GEODESY AND CARTOGRAPHY
Vol. 60, No 2, 2011, pp. 123-134
DOI No 10.2478/v10277-012-0011-z

Sensitivity of robust estimators applied in strategy for testing stability of reference points. EIF approach

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Received: 25 May 2011/Accepted: 13 October 2011

Abstract: In deformation analyses, it is important to find a stable reference frame and therefore the stability of the possible reference points must be controlled. There are several methods to test such stability. The paper's objective is to examine one of such methods, namely the method based on application of *R*-estimation, for its sensitivity to gross errors. The method in question applies three robust estimators, however, it is not robust itself. The robustness of the method depends on the number of unstable points (the fewer unstable points there are, the more robust is the proposed method). Such property makes it important to know how the estimates applied and the strategy itself respond to a gross error. The empirical influence functions (EIF) can provide necessary information and help to understand the response of the strategy for a gross error. The paper presents examples of EIFs of the estimates, their application in the strategy and describes how important and useful is such knowledge in practice.

Keywords: displacement, stability, *R*-estimation, empirical influence function

1. Introduction

Analysis of deformation or displacement is a complex surveying problem. The foundations and methodology of such analyses are known well (see, e.g. Prószyński and Kwaśniak, 2006), however, the theory as well as practical techniques are still developed (e.g. Hekimoglu et al., 2010), and new methods are proposed to solve certain problems that also concern deformation (e.g. Denli, 2008; Wiśniewski, 2009; Duchnowski, 2010). It follows the increasing accuracy of surveying measurements, new technologies applied as well as the requirement of analysing deformation more frequently and with higher precision.

Deformation or displacement of certain points can be determined and analysed on the basis of measurement results obtained at least at two different epochs. Usually the observation sets are required and assumed to be free of outliers, namely observations that are affected with gross errors. However, gross errors may sometimes occur and spoil deformation analyses. Therefore, it is necessary to apply special procedures to find and reject outliers (e.g. Baarda, 1968; Ding and Coleman, 1996; Gui et al., 2007) or to use robust methods of adjustment. The problem of how to identify and cope with outliers is especially complex in the case of deformation analysis. This is due to the fact that tested points can really displace, which results in changes of observation at the second epoch. The problem is how to distinguish between such expected changes and unexpected gross errors (Shaorong, 1990). Another problem is that the robust procedures or methods are not always efficient, and their robustness, effectiveness or success depend on many factors (see, e.g. Hekimoglu and Erenoglu, 2007; Prószyński, 2010). Thus, it may happen that gross errors still influence results of analyses, regardless of methods applied. Therefore, it is important to know and understand how such errors may influence estimation process (e.g. Gui et al., 2011) or its results (e.g. Duchnowski, 2011).

Duchnowski (2008, 2009) proposed to apply estimates based on rank tests (*R*-estimates) to analyse deformation of geodetic networks. The main advantage of the estimators in question is their robustness against outlying observations. This property was the basis for the strategy for testing of stability of possible reference marks presented in (Duchnowski, 2010). Since this problem becomes complicated when some possible reference marks are not stable then application of *R*-estimate only is good enough in the simplest cases. Generally, *R*-estimates of point displacements should be supported by some robust estimates of standard deviation to find stable points properly. Duchnowski (2010) proposed to use two such estimates, namely MAD (median absolute deviation) and ADM (average distance to the median). They both are classified as estimates robust against outliers but they are robust in different ways (Rousseeuw and Verboven, 2002; Duchnowski, 2010, 2011).

Robustness of an estimate can be studied by applying two main statistical tools, i.e. breakdown values or influence functions. The breakdown values give an answer to the question of how many outliers can make the estimate fail. There are several kinds of breakdown values that can be applied: contamination breakdown values, replacement breakdown values or subjective breakdown points (see, e.g. Rousseeuw and Croux, 1993; Rousseeuw and Verboven, 2002; Xu, 2005). Duchnowski (2011) applied the most convenient kind of breakdown values, namely replacement ones, to investigate robustness of all estimates applied in the strategy. All the estimates in question are robust against gross errors, however, they do not guarantee robustness of the strategy itself. Generally, the higher number of unstable points, the less gross errors the strategy can withstand. Thus, the robustness of the strategy depends on the number of unstable reference marks (Duchnowski, 2011). For that reason it is important to understand how gross errors may influence the final strategy results.

The second way to investigate robust estimates is application of influence functions (IF). From a theoretical point of view such functions can be based on some assumptions concerning distributions of random variables and properties of estimates that are investigated (see, e.g. Huber, 1981; Hampel et al., 1986; Rousseeuw and Croux, 1993). In practice, it is also very useful to apply some versions of IF, namely empirical influence function (EIF) or its stylised variant (SEIF). In general, all these functions describe what happens with the estimate if an observation set is disturbed by a gross error (see, e.g. Huber, 1981; Hampel et al., 1986; Rousseeuw and Verboven, 2002). Thus, EIF can be applied to study how estimates that are used in the strategy respond to a gross error that may affect measurement results.

2. Strategy for testing stability of reference marks based on R-estimates

The main aim of the present paper is to investigate how the strategy for testing stability of reference marks, which was proposed in (Duchnowski, 2010), responds to a single gross error. This is very important from the practical point of view, especially when considering robustness of the estimates that are applied, namely R-estimate of the expected value, MAD and ADM, which was examined from a theoretical point of view in (Duchnowski, 2011). Before the strategy is tested the brief review is presented.

The basis for the strategy is the following R-estimate of the vertical displacement (Duchnowski, 2009, 2010):

$$\hat{\Delta}_k^R = med\left(\left[\tilde{\mathbf{v}}_2^k\right]_i - \left[\tilde{\mathbf{v}}_1^k\right]_j\right) \tag{1}$$

where $1 \leqslant i \leqslant n$, $1 \leqslant j \leqslant$ n, and $\tilde{\mathbf{v}}_1^k \in R^{n\times 1}$ and $\tilde{\mathbf{v}}_2^k \in R^{n\times 1}$ are the vectors of the initial residuals of the observations that concern the k^{th} element of the parameter vector in two measurement epochs, respectively; here $[\tilde{\mathbf{v}}_2^k]_i$ is the i^{th} element of the vector $\tilde{\mathbf{v}}_2^k$. The initial residuals are computed on the basis of the initial parameter vector $\tilde{\mathbf{x}} \in R^{m\times 1}$ and the functional model of a geodetic network in the following form:

$$\tilde{\mathbf{v}} = \mathbf{y} - \mathbf{A}\tilde{\mathbf{x}} \tag{2}$$

where $\mathbf{y} \in R^{n\times 1}$ is the observation vector, $\mathbf{A} \in R^{n\times m}$ is a known rectangular matrix. Of course to compute the vectors of the initial residuals $\tilde{\mathbf{v}}_1$ and $\tilde{\mathbf{v}}_2$ one should apply two vectors of the observations $\mathbf{y}_1$ and $\mathbf{y}_2$ that are obtained at two different measurement epochs, respectively. Usually, if a levelling network is considered, the parameter vector $\mathbf{x}$ contains the heights of the network points. The vector $\tilde{\mathbf{x}}$ of their initial values can be taken from the former computations or computed from the first epoch observations (Duchnowski, 2009). Thus, vectors $\tilde{\mathbf{v}}_1^k$ and $\tilde{\mathbf{v}}_2^k$ contain the initial residual to these height differences, for which the k^{th} point is one of the network vertices. Such initial residuals fulfil some theoretical assumptions concerning their distribution and enable to apply R-estimation in deformation analyses (Duchnowski, 2008, 2010).

In general, the stability of a potential reference point is tested on the basis of the estimate of Eq. (1). If the value of such estimated vertical displacement is acceptable considering random errors of the measurements then such point can be regarded as stable. Such approach is of course sufficient only in simple cases (Duchnowski, 2010). Generally, the R-estimates must be supported by some robust estimates of the standard deviation to correctly identify the stable reference frame. Here, such estimates are applied to analyse the standard deviation of the sample created from the elements of the vector $\tilde{\mathbf{v}}_2^k$. From a theoretical point of view, the standard deviations of such samples should be equal to the known or assumed accuracy of the measurements unless there are some outliers in the sample (or if it is too many outliers considering robustness of the estimates). Thus, they provide information about contaminations of vectors $\tilde{\mathbf{v}}_2^k$ with outlying observations which helps to find unstable points (see next section). Of course, such knowledge is essential in more complicated cases to verify if the estimated vertical displacement is a true one, or it is a consequence of the vertical displacements of the other points (Duchnowski, 2010, 2011). The estimates in question can also be computed for the vectors $\tilde{\mathbf{v}}_1^k$ to check if these vectors are free of outliers.

Duchnowski (2010) proposed to use two estimates of standard deviation, namely MAD (median absolute deviation, or more formally the median distance to the median)

$$MAD(Z) = 1.4826 \underset{i=1}{\overset{n}{med}} |z_i - med(z_i)| \tag{3}$$

and ADM (the average distance to the median)

$$ADM(Z) = \underset{i=1}{\overset{n}{ave}} |z_i - med(z_i)| \tag{4}$$

where Z is a random variable and $z_1, z_2, \ldots, z_n$ is a sample containing its realisations. Such choice resulted from properties of both estimates. In some critical cases these estimates are good supplements for each other, and comparison of their values can provide valuable information about outliers and their locations (see, e.g. Rousseeuw and Verboven, 2002; Duchnowski, 2010, 2011). Detailed description of the strategy can be found in (Duchnowski, 2010).

3. Outliers, their role and influences on the strategy results

It was already mentioned that the strategy is based on robustness against outliers of three estimates applied. In fact, it is sometimes based on the lack of the robustness, which helps to analyse more complicated cases, for example when two of four possible reference marks are not stable (Duchnowski, 2010). To understand the role of outliers, which may occur in the samples created from the elements of the vectors $\tilde{\mathbf{v}}_2^k$, one should first consider their sources (the vector $\tilde{\mathbf{v}}_1^k$ is assumed to be free of outliers). The obvious source of outlying observations are gross errors, however, it is not the only one. Let us now assume that all height differences between possible reference marks are measured twice, namely once at every epoch. In such case, if one of the possible reference marks, for example point number i, is unstable then it becomes a source of outlying observations in all vectors $\tilde{\mathbf{v}}_2^k$ (where $k \neq i$) (Duchnowski, 2011). This is a very important property that helps to identify unstable points correctly. For example, if there is only one unstable point, say the point number m, then the vector $\tilde{\mathbf{v}}_2^m$ is free of outliers that result from such instability while other vectors $\tilde{\mathbf{v}}_2^k$ (where $k \neq m$) contain one such type outlier. Thus, if one knows the number of outlying observations in all vectors $\tilde{\mathbf{v}}_2^k$ then the unstable point can be found much easier. However, in some cases it may also result in increased sensitivity of the strategy to gross errors, namely it might be hard to distinguish two types of outliers from each other. This can happen when there are more unstable points, and therefore there are many outlying observations of the second kind, namely resulting from instability of certain reference marks. Then neither the estimates nor the strategy can absorb any more outliers from the first source (resulting from gross errors). In other words, in such cases even one gross error can spoil the strategy results. The observation that is affected by a gross error might disturb estimation of the vertical displacement, especially if it coincides with other outlying observations, and also might spoil the MAD and ADM.

Let us now describe how the estimates respond to a gross error, which can be done by application of empirical influence functions. Consider a levelling network with five possible reference points and let the theoretical point heights be as follows: $H_1 = 0.000\ m$, $H_2 = 1.000\ m$, $H_3 = 2.000\ m$, $H_4 = 2.000\ m$ and $H_5 = 1.000\ m$ (these heights will also be assumed as the initial values of the parameters $\tilde{\mathbf{x}} \in R^{5\times1}$). Let us assume that all combinations of height differences h_{ij} (where i and j are the numbers of the network vertices) be measured twice (once in each measurement epoch) with a standard deviation of $\sigma = 1\ mm$. Thus, in general the observation vector can be written as $\mathbf{y} = [h_{12}, h_{13}, h_{14}, h_{15}, h_{23}, h_{24}, h_{25}, h_{53}, h_{54}, h_{43}]^{\mathrm{T}}$ (see, Fig. 1; similar example but in a different context was also considered in Duchnowski (2011)).

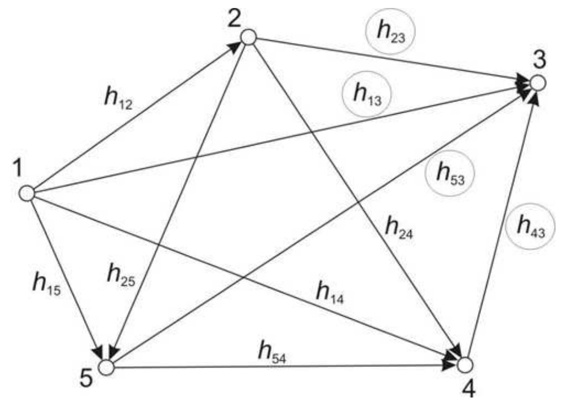

Fig. 1. Levelling network of the possible reference points

If all five points are stable then under such assumptions two vectors of measurement results can be simulated, namely $\mathbf{y}_1$ and $\mathbf{y}_2$, as follows (the estimated measurement accuracy is 0.6 mm and 0.7 mm, respectively):

$$\mathbf{y}_1 = [1.0002,\ 2.0007,\ 2.0008,\ 0.9998,\ 0.9988,\ 0.9999,\ -0.0005,\ 1.0003,\ 0.9996,\ -0.0004]^{\mathrm{T}}$$

$$\mathbf{y}_2 = [0.9993,\ 1.9995,\ 2.0006,\ 0.9998,\ 0.9996,\ 1.0010,\ -0.0007,\ 1.0001,\ 1.0007,\ 0.0009]^{\mathrm{T}}$$

These two vectors together with the theoretical heights of the network points, which are presented above, are the basis for the computations of the vectors $\tilde{\mathbf{v}}_1^k \in R^{4\times1}$ and $\tilde{\mathbf{v}}_2^k \in R^{4\times1}$ $(1 \leqslant \mathrm{k} \leqslant 5)$. For example, in Figure 1, the observations that concern the point 3 are marked with a circle. These four observations take part in computation of the vectors $\tilde{\mathbf{v}}_1^3$ and $\tilde{\mathbf{v}}_2^3$ for respective observation vectors. However, it is always important to check the direction of the observations, namely, whether the height difference was measured from, for example, the point number 1 towards the point 3 or in the opposite direction, from the point 3 to the point 1 (if the direction is opposite then the sign of the certain initial residual should be changed). One should notice that each initial residual is used twice, namely in computations for each of the network vertices.

Let us now describe how the estimates applied in the strategy respond to a gross error. It was mentioned that this can be done by application of EIF that is generally defined for the estimate T_n and the sample $z_1, z_2, \ldots, z_{n-1}$ as (see, Huber, 1981; Rousseeuw and Verboven, 2002)

$$\mathrm{EIF}\,(x) = T_n\,(z_1, z_2, \ldots, z_{n-1}, x) \tag{5}$$

This function can be adapted for the purpose of the present paper in the following form:

$$\mathrm{EIF}\,(x) = T_n^k\,(\mathbf{y}_1, \mathbf{y}_2 + \mathbf{g}) \tag{6}$$

where $\mathbf{g} = [x\ 0\ \cdots\ 0]^{\mathrm{T}}$ thus, here we assume that the first observation at the second epoch is affected with a gross error of x; T_n^k is $R-$estimate, namely $\hat{\Delta}_k^R$, of the displacement of the point number k (it may also be other estimates like for example MAD or ADM). Of course, several appropriate variants must be considered to study the robustness and responses of the estimates properly. Thus, several EIFs will be created for different variants in which certain points are not stable. Note that if some points are not stable then values of the certain height differences at the second epoch, i.e. the elements of the vector $\mathbf{y}_2$, change. For comparison, EIFs are also created for conventional estimates of the least-squares method (LSE), which are results of free adjustment of the levelling network.

Let us now consider some variants when two of five possible reference points are not stable. In such variants, there are many outliers that result from instability of reference points and hence such cases are the most complicated and the hardest to be analysed (Duchnowski, 2010, 2011). Furthermore, they may also be the most sensitive to a gross error. So let us examine three following variants: Variant 1, where points 1 and 2 are unstable and their assumed displacements are 20 mm and 10 mm, respectively; Variant 2, where points 1 and 5 are unstable with the theoretical displacements of 20 mm and -10 mm; and Variant 3, where points 4 and 5 are unstable with the theoretical displacements of 20 mm and -10 mm, respectively. Note that the gross error x affects the first observation, namely the height difference h_{12}, at the second epoch.

The empirical influence functions of the R-estimates of Eq. (1) and LSEs, for all proposed Variants, are presented in Figures 2, 3, and 4, respectively. The EIFs of LSE of vertical displacement are straight lines with the constant, for each of Variants, slope (in fact, the slope depends on a number of observations that is also constant for each Variant (see, e.g. Rousseeuw and Verboven, 2002)). Such shape of EIFs reflects sensitivity of the least squares method to gross errors.

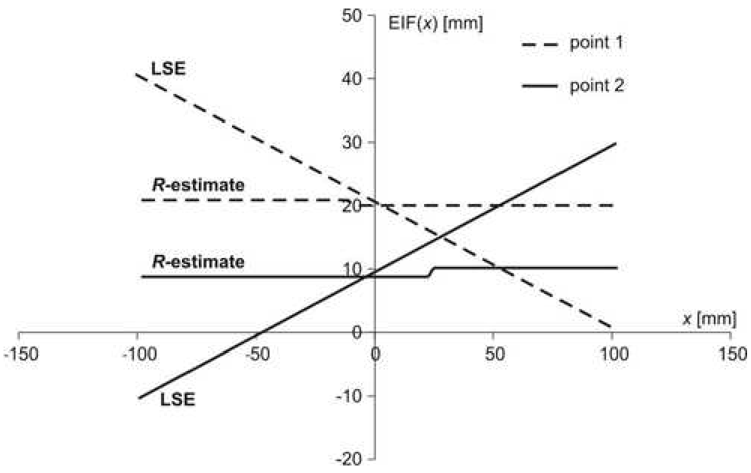

Fig. 2. EIFs of *R*-estimates and LSE of the vertical displacements when the points 1 and 2 are unstable

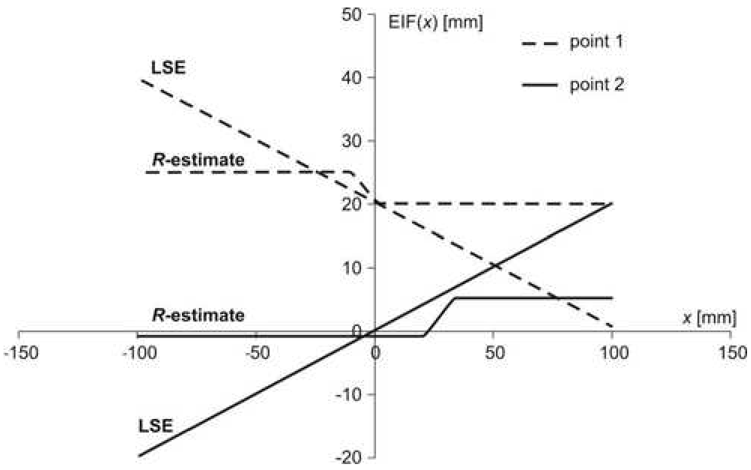

Fig. 3. EIFs of *R*-estimates and LSE of the vertical displacements when the points 1 and 5 are unstable

The EIFs for *R*-estimates are bounded, however, they show that the response of *R*-estimate of the vertical displacement to a gross error depends on configuration of unstable points. The gross error hardly influences the estimation results if the points 1 and 2 are not stable (note that the gross error affects the height difference between those two points). In such case, the estimate as well as the strategy itself is robust against the gross error.

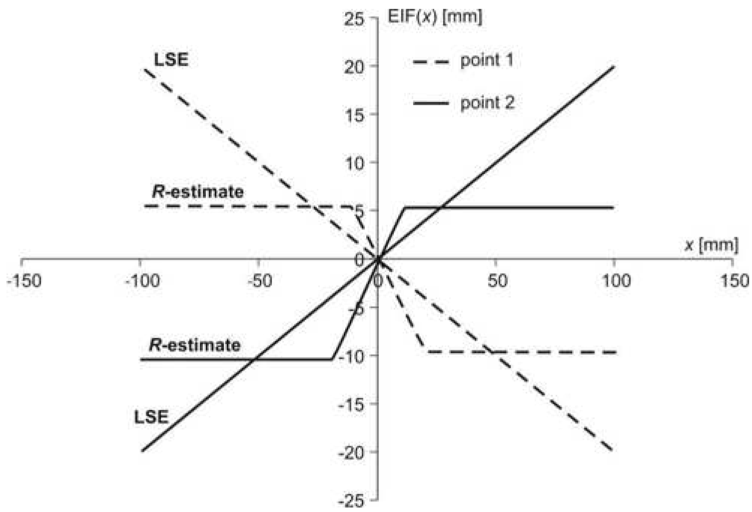

Fig. 4. EIFs of *R*-estimates and LSE of the vertical displacements when the points 4 and 5 are unstable

The influence of a gross error becomes more significant if the points 1 and 2 are stable and the other points are not. It is especially evident in Figure 4, however, even then the influence of the gross error is bounded, and the maximum influence is about 1 cm. It is also worth noting that for small values of gross errors the EIFs of the *R*-estimates have bigger slope than the EIFs of the LSE, which means that in such cases *R*-estimates are more sensitive to a gross error than the traditional estimates of displacement.

Now, let us examine the estimates of the standard deviation which are also applied in the strategy. Consider the same three variants and let us create EIFs of the MAD and the ADM, Eqs (3) and (4), respectively, and for comparison EIFs of the traditional estimate, namely the sample standard deviation (SD). Figures 5, 6 and 7 present the EIFs of the estimates in question, and obtained for the sample created from the elements of the vector $\tilde{\mathbf{v}}_2^1$ (computation for the point 1). One can create similar EIFs for the estimates that are obtained for the samples created from the elements of the vector $\tilde{\mathbf{v}}_2^2$ (computation for the point 2), (these functions are omitted here just in order not to obfuscate the figures).

The EIFs of the MAD are always bounded in contrast to the EIFs of the ADM and SD. When points 1 and 2 are not stable then a gross error does not affect the MAD.

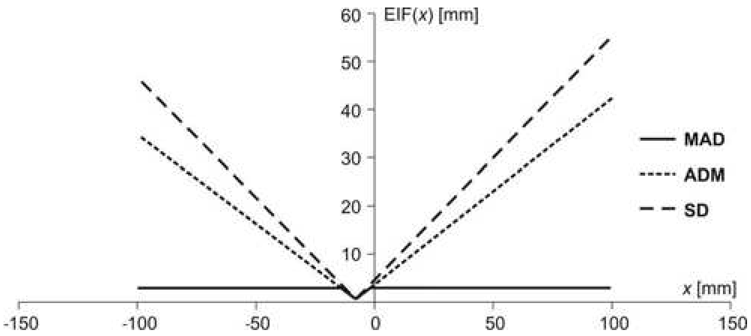

Fig. 5. EIFs of MAD, ADM and SD when the points 1 and 2 are unstable

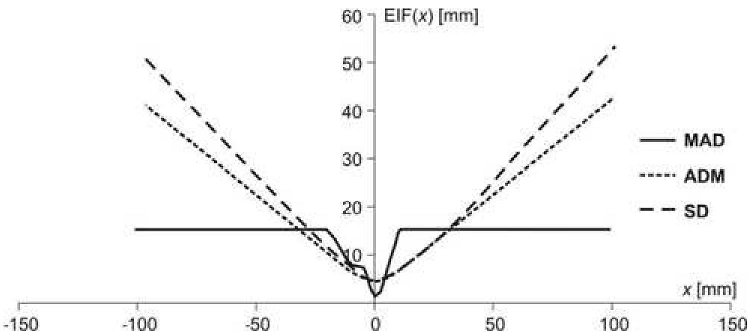

Fig. 6. EIFs of MAD, ADM and SD when the points 1 and 5 are unstable

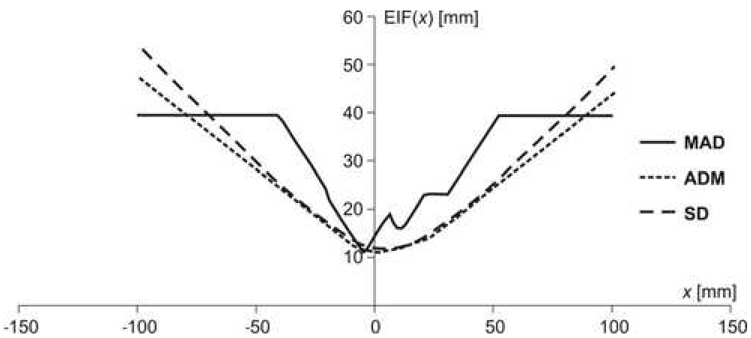

Fig. 7. EIFs of MAD, ADM and SD when the points 4 and 5 are unstable

The situation changes when other points are unstable. The most significant influence of a gross error on the MAD is in Variant 3. Note that the EIF of the MAD has the most complicated shape in this Variant too. The EIFs of the ADM and SD are similar to each other, however, the ADM is always less affected by a gross error than SD. It is also worth noting that in Variant 3, and to a lesser extent in Variant 2, the EIF of the MAD has bigger values than EIFs of the ADM and SD for smaller values of a gross error. Thus the MAD is more sensitive to a gross error than the traditional estimate in such a case (note that the similar conclusion concerns the robust and conventional estimates of the vertical displacement, see Figure 4). Such disadvantage of the MAD might become an advantage of the strategy itself, namely this may help to analyse the estimation results. Generally, if an observation set is free of outliers resulting from gross errors and there is at least one unstable point in the network then values of the MAD are smaller than or similar to values of the ADM (see, Duchnowski, 2010). Thus, if it happens that the value of the MAD is bigger than the value of the ADM then it is a suggestion, or a warning, that some observation must have been affected by a gross error (of course, the contrary conclusion is not true). This conclusion is especially important for relatively small gross errors which are hard to detect.

4. Conclusions

The strategy for testing stability of possible reference points proposed in (Duchnowski, 2010) applies robust estimates, however, this does not guarantee the robustness of the strategy itself. Thus, it is very important to know and understand how the estimates and the strategy might respond to a gross error that may affect an observation. It is no doubt that if all reference points are stable then the strategy cannot be spoiled by a gross error. Such case is easy to be analysed even if a gross error occurs. If some points are unstable the situation is more complicated because certain vectors $\tilde{\mathbf{v}}_2^k$ contain outlying observations that result from such instabilities. Note that in the case at hand, if there are no gross errors then analyses are also easy to be carried out, and the strategy can identify unstable reference points. However, if a gross error occurs then the estimates might respond in different ways. The responses of the estimates in case of two unstable points are described by the EIFs. In case of one unstable point the EIFs would be similar but have less complicated shapes. These functions show that the most significant influence of a gross error on the estimates is when such an error affects an observation, namely a height difference, of which network vertices are stable. Then the effect of a gross error might spoil the estimation and the strategy results. The EIFs, which describe how the estimates change in a presence of a gross error, can provide important information to identify "suspicious" observations. Such observation may be, for example, rejected from the observation set. Usually, this leaves enough observations to repeat the process, and thus to find stable reference frame.

The strategy for testing stability of possible reference marks that is based on the application of *R*-estimates gives good results if the observations are not affected by a gross error. Thus, the application of empirical influence functions is advisable when the results of estimation are not satisfactory, i.e. when the analyses of the estimation results cannot provide enough information about stability of all possible reference points. The EIFs presented in this paper apply to the case of a levelling network that contains five possible reference points, and all height differences are measured twice, at two different epochs. Of course, this is only an example network, however, the idea of application of EIF and the general conclusions have wider significance. When another network is analysed, then one should create separate EIFs which reflect the geometric structure of the network well.

Acknowledgments

The paper was prepared within the statutory research of the Institute of Geodesy, University of Warmia and Mazury in Olsztyn.

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Wykorzystanie empirycznych funkcji wpływu do badania wrażliwości odpornych estymatorów zastosowanych w strategii badania stabilności punktów nawiązania

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Streszczenie

Ważnym etapem badania deformacji jest wyznaczenie stabilnej bazy odniesienia a więc także badanie stabilności potencjalnych punktów odniesienia. Istnieje kilka metod badania stabilności, jedną z nich jest metoda wykorzystująca *R*-estymatory. Celem niniejszej pracy jest zbadanie wrażliwości na błędy grube estymatorów stosowanych w wymienionej metodzie. Jakkolwiek zastosowane estymatory są odporne, to sama metoda nie ma tej własności a jej odporność zależy w dużej mierze od liczby punktów niestabilnych (ogólnie mówiąc, im mniej jest punktów niestabilnych tym strategia jest odporniejsza na błędy grube). Z tego powodu ważnym jest by rozumieć w jaki sposób błędy grube wpływają na zastosowane estymatory i na wyniki samej strategii. Powyższy problem może być rozwiązany z zastosowaniem empirycznych funkcji wpływu (EIF). W pracy przedstawiono przykładowe funkcje EIF, ich zastosowanie w strategii oraz omówiono jak pozyskane informacje mogą być ważne i przydatne w praktyce.