GEODESY AND CARTOGRAPHY
Vol. 63, No 2, 2014, pp. 183-198

DOI: 10.2478/geocart-2014-0013

A new geoid for Brunei Darussalam by the collocation method

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Received: 20 March 2014 / Accepted: 23 June 2014

Abstract: Computation of a new gravimetric geoid in Brunei was carried out using terrestrial, airborne and altimetric gravity data and the EGM08 geopotential model by the collocation method. The computations were carried out by the „*remove-restore*" technique. In order to have better insight in the quality of input data the estimation of accuracy of the gravity data and geoid undulations from GPS/levelling data was carried out using EGM08 geopotential model. It shows a poor quality of GPS/levelling data. Result of the computation is the gravimetric geoid for the territory of Brunei computed by collocation method with an accuracy estimated below of ±0.3 m.

Keywords: geoid, altimetric gravity, collocation method

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1. Introduction

A local geoid is needed for developing a geometrical relationship between the Earth's surface and a reference ellipsoid. This relationship is required for numerous geodetic, geophysical and oceanographic applications. The knowledge of a local geoid became substantial in the last decades due to the extensive use of satellite-based methods in surveying, and in particular, for height difference determination. The modeling of a regional geoid is one of the major tasks of a number of geodetic research groups. Recently local geoids were calculated e.g. for South Korea (Hwang et al., 2012), Greece (Daras et al., 2010), Japan (Kuroishi, 2009), Argentina (Corchete and Pacino, 2007), Egypt (Dawod, 2008), Tanzania (Olliver, 2007), New Zealand (Claessens et al., 2011), and Europe (Denker at al., 2009). Various methods and types of gravimetric data sets were used to calculate these geoids (Sideris, 1994). Achieved accuracy of these geoids varies in a range of centimeters to decimeters.

The first gravimetric geoid model in the area of Brunei was computed in 2002––2003 (Morgan et. al., 2004) using *ring integration* method with the use of GRAV software (Kearsley et al.,1988).

The second geoid model in the area around Brunei has been constructed from gravity data measured on land, sea (altimetry) and aircraft, along with the global gravity field model EGM08. The geoid was computed by the remove restore technique using Fast Fourier Transform technique and GRAVSOFT software. Geopotential model EGM08 was truncated to degree 720 since "*there is no good information on the quality of the errors in EGM08 at the high wavelength*" in Brunei (KT Ryan, 2009, p. 37). Accuracy of computed gravimetric model for the territory of Brunei estimated by the authors is ±0.031 m (ibid. p.43).

Since Brunei is a relatively small country (just 5 765 km^2), and is covered by not numerous gravity data, determination of a geoid using the collocation method is quite feasible. because the inverse of the normal equations can be computed easily. The major advantages of the collocation method include its capability to use data from various measurement sessions that are characterized by different accuracy levels. Besides of that, the collocation method yields the accuracy estimation of the calculated geoid. Based on the above mentioned local geoid determinations, one may expect that the collocation method applied for these calculations will furnish better insight into the computation process and produce more realistic results.

2. Summary of the collocation methods for geoid calculation

Regional geoids are calculated using the „*remove-restore*" technique. This method is based on the following formula

$$N = N_{GM} + N_{\Delta g_{rez}} + N_H \tag{1}$$

where N_{GM} is computed from a geopotential model, $N_{\Delta g_{res}}$ is computed from the residual Faye's gravity anomalies, and N_H express the influence of topography known as an indirect effect.

The displacement of the topographic masses in gravity reductions changes the gravitational potential and thus the geoid. Therefore, the computed surface is not the 'true' geoid, rather a slightly different surface known as cogeoid. The vertical distance between the geoid and cogeoid can be computed from (e.g. Wichiencharoen, 1982)

$$N_H \approx \frac{\pi G\rho}{\gamma_m} H_P^2 \tag{2}$$

where G is gravitational constant, ρ is the earth's mean mass density, H_P is elevation of point P, and γ_m is the mean normal gravity.

The residual Faye's gravity anomaly is the difference

$$\Delta g_{res} = \Delta g_F - \Delta g_{GM} \tag{3}$$

where Δg_F is Faye gravity anomaly, and Δg_{GM} is gravity anomaly computed from geopotential model. Faye anomaly is defined

$$\Delta g_F = g + \delta g_T + \delta g_F - \gamma_0 \tag{4}$$

where g is gravity acceleration measured on the surface of the Earth, δg_T is terrain correction, δg_F is free air reduction and γ_0 is normal gravity on the ellipsoid.

The term N_{GM} is computed from the formula (Torge, 2001):

$$N_{GM}(r,\phi,\lambda) = N_0 + \frac{GM}{r\gamma}\sum_{n=2}^{n_{\max}}\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\sin\phi) \tag{5}$$

and the term Δg_{GM} is given by

$$\Delta g_{GM}(r,\phi,\lambda) = \frac{GM}{r^2}\sum_{n=2}^{n_{\max}}(n-1)\left(\frac{a}{r}\right)^n \sum_{m=0}^{n}\left(\overline{C}_{nm}\cos m\lambda + \overline{S}_{nm}\sin m\lambda\right)\overline{P}_{nm}(\sin\phi) \tag{6}$$

where $\overline{C}_{nm}$, $\overline{S}_{nm}$ are fully normalized spherical harmonic coefficients of degree n and order m, n_{max} is the maximum degree of geopotential model, GM is product of the Newtonian gravitational constant times the mass of the earth, r, ϕ, λ are spherical coordinates, a is the equatorial radius of geopotential model, and $\overline{P}_{nm}$ are the fully normalized associated Legendre's functions.

The term N_0 is the zero term due to the difference between the Earth mass per IERS convention and GRS80 ellipsoid. It is calculated from the following formula

$$N_0 = \frac{GM - GM_0}{R\gamma} - \frac{W_0 - U_0}{\gamma} \tag{7}$$

where the parameters GM_0 and U_0 correspond to the normal gravity field on the surface of the reference ellipsoid.

For the GRS80 ellipsoid GM_0 is equal to 398 600.5000 x $10^9 \mathrm{m}^3\mathrm{s}^{-2}$ and U_0 is equals to 62 636 860.85 $\mathrm{m}^2\mathrm{s}^{-2}$.

The following values were assumed for the Earth's GM term and the gravity potential W_0 on the geoid, as specified by the IERS Conventions: GM = 398 600.4415 x 10^9 $\mathrm{m}^3\mathrm{s}^{-2}$, and W_0 = 62 636 856.00 $\mathrm{m}^2\mathrm{s}^{-2}$.

The mean Earth's radius R and the mean normal gravity γ on the reference ellipsoid (GRS80 values) were 6 371 008.771 m and 9.798 $\mathrm{m\,s}^{-2}$, respectively. Based on the above assumptions the N_0 term (see Eq.(7)) is -0.442 m. This value has been added to the geoid's heights obtained from the corresponding spherical harmonic coefficients series expansions of geopotential model, equation (5).

The term $N_{\Delta g_{res}}$ can be computed from the Stokes' integral or by the least squares collocation method. When the geoid is computed by the least squares collocation method then the term $N_{\Delta g_{res}}$ can be computed from the formula e.g. (Moritz, 1989)

$$N_{\Delta g_{res}} = \mathbf{C}_{Nl}\left(\mathbf{C}_{ll} + \mathbf{D}_{ll}\right)^{-1}\mathbf{l} \tag{8}$$

where $\mathbf{l}$ is the vector of observations, $\mathbf{C}_{ll}$ is the auto covariance matrix of observations, $\mathbf{C}_{Nl}$ is the cross-covariance matrix between the $\mathbf{l}$ and N, $\mathbf{D}_{ll}$ is the covariance matrix of the observations errors. The vector $\mathbf{l}$ includes the gravity anomalies of the considered region.

To determine the term $N_{\Delta g_{res}}$ the matrix $\mathbf{C}_{ll}$ and $\mathbf{C}_{Nl}$ are needed first. They can be estimated assuming a certain model of the covariance function. In this case, a logarithmic function was used (Forsberg, 1987). The logarithmic model consists of three parameters, i.e. variance C_0 of the gravity anomalies and parameters D and T that determine the degree of damping of high and low frequencies of the gravity signal.

A convenient tool for the estimation of the parameters of the covariance function C_0, D and T is the *gpfit.for* module of the *GRAVSOFT* software package (Tscherning et al., 1992). The altimetry gravity anomalies over the area of interest were used for the estimation of these parameters.

3. Data

In this project four data sets (Fig. 1) of the gravity anomalies were used, that are available for the area of interest (Ryan, 2009).

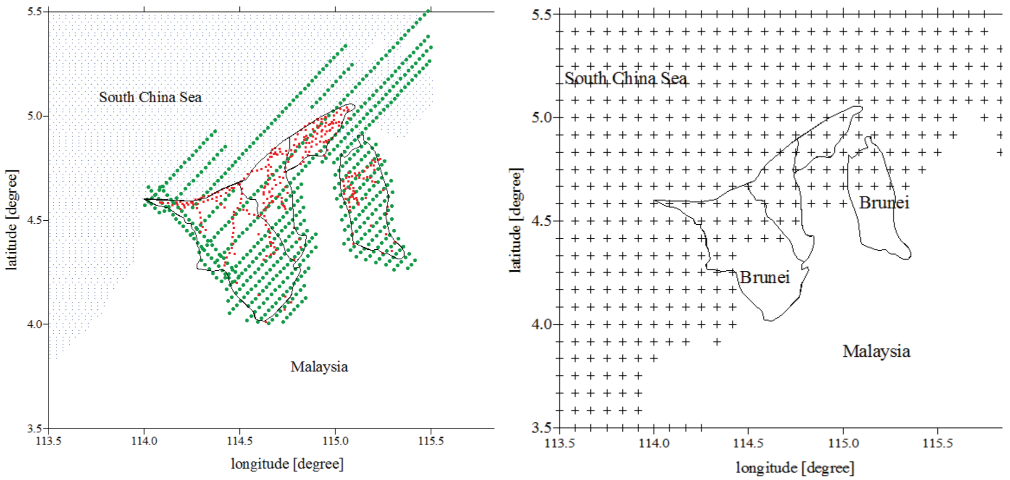

Fig. 1. Distribution of terrestrial (red), airborne (green),altimetric (blue) gravity data (left), and GETECH gravity data (right) on the territory of Brunei and adjacent areas

3.1. Terrestrial gravimetric data

The terrestrial gravimetric data set consists of 306 measurements of the gravity anomalies carried out in 2003 (Ryan, 2009). The gravity meter Lacoste & Romberg model G was used for the measurements. These observations were referenced to the JKR absolute gravity station on the floor of the National Seismological Centre. Since absolute observation using the free fall techniques provide access to the International Gravity Standardization Network 1971, the gravity network of Brunei is in IGSN71 gravity reference frame. Accuracy of gravity data estimated from repeatability on multiple occupation stations is ±4.0 (μms^{-2}) (ibid. p.13).

The position of the points was determined using the GARMIN GNSS receiver. The determination of the best value of the orthometric heights of the gravity stations where there was no benchmark was a difficult iterative process which is described in (Morgan at al., 2004). The standard deviation of the orthometric height of a gravity station is less than 0.1 meter when benchmark value is available. This is the situation for the majority of stations in Muara district. If the height is determined from two or more receivers the estimated value is better than 3 meters while it is worse than 3 meters if there is only one hand held GPS receiver (ibid).

The "GETECH" gravity anomaly data consists of 462 measurements (Fig. 3-1). Gravity data covers mainly offshore area and part of Brunei, Sarawak and Sabah mainland and are in gridded form with a spacing $5^{'}$ x $5^{'}$. No further details on the data set were available from the source that is (Ryan, 2009).

3.2. Airborne gravity data

Airborne gravity data published in (Ryan, 2009) includes 1274 gravimeter anomalies. The discussion of the methodology associated with the reduction of airborne gravimetry has been done by Olesen (Olesen et al., 2002) who managed the Malaysian airborne campaign from which this data is extracted. Unfortunately, the authors (ibid.) do not provide essential information on the characteristics of these measurements. More information can be found in (Nordin et al., 2005). According to (ibid.) airborne gravity data has been acquired at a flight speed of 150–250 km/hr with aircraft altitude typically at 300–1000 m above topography. The quality of individual flight data are checked by field computations of single GPS baselines (one rover to one static), combined with results from one gravimeter. Visual inspection/plotting will quickly identify problematic data and make the necessary background information for decisions to re-fly a line. Airborne gravity data has been processed using AG-Suites software from KMS and accuracy 20 μms^{-2} of gravity anomaly data at 5 km spacing was obtained.

3.3. Altimetric data

To compute the precise gravity geoid, data not only from territory of Brunei but also from surrounding areas are needed. Therefore, the possibility of marine gravity data derived from altimetry was considered. Satellite altimetry has provided the most comprehensive images of the gravity field of the ocean basins with accuracies and resolution approaching typical shipboard gravity data. In analysis were used three approaches to reduce the error in the satellite-derived gravity anomalies to 20–30 (μms^{-2}).

The satellite altimetry data set used in our computations was downloaded from the Scripps Institution of Oceanography, University California, San Diego website (http://topex.ucsd.edu/WWW_html/mar_grav.html). These gravity anomaly data cover both the ocean and land at the grid 1 x 1 arc minutes and were derived from the Geosat and ERS-1 satellite missions (Sandwell and Smith, 2009). Authors (ibid.) have used the recently published EGM2008 global gravity model as a reference field to provide a seamless gravity transition from land to ocean. The accuracy of altimetric data is estimated by (ibid.) as 20–30 μms^{-2}.

As the altimetric gravity data covers both oceans and land and so, the data from the terrestrial part of Brunei should be removed. Therefore, the altimetric gravity data set which was used in our geoid computation consists of 6 255 points only. Table 1 shows a summary of the gravimetric anomaly data used in this project.

Table 1. Summary of the gravity data used in this project

Type of gravity data	Coverage	Gravity anomalies			
		No. of points	Mean (μms^{-2})	Std. dev. (μms^{-2})	Min, Max (μms^{-2})
Terrestrial	$4^o < \varphi < 5^o$ N $114^o < \lambda < 115.3^o$ E	306	133	121	-153 118
Airborne	$4^o < \varphi < 5.5^o$ $114^o < \lambda < 115.5^o$	1 274	136	113	-62 749
GETECH	$3.5^o < \varphi < 5.5^o$ $113.5^o < \lambda < 116^o$	462	360	179	137 1014
Altimetry	$3.5^o < \varphi < 5.5^o$ $113.5^o < \lambda < 115.5^o$	6 255	246	172	-140 620

Terrestrial and airborne gravimetric anomalies for the area of Brunei are of different size and density however their mean values and standard deviations are almost the same. This means a significant compliance data.

Similarly, altimetric and “GEOTECH” data from almost identical areas are of different size and their mean values vary considerably while the standard deviations are almost identical. This means the existence of certain systematic terms in these data sets.

On the basis of the information contained in Table 1 it is difficult to assess the quality of these gravity data. The authors of the work (Morgan et al., 2004) conducted inter comparisons of the available data set by the block mean approach. This simple approach averages the available data over the blocks. The comparison shows a good agreement between the three gravity anomaly fields (ibid. p. 57).

3.4. GPS and levelling data

The GPS coordinates, ellipsoidal h and orthometric heights H, for 86 stations were supplied by the Survey Department of Brunei Darussalam. Locations of the stations follow the pattern of transportation routes of the country. First class of spirit levelling was used to estimate the orthometric heights of the stations. There is lack of information concerning vertical datum in Brunei. Further study reveals that used in this study vertical datum is shifted significantly from the mean sea level.

The GPS observations on these stations were carried out from 19 April to 6 May 2010. The data were collected for at least three hours at 15 second intervals for each station. The adjustment of the network yielded the semi-major axis of the error ellipses from 4 to 17 mm, with the average value of 9.6 mm. The errors of the vertical component were from 4 to 22 mm, with the average value of 11.1 mm (Ryan, 2009)**.**

3.5. Topography data

The topography data are needed to determine the indirect effect of the geoid according to equation (2). Topographic data SRTM30 PLUS were obtained from the above mentioned web site of the University of California. Land data are based on the 1-km averages of topography derived from the USGS SRTM30 gridded DEM data product created with data from the NASA Shuttle Radar Topography Mission. GTOPO30 data are used for high latitudes where SRTM data are not available. Detail description of these data is in the paper (Becker et al., 2009).

From DEM can be concluded that the area of interest is flat with exception of the south-eastern fragment where terrain elevates up to 1800 m. This indicates that the indirect effect on the geoid will be small.

4. Accuracy evaluation of the data

In the paper (Morgan et al., 2004) authors roughly evaluated their sets of gravity data and came to conclusion, that they agree quite good. The differences between the three gravity anomaly data sets do not excide the value of standard deviation of 32.2 μms^{-2} (3.22 mGal). Considerably better accuracy evaluation can be done by the use of the EGM08 geopotential model. This model includes all the terms to 2100s level (Pavlis et al., 2012) and is considered a truly remarkable achievement, considering that its accuracy is comparable with that of the time-consuming traditional quasigeoid calculations.

The EGM08 is available from the following website: *http://earth-info.nima.mil/GandG/wgs84/gravitymod/egm2008/index.html*. The EGM08 data in form of the quasigeoid separations and gravity anomalies is available at 1 x 1 and 2.5 x 2.5 arc minutes resolutions for the whole globe. Suitable interpolations tools are also available from the site. On the other hand, the quasi geoid separations and gravity anomalies can be computed from EGM08 geopotential model using e.g. *geocol17* software (Tscherning et al., 1992).

Our study included an assessment of the performance of the geopotential model over the area of interest. This was done by analyzing the differences between the values of the gravity anomalies taken from the data sets described above and gravity anomalies from geopotential model, that is:

$$\delta g = \Delta g - \Delta g_{GM} \tag{9}$$

where Δg_{GM} is the gravity anomaly calculated from the geopotential model, and Δg is the corresponding value of the gravity anomaly taken from one of the data sets available for the project.

For EGM08 model and each data set the mean value of the differences and the standard deviation was calculated. The good data sets of residual gravity anomalies should have mean value close to zero and standard deviation as small as possible. For example similar calculations carried out for the territory of Poland gives mean value of -6 μms^{-2} and standard deviation of 25 μms^{-2} (Łyszkowicz, 2012).

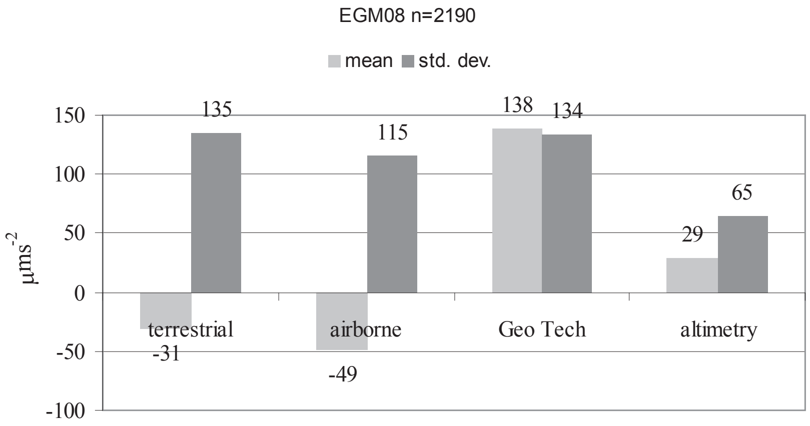

Fig. 2. Mean value and standard deviation of the differences δg between each data set of the gravity anomalies and EGM08 model

Our study also included the accuracy estimation of geoid undulations calculated from the geopotential models versus the GPS and orthometric elevations. This was facilitated by calculating the mean value and the standard deviation of the following differences for 86 benchmarks

$$\delta N = N_{GPS/lev} - N_{GM} \tag{10}$$

where N_{GM} is the geoid-ellipsoid separation calculated using equation (5) and $N_{GPS/lev}$ is the corresponding geoid-ellipsoid separation calculated as a difference between and GPS and the orthometric elevation.

The relevant calculations and subsequent analysis of the results allowed for identification of four benchmarks with extreme values of δN of approximately 10 m. These benchmarks were removed from further considerations. It is suspected that a source of the outliers is erroneous levelling of benchmarks.

According to our calculations mean value is -39.72 m and the standard deviation of the geoid ellipsoid separation is 1.57 m (Table 2). Mean value shows significant difference between "global" geoid implied by the geopotential model and the local vertical datum. The standard deviation however, is significantly larger than 0.914 m provided by KT Ryan, 2009. However, the authors (ibid) in their calculations used the EGM08 geopotential model to degree and order 720 only. A corresponding value for Malaysia is 0.50 m (Wan Mohd. Akib et al. 1998), (Tahir et al., 2009) and for the territory of Poland the standard deviation of such discrepancies is 0.04 m (Krynski and Kloch-Główka, 2009).

Table 2. Accuracy characteristics of the EGM08 geopotential model tested on benchmarks (four outliers were excluded). Results are in (m). It was assumed that in eq. (5) term N_0 is equal – 0.442 m

		after removing mean value
Mean	-39,72	0,00
Standard deviation	1,57	1,57
Minimum	-43.09	-3,38
Maximum	-36.27	3,45

This fact suggests a rather poor accuracy of the input GPS/levelling data, which were used in the computation of the separation between the geoid and ellipsoid on 86 benchmarks in Brunei.

5. Calculation of a residual geoid using the collocation method

Calculations of a residual geoid were carried out for the area of interest extending between 3.5° and 5.5° North and 113.5° and 116° East, at 2 by 2 arc minutes grid points. The first term N_{GM} in equation (1) was calculated for the EGM08 model to degree and order 2100 according to equation (5). The *geocol.for* routine of the GRAVSOFT software package was used to carry out the computations.

To calculate the second term $N_{\Delta g_{res}}$ in equation (1), the covariance function of the residual gravity anomalies must be known.

An estimate of the covariance function was computed using the *geocol.for* routine. In this routine, the gravity covariance model between gravity anomalies at two altitudes is of form (Forsberg, 1987)

$$C\left(\Delta g^{h_1}, \Delta g^{h_2}\right) = -\sum \alpha_k \log\left(D_k + \sqrt{s^2 + \left(D_k + h_1 + h_2\right)^2}\right) \tag{11}$$

where α_k are weight factors combining terms relating to depth value terms ($D_k = D + kT$), with the "free parameters" D and T taking the role analogous to the Bjerhammar sphere depth of spherical collocation and a "compensating depth" attenuation factor. The attenuation of long wavelengths in the model are necessary when a spherical harmonic reference model is used.

Figure 3 shows resulting curve (empirical) and a fitted analytical curve of the form describe in (ibid.) for the residual terrestrial data. The following parameters were obtained for this case: $\sqrt{C_0} = 136$ (μms^{-2}), $D = 3$ km, $T = 51$ km.

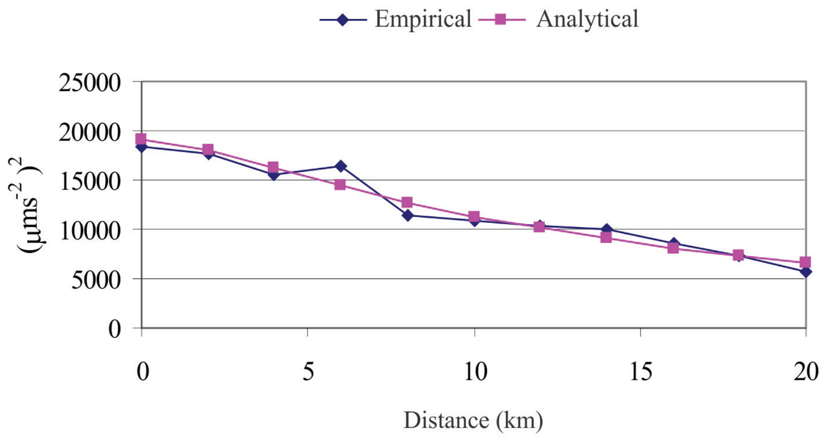

Fig. 3. Empirical covariance function of the residual terrestrial gravity. A logarithmic regression function is also shown

The second term in equation (1), – i.e. $N_{\Delta g_{res}}$, was calculated using three different combinations of the input data sets. The following combinations of the data sets were used.

- Case 1: Gravity anomalies from the terrestrial surveys;
- Case 2: Gravity anomalies from the aerial survey;
- Case 3: All available data sets were used.

Table 3 shows a summary of these calculations. The precision of the determination of the residual geoid ellipsoid separations depends on the accuracy and the amount of gravity data. In the case of 306 point terrestrial gravity data and the term (Table 3, Fig. 3) $N_{\Delta g_{res}}$ in equation (1) is calculated with the error 0.25-0.30 meters. Similar results were obtained in the case of airborne data. The accuracy of the residual geoid ellipsoid separations has been significantly improved by using all available data sets in the calculation. This is due to the fact that the predominant residual altimetric data are characterized by a small standard deviation (71 μms^{-2}). In the case of Poland the residual gravimetric anomalies are characterized by the standard deviation of the 26 μms^{-2} which gives the term $N_{\Delta g_{res}}$ with an accuracy between ± 3 and ± 4 mm (Łyszkowicz, 2010).

Table 3 Standard deviation of residual geoids computed from different data sets

Case	Data set used	Accuracy	$\sqrt{C_0}$, D, T	Standard deviation of the residual geoid – ellipsoid separation in Brunei (m)
1	306 gravity anomalies from the terrestrial survey	4 μms^{-2}	136 μms^{-2} 3 km 51 km	0.25 – 0.30
2	1274 gravity anomalies from aerial survey	10 μms^{-2}	115 μms^{-2} 5 km 50 km	0.20 – 0.23
3	As in case 1+ case 2 + 6255 altimetric data	as in case 1, 2 and 30 μms^{-2} for altimetric data	71 μms^{-2} 7 km 16 km	0.04 – 0.06

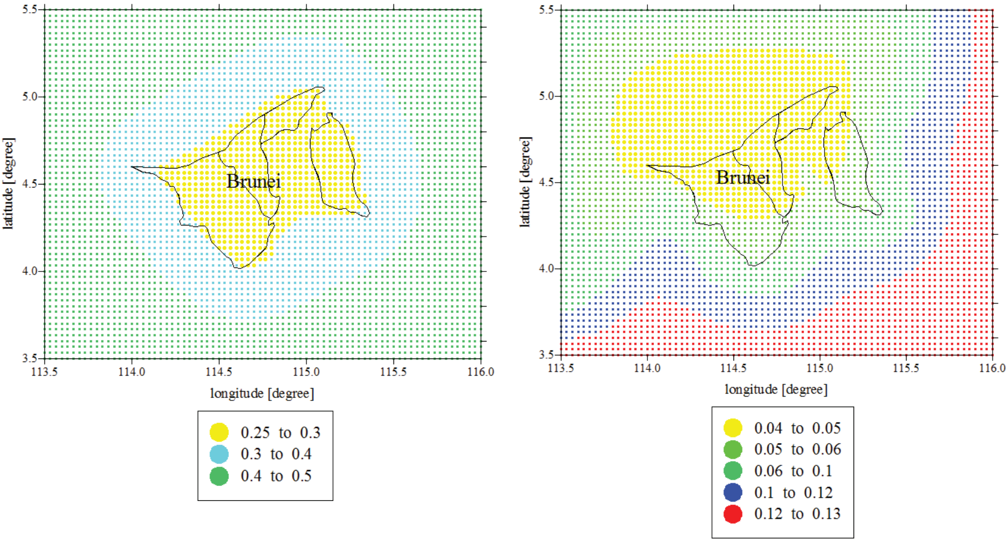

Fig. 4. Distribution of standard deviations in meters of residual geoids computed from terrestrial data (left) and from all data (right)

Spatial distribution of accuracy of residual term $N_{\Delta g_{res}}$ for different gravity data sets is shown in the Figure 4. It gives the possibility once more to assess quality of gravity data. From Figure 2 appears that present gravity data sets do not permit to determine the residual geoid with accuracy better than ±0.05 meters. Particularly terrestrial point data have little impact on a good accuracy of gravimetric geoid in Brunei. In order to increase the significance of these data, Brunei should be covered with terrestrial gravity data with a density of at least 2-3 points per 1 km^2.

The indirect effect necessary to calculate the final geoid, was calculated from equation (2). It was computed assuming $G = 6.673\ 10^{-11}\ \mathrm{m^3kg^{-1}s^{-2}}$, $\rho = 2700\ \mathrm{kg\ m^{-3}}$, $\gamma_m = 9.798\ \mathrm{m\ s^{-2}}$. The computations were done for the topographical heights i.e. for the $H > 0$. In the case of $H < 0$ (ocean, South China Sea) the indirect effect was assumed to be zero. The magnitude of the indirect effect for the majority of Brunei is very small and does not exceed one millimeter.

The final geoid calculation using the collocation method was carried out using residual geoid from all data (case 3), the undulation computed from EGM08 model and "corrections" due to indirect effect. Computations were done according to equation (1) and the final collocation geoid is shown on Fig. 5.

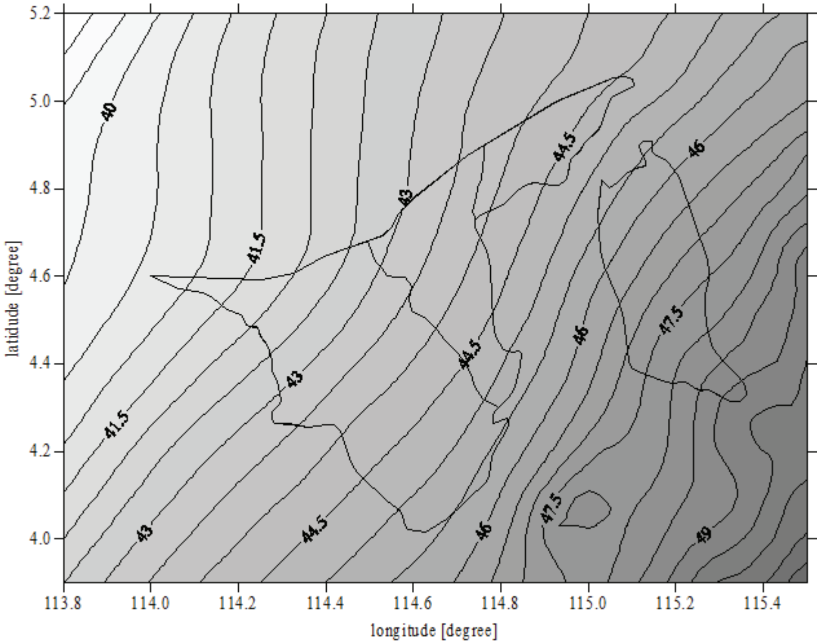

Fig. 5. Gravimetric geoid for the area of Brunei computed by the collocation method

Computed geoid (Fig. 5) on the territory of Brunei is an almost plane surface which is inclined in the north-west direction and attains values from 41 m to 49 m.

Table 4. Accuracy characteristics of the geoid computed by collocation method tested on benchmarks (four outliers were excluded). Results are in (m)

		after removing the mean value
Mean	-40.41	0.00
Standard deviation	1.58	1.58
Minimum	-43.80	-3.40
Maximum	-36.94	4.46

Because of poor quality of GPS/levelling data the estimation of the accuracy of the so computed geoid using the points of the GPS/levelling network (Table 4) is not credible. Also, it was not possible to assess the consistency of the calculated collocation geoid with the model published in work (Ryan, 2009).

It should be expected, that the accuracy of the first term in the formula (1) is of the order of 1 – 2 decimeters, the accuracy of second term is of the order of several centimeters, that means that the total accuracy of the geoid computed in this way should be below ± 0.3 m, while its accuracy estimated on the GPS/levelling points gives a value of ± 1,58 m.

6. Conclusions

The major objective of this study was to calculate a local geoid for Brunei Darussalam using the collocation method with all available gravity data. The collocation method is best suited for the particular project area because of its small territory, and also allows for the accuracy assessment of the resulting geoid, which accordingly to our estimates is below ±0.3 m.

Accuracy and resolution of gravity data have significant impact on the precision of gravimetric geoid. The gravity data sets, except the altimetry data set, used in this project were poorly documented as we did not have access to the sources of the data. To mitigate the issue, we compared the data sets with the corresponding values of the gravity anomalies calculated from the EGM08 geopotential model. Assessment of accuracy of three sets of the gravity anomaly indicates that these data are not as good as suggest the authors in the work (Morgan et al., 2004) which in turn gives a significant error of residual geoid (± 0.05 m). Studies suggest that in order to get the geoid with centimeter accuracy terrestrial gravimetric data are necessary with the density of 2-3 points per 1 km^2.

We have also estimated the accuracy of 86 GPS survey marks tied to the local vertical datum by spirit levelling. We found that they are characterized by an accuracy of ±1.6 m. This result is quite surprising, considering the EGM08 model being significantly better anywhere else in the world. Our result is also significantly different from the value of 0.914 m published by KT Ryan, 2009. In our opinion their standard deviation is too optimistic. In addition, we were able to identify four GPS marks that must be considered as outliers. It suggests that the accuracy of ellipsoidal height computed from GPS observations and precise levelling is very poor.

Because of poor quality of GPS/levelling data the estimation of the accuracy of computed geoid on the points of the GPS/levelling network is not credible. It should be expected that the accuracy of the collocation geoid should be slightly below ±0.3 m.

We suspect that overall the accuracy ±0.031 m of the geoid undulation calculated by KT Ryan, 2009 is not correct if they used the same, as in this paper, GPS/levelling data.

Acknowledgments

This work was carried out within the statutory studies of the University of Warmia and Mazury in Olsztyn, Faculty of Geodesy and land Management and Wroclaw University of Environmental and Life Sciences, Faculty of Environmental Engineering and Geodesy.

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Wyznaczenie przebiegu nowej geoidy na obszarze Brunei Darussalam metodą kolokacji

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Streszczenie

Wyznaczenie przebiegu nowej geoidy na obszarze Brunei zostało zrealizowane z wykorzystaniem lądowych, lotniczych i altimetrycznych danych grawimetrycznych oraz modelu geopotencjału EGM08 metodą kolokacji. Obliczenia zostały przeprowadzone z wykorzystaniem techniki „*remove-restore*". W celu uzyskania lepszego wglądu, w jakość danych wejściowych oszacowano dokładność danych grawimetrycznych i geometrycznych odstępów geoidy od elipsoidy na punkach sieci GPS wykorzystując do tego celu model geopotencjalu EGM08. Z przyprowadzonych oszacowań wynika przede wszystkim niska dokładność danych GPS/niwelacja. Wynikiem przeprowadzonych obliczeń jest grawimetryczna geoida dla obszaru Brunei, obliczona metodą kolokacji, której dokładność szacuje się poniżej ±0.3 m.