%	92,07%	20000
25	12606	7528	7558	7980	7750	8420	7240	13404	17170	67,02%	85,85%	20000

The table gives information how many time there were no bases (for particular test) for rejection of the hypothesis of exponentiality of the distribution and how many times respectively all 7 or all 6 test (excluding K test) gave identical result. Designation of the tests identical as in Table 1.

sensitive to the exeptions in the boundary values and requires samples with high quantity – it is often stated that the Cramer – von Mises test is "better" for small samples and the Anderson-Darling test is "better" for large samples (the descriptions make reservation, that the sample should be greater than 20).

An important conclusion is that for small samples and composite hypothesis (tests with simple hypothesis were not performed), with selected statistical tests there is no possibility to differentiate the Weibull, gamma and exponential distributions – and these are the basic consistency tests for testing the hypothesis of exponentiality of the distribution for composite hypothesis (and simple too) available in the literature. It is important problem not only for theoretical discussion, but also for practice. There arises the question what should be the minimal quantity of samples to differentiate those distributions statistically (probably as a function of parameters) and is there, in practical research, any possibility to obtain samples with such quantities. The situation is presented in the Table 9, which shows in how many cases there were no bases do reject the hypothesis of exponentiality of the distribution for 20 000 samples (for each quantity of n = 100, 75, 50, 25), when samples were generated from other than exponential distributions (the structure consistent with tables 4-8). The tests with least number of faulty results were A-D and K-S1, the tests K-S, C-M and C-M1 had similar effectiveness. The K test stand out significantly – for n = 25 there is over 60% of faulty results. Despite the fact that 25% to 30% of results of tests for quantities 50, 75, 100 (excluding K test) are faulty, the consistency of all 6 tests is suprisingly high – more than 92%.

The undoubted virtue of the classic λ Kolmogorov test is a fact that in case of rejection of the hypothesis of the exponentiality of the distribution it would be hard to find the justification for using the exponential distribution. Let us emphasise that the above tests – for the Anderson-Darling test the adjustment is necessary – might be used for verification of the hypothesis of the shifted exponential distribution. The maximum likelihood estimator of the shift is the minimum value in the sample – in case of the shift value specified in the hypothesis the Anderson-Darling test does not require the adjustment.

Another interesting and in the long run important problem is the fact that there was about 10% of faulty results in case when samples originated from the exponential distibution (and were rejected as originated from other distribution by most of the tests) even with the quantity of the sample 100. It is then double of what one would expect based on the assumed significance level α= 0,05. The question is if it is a "property" of the test related to testing the composite hypothesis or it is the low quality of the generator of the uniform distribution built in the Excel spreadsheet (author is not familiar to any research about that problem). Similar results were obtained by author while testing the hypothesis of uniformity of the distribution (composite hypothesis) – also one of the conclusions stated that the classic Kolmogorov test was not suitable for testing the hypotheses about the uniformity of the distribution for the composite hypothesis. The simulation showed also that the modified Kolmogorov test (for that type of hypotheses) – Kolmogorov-Lilliefors test is less effective than for example Epps-Pulley test or Shapiro-Wilk test [9].

References

1. Bester L.: Analiza zintegrowanego systemu bezpieczeństwa w transporcie lądowym na przykładzie przejazdów kolejowych. Radom 2012, rozprawa doktorska, promotor prof. Andrzej Lewiński.
2. Domański Cz.: Statystyczne testy nieparametryczne. Państwowe Wydawnictwo Ekonomiczne, Warszawa 1979.
3. Firkowicz S.: Statystyczne badanie wyrobów. WNT Warszawa 1970.
4. Kisilowski J.: Podstawy technik pomiarowych. Skrypty Naukowe WSTE, Wydawnictwo Naukowe OBSERWACJE, Warszawa 2005.
5. Kopociński B.: Zarys teorii odnowy i niezawodności. PWN Warszawa 1973.
6. Koźniewska I., Włodarczyk M.: Modele odnowy, niezawodności i masowej obsługi. PWN Warszawa 1978.
7. Krawczyk G.: Ocena efektywności zastosowania zasobników w systemie zasilania linii metra. Radom 2012, rozprawa doktorska, promotor dr hab. inż. Marek Pawełczyk.
8. Kuśmińska-Fijałkowska A.K.: Zmodyfikowana metoda procesów przeładunkowych intermodalnych jednostek transportowych w terminalu lądowym. Radom 2008, rozprawa doktorska, promotor prof. Z.S. Łukasik.
9. Pearson E.S., Hartley H.O. (Editors): Biometrika Tables for Statisticians. Cambridge University Press, Vol. 2, 1976.
10. Perzyński T.M.: Problemy bezpieczeństwa sieci komputerowych stosowanych w sterowaniu ruchem kolejowym. Radom 2009, rozprawa doktorska, promotor prof. Andrzej Lewiński.
11. Rogowska R., Rogowski A.: Zgodność wyników testowania hipotezy o rozkładzie normalnym dla wybranych testów statystycznych. Logistyka nr 3/2011 (Logistyka – nauka, materiały VIII Konferencji Naukowo-Technicznej „Logitrans" – Logistyka, Systemy Transportowe, Bezpieczeństwo w Transporcie, s. 2317-2338).
12. Szczuraszek T., Macioszek E.: Analiza rozkładów odstępów pomiędzy pojazdami czasu na obwiedni małych rond, „Drogi i Mosty" 3/2010.
13. Woch J.: Statystyka procesów transportowych: Wydawnictwo Politechniki Śląskiej, Katowice 2001.
14. Woch. J.: Teoria potoków ruchu, wyd. internetowe.
15. Zeszyty Naukowe Politechniki Śląskiej. Seria Transport, zeszyt 50/2003, Wydawnictwo Politechniki Śląskiej, Gliwice 2003.