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Theoretical geodesy

Andrzej Borkowski[1], WiesławKosek[2]

[1] Wroclaw University of Environmental and Life Sciences
Institute of Geodesy and Geoinformatics
53 Grunwaldzka St., 50-357 Wroclaw, Poland
e-mail: andrzej.borkowski@igig.up.wroc.pl

[2] University of Agriculture in Krakow
Faculty of Environmental Engineering and Land Surveying
24-28 Al. Mickiewicza, 30-059 Krakow, Poland
e-mail: kosek@cbk.waw.pl

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ABSTRACT: The paper presents a summary of research activities concerning theoretical geodesy performed in Poland in the period of 2011–2014. It contains the results of research on new methods of the parameter estimation, a study on robustness properties of the *M*-estimation, control network and deformation analysis, and geodetic time series analysis. The main achievements in the geodetic parameter estimation involve a new model of the *M*-estimation with probabilistic models of geodetic observations, a new Shift-M_{split} estimation, which allows to estimate a vector of parameter differences and the Shift-$M_{split}^{(+)}$ that is a generalisation of Shift-M_{split} estimation if the design matrix **A** of a functional model has not a full column rank. The new algorithms of the coordinates conversion between the Cartesian and geodetic coordinates, both on the rotational and triaxial ellipsoid can be mentioned as a highlights of the research of the last four years. New parameter estimation models developed have been adopted and successfully applied to the control network and deformation analysis.

New algorithms based on the wavelet, Fourier and Hilbert transforms were applied to find time-frequency characteristics of geodetic and geophysical time series as well as time-frequency relations between them. Statistical properties of these time series are also presented using different statistical tests as well as 2nd, 3rd and 4th moments about the mean. The new forecasts methods are presented which enable prediction of the considered time series in different frequency bands.

Keywords: *M*-estimation, robust estimation, reliability, time series, polar motion

1. Introduction

In the last decades some new measurement technologies have been developed, e.g. laser scanning technology and several variants of its technical realization. Properties and a structure of data collected by these new sensors have triggered the evolution of the existing data modelling methods and the development of new ones, especially for parameter estimation, in order to exploit all information contained in the data. A good example to illustrate this process is M_{split} estimation introduced by Wiśniewski in his previous research. This method allows an estimation of parameters in a split functional model of observations. In the period from 2011 to 2014 which is a subject of this review paper some new variants of *M*-estimation have been proposed. In the section 2 the *M*-estimation with probabilistic model of geodetic observation and the Shift-M_{split} estimation are featured. The last mentioned parameter estimation model was a subject of modification and adoption for geodetic control network analysis. All these aspects are discussed in detail in this paper. The robustness of the parameter estimation methods have been studied by the Polish researcher for many years. This issue have been also considered in the last four years. Another issue reported in the section 2 is a reliability analysis of observation systems, which is deeply studied in case of Errors-in-Variables models.

Development during last decades of the time-frequency analysis methods based on the wavelet and Fourier as well as Hilbert transforms enable computation of variable amplitudes and phases of oscillations with different frequencies in geophysical time series, e.g. the Earth orientation parameters and their fluid excitation functions, sea level anomalies data, Earth centre of mass coordinates, the coordinates of sites of permanent networks, zenith tropospheric delay and gravimetric measurements at permanent GGP stations. The semblance function introduced in the beginning of this century, as a modification of the coherence function, enable estimation of time-frequency correlation coefficients between two time series. It has also been very useful to compute a common signal in two time series. One of the problems of forecast algorithms which is simultaneous prediction of time series in different frequency bands can be solved using the combination of polynomial harmonic extrapolation and the autoregressive based methods which enable predictions of extrapolation residuals. This problem can be also solved using combination of the discrete wavelet transform band pass filter with a prediction method, e.g. autocovariance or autoregressive predictions.

In the last part of this review paper some original algorithms of the computational geodesy, especially related to the rotational and triaxial ellipsoid are briefly presented.

2. Geodetic parameter estimation

2.1. M-estimation and its variants

M-estimation is a well-studied standard method for parameter estimation in the presence of outlying observations. In order to reduce the impact of outlying observations a proper weighting function is applied and the estimation is performed iteratively. In addition to multiple weighting functions in use Banaś and Ligas (2014) introduced six new weighting functions, which have been derived using the following base functions: Wigner semicircle distribution, Epanechnikov kernel, Tricube kernel and Jacobi orthogonal polynomials (1st, 2nd and 3rd degree). For these functions the authors computed also tuning constants assuring 95% efficiency with respect to the standard normal distribution. The performance of the introduced weighting functions was tested empirically together with the M-estimators of Huber, Tukey and Hampel. As the numerical example, a simple levelling network was used, wherein the original observations were sequentially contaminated with blunders. The overall number of performed tests was 432 and the tests showed that the performance of M-estimators depends highly on the dataset considered. The presented research showed the imperfection of M-estimation due to its dependence on model residuals only.

Wiśniewski (2014) developed a new variant of M-estimation with probabilistic models of geodetic observations that is called M_P estimation. This new model depends on the variance but also on the kurtosis, excess and asymmetry of the observation error distribution. To consider these characteristics of observation in the adjustment procedure, a proper probabilistic model of observation errors has to be assumed. In the paper (Wiśniewski, 2014) the Pearson distributions of type IV or VII are assumed to describe leptokurtic distributions with asymmetry. As an alternative approach the approximation of the Pearson distributions by Gram-Charlier series ($M_{G\text{-}C}$ method) is also considered.

Since the influence function of M_P estimation is based on the differential equations of Pearson distribution, the central moments μ_k, $k = 2, 3, 4$, are the parameters of this function. According to the assumed models of influence functions, the weight functions were derived for different cases of M_P estimation. It was shown that the M_P estimation belongs to the class of robust estimation. The performance and properties of the new estimation method were tested numerically on simulated levelling network in comparison to the least squares and M-estimation of Huber. Both the theoretical analysis and the numerical tests showed that the $M_{G\text{-}C}$ estimation method loses their robustness when large gross errors occur. The M_P estimation that includes Pearson distribution is especially effective as a robust estimator. The robustness of the method increases with the growing values of kurtosis, if the asymmetry coefficient is constant.

Let the observation set be a mixture of realizations of two random vectors $\mathbf{Y}_\alpha$ and $\mathbf{Y}_\beta$ with the respective expected values $\mathrm{E}\{\mathbf{Y}_\alpha\}$ and $\mathrm{E}\{\mathbf{Y}_\beta\}$. The observation vector $\mathbf{y}$, which contains disordered elements of the considered vectors may be assigned either to the two competitive vectors of expected values $\mathrm{E}_\alpha\{\mathbf{y}_i\} = \mathbf{A}\mathbf{X}_\alpha$ or $\mathrm{E}_\beta\{\mathbf{y}_i\} = \mathbf{A}\mathbf{X}_\beta$, with the design matrix $\mathbf{A}$. The simultaneous estimation of different parameter vectors $\mathbf{X}_\alpha$ and $\mathbf{X}_\beta$ was proposed by Wiśniewski in his previous works and is called as M_{split} estimation. For some practical applications (e.g. deformation analysis of control networks) it is interesting to know the difference $\boldsymbol{\Delta}_\mathbf{X} = \mathbf{X}_\beta - \mathbf{X}_\alpha$ between the parameter vectors. $\boldsymbol{\Delta}_\mathbf{X}$ can be estimated directly from $\boldsymbol{\Delta} = \mathbf{A}\mathbf{X}_\alpha - \mathbf{A}\mathbf{X}_\beta = -\mathbf{A}\boldsymbol{\Delta}_\mathbf{X}$ by applying M_{split} estimation. For this reason Duchnowski and Wiśniewski (2011) developed so called Shift-M_{split} estimation method, which allows to estimate a vector of parameter differences without estimating the particular parameter vectors. The optimization problem of M_{split} estimation in relation to the shift of parameters was solved by the use of two equivalent systems of normal equations. Possible application of Shift-M_{split} estimates in deformation analyses was presented on exemplary illustration. The robustness property was also discussed in the paper and it was shown that Shift-M_{split} estimates are usually affected by gross errors and the method is robust in special cases only.

A generalization of the above mentioned method is given by Duchnowski and Wiśniewski (2012). In this case it was assumed that the design matrices **A** of functional models may not have a full column rank. The proposed new estimation procedure is called Shift-$M_{split}{}^{(+)}$ and the optimization problem is solved iteratively using Moore-Penrose inverses. The iterative process ends usually after six or seven iterations.

2.2. Control networks and deformation analysis

The Shift-$M_{split}{}^{(+)}$ (Duchnowski and Wiśniewski, 2012) has been developed primary for a deformation analysis of free networks or for testing the stability of network points. In this case the shift vector $\boldsymbol{\Delta}_{\mathbf{X}}$ to be estimated is related to the changes in the coordinates of the network points at two different measurement epochs. The authors tested the $M_{split}{}^{(+)}$ method on a levelling control network and showed that the new method is not only an alternative to the conventional determination of point displacements but in some cases it is better than the traditional approaches.

Classical robust methods used in the deformation analysis are based on the results of separate adjustments by the least squares method of two measurement epochs, i.e. on the adjusted coordinates, wherein the term "robust" does not refer to outliers but to single-point movements. The main step of these methods is the *S*-transformation (Helmert similarity transformation) of the coordinate differences with the optimization conditions of a robust estimation, which leads to iterative weighted similarity transformation method.

Deformation measurements are carried out as repeated campaigns with the same instruments, methods, geometrical conditions and in similar environmental conditions.

Therefore it is well-founded to assume that the results of deformation measurements can be distorted by both random and some no-random errors, which are constant in both measurement epochs. Such errors are not completely eliminated, when the displacement vector is determined by means of robust *S*-transformation of adjusted coordinate differences. The existence of constant errors reduces the quality of deformations analysis. To eliminate effects of those errors in deformation analysis Nowel and Kamiński (2014) proposed a new robust alternative method called robust estimation of deformation from observation differences (REDOD). This method is based on the assumption, that the vector **d** is a function of differences between independent observations, $\mathbf{d} = f(\mathbf{l}_{(2)}^{obs} - \mathbf{l}_{(1)}^{obs})$, and

$$\left(\mathbf{l}_{(2)}^{obs} - \mathbf{l}_{(1)}^{obs}\right) \xrightarrow{\text{robust estimation}} \hat{\mathbf{d}}^{R}$$

where $\hat{\mathbf{d}}^{R}$ is the robust estimator of **d**. The proposed estimation procedure was tested on both levelling and horizontal control networks. The conducted numerical test showed that:

– the REDOD method completely eliminates effects of additional constant errors from deformation analysis results,
– results of deformation analysis using the REDOD method estimates are very similar to those obtained with classical iterative weighted similarity transformation method if the results of deformation measurements are distorted only by random errors.

The REDOD method is highly recommended in case of automated, continuous control measurements, where there is a high risk that the errors of measurement of the same geometric elements in both epochs may contain a constant factor in both epochs.

One of the basic approaches to the robust estimation is *R*-estimation (also known as Hodges-Lehmann estimates) distinguished by its high robustness against outliers. Application of this method to the deformation analysis was successful, but there are some disadvantages. One of them is a limitation of its application to the cases when all observations are of the same accuracy. Another disadvantage of the method is a need for using initial residuals.

Let us consider two independent samples x_i, i = 1, 2,..., m and y_j, j = 1, 2, …, n, which are realizations of random variables X_i and Y_j, respectively. Furthermore let the distributions of these variables differ from each other in a shift Δ only. When applying Wicoxon test in order to estimate the shift one can get the well-known form of *R*-estimates of the shift between two samples

$$\hat{\Delta}^R = \mathrm{med}(y_j - x_i)$$

To overcome the above mentioned limitation of *R*-estimates Duchnowski (2013) proposed to use weighted median $\hat{\Delta}^W = \mathrm{medw}(y_j - x_i)$ instead of median (med), wherein weights are determined using proper statistic. This allows taking into account the differences in measurement accuracy of the observations. It has been shown how to apply the new approach in surveying problems. The coordinates of objective points are to be computed by using all possible independent ways and such created set to Hodges-Lehmann weighted estimates is to be applied instead of initial residuals, which are used in the typical *R*-estimates. The proposed method holds the property of high robustness against outliers, which was confirmed by numerical tests performed on horizontal control network.

An important task in the deformation analyses is to find a stable reference frame and therefore the stability of the possible reference points has to be controlled. To test such stability Duchnowski (2011a) used the *R*-estimation investigating several strategies and several empirical influence functions. The tests showed that the robustness of the method depends on the number of unstable network points and it increases if their number decreases.

The similar problem was studied by Duchnowski (2011b) on a levelling control network. The *R*-estimate and two other robust estimates of the standard deviation were used for the examination of a strategy of monitoring the stability of levelling reference marks. It has been shown that a robustness of the estimates does not result in a robustness of the strategy itself. This is due to the fact that beside gross errors also unstable points cause outliers. The research indicated that the robustness of the strategy depends strongly on the number of unstable points.

The classical deformation analysis based on Gauss-Markov adjustment model provides correct results if gross and systematic errors are previously eliminated. In this case the quality of deformation analysis is determined by the precision of measurements only. In case of non-homogenous monitoring network, i.e. networks in which measurements were carried out with different equipment, in different atmospheric conditions, there is an additional risk of erroneous a priori estimation of weights. This can lower quality of deformation analysis. For deformation analysis of non-homogenous monitoring networks Kamiński and Nowel (2013) proposed a modification of classical algorithm and analysis procedure by introducing local variance factor estimators assigned to distinguished homogenous groups of observations. The numerical experiment performed leads to the following recommendations:

- in case of non-homogenous monitoring networks, deformation analysis should be carried out using local variance factor estimators,
- the global variance factor estimator can be applied for deformation analysis in case of homogenous monitoring networks only.

The consequences of erroneous weighting and their impact on displacement analysis in non-homogenous monitoring network have also been studied by Nowel and Kamiński (2103). The conclusions of the study were the same as the above mentioned (Kamiński and Nowel, 2013).

One of the most important tasks of engineering surveying is measurements of displacements and strains of engineering objects and their environment. Due to the technological process of the construction, it may happen that the measurements are performed from temporary sites and it will be not possible to continue the observations measurement on those sites in future. This leads to the problem of determination of displacements and strains, when measurements were carried out in an unstable reference system. In addition, observations can be distorted by gross errors. Kamiński (2011) introduced the displacement and strain using transformation method robust to gross errors (DiSTFAG), which allows to determine the displacement and strain parameters in above indicated situations. The method based on an estimation of two vectors, where one of them corresponds to the distance between the random surface and optimal plane and the other consists of rotation angles. These vectors estimated using robust approach are later used to calculate displacements and strains. The method proposed consists of two stages:

- stage 1 is the initial adjustment, that allows to point out the observations affected by gross errors and to correct of measurement results,
- stage 2 is the use of the method in question to determine displacements and strains.

The issue of determination of an optimal structure of horizontal control network consisting of directional measurements was studied by Mrówczyńska (2013) using entropy concept. According to this study an optimal number of observation results from the entropy difference of parameters vector corresponding to one additional observation.

2.3. Reliability analysis of observation systems

The problem of an internal reliability of observation systems was studied by Prószyński (2013a, 2013b, 2014) in different aspects and for several parameter estimation models. Prószyński, (2013a) presents an approach to internal reliability analysis of observation systems known as Errors-in-Variables (EIV) models with the parameters estimated by least squares method. The total least squares adjustment or orthogonal regression are typically used for parameter estimation in case of EIV. Starting from a functional model of the total least squares adjustment (also known as the Gauss-Helmert model) and using standardized observations the author derived a formula for a matrix **H**. The rank of **H** is crucial for the internal reliability analysis. Since the matrix **H** is an oblique projector, for EIV models with correlated observations, a two-parameter measure for *i*th observation is proposed,

$$h_{(i)} = (h_{ii} - w_{ii})$$

where h_{ii} is the *i*th diagonal element of **H** and w_{ii} is the asymmetry index for *i*th row and *i*th column of **H**. The index h_{ii} is called a "local response of the model", what means the response in the *i*th residual to a possible gross error in that observation. In the mentioned paper, formulae for reliability analysis of specific cases of quasi-linear EIV models, i.e. multiple linear regression, 2D similarity transformation and 3D affine transformation are given in comparison to the ordinary Gauss-Markov (GM) model. Theoretical consideration and numerical experiments concluded that in terms of average reliability indices EIV models are at least twice weaker then GM models. The reliability criteria for EIV models should thus be set at a lower level than for GM models. Moreover it should be noticed that the relatively low response–based reliability of EIV models may indicate lower effectiveness of outlier detection in comparison to GM model.

In Prószyński (2014) the problem of determination of realistic upper-bounds for internal reliability of systems with uncorrelated observation was studied. Based on an experimental material, i.e. post-adjustment documentation of 63 networks of different size Adamczewski has developed in his previous work so called "law of gross errors". This law was a subject of further investigation with help of probabilistic theory and Prószyński (2014) formulated the probability-derived formula for occurrence of Gaussian-type gross errors:

$$P(Y = k) = \binom{n}{k} p^k (1-p)^{n-k}, \quad k = 1, 2, \ldots, n$$

where $P(Y = k)$ is the probability that k gross errors occur in n observations and p is determined from the normal distribution $N(0, \sigma)$ for a specific interval $r\sigma$, e.g. $p = 0.0027$ if $r = 3$. According to the discussed research:

– increasing the number of observations raises the network internal reliability without increasing number of potential gross errors providing that, the network consists of up to 50 parameters and no more than 100 observations, a proper level of internal reliability cannot be practically secure if the number of observations increases and networks consist of more than 100 parameters.

The links of internal and external reliability with system conditionality in Gauss-Markov models with uncorrelated observations were investigated by Prószyński (2013b). The external reliability was expressed both as a vector of parameter distortions (according to Baarda) and in the form of L_2 norm of this vector. All considered values, condition numbers of the design matrix, internal and external reliability were decomposed using the Singular Value Decomposition (SVD) and compared. Following this research the internal reliability and the conditionality are defined on different components of the SVD of a design matrix in GM models, so these characteristics are not interrelated. Meanwhile the external reliability depends on both, the conditionality and the internal reliability of a model.

The effectiveness of some chosen robust estimation methods was investigated by Kwaśniak (2011) in comparison to a level of network reliability. The study was performed on a levelling network using computer-simulated observation systems and it has showed that the effectiveness of gross error detection by means of robust estimation methods depends on the level of internal reliability of a network. Furthermore for effective detection of a single gross error, the surveying network should be designed in such way that for each observation the standard deviation of a residuum should fulfil $\sigma_v \geq 0.71$.

3. Geodetic time series analysis

Pole coordinates data were predicted using the combination of least squares (LS) extrapolation and autoregressive (AR) prediction. In this LS+AR prediction algorithm first the LS model which consists of the Chandler circle, annual and semi-annual ellipses and linear trend is fit to the complex-valued pole coordinates data. The difference between pole coordinates data and its LS model is equal to the LS residuals. Prediction of pole coordinates data is the sum of the LS extrapolation model and the AR prediction of the LS residuals. The smallest mean prediction errors of such combination are obtained when the length of pole coordinates data to fit the LS model is equal to 10 years and the length of the AR model to fit the LS residuals is equal to 850 days. The problem of any prediction algorithm is to predict time series in all its frequency bands. To compute the prediction of UT1 – UTC data the combination of discrete wavelet transform (DWT) and autocovariance (AC) prediction was applied to the length of day (LOD) designed as $\Delta - \delta\Delta$ data which is the first derivative of the UT1 – UTC data from which the leap seconds and tidal model were removed. In this prediction algorithm the $\Delta - \delta\Delta$ data are decomposed into frequency components using the discrete wavelet transform band pass filter (DWT BPF) with the Meyer wavelet functions and each frequency component is predicted by the AC prediction. The DWT BPF enables decomposition of any time series into frequency components in such a way that their sum is exactly equal to these series. It was also found that the mean LS+AR prediction errors for a few days in the future of the pole coordinates model data obtained after removal two highest frequency components computed by the DWT BPF with Shannon wavelet functions are several times smaller than the mean prediction errors of the pole coordinates data. The short term mean prediction errors of the UT1 – UTC model data obtained after removal two highest frequency components computed by the DWT BPF with Meyer wavelet functions are also several times smaller than the mean prediction errors of the UT1 – UTC data (Kosek, 2012).

The pole coordinate data predictions from different prediction contributors of the Earth Orientation Parameters Combination of Prediction Pilot Project (EOPCPPP), initiated by the IERS in 2010 were studied to determine the statistical properties of polar motion forecasts by looking at third and fourth moments about the mean. The differences between the future pole coordinates data and their predictions are the prediction residuals which can be checked whether they satisfy normal distribution. The skewness values of these prediction residuals for different participants of EOPCPPP show that their probability distribution becomes more nonsymmetrical when the prediction length increases. The kurtosis values usually decrease with the prediction length, which means that the probability distribution becomes more flat and has larger tails than a normal distribution. Both statistics show that the prediction residuals are not of normal distribution and their distribution becomes more uniform than normal when the prediction length increases (Kosek et al., 2011b).

The wavelet technique enables comparison of the complex-valued geodetic and fluid excitation functions of polar motion in different frequency bands, e.g. by looking at the semblance function of the order of r between them. The semblance function is the product of the coherence function and the cosine of the phase synchronization function $\Delta\hat{\phi}_{xy}(t, a)$ to the even power of r

$$\hat{\vartheta}^{r}_{xy}(t,a)=\frac{\left|\hat{S}_{xy}(t,a)\right|}{\sqrt{\hat{S}_{xx}(t,a)\hat{S}_{yy}(t,a)}}\cdot\cos^{r}[\Delta\hat{\phi}_{xy}(t,a)],\quad t=m_0+m/2,\quad m_0=0,1,\ldots,n-1-m,\quad r=1,3,5\ldots$$

Such function is interpreted as the frequency- and time-dependent correlation coefficient between two complex-valued time series. It varies within the interval $<-1, 1>$ which shows oscillations with different frequencies in two time series, which are out of phase (−1) and in phase (1). The coherence and phase synchronization functions are computed from the wavelet spectra $\hat{S}_{xx}(t,a)$ and $\hat{S}_{yy}(t,a)$ of two time series and the cross-spectrum $\hat{S}_{xy}(t,a)$ between them, which are estimated from the wavelet transform coefficients obtained by computation of the convolution of time series and modified Morlet wavelet function. To speed up computations such convolution was performed in the frequency domain using the inverse Singleton fast Fourier transform (FFT) of the product of the FFT of time series and frequency domain modified Morlet wavelet function. The considered fluid excitation functions χ consist of the atmospheric angular momentum (AAM) or the sums of atmospheric and ocean angular momentum (AAM+OAM), and atmospheric, ocean and land hydrology angular momentum (AAM+OAM+HAM) excitations. The geodetic excitation functions ψ were computed from the IERS pole coordinates data. The semblance functions between these fluid and geodetic excitation functions increase when OAM excitation is added to the AAM one, and become still greater when the HAM excitation function is taken into account. Moreover, adding hydrology excitation function to the sum of atmospheric and ocean terms improves the phase agreement between the geodetic and fluid excitation in the annual frequency band which can be seen better when the order *r* of semblance function increases. Figure 1 shows the mean (averaged in time) semblance functions between the geodetic and fluid excitation functions for the semblance orders equal to zero (which gives the mean coherence) and equal to 3.

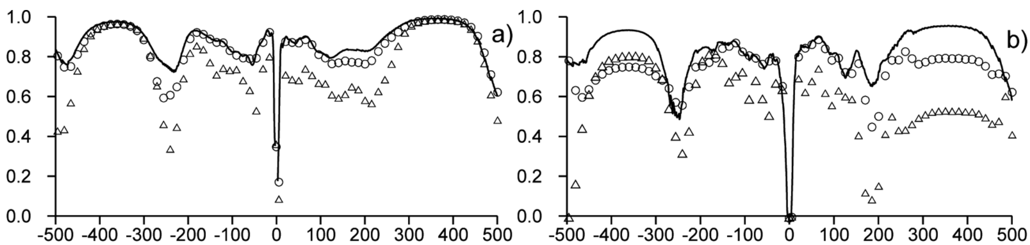

Fig. 1. The semblance functions of the orders 0 a) and 3 b) between the polar motion geodetic and fluid excitation functions AAM (triangles), AAM+OAM (circles) and AAM+OAM+HAM (solid line) (Kosek et al., 2011a)

The wavelet based semblance filtering enables the determination of the common signals in both geodetic and fluid excitation functions of polar motion (Fig. 2). In the semblance filtering first the wavelet transform coefficients are computed of both time series using Shannon wavelet functions. Using these wavelet transform coefficients both time series can be reconstructed using the inverse wavelet transform. Semblance filtering is performed by keeping in the reconstruction formulae of both time series only the wavelet transform coefficients for which the semblance exceeds a given threshold value, e.g. equal to 0.9. Other wavelet transform coefficients of both time series, for which the semblance is below the adopted threshold, are set to zero. It was found that increase of this threshold value makes filtered oscillations in two time series more similar, however at the same time the amplitudes of them decrease (Kosek et al., 2011a).

Similar algorithm of wavelet based semblance filtering was applied to the 3D centre of mass time series determined by the space geodetic techniques such as SLR, GNSS and DORIS, which were projected into the planes of the International Terrestrial Reference Frame (ITRF) to get three complex-valued time series in each plane. This algorithm enables computation of common oscillations in these series and it was found that there exists a common retrograde annual oscillation in the equatorial plane between the GNSS and SLR geocenter time series. Such result was confirmed using the spectro-temporal semblance function between these centre of mass time series projections as well as spectro-temporal polarization functions computed for each space geodetic technique. The spectro-temporal polarization function detects turning directions of ellipses in complex-valued time series for different oscillation periods (Kosek et al., 2014).

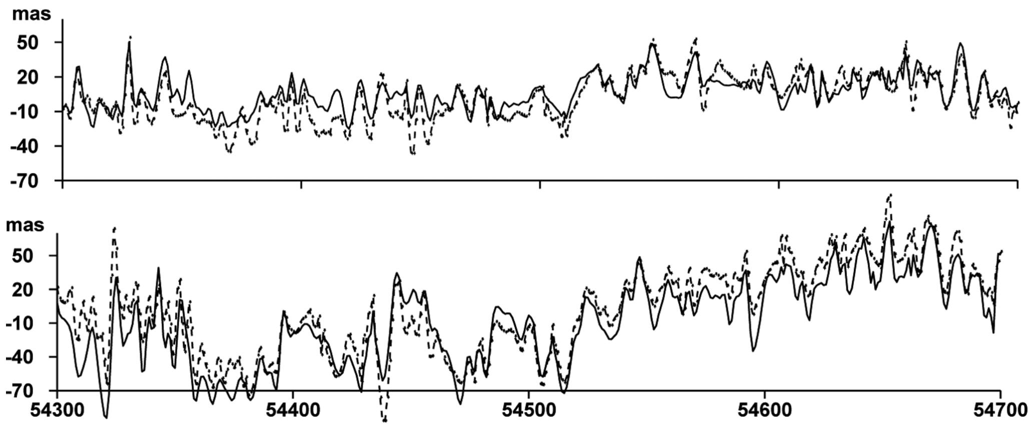

Fig. 2. An example of common oscillations in the equatorial components (ψ_1/χ_1 – upper graph, ψ_2/χ_2 – lower graph) of geodetic (solid line) and fluid AAM+OAM+HAM (dashed line) excitation functions, computed using wavelet based semblance filtering with threshold value equal to 0.90 (Kosek et al., 2011a)

The combination of the Fourier Transform Band Pass Filter (FTBPF) with the Hilbert transform (HT) was applied to compute variable amplitudes and phases of seasonal and subseasonal oscillations in real-valued time series, e.g. altimetric sea level anomaly (SLA) data as a function of geographic location. In this algorithm the complex-valued time series is created from real-valued oscillation filtered by the FTBPF and the HT of this oscillation put into the imaginary part. It was found that this algorithm is slightly modified FTBPF method in which the parabolic transmittance function is multiplied by [1+ sign(ω)] where ω is the central frequency of the filtered oscillation. To speed up computation the Singleton FFT and its inversion were applied to perform the FTBPF algorithm. Such seasonal and subseasonal variations are mostly irregular and cause the increase of prediction errors of the SLA data for a few weeks in the future. In order to detect the impact of these irregular variations on the SLA prediction errors, standard deviations maps of both amplitude time differences of the products of phase time differences and amplitudes were examined. The SLA data prediction errors in certain geographic regions of the ocean seem to be caused mainly by nonlinear behaviour of the broadband annual oscillation. The broadband character of the annual oscillation is manifested by the peaks in the SLA spectra which correspond to the integer multiplicities of the annual frequency, e.g. the semi-annual, 120-day and quarter-annual oscillations. The amplitude maxima of the semi-annual oscillation and other shorter period oscillations are located in geographic regions where the amplitude maxima of the annual oscillation occur. The mean prediction errors of the SLA data for two weeks in the future are usually considerable in geographic regions where amplitude maxima of the annual oscillation are the largest (Kosek et al., 2013).

Bogusz et al. (2013) applied wavelet decomposition with symmetric Meyer mother-wavelet to process data from two GGP sites (The Global Geodynamics Project). The time series that reflected Earth›s gravity field changes with 5-second sampling rate were recorded at the time of earthquake and at the time when the gravimeters worked without any disturbances. The authors found that the wavelet decomposition may be considered as a good method of data interpolation and noise reduction for earthquake periods. The signal approximation at the highest level of the decomposition after removing tidal effects may be a good representation of the SG drift.

Klos et al. (2014a) showed how the unmodeled effects influence the estimated character of probability density function. They simulated daily time series with white plus flicker noise which is widely recognized to characterize the GPS time series to compare the Gaussian and non-Gaussian data. They added then different values of trend as well as seasonal terms and described how the skewness and kurtosis values change with each of them. The analyses were then performed for real data in the ETRF2000 reference frame. The authors found the lower bound for relationship between skewness and kurtosis to be equal to quadratic function. They concluded, that the uncertainties of velocities determined from ETRF2000 time series are underestimated by up to 5 mm/year when comparing the power-law noise model with white noise only.

Klos et al. (2014b) showed that a median absolute deviation is an optimum criterion for outliers removal in the GNSS time series. They used 12 most noisy EPN time series and compared different criteria commonly used for outliers detection. The results show that the removal of outliers is necessary before any further analysis, otherwise one may obtain quite odd and unrealistic values.

Klos et al. (2014c) used daily time series from the EPN stations in Poland to investigate the effect of the type of GPS antenna monument on the type and amplitude of noise. They analyzed the 5-years time series with Maximum Likelihood Estimation (MLE). The noise caused by the monument is thought to follow the random-walk character, therefore the authors analyzed noises twofold. Firstly as the combination of white, flicker and random-walk models what resulted in no conclusions on the monuments' stability level due to the domination of flicker noise in the time series and then as the sum of white plus random-walk noise characters, what showed that concrete pillars are better than buildings as GPS monuments.

Klos et al. (2014d) analyzed the daily GPS time series from the area of Sudeten with three different assumptions of noise models: a white noise – to show how the omitting of correlations in the stochastic part of the GPS time series results in underestimation of the velocity uncertainties, a white plus flicker plus random-walk noise – to estimate which of the power-law noise with the known spectral index prevails in the GPS time series, and white plus power-law noise – to estimate which coloured noise fits best the stochastic part. On the basis of MLE's values they showed that the combination of white plus power-law noise fits best the data. The spectral indices of power-law range from –1.6 to –0.4 for the horizontal components and from –1.0 to –0.4 for the vertical component, which confirms the prevalence of fractional white and Brownian motion quite close to flicker noise with amplitudes of power-law noise of 2–5 mm $\cdot$ yr$^{\kappa/4}$ and 4–12 mm $\cdot$ yr$^{\kappa/4}$, respectively. For this combination, the uncertainties of the estimated velocities reach in the most extreme case even 0.8 mm/year for the Up component.

Bogusz et al. (2014b) used a locally weighted scatterplot smoothing (LOESS) method which parameters are smoothing parameter and polynomial degree to analyze the autocorrelations in the daily changes of topocentric components of EPN stations in Poland. The authors showed that the trend-related behaviour due to plate motion is modelled best by both smoothing parameter and polynomial of degree 1. The polynomial of degree 2 with smoothing parameter close to 0.1 fits seasonal components quite well. Larger values of smoothing parameter flatten time series too much, while smaller ones detect higher frequency variations. The LOESS function has also shown to be a good approximation of time series residua and helpful in long-range dependencies analyses.

Bogusz et al. (2014c) showed how the improper data pre-analysis may affect the quality of fit of trend line interpreted as velocity into GPS time series. The unremoved outliers and offsets, although giving quite satisfying fit, change the velocity values up to 1.3 mm/year. The velocities suffer not only from improperly removed factors at the pre-processing stage, but also from seasonal components not sufficiently modelled.

Bogusz et al. (2014a) analyzed how the different systematic errors due to mis-modelling of satellite orbits or satellite antenna phase centre corrections affect the regionally correlated errors, called common mode errors (CME). They implemented the stacking method for ASG-EUPOS permanent stations with daily time series and showed that the time series residua (data with no trend and seasonal terms) are correlated for stations located even 200 km apart. After removal of CME values from time series, the correlations were estimated again. Their values did not exceed 0.01 what stands for the decrease of correlation value of 300% in comparison to correlation coefficients before spatial filtering and practically irrelevant spatial correlation of analysed ASG-EUPOS network. The obtained improvement (decrease) of the correlation coefficient between the ASG-EUPOS permanent stations testifies that the CME can be treated as the homogeneous for the area of Poland.

The problem of the impact of environmental effects on observed gravity is crucial. Satellite systems are not as susceptible to changes in local hydrology or atmospheric effects, although significant influences are clearly visible in the change of coordinates. Bogusz et al. (2011) showed potential environmental influences to GNNS coordinates on the example of the ASG-EUPOS network. Two solutions: daily and sub-daily were considered in order to identify the factors that may cause degradation of precision of coordinates and velocity determined. Separate analyses were performed for GPS and GPS+GLONASS solutions. GPS time series (by means of North, East and Up components) are the sum of the deterministic and stochastic part. The first one is the sum of the initial value, linear velocity and seasonal components. The annual curves determined by least squares estimation with assumption of stationarity of amplitude and phase, and uncertainties calculated with the coloured noise assumption using the First Order Gauss Markov model were shown (Bogusz and Figurski, 2014). The velocity bias due to annual oscillation could range from –0.6 to +0.5 mm/y. Some part of periodic changes in the deterministic part of GPS-derived time series could be of artificial reasons (numerical and observational). In the papers by Bogusz and Hefty (2011) and Bogusz and Figurski (2012) the unmodelled or mismodelled short-period (e.g. tidal) oscillations in Up components at more than 100 ASG-EUPOS stations were analysed. The analysis of the residua from IERS2003 tidal model was performed using least squares method with the ETERNA software upon the idea of Chojnicki. It confirmed the existence of significant energy in the frequencies corresponding to S1 (thermal influences), K1 and K2 (dynamic changes of satellites' constellation and network geometry or multipath). Bogusz and Kontny (2011) analysed the stochastic part of the 3-hour sampled time series with the use of the autoregressive integrated moving average (ARIMA) model providing Box-Pearce and Ljung-Box tests for rejecting null hypothesis about "whiteness" of the residua. As the processed network contained 130 sites the spatial distribution of the noise parameters was also investigated. Szafranek et al. (2014) analysed Zenith Tropospheric Delay (ZTD) time series resulted from the EUREF Permanent Network reprocessing performed by the Military University of Technology. Time series were analyzed in order to find short- and long-term trends which could be associated with the meteorological conditions and climate change. In particular linear trend and seasonal components (annual and semi-annual) were estimated on the basis of IWV (Integrated Water Vapour) time series for Matera (Italy) permanent station using least squares estimation.

A few time series analysis techniques have been used for analyzing and predicting geodetic time series, namely length of day (LOD) data, axial component of atmospheric angular momentum (AAM X_3), global mean sea level (MSL) as well as local and regional sea level anomalies (SLA). These techniques can be divided into interpretation-oriented approaches, the aim of which is to understand governing processes, and prediction-oriented ones that lead to the anticipation of future values of geodetic time series. Amongst the first group of methods, the cross-correlation analysis interpreted along with the wavelet coherence has been used to show El Niño/Southern Oscillation (ENSO) teleconnections, and LOD and AAM X_3 have been used as ENSO indices (Niedzielski, 2011, 2014). Wavelets are widely used not only in LOD/AAM X_3 data analysis (Kosek et al., 2011a), but also in the process of understanding deterministic harmonic oscillations in SLA time series (Świerczyńska et al., 2012, 2014). In order to analyze SLA residuals, in the form of time series free of deterministic signals, moment-based statistics have been found as efficient tools for evaluating SLA data nonlinearity (Niedzielski and Kosek, 2012a). Another method utilized for the purpose of inference about data variability and its characteristics was a sea level trend analysis, performed using the Cox-Stuart test on altimetric MSL time series (Niedzielski and Kosek, 2011).

The prediction-oriented time series techniques are closely associated with autoregression, in its numerous versions. In general, the deterministic signal is predicted with extrapolating the polynomial-harmonic model, and the residuals are satisfactorily predicted when autoregression is applied for residuals. Indeed, multivariate (vector) autoregressive model (MAR) have been shown to perform better than the autoregressive model (AR) in the process of forecasting LOD time series (Niedzielski and Kosek, 2012b). The same methods, together with the threshold autoregressive model (TAR) and generalized space-time autoregressive (GSTAR) model, are used to forecast sea level dynamics. In the first version of the Prognocean system implemented at the University of Wroclaw (Niedzielski and Miziński, 2013), these methods are used to predict SLA data in real time.

4. Diverse algorithms

The conversion between Cartesian and geodetic coordinates $(x, y, z) \leftrightarrow (\varphi, \lambda, h)$ on the rotational ellipsoid is a basic, well investigated issue in geometrical geodesy. There are few algorithms that can be used for this purpose. Ligas and Banasik (2011) proposed a new approach based on the solution of the system of nonlinear equations. The proposed algorithm consists of two steps. The first step is a projection of a point onto the ellipsoid and computation of the coordinates of the projected point by solving nonlinear equations with the use of the generalized Newton method. The second step is a straightforward computation of latitude and height. The performance of the proposed algorithm was compared in numerical tests with six existing algorithms of the coordinate conversion.

The same concept underlies a new method of coordinates conversion on a triaxial ellipsoid proposed by Ligas (2012). The method is based on solving a nonlinear system of equations for coordinates of the point being the projection of a point located outside or inside a triaxial ellipsoid along the normal to the ellipsoid. The nonlinear system of equations is solved by means of generalized Newton method. The numerical tests were performed for several celestial bodies and compared with the only existing Feltens's method. These tests showed that, the new approach is more universal and applicable to broad class of objects

Determination of the covariance function and its parameters is a crucial step in least squares collocation. Jarmołowski and Bakuła (2014) have studied the issue of precise covariance parameter estimation in least squares collocation using restricted maximum likelihood (LEML) method. The authors have implemented REML method for estimation of the variance, correlation length and a noise of the Gauss-Markov second order function, which is frequently used as a correlation function when interpolating gravity anomalies. The method proposed was tested on two data sets. The first set includes terrain elevations measured with GNSS and the second includes terrestrial free-air gravity anomalies. A hold-out method was used for cross validation of the parameters estimated by REML.

Keller and Borkowski (2011) proposed a new algorithm for building identification and segmentation in laser scanning data. The algorithm bases on multi-resolution analysis in wavelet domain and works on gridded data. The wavelet-based segmentation proceeds in the following main steps: wavelet decomposition up to appropriately chosen level, thresholding on the chosen and adjacent levels, removal of all coefficients in the so-called influence pyramid and wavelet reconstruction. If buildings on several scaling spaces have to be segmented, the procedure has to be applied iteratively.

5. Summary and conclusions

In this review paper an outline of research activities concerning theoretical geodesy performed in Poland in the period from 2011 to 2014 is presented. The studies reported in this review are partially continuation of studies started in previous years. Several fundamental achievements have been gained in the last four years. The new models of parameter estimation, new results related to the robustness of M-estimation and related to reliability of observations systems can be mentioned as the main achievements of the Polish researchers. Especially, it can be emphasize as the highlights:

– The new model of M-estimation with probabilistic models of geodetic observations, which depends on the variance, kurtosis, excess and asymmetry of the observation errors distribution.
– The new Shift-M_{split} estimation, which enables to estimate a vector of parameter differences without estimating the particular parameters vectors.
– The Shift-$M_{split}^{(+)}$ as a generalisation of Shift-M_{split} estimation if the design matrix **A** of a functional model has not a full column rank.
– The new algorithms of the coordinates conversion between the Cartesian and geodetic coordinates, both on the rotational and triaxial ellipsoid.
– The algorithm of computation of common oscillations in two complex-valued time series using wavelet semblance filtering with discrete Shannon wavelet functions.
– The algorithm of computation of spectro-temporal semblance function between two complex-valued time series with application of the modified Morlet wavelet function.
– The algorithm to compute variable amplitudes and phases in real-valued time series based on the combination of the Fourier Transform Band Pass Filter (FTBPF) with the Hilbert transform (HT). This algorithm utilizes the Singleton Fast Fourier Transform to speed up computations.
– The algorithms for prediction by combination of extrapolating the polynomial-harmonic model with the multivariate (vector) autoregressive model (MAR) and threshold autoregressive model (TAR) as well as the generalized space-time autoregressive model (GSTAR).
– The algorithm for forecasting by combination of the discrete wavelet transform band pass filter with discrete Shannon wavelet functions and the autocovariance prediction.

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